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On intertemporal poverty measures: the role of affluence and want

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Abstract This paper proposes classes of intertemporal poverty measures which take into account both the debilitating impact of prolonged spells in poverty and the mitigating effect of periods of affluence on subsequent poverty. The weight assigned to the level of poverty in each time period depends on the length of the preceding spell of poverty or of non-poverty. The proposed classes of intertemporal poverty measures are quite general and allow for a range of possible judgements as to the overall impact on a poor period of preceding spells of poverty or affluence. We discuss the properties of the proposed classes of measures and axiomatically characterize these measures.

1 Introduction

The important question of how poverty should best be measured has generated wide interest both in policy circles and in academia, leading to a large discourse on the level of poverty and the different poverty measures. Most of these poverty indices, however, capture poverty only at a given point in time (Watts 1968; Sen 1976; Clark et al. 1981; Chakravarty 1983; Foster et al. 1984; see also Zheng 1997). An increasing number of studies, however, indicate that measuring poverty at any single point in time is inadequate for capturing the true level of poverty, since a far greater proportion of people may experience poverty when observed over a longer term (Baulch and Hoddinott 2000).

In this paper, we develop classes of intertemporal poverty measures which take into account previous poverty experiences of individuals. Our focus is exclusively on the measurement of intertemporal poverty at the individual level, rather than at the societal level. Thus, we attempt to address the question of distinguishing between two

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individuals who may have the same level of poverty in the current period, but had different levels of poverty, or were poor at different times from one another, in the past.

Given this objective, the paper adds to a recent literature with notable contributions by Foster (2009), Hoy and Zheng (2011), Bossert et al. (2012), Mendola et al. (2011) and Zheng (2011), among others.[1] In Foster (2009), the spread of poor episodes over time is of no importance; only the proportion of spells in which the individual is in poverty is taken into account. Overall poverty for each individual is a simple average of the generalized poverty gaps in each period. Bossert et al. (2012), on the other hand, assign particular importance to the extent to which individuals are in a state of poverty for consecutive periods. They argue for a principle in which people facing periods of poverty which are interrupted by relatively affluent periods are deemed as being able to manage more easily than those who are exposed to longer consecutive periods of poverty (even if the total number of periods of poverty and non-poverty are the same in each case). In this context, one can argue that while the bunching of poor episodes is important in its own right, there is also a case for explicitly taking into account the role of affluent periods in mitigating subsequent poverty.

More recently, Mendola et al. (2011) and Zheng (2011) have proposed measures which, by considering each pair of poor episodes, take into account the damaging impact of consecutive poor episodes as well as the mitigating effect of affluent spells. In both these works, proximity of one poor episode to another serves to intensify the overall experience of poverty. While Mendola et al. (2011) consider the number of non-poor periods between two periods of poverty as an indicator of the mitigating effect, Zheng (2011), in a general framework, allows for non-poor periods to directly interact with the poor periods. In both papers, the mitigation depends on the distance between two poor episodes. Thus, it is difficult to disentangle whether affluent periods have an independent mitigating impact, aside from simply indicating that the poor episodes are not close together. A similar issue arises in Hoy and Zheng (2011), where a person who is poor in periods which are either consecutive, or separated by only short spells of relative wellbeing, is worse off than a person with similar incomes but more widely dispersed poor episodes.

The classes of measures proposed in this paper are motivated by the relevance of consecutive poor periods, but afford a richer interpretation of the dynamics which cause closely bunched poor spells to be debilitating. The method used differs from that of the existing literature in how both the mitigating effect of affluent periods and the debilitating impact of consecutive episodes of poverty are accounted for. It is motivated by the observation that the longer the spell of relative affluence experienced prior to becoming poor, the better equipped an individual is to deal with that period of poverty. In our measures, the impact of a poor period is discounted according to the number of affluent periods directly preceding it.

This approach allows us to account for the mitigating impact of affluence independently from the intensification of poverty arising from the bunching of poor episodes. To illustrate the advantage of our methodology, consider the following stylized exam-

[1] See also Cruces (2005), Calvo and Dercon (2009), Grab and Grimm (2007), Carter and Ikegami (2007), Porter and Quinn (2008), Foster and Santos (2012) and Gradín et al. (2012).

ple. Suppose that there are two individuals who both live over three time periods. Each is poor, and to a similar extent, in only one of the three periods. The first person is poor in the first period, while the second person is poor in the last period. Our measures would indicate that the second person is better off since he had an opportunity to accumulate resources before facing the poor episode. However, most of the existing measures, such as those by Foster (2009), Bossert et al. (2012), Mendola et al. (2011) and the Newtonian measure by Zheng (2011), are unable to distinguish between such cases.[2]

We capture the intensification of poverty due to bunching by weighting each poor period according to the number of directly preceding poor periods. Consider another example, where two individuals live for two time periods and are poor in each period. Suppose that the first person is poor in the first period, by some certain amount, but only half as poor in the second period. Suppose that it is the other way around for the second person. Intuitively, it seems reasonable to expect the two profiles to be ranked differently, since in one case poverty is decreasing, while in the other case it is increasing. In contrast to our measures, most existing measures, including those of Foster (2009), Bossert et al. (2012), Mendola et al. (2011) and Zheng (2011), are unable to distinguish between these two profiles.

Like Foster (2009), Bossert et al. (2012), Mendola et al. (2011) and others, we do not allow the level of income in affluent periods to mitigate poverty in other periods. Yet one can argue that affluent periods should have some mitigating effect on poverty. Our measures are general and do not specify the mitigating attributes explicitly [e.g., income as in Hoy and Zheng (2011) and Zheng (2011)]. Implicit in our approach is that, in the absence of income smoothing opportunities, the mitigating effect of affluent periods is transmitted through non-income dimensions such as assets, health, social networks, human capital and so on.[3] For instance, in a rich and detailed study, Narayan et al. (2009) have found that, in India, those who move out of poverty have "almost always made investment in land." Typically these investments would also include acquiring education and building social networks, the latter of which is regarded by Woolcock and Narayan (2000) as being one of the primary resources the poor have for managing risk and vulnerability. There is also strong evidence that in rural areas of developing countries body weight varies significantly between peak and off-peak seasons (Behrman and Deolalikar 1989; Dercon and Krishnan 2000). People who have a possibility of falling into hardship often use their "body as a store of energy" during affluent times by employing a "feast now fast later" strategy (see Dercon and Hoddinott 2003, pp. 7–8). There is significant evidence that these factors do indeed, in general, have some impact on mitigating poverty (Narayan et al. 2000; Sen and Hulme 2004). Thus, from a social planner's perspective, when evaluating intertemporal poverty, there

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[2] Hoy and Zheng (2011) would rank these cases differently. However, the motivation for doing so is very different from that here. They explicitly consider poverty early in life to be more damaging than poverty later on. We have no such assumption here.

Zheng (2011) proposes several classes of measures. Here, and for the rest of the paper, we are concerned mainly with his Newtonian poverty measure (p. 10).

[3] Our notion of income smoothing is based on Morduch (1995, p. 104) where households can smooth income by "making conservative production and employment choices and diversifying economic activities."

might be sufficient reason to take into account the mitigating impact of affluent periods on subsequent poverty even in the absence of income smoothing.

In our measures, the mitigating impact of a spell of affluence does not last long. There is strong evidence that, in the face of poverty, households draw down their existing resources quite significantly (Hulme 2003; Moser 1996). Davis (2006) and Davis and Baulch (2009), for example, provide empirical evidence that in Bangladesh, while improvements in life conditions of individuals typically occur slowly, over long periods of time, they decline suddenly following shocks. With regard to social networks, Beall (2001) points out that such social resources can be quickly eroded by poverty. In contrast, both Mendola et al. (2011) and Zheng (2011) allow a single episode of affluence to have a mitigating impact on all subsequent episodes of poverty.

Our approach retains much of the appealing intuition of Bossert et al. (2012), with respect to the exacerbating impact of bunching of poor episodes, but adds to this the characteristics of mitigation of poverty by preceding non-poor periods and an increasing intensification of the impact of consecutive episodes of poverty. The proposed intertemporal poverty measures are quite general and allow for a range of possible judgements as to the overall impact of a poor period that is preceded by a non-poor spell and also to the overall effect of consecutive periods of poverty. When no significance is attached to either the relatively affluent periods or to the exacerbating impact of consecutive periods of poverty, our class of relative intertemporal poverty measures reduces to the simple average of per period (static) poverty measures advocated by Foster (2009). In this sense we provide axiomatic foundations for a class of measures that encompasses the latter.[4]

The remainder of the paper is organized as follows. Section 2 introduces our notation and the basic framework. Section 3 formally introduces our classes of individual intertemporal poverty measures and provides axiomatic characterizations. The following section discusses two important extensions of the measures presented in Section 3. Section 5 concludes the paper. All proofs are deferred to the Appendix.

2 Notation and basic framework

We focus on the measurement of an individual's aggregate poverty over finitely many time periods. This requires the determination of a static poverty index for each time period and an aggregation of the latter across time. Subsequently, one can construct measures of poverty for an entire society by aggregating across individuals. We focus on the former two steps.

For $T \in \mathbb{N}$ let $t \in \{1, \ldots, T\}$ denote a particular time period. An individual has income $x_t \geq 0$ in each period $t = 1, \ldots, T$. The income is net of any taxes and

[4] Note that here, and throughout the paper, we are referring to Foster (2009)'s total intertemporal poverty measure, not his chronic poverty measure. Foster (2009) defines a poverty duration cut-off line as the minimum proportion of time periods a person must be poor in order to be deemed chronically poor; individuals who are in poverty for a proportion of periods less than this threshold are considered transiently poor. Foster (2009)'s total intertemporal poverty measure is obtained by choosing a poverty duration cut-off line of zero.

transfers.[5] It is important to note that since this is an ex-post measure the income in each period is the realised income for that period.

As usual, there is an exogenously determined poverty line z_t for each time period t, where $0 < z_t < \infty$, and $\mathbf{z} = (z_1, \ldots, z_T) \in \mathbb{R}^T_{++}$ denotes the profile of poverty lines. If $x_t < z_t$ in period $t \in \{1, \ldots, T\}$, the individual is *poor* and has an income shortfall $p_t \in [0, 1]$.[6] If $x_t \geq z_t$, the individual is non-poor and $p_t = 0$.

The individual's poverty profile is $\mathbf{p} = (p_1, \ldots, p_T)$, representing the income shortfalls that the individual faces in each of the T time periods. Thus, a poverty profile is a T-vector where $\mathbf{p} \in [0, 1]^T$. We use $\mathbf{0}^T$ to represent the poverty profile in which there is no poor period, i.e. $p_t = 0$, for all $t \in \{1, \ldots, T\}$. Further, a T-period poverty profile with only one poor period such as $\mathbf{p} = (0, \ldots, 0, p_s, 0, \ldots, 0)$, $1 \leq s \leq T$, is represented as $\mathbf{p} = p_s \cdot \mathbf{e}^T_s$, where $\mathbf{e}^T_s$ is the profile with $e_t = 0$ for all $t \in \{1, \ldots, T\} \setminus \{s\}$ and $e_s = 1$.

For a profile $\mathbf{p}$ we define n_t to be the number of consecutive non-poor periods immediately prior to a poor period t, and we let k_t be the number of preceding periods of uninterrupted poverty, up to and including the poor episode in period t. Formally,

$$n_t := \begin{cases} 0, & \text{if } t = 1 \text{ or } p_{t-1} > 0 \\ t - \min\{s : s < t \text{ and } p_s = \cdots = p_{t-1} = 0\}, & \text{otherwise,} \end{cases}$$

and

$$k_t := \begin{cases} 1, & \text{if } t = 1 \text{ or } p_{t-1} = 0, \\ t - \min\{s - 1 : s < t \text{ and } p_{t'} > 0, \forall t' = s, \ldots, t\}, & \text{otherwise.} \end{cases}$$

For example, for $T = 5$, the poverty profile $\mathbf{p} = (p_1, 0, p_3, p_4, 0)$ has $n_1 = 0, k_1 = 1, n_3 = 1, k_3 = 1$, and $n_4 = 0, k_4 = 2$. It will later become clear that there is no need to define n_t and k_t for non-poor periods.

3 Individual intertemporal poverty indices

An intertemporal poverty measure for an individual is a function that assigns to each poverty profile a non-negative number. Thus, $P : \cup_{T \in \mathbb{N}} [0, 1]^T \longrightarrow \mathbb{R}_+$. The class of individual intertemporal poverty measures that we consider are close in structure to the measures of Foster (2009) and Bossert et al. (2012).

We propose a class of measures that takes into account both the poverty mitigation arising from the presence of affluent periods (n_t) and the intensification of poverty due

[5] Net income can be thought of as consumption, but then our interpretation of the measure has to change in line with this. If we assume consumption as our primitive, then the mitigating impact of affluent periods reflects non-consumption smoothing mechanisms since presumably any consumption smoothing is already reflected in the consumption vector.

[6] For example, p_t could be any static poverty measure from the literature, such as a normalized poverty gap. In fact, with some minor amendments, our results will go through for a more general definition, where $p_t \in \mathbb{R}_+$.

to consecutive poor periods (k_t). The *constant-relative affluence-dependent intertemporal poverty measure* P_R is defined as

$$P_R(\mathbf{p}) = \frac{1}{T}\sum_{t=1}^{T}\frac{k_t^{\alpha}}{(1+n_t)^{\beta}}p_t^{\theta}, \text{ where } \alpha, \beta, \theta \geq 0. \tag{1}$$

The parameter θ captures the sensitivity of the poverty experienced in each time period to the income shortfall. The damaging impact of consecutive periods of poverty, which serve to intensify the overall impact of poverty, is captured by k_t. The parameter α determines the extent of this intensification of poverty. If $\alpha = 0$, there is no intensification. Similarly, β can be interpreted as an index representing how much one chooses to discount the impact of an individual's poor episodes according to preceding uninterrupted spells of non-poverty. When $\beta = 0$, there is no mitigation.[7]

The mitigating effect of an affluent spell on subsequent poverty is determined primarily by its duration. In the example below, we evaluate an individual's intertemporal poverty level for a four period poverty profile $\mathbf{p} = (p_1, p_2, p_3, p_4)$, where $\alpha = 1$, $\beta = 1$ and $\theta = 1$.

Example 1 For $\mathbf{p}_1 = (1/2, 0, 1/4, 0)$ we have

$$P_R(\mathbf{p}_1) = \frac{1}{4}\left(1\cdot\frac{1}{2} + 0 + \frac{1}{2}\cdot\frac{1}{4} + 0\right) = \frac{5}{32}(\approx 0.156)$$

and for $\mathbf{p}_2 = (1/2, 0, 0, 1/4)$ we have

$$P_R(\mathbf{p}_2) = \frac{1}{4}\left(1\cdot\frac{1}{2} + 0 + 0 + \frac{1}{3}\cdot\frac{1}{4}\right) = \frac{7}{48}(\approx 0.146).$$

The measure P_R differentiates between these two profiles by attaching lower weight to the snapshot poverty level 1/4 in the second profile, thereby indicating that the second profile represents less intertemporal poverty. While Mendola et al. (2011) and Zheng (2011) would rank the two profiles in a similar manner to P_R, the measures of Foster (2009) and Bossert et al. (2012) rank the two profiles as being equally poor.

Implicit in the importance of the duration of affluent spells is the 'focus axiom,' which states that if income in a non-poor period is increased, this has no effect on overall poverty (see Foster 2009).[8] Although this may, at first glance, seem limiting in an intertemporal context, opportunities for income (or consumption) smoothing between periods is limited, particularly in a developing country context (Hulme and McKay 2005). Furthermore, since this is an ex-post measure, in each time period we observe the realised income, which incorporates any income smoothing that has taken

[7] If both $\alpha = 0$ and $\beta = 0$, provided p_t is a normalized poverty gap, the measure reduces to the simple average of static poverty measures advocated by Foster (2009).

[8] The approach of effectively censoring the income in each time period at the poverty line is common in the literature on intertemporal poverty measurement and is also adopted by Bossert et al. (2012) and Mendola et al. (2011), among others. However, this leads to a discontinuity in the measure at the poverty line. For a continuous measure see Hoy and Zheng (2011).

place. Thus, if an individual had a high income (well above the poverty line) in one period followed by a low income (just below the poverty line) in the next, this reflects the constraints that the individual faced in those periods. Allowing further income smoothing between these two periods to take place, which would be the result if the focus axiom were discarded, would be quite difficult to justify, especially when we know that in reality such smoothing has not occurred. As in our approach, Mendola et al. (2011) also adopt the focus axiom and resort to the duration of affluent spells as a mitigating factor. On the other hand, there are also some compelling arguments for why income levels during affluent periods may matter, in particular through investment in assets (McKay 2009). Hence, we will later relax this assumption.

The mitigating impact of a spell of affluence dissipates after the initial period of poverty. Individuals may have non-income resources left after the first period, but these may be so limited that their effectiveness for mitigating poverty further is considered to be negligible. This follows evidence gathered by Hulme (2003) and Moser (1996), among others, that households draw down their existing resources quite significantly when faced with poverty.

The mitigation due to an affluent spell is a *proportion* of the poverty level, irrespective of the depth of poverty. In other words, if we consider two different poverty levels, immediately following the same number of affluent periods, then they should have the same constant proportion of their poverty levels mitigated. In this sense, our measures belong to a class based on relative mitigation. Note that the mitigation of poverty is not occurring through changing the person's income in the poor period—if such a change had occurred, this would already be reflected in the person's income for that period.

Next we illustrate some additional properties of P_R through an example. Again let $\alpha = \beta = \theta = 1$.

Example 2 For $\mathbf{p}_3 = (0, 3/4, 1/2, 1/4)$ we have

$$P_R(\mathbf{p}_3) = \frac{1}{4}\left(0 + \frac{1}{2} \cdot \frac{3}{4} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4}\right) = \frac{17}{32},$$

for $\mathbf{p}_4 = (0, 1/4, 1/2, 3/4)$ we have

$$P_R(\mathbf{p}_4) = \frac{1}{4}\left(0 + \frac{1}{2} \cdot \frac{1}{4} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{3}{4}\right) = \frac{27}{32},$$

and for $\mathbf{p}_5 = (3/4, 1/2, 1/4, 0)$ we have

$$P_R(\mathbf{p}_5) = \frac{1}{4}\left(1 \cdot \frac{3}{4} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4} + 0\right) = \frac{20}{32}.$$

One might reasonably expect the three profiles considered here to be ranked differently, yet the measures of Foster (2009), Bossert et al. (2012) and Mendola et al. (2011), and the Newtonian measure of Zheng (2011), all rank them as equal. Our measures distinguish between the poverty profiles $\mathbf{p}_3$ and $\mathbf{p}_5$. Thus, in our framework, it matters whether the mitigating affluence occurs early, or later on. This, however, does not imply that it is better for less severe poverty to occur early on, and more severe poverty to occur later, as is the case in Hoy and Zheng (2011). We do not make any such assumption and this is evident from a comparison of $\mathbf{p}_3$ and $\mathbf{p}_4$. Here, although the latter profile has less severe poverty in early periods, it is deemed by our measures to have greater intertemporal poverty because the episode with the highest severity of poverty in $\mathbf{p}_4$ occurs after two periods of poverty while in $\mathbf{p}_3$ it occurs after a (mitigating) period of affluence. What is important in our measures is the interplay between the mitigating effect and the intensification effect. The ranking of poverty profiles depends on the overall result of this interaction. The proposed measures can also distinguish between $\mathbf{p}_4$ and $\mathbf{p}_5$, where in one case poverty has steadily increased and in the other case it has steadily declined.

We now turn to the axiomatic foundations for the proposed affluence-dependent intertemporal poverty measures.

3.1 A foundation for P_R

Our first requirement for an individual intertemporal poverty measure is that in trivial cases, where there is only one time period, the individual intertemporal poverty measure is a reflection of the income shortfall in that period. This axiom is similar to the single period equivalence axiom proposed by Bossert et al. (2012).

Axiom 1 *Single period equivalence* holds if $\forall\ p \in (0, 1]$, $P(p) = p^\theta$, where $\theta \geq 0$.

Even in the context of a single period, there is scope for a range of judgements as to the relationship between an income shortfall and poverty. By allowing for a broad range of possible values of θ, we allow for a broad range of judgements as to the precise nature of this relationship.

The second axiom considers the possibility of partitioning a longer poverty profile into shorter ones and the relation of the sub-profile measures with the overall measure. This is an additive separability condition. It is clear from our objective that different periods of poverty will be given different weights when poverty is aggregated over time. Hence, only a restricted version of separability into specific sub-profiles is permitted and the timing of the periods which have a non-zero income shortfall is critical. Similar restrictions on separability were used by Bossert et al. (2012).

Axiom 2 *Time decomposability* holds if for all periods of length $T \in \mathbb{N}\backslash\{1\}$, all poverty profiles $\mathbf{p} \in [0, 1]^T$ and all $t \in \{1, \ldots, T-1\}$ such that $p_t > 0$ and $p_{t+1} = 0$ then

$$P(\mathbf{p}) = \frac{t}{T} P(p_1, \ldots, p_t) + \frac{T-t}{T} P(p_{t+1}, \ldots, p_T).$$

The axiom means that intertemporal poverty must be equal to a weighted average of two sub-profiles, where the weights are proportional to the lengths of the two sub-profiles. However, it can be applied only to situations in which the first sub-profile ends with a poor period and the second sub-profile starts with a non-poor period.

The next requirement fixes the location of our measurement scale. It concerns only the desirable case of non-poor profiles.

Axiom 3 *Normalization* holds if for all $T \in \mathbb{N}$ we have $P(\mathbf{0}^T) = 0$.

Next we focus on axioms which reflect our motivation that poverty in some periods can be mitigated by preceding periods of affluence. Recall Examples 1 and 2, which demonstrated how an additional preceding period of non-poverty leads to smaller weights being given to an immediately subsequent period of poverty. Our next axiom formalizes the intuition that the discounting of a poor period's poverty is proportional to the length of the immediately preceding non-poor spell.

Axiom 4 *Constant-relative poverty mitigation* holds if for all $T \in \mathbb{N}\backslash\{1\}$ and all $p \in (0, 1]$ we have $P(p \cdot \mathbf{e}_t^T) = P(p \cdot \mathbf{e}_1^T)/t^{\beta}$ for some $\beta \geq 0$ and any $t \in \{1, \ldots, T\}$.

When $\beta = 0$, there is no mitigating effect of periods of affluence. On the other hand, $\beta > 0$ ensures that non-poor episodes will have an impact on immediately subsequent poor episodes. The axiom also implies that the greater the length of non-poor spells, the larger will be the discount. However, the incremental benefit arising from each additional period of affluence (non-poverty) diminishes.

To help clarify these concepts, consider three profiles $\mathbf{p} = (0, 0, 2/3)$, $\overline{\mathbf{p}} = (0, 2/3, 0)$, and $\widetilde{\mathbf{p}} = (2/3, 0, 0)$. For $\mathbf{p}$ there are two affluent periods ($n_3 = 2$) before the poor episode, for $\overline{\mathbf{p}}$ there is only one affluent period ($\overline{n}_2 = 1$) and for $\widetilde{\mathbf{p}}$ there are none. Thus, unlike the profiles $\mathbf{p}$ or $\overline{\mathbf{p}}$, the poor episode in $\widetilde{\mathbf{p}}$ has no possibility of being mitigated by previous affluent periods. Note that for any profile $\mathbf{p} = p \cdot \mathbf{e}_t^T$ and $p > 0$, it will always be the case that $t = n_t + 1$. The axiom of constant-relative poverty mitigation would then say that the poverty of $\mathbf{p}$ and $\overline{\mathbf{p}}$ should be less than that of $\widetilde{\mathbf{p}}$, and given by the following rule (when $\beta = 1$):

$$
\begin{aligned}
P(\mathbf{p}) &= \frac{1}{(1+n_3)} P(\widetilde{\mathbf{p}}) = \frac{1}{3} P(\widetilde{\mathbf{p}}), \\
P(\overline{\mathbf{p}}) &= \frac{1}{(1+\overline{n}_2)} P(\widetilde{\mathbf{p}}) = \frac{1}{2} P(\widetilde{\mathbf{p}}).
\end{aligned}
$$

Although the effective rate of discount will depend on the value of β, what this axiom essentially proposes is that the level of discount should depend on the number of immediately preceding affluent periods. Importantly, it does not take into consideration the amount of income during the non-poor episodes. As discussed above, there may be legitimate reasons for ignoring the amount of income in non-poor periods, based on lack of income-smoothing opportunities across periods, particularly in a developing country context, but this is quite a restrictive assumption and we shall later relax this condition.

It should be noted that there are a number of other possible ways of capturing the mitigating effect of affluent periods on subsequent poor episodes. A more general axiom could easily be provided that would accommodate a broader range of methodologies.[9] Our goal here, however, is to provide an axiomatic basis for a specific functional form. Thus we concentrate on a particular form of discounting, as stipulated in the above axiom.

[9] See Dutta et al. (2011) for a more general stucture for poverty mitigation.

If non-poor episodes have a mitigating effect on poverty, then by a similar intuition, poor episodes should serve to intensify the experience of subsequent poverty. Our next axiom captures the intuition that not only do spells of poverty have an exacerbating impact on subsequent poverty, but that the detrimental impact increases as the length of the poor spell increases. Recall that we use k_t to denote the number of consecutive poor periods, up to and including period t.

Axiom 5 *Relative poverty intensification* holds if for all $T \in \mathbb{N}\backslash\{1\}$, all $\mathbf{p} \in [0, 1]^T$ such that $p_{T-1} > 0$ and $p_T > 0$, and $\alpha \geq 0$,

$$P(\mathbf{p}) = P(p_1, \ldots, p_{T-1}, 0) + k_T^{\alpha} P(p_T \cdot \mathbf{e}_1^T).$$

Consider a poverty profile where the penultimate period is a poor one. This axiom ensures that the difference in impact between having a poor and having a non-poor episode in the last period is not determined solely by the size of the income shortfall in the last period—it is also weighted by a factor depending on the number of poor periods preceding it. Thus the impact of being poor in the last period, when there was also poverty previously, is greater than the poverty corresponding to the income shortfall that is being added. The parameter α allows for a range of judgements as to the precise impact of the preceding poor periods. For $\alpha = 0$, previous poor episodes should carry no additional weight. The larger the value of α, the greater the exacerbating impact of the previous episodes of poverty.

Combining the above axioms we obtain the following result.

Proposition 1 *An intertemporal poverty measure satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3), constant-relative poverty mitigation (Axiom 4) and relative poverty intensification (Axiom 5) if and only if it is the constant-relative affluence-dependent intertemporal poverty measure P_R.*

The following proposition demonstrates that the axioms in Proposition 1 are independent.

Proposition 2 *The axioms single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3), constant-relative poverty mitigation (Axiom 4) and relative poverty intensification (Axiom 5) are independent.*

Before moving to the next section, we briefly draw attention to the important issue of truncation. In general, an individual will have lived prior to the first period for which we have data, and he or she could have been either affluent or poor in those periods. In our paper, however, we do not take into consideration any such information. Consider any poverty profile $\mathbf{p} = (p_1, \ldots, p_T)$. If the poverty in the first time period $p_1 > 0$, then, by definition, $n_1 = 0$ and $k_1 = 1$. Together they imply that poverty in the first time period will not be mitigated, and neither will it be intensified.

4 Discussion

In this section we discuss two broad extensions of the proposed measures P_R. Firstly, we relax the focus axiom and allow for income levels in affluent periods to have some mitigating effect on subsequent poverty. Secondly, we present a class of measures where the mitigation due to affluent periods is an absolute amount, rather than a constant proportion of subsequent poverty levels.

4.1 Focus axiom

Previously we have highlighted that the P_R measures implicitly adopt the focus axiom and have provided arguments as to why it may be reasonable to do so. Nevertheless, it is also possible to give a justification for the amount of income in affluent periods to have a role in mitigating subsequent poverty. As we have discussed earlier, the mitigating effect in our paper is transmitted through non-income resources, the quantity of which may clearly depend on the levels of income received. The amount of assets one can purchase, the level of skills one can acquire or the kind of location that one may be able to move to clearly may depend on the level of income one has during affluent times. Thus individuals with higher levels of income may be 'better prepared' for future hardships. It is this aspect that we now try to capture by relaxing the focus axiom.[10]

Although the amount of income above the poverty line no doubt helps, it does not necessarily have a strong correlation with achievements along non-income dimensions (see Sen 1985, 1987; UNDP 1990). Thus while we want to capture some impact of income levels in non-poor periods, we wish to ensure that it does not have a one-to-one effect on mitigating subsequent poverty. We would also like to relax the focus axiom in such a way that we do not lose the duration aspect inherent in our P_R measure.

With this in mind, for any profile $\mathbf{p}$, let us define,

$$\tilde{n}_t = \begin{cases} \sum_{t=s}^{t'} \lambda_t, & \text{if } p_t = 0 \text{ for all } t = s, \ldots, t' \\ 0 & \text{otherwise} \end{cases}$$

where

$$\lambda_t = \begin{cases} \gamma, & \text{if } x_t > \delta z \\ 1 & \text{otherwise} \end{cases}$$

and $\gamma \geq 1$ and $\delta > 1$. If $\gamma = 1$, then $\tilde{n}_t = n_t$, i.e. it is the sum of all the non-poor periods directly preceding period t. On the other hand, if $\gamma > 1$, then the mitigating factor is the weighted sum of all those periods, with the weights being higher in periods where income was more than δ times the poverty line. The parameter δ captures how far above the poverty line income must be for it to carry a higher weight in the mitigating factor. An alternative axiom, in the spirit of our constant-relative poverty mitigation axiom (Axiom 4) is the following:

Axiom 6 *Relative poverty mitigation* holds if for all $T \in \mathbb{N}\backslash\{1\}$ and all $p \in (0, 1]$ we have $P(p\mathbf{e}_t^T) = P(p\mathbf{e}_1^T)/(1 + \tilde{n}_t)^\beta$ for $\beta \geq 0$ and any $t \in \{1, \ldots, T\}$.

[10] Note that explictly incorporating the non-income dimensions is not feasible in this context, since all we observe is the ex-post income distribution for the individual across time.

Note that, in contrast to Axiom 4, if income in non-poor periods is higher than a certain amount, the discount will be greater.

By replacing the constant-relative poverty mitigation axiom (Axiom 4) in Proposition 1 with Axiom 6, we can derive the *relative affluence-dependent intertemporal poverty measure* $\widetilde{P}_R$, which is defined as follows.

Proposition 3 *An intertemporal poverty measure satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3), relative poverty intensification (Axiom 5) and relative poverty mitigation (Axiom 6), if and only if it is*

$$\widetilde{P}_R(\mathbf{p}) = \frac{1}{T}\sum_{t=1}^{T} \frac{k_t^{\alpha}}{(1+\tilde{n}_t)^{\beta}} p_t^{\theta}, \textit{ where } \alpha, \beta, \theta \geq 0.$$

4.2 Absolute mitigation

One may raise two other valid objections with the measure P_R that we have proposed in this paper. The first issue is that the extent of the mitigation, in absolute terms, depends on the level of poverty. Consider the profile $\mathbf{p} = (0, p)$. When $p = 1$, the absolute size of the mitigation under P_R (given $\alpha = \beta = \theta = 1$) is 0.5. On the other hand when $p = 0.5$, under the same parameters, P_R would determine the mitigation to be only 0.25. The greater the size of p is, the greater is the size of the mitigation.[11]

A second possible criticism is that there is no possibility of the mitigating effect of affluence being able to wipe out subsequent poverty completely, no matter how small the income shortfall is and no matter how many non-poor episodes precede it. There is room for differing judgements on this point too. It might instead be argued that the fact that there is still poverty in period t means that, notwithstanding the mitigating effects of the preceding affluence, there remains a definite problem in period t that cannot be alleviated completely. These issues are not unique to P_R and any measure belonging to the relative mitigation class of measures would be prone to the same criticisms.

To obtain the desired relaxation for the manner in which a poor period can be discounted if preceded by periods of affluence, we need to replace constant-relative poverty mitigation (Axiom 4) with an axiom in which there is an absolute mitigating effect.

Before stating the axiom, let us define a function $h : \mathbb{Z}_+ \longrightarrow \mathbb{R}_+$, such that $h(1) = 0$. The 'absolute' poverty mitigation axiom can be formally stated as follows.

Axiom 7 *Absolute poverty mitigation* holds if for all $T \in \mathbb{N}\backslash\{1\}$ and all $p \in (0, 1]$ we have, for any $t \in \{1, \ldots, T\}$, $P(p\mathbf{e}_t^T) = \max\left(P(p\mathbf{e}_1^T) - h(t), 0\right)$.

Thus we allow for the possibility that the mitigation from affluent periods may be so high that poverty arising from a subsequent income shortfall can be completely

[11] A natural counter-argument of course is that the proportion of poverty which is mitigated remains the same, regardless of the poverty level. This possible criticism is somewhat reminiscent of the charge often made against relative inequality measures, which register no change in inequality when all incomes are increased by the same proportion. Those who regard absolute differences in income to be important with respect to inequality would reject such measures.

mitigated. The condition $h(1) = 0$ ensures that in the absence of any prior periods, there is no discounting.

Our final axiom is a monotonicity condition. It considers two poor profiles of length T, each with only one episode of poverty, of level p. In one profile the poor episode occurs in period $t \leq T$ and in the other profile the poor episode takes place in period $(t-1) \geq 1$. Note that the static level of poverty in the poor period is the same in each profile. The axiom stipulates that $t-1$ directly preceding periods of affluence have a greater mitigating impact than $t-2$ periods.

Axiom 8 *Monotonic poverty mitigation* holds if for all $T \in \mathbb{N}\backslash\{1\}$ and all $p \in (0, 1]$ we have $P(p \cdot \mathbf{e}^T_{t-1}) \geq P(p \cdot \mathbf{e}^T_t)$ for any $t \in \{2, \ldots, T\}$.

The above two axioms along with single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3) and relative poverty intensification (Axiom 5), characterize a class of *absolute affluence-dependent intertemporal poverty* measures.

Proposition 4 *An intertemporal poverty measure satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3), relative poverty intensification (Axiom 5), absolute poverty mitigation (Axiom 7), and monotonic poverty mitigation (Axiom 8) if and only if it is*

$$P_A(\mathbf{p}) = \frac{1}{T} \sum_{\substack{t=1 \\ p_t \neq 0}}^{T} \max\left(k_t^\alpha p_t^\theta - f(n_t), 0\right), \ \textit{where } \alpha, \theta \geq 0$$

and $f : \mathbb{Z}_+ \to \mathbb{R}_+$ *such that* $f(0) = 0$ *and* $f(n_t + 1) \geq f(n_t)$.[12]

5 Conclusions

In this paper we have proposed and characterized new classes of individual-level intertemporal poverty measures. Our main objective was to account for the mitigating effect of affluent periods, as well as the debilitating impact of prolonged spells of poverty. We have proposed three broad classes of measures which, collectively, incorporate a broad range of views as to the impact of affluent periods on subsequent poor episodes. The first proposed class of measures, P_R, incorporates an intertemporal version of the focus axiom, as used by Foster (2009) and Bossert et al. (2012), where the mitigating impact of non-poor episodes is the same, irrespective of the level of affluence in those periods. We relax this stringent restriction in another class of measures, $\widetilde{P}_R$, and allow for the level of income in non-poor periods to have a role in determining the amount of mitigating impact such periods can bring. Within the classes of measures proposed, we also allow for two broad views as to the way in which affluent periods mitigate the level of poverty. In P_R, the mitigating impact is

[12] Note that the summation in P_A is over only those periods where $p_t \neq 0$. This is a technical requirement since in our paper n_t (the number of immediately preceding affluent periods) is not defined when $p_t = 0$.

'proportional' to the level of poverty and, as such, can never alleviate poverty fully. We also characterize a class of measures, P_A, where the mitigating impact is 'absolute' and thus has the potential to completely eradicate the impact of subsequent poverty.

Our measures build on the individual-level measures of Bossert et al. (2012), which also evaluate individual intertemporal poverty as a weighted average across time of snapshot poverty. The central innovation in our paper lies in using both the number of preceding affluent periods and the number of consecutive poor periods to determine the weights. We allow for this to be conducted in a quite general way, so that different judgements regarding the precise composition of the weights can be accommodated. Our approach provides added sensitivity, allowing one to distinguish between poverty profiles which other measures in the literature are unable to. This enhanced sensitivity can be useful for policy purposes as it allows for a more diversified approach when it comes to allocation of resources. How precisely a redistribution of wealth or consumption should be implemented in an intertemporal framework, among the poor and also between the time periods, is an important issue to which a solution is not immediately apparent. This issue is left for future research.

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Appendix: Proofs

Proof of Proposition 1 We concentrate on the "only if" part of the proof, as it is immediate to verify that P_R satisfies the axioms stated in Proposition 1.

Suppose that P satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3), constant-relative poverty mitigation (Axiom 4) and relative poverty intensification (Axiom 5). We need to show that for any time period $T \in \mathbb{N}$ and any poverty profile $\mathbf{p} \in [0, 1]^T$ we have $P(\mathbf{p}) = P_R(\mathbf{p})$ to for an exogenously determined $\alpha, \beta \geq 0$ and $\theta \geq 1$. So, take any $T \in \mathbb{N}$ and any poverty profile $\mathbf{p} \in [0, 1]^T$.

Suppose $T = 1$. In this case single period equivalence holds and $P(\mathbf{p}) = P(p) = P_R(p)$ for some $p \in (0, 1]$. Hence, $P(\mathbf{p}) = P_R(\mathbf{p})$ follows.

Assume now that $T > 1$. If $\mathbf{p} = \mathbf{0}^T$, then by normalization we obtain $P(\mathbf{p}) = P(\mathbf{0}^T) = 0 = P_R(\mathbf{0})$. Thus, $P(\mathbf{p}) = P_R(\mathbf{p})$ follows.

Next we proceed by induction on the number of poor periods, when $\mathbf{p} \neq \mathbf{0}^T$. Consider first the case where there is exactly one poor period. Recall that we represent a T-period poverty profile $\mathbf{p} =(p_1, \ldots, p_s, \ldots p_T)$, where $\forall t \neq s,\ p_t = 0$ and $p_s = 1$ as $\mathbf{e}_s^T$. Thus $\mathbf{p} = p \cdot \mathbf{e}_s^T$ would stand for a T-period poverty profile $\mathbf{p} =(p_1, \ldots, p_s, \ldots p_T)$, where $\forall t \neq s,\ p_t = 0$ and $p_s = p$.

Without loss of generality, we can write $\mathbf{p} = p \cdot \mathbf{e}_t^T$ where $p > 0$ and $t \in \{1, \ldots, T\}$. Thus $P(\mathbf{p}) = P(p \cdot \mathbf{e}_t^T)$. Then

$$\begin{aligned}
P(p\cdot \mathbf{e}_t^T) &= \frac{t}{T}P(p\cdot \mathbf{e}_t^t) + \frac{T-t}{T}P(\mathbf{0}^{T-t}), \text{ by time decomposability,}\\
&= \frac{t}{T}P(p\cdot \mathbf{e}_t^t), \text{ by normalization,}\\
&= \frac{t}{T}\frac{P(p\cdot \mathbf{e}_1^t)}{t^\beta}, \text{ by constant-relative poverty mitigation,}\\
&= \frac{1}{Tt^{\beta-1}}\cdot\left[\frac{1}{t}P(p) + \frac{t-1}{t}P(\mathbf{0}^{t-1})\right], \text{ by time decomposability,}\\
&= \frac{P(p)}{Tt^\beta}, \text{ by normalization,}\\
&= \frac{p^\theta}{Tt^\beta}, \text{ by single period equivalence.}
\end{aligned} \tag{2}$$

Since $k_t = 1, t = (1 + n_t)$ and $\forall\, i \neq t$, $p_i = 0$ we can write (2) as

$$P(\mathbf{p}) = P(p\cdot \mathbf{e}_t^T) = \frac{1}{T}\sum_{i=1}^{T}\frac{k_t^\alpha p_i^\theta}{(1+n_t)^\beta} = P_R(\mathbf{p}).$$

Thus we obtain $P(\mathbf{p}) = P_R(\mathbf{p})$.

Suppose now that $P(\hat{\mathbf{p}}) = P_R(\hat{\mathbf{p}})$ whenever $\hat{\mathbf{p}}$ contains m poor periods, for some $m \in \{1, \ldots, T-1\}$. Let $\mathbf{p} \in [0,1]^T$ be any poverty profile, such that the number of poor periods is $m+1$. Let $t \in \{2, \ldots, T\}$ be such that the final poor period is period t. Thus $t = \max\{s : 2 \leq s \leq T, p_s > 0\}$. From time decomposability we derive

$$\begin{aligned}
P(\mathbf{p}) &= P(p_1, \ldots, p_t, \ldots, p_T)\\
&= \frac{t}{T}P(p_1, \ldots, p_t) + \frac{T-t}{T}P(p_{t+1}, \ldots, p_T).
\end{aligned}$$

Now t being the final poor period means that by normalization we get

$$P(\mathbf{p}) = \frac{t}{T}P(p_1, \ldots, p_t). \tag{3}$$

Let $s \neq t$ be maximal with $p_s > 0$. So s is the last poor period prior to t. Suppose $s \neq t-1$. Then by time decomposability we obtain

$$P(p_1, \ldots, p_s, \ldots, p_t) = \frac{s}{t}P(p_1, \ldots, p_s) + \frac{t-s}{t}P(p_{s+1}, \ldots, p_t). \tag{4}$$

Further, $P(p_{s+1}, \ldots, p_t) = P(p_t \cdot \mathbf{e}_{t-s}^{t-s})$ since $p_i = 0$ for all $i \in \{s+1, \ldots, t-1\}$. Applying single period equivalence, time decomposability, normalization and constant-relative poverty mitigation and noting that $k_t = 1$ and $n_t = t - s - 1$, we obtain

$$P(p_{s+1}, \ldots, p_t) = \frac{k_t^\alpha}{(1+n_t)^{1+\beta}}p_t^\theta. \tag{5}$$

Now consider the case when $s = t - 1$. Then using relative poverty intensification we get

$$P(p_1, \ldots, p_s, \ldots, p_t) = P(p_1, \ldots, p_s, 0) + k_t^{\alpha} P(p_t \mathbf{e}_1^t). \tag{6}$$

Using time decomposability, single period equivalence and noting that $n_t = 0$, (6) can be written as

$$P(p_1, \ldots, p_s, \ldots, p_t) = \frac{s}{t} P(p_1, \ldots, p_s) + \frac{k_t^{\alpha}}{(1+n_t)^{\beta}} \frac{p_t^{\theta}}{t} \tag{7}$$

Now $(p_1, \ldots, p_s)$ contains m poor periods. Thus, by the induction hypothesis, we have

$$P(p_1, \ldots, p_s) = P_R(p_1, \ldots, p_s) = \frac{1}{s} \sum_{i=1}^{s} w_i p_i^{\theta} \text{ where } w_i = \frac{k_i^{\alpha}}{(1+n_i)^{\beta}}. \tag{8}$$

Substituting (8) and (5) into (4) (for the $s \neq t-1$ case) or substituting (8) into (7) (for the $s = t-1$ case) yields

$$P(p_1, \ldots, p_s, \ldots, p_t) = \left[\frac{1}{t} \sum_{i=1}^{s} \frac{k_i^{\alpha} p_i^{\theta}}{(1+n_i)^{\beta}} \right] + \frac{k_t^{\alpha}}{(1+n_t)^{\beta}} \frac{p_t^{\theta}}{t}. \tag{9}$$

Further, substituting (9) into (3) we obtain

$$P(\mathbf{p}) = \frac{1}{T} \left[\left(\sum_{i=1}^{s} \frac{p_i^{\theta}}{(1+n_i)^{\beta}} \right) + \frac{k_t^{\alpha} p_t^{\theta}}{(1+n_t)^{\beta}} \right].$$

Finally, since $p_i = 0$ for all $i \in \{s+1, \ldots, t-1\}$ and all $i \in \{t+1, \ldots, T\}$, we have

$$P(\mathbf{p}) = \frac{1}{T} \sum_{i=1}^{T} \frac{k_i^{\alpha} p_i^{\theta}}{(1+n_i)^{\beta}} = P_R(\mathbf{p}).$$

This concludes the proof for the case of $m + 1$ poor periods, and by induction it follows that $P(\mathbf{p}) = P_R(\mathbf{p})$ for any poverty profile $\mathbf{p}$. This completes the proof of Proposition 1. □

Proof of Proposition 2 We demonstrate that axioms single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3), absolute poverty mitigation (Axiom 7), constant-relative poverty mitigation (Axiom 4) and relative poverty intensification (Axiom 5) are independent by presenting a separate poverty measure that satisfies all the axioms except one. We do this one at a time for each of the five axioms.

Consider a poverty profile $\mathbf{p} \in [0, 1]^T$. Then the following measure violates single period equivalence (Axiom 1) but satisfies the other axioms.

$$P_1(\mathbf{p}) = \frac{1}{T} \sum_{i=1}^{T} \frac{k_i^{\alpha} 2 p_i}{(1 + n_i)^{\beta}}.$$

The next measure violates time decomposability (Axiom 2) but satisfies the other axioms.

$$P_2(\mathbf{p}) = \sum_{i=1}^{T} \frac{k_i^{\alpha} p_i^{\theta}}{(1 + n_i)^{\beta}}.$$

A measure which violates normalization (Axiom 3) but satisfies the other axioms is given by

$$P_3(\mathbf{p}) = \begin{cases} 10 & \text{if} \, \mathbf{p} \in \mathbf{0}^T \\ \frac{1}{T} \sum_{i=1}^{T} \frac{k_i^{\alpha} p_i^{\theta}.}{(1+n_i)^{\beta}} & \text{otherwise.} \end{cases}$$

A measure which violates constant-relative poverty mitigation (Axiom 4) but satisfies the others is

$$P_4(\mathbf{p}) = \frac{1}{T} \sum_{i=1}^{T} \frac{k_i^{\alpha} p_i^{\theta}}{\ln(1 + n_i)}.$$

A measure which violates relative poverty intensification (Axiom 5) and satisfies the rest is as follows.

$$P_5(\mathbf{p}) = \frac{1}{T} \sum_{i=1}^{T} \frac{p_i^{\theta}}{(1 + n_i)^{\beta}}.$$

□

Proof of Proposition 3 The proof is similar to that of Proposition 1 and is omitted. □

Proof of Proposition 4 We concentrate on the "only if" part of the proof, as it is immediate to verify that P_A satisfies the axioms stated in Proposition 4.

Suppose that P satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3), relative poverty intensification (Axiom 5), absolute poverty mitigation (Axiom 7), and monotonic poverty mitigation (Axiom 8). We need to show that for any time period $T \in \mathbb{N}$ and any poverty profile $\mathbf{p} \in [0, 1]^T$ we have $P(\mathbf{p}) = P_A(\mathbf{p})$ for a monotonically increasing function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $f(0) = 0$ and $f(n_t + 1) \geq f(n_t)$.

Take any $T \in \mathbb{N}$ and any poverty profile $\mathbf{p} \in [0, 1]^T$. Suppose $T = 1$. Note that $k_1 = 1$ and $n_1 = 0$. By absolute poverty mitigation, we have

$$P(p\mathbf{e}_1^1) = \max\left(P(p\mathbf{e}_1^1) - h(1), 0\right). \tag{10}$$

By construction let $f(n_t) = th(t)$, $t = n_t + 1$. Given $h(t) \in \mathbb{R}_+$ and $t \in \{1, \ldots, T\}$, $f : \mathbb{Z}_+ \to \mathbb{R}_+$. When $t = 1$, it implies $f(0) = h(1) = 0$. Due to single period equivalence and $k_1 = 1$ we can write Eq. (10) as

$$P(\mathbf{p}) = \max\left(k_1^\alpha p^\theta, 0\right) = P_A(\mathbf{p}).$$

Assume now that $T > 1$. If $\mathbf{p} = \mathbf{0}^T$, then by normalization we obtain $P(\mathbf{p}) = P(\mathbf{0}^T) = 0 = P_A(\mathbf{p})$. Thus, $P(\mathbf{p}) = P_A(\mathbf{p})$.

Next, we proceed by induction on the number of poor periods when $\mathbf{p} \neq \mathbf{0}^T$. Consider the case where there is exactly one poor period. Without loss of generality, we can write $\mathbf{p} = p \cdot \mathbf{e}_t^T$ where $p \in (0, 1]$ and $t \in \{1, \ldots, T\}$. Thus $P(\mathbf{p}) = P(p \cdot \mathbf{e}_t^T)$. If $t = 1$ then, applying single period equivalence, time decomposability and normalization, and noting that $k_1 = 1$ and $f(0) = 0$, yields

$$\begin{aligned} P(p \cdot \mathbf{e}_1^T) &= \frac{1}{T} p^\theta, \\ &= \frac{1}{T} \max\left(k_1^\alpha p^\theta - f(0), 0\right). \end{aligned}$$

For $t > 1$ applying time decomposability and normalization, we obtain

$$\begin{aligned} P(p \cdot \mathbf{e}_t^T) &= \frac{t}{T} P(p \cdot \mathbf{e}_t^t). \\ &= \frac{t}{T} \max\left(P(p \cdot \mathbf{e}_1^t) - h(t), 0\right), \text{ by absolute poverty mitigation,} \\ &= \frac{1}{T} \max\left(t(P(p \cdot \mathbf{e}_1^t) - h(t)), 0\right) \\ &= \frac{1}{T} \max\left(t\left(\frac{1}{t} P(p) - h(t)\right), 0\right), \text{ by time decomposability,} \\ &= \frac{1}{T} \max\left(P(p) - f(n_t), 0\right), \text{ where } f(n_t) = th(t) \text{ by construction.} \end{aligned} \tag{11}$$

To show that $f(n_t)$ is monotonic, consider another profile $P(p \cdot \mathbf{e}_{t-1}^T)$. Applying (11) we get

$$P\left(p \cdot \mathbf{e}_{t-1}^T\right) = \frac{1}{T} \max\left(P(p) - f(n_{t-1}), 0\right). \tag{12}$$

By monotonic poverty mitigation it must be the case that $P(p \cdot \mathbf{e}_{t-1}^T) \geq P(p \cdot \mathbf{e}_t^T)$. Comparing (11) and (12) and noting that $n_t = n_{t-1}+1$, we can show $f(n_t) \geq f(n_{t-1})$. Applying single period equivalence in (11) and noting that $k_t = 1$, we can show

$$P\left(p \cdot \mathbf{e}_t^T\right) = \frac{1}{T} \max\left(k_t^\alpha p^\theta - f(n_t), 0\right). \tag{13}$$

Since $p_i = 0$ for all $i \neq t$, we can write (13) as

$$P(\mathbf{p}) = P\left(p \cdot \mathbf{e}_t^T\right) = \frac{1}{T} \sum_{\substack{i=1 \\ p_i \neq 0}}^{T} \max\left(k_i^\alpha p_i^\theta - f(n_i), 0\right) = P_A(\mathbf{p}). \tag{14}$$

Suppose now that $P(\hat{\mathbf{p}}) = P_A(\hat{\mathbf{p}})$ whenever $\hat{\mathbf{p}}$ contains m poor periods, for some $m \in \{1, \ldots, T-1\}$. Let $\mathbf{p} \in [0, 1]^T$ be any poverty profile, such that the number of poor periods is $m+1$. Let $t \in \{2, \ldots, T\}$ be such that the final poor period is period t. Thus $t = \max\{s : 2 \leq s \leq T, p_s > 0\}$. From time decomposability we derive

$$\begin{aligned} P(\mathbf{p}) &= P(p_1, \ldots, p_t, \ldots, p_T), \\ &= \frac{t}{T} P(p_1, \ldots, p_t) + \frac{T-t}{T} P(p_{t+1}, \ldots, p_T). \end{aligned}$$

Now t being the final poor period means that by normalization we get

$$P(\mathbf{p}) = \frac{t}{T} P(p_1, \ldots, p_t). \tag{15}$$

Let $s \neq t$ be maximal with $p_s > 0$. So s is the last poor period prior to t. Suppose $s \neq t-1$. Then by time decomposability we obtain

$$P(p_1, \ldots, p_s, \ldots, p_t) = \frac{s}{t} P(p_1, \ldots, p_s) + \frac{t-s}{t} P(p_{s+1}, \ldots, p_t). \tag{16}$$

Further, $P(p_{s+1}, \ldots, p_t) = P(p_t \cdot \mathbf{e}_{t-s}^{t-s})$ since $p_i = 0$ for all $i \in \{s+1, \ldots, t-1\}$. Using (14) and noting that $k_t = 1$ and $n_t = t - s - 1$, we obtain

$$P(p_{s+1}, \ldots, p_t) = \frac{1}{t-s} \sum_{\substack{i=s+1 \\ p_t \neq 0}}^{t} \max\left(k_i^\alpha p_i^\theta - f(n_i), 0\right). \tag{17}$$

Substituting (17) in (16) we get

$$P(p_1, \ldots, p_s, \ldots, p_t) = \frac{s}{t} P(p_1, \ldots, p_s) + \frac{1}{t} \sum_{\substack{i=s+1 \\ p_t \neq 0}}^{t} \max\left(k_i^\alpha p_i^\theta - f(n_i), 0\right). \tag{18}$$

Now consider the case when $s = t - 1$. Using relative poverty intensification we get

$$P(p_1, \ldots, p_s, p_t) = P(p_1, \ldots, p_s, 0) + k_t^\alpha P(p_t \cdot \mathbf{e}_1^t). \tag{19}$$

Applying time decomposability, single period equivalence and using (11) we can write (19) as

$$\begin{aligned} P(p_1, \ldots, p_s, p_t) &= \frac{s}{t} P(p_1, \ldots, p_s) + \frac{k_t^{\alpha}}{t} \max\left(p_t^{\theta} - f(n_1), 0\right), \\ &= \frac{s}{t} P(p_1, \ldots, p_s) + \frac{1}{t} \max\left(k_t^{\alpha} p_t^{\theta} - f(n_1), 0\right). \end{aligned} \tag{20}$$

Note that in (20) $f(n_1) = 0$, since $n_1 = 0$ by definition. Now $(p_1, \ldots, p_s)$ contains m poor periods. By the induction hypothesis, we have

$$P(p_1, \ldots, p_s) = P_R(p_1, \ldots, p_s) = \frac{1}{s} \sum_{\substack{i=1 \\ p_i \neq 0}}^{s} \max\left(k_i^{\alpha} p_i^{\theta} - f(n_i), 0\right). \tag{21}$$

Substituting (21) into (18) (for the $s \neq t - 1$ case) or substituting (21) into (20) (for the $s = t - 1$ case) yields

$$\begin{aligned} P(p_1, \ldots, p_s, \ldots, p_t) \;=\; & \frac{1}{t} \sum_{\substack{i=1 \\ p_i \neq 0}}^{s} \max\left(k_i^{\alpha} p_i^{\theta} - f(n_i), 0\right) \\ & + \frac{1}{t} \sum_{\substack{i=s+1 \\ p_i \neq 0}}^{t} \max\left(k_i^{\alpha} p_i^{\theta} - f(n_i), 0\right). \end{aligned} \tag{22}$$

Further, substituting (22) into (15) we obtain

$$P(\mathbf{p}) = \frac{1}{T} \left[\sum_{\substack{i=1 \\ p_i \neq 0}}^{s} \max\left(k_i^{\alpha} p_i^{\theta} - f(n_i), 0\right) + \sum_{\substack{i=s+1 \\ p_i \neq 0}}^{t} \max\left(k_i^{\alpha} p_i^{\theta} - f(n_i), 0\right) \right].$$

Finally, since $p_i = 0$ for all $i \in \{t + 1, \ldots, T\}$, we have

$$P(\mathbf{p}) = \frac{1}{T} \sum_{\substack{t=1 \\ p_t \neq 0}}^{T} \max\left(k_t^{\alpha} p_t^{\theta} - f(n_t), 0\right) = P_A(\mathbf{p}).$$

This concludes the proof for the case of $m+1$ poor periods, and by induction it follows that $P(\mathbf{p}) = P_A(\mathbf{p})$ for any poverty profile $\mathbf{p}$. It therefore follows that $P = P_A$, which completes the proof of Proposition 4. □

References

Baulch B, Hoddinott J (2000) Economic mobility and poverty dynamics in developing countries. J Dev Stud 36:1–24

Beall J (2001) Valuing social resources or capitalising on them? Limits to pro-poor urban governance in nine cities of the south. Int Plan Stud 6:357–375

Behrman J, Deolalikar A (1989) Wages and labor supply in rural India: the role of health, nutrition and seasonality. In: Sahn D (ed) Causes and implications of seasonal variability in household food security. Johns Hopkins University Press, Baltimore

Bossert W, Chakravarty S, D'Ambrosio C (2012) Poverty and time. J Econ Inequal 10:145–162

Calvo C, Dercon S (2009) Chronic poverty and all that: the measurement of poverty over time. In: Addison T, Hulme D, Kanbur R (eds) Poverty dynamics: interdisciplinary perspectives. Oxford University Press, Oxford, pp 29–58

Carter M, Ikegami M (2007) Looking forward: theory-based measures of chronic poverty and vulnerability. CPRC Working Paper No. 94, Chronic Poverty Research Centre (CPRC), Manchester

Chakravarty S (1983) A new index of poverty. Math Soc Sci 6:307–313

Clark S, Hemming R, Ulph D (1981) On indices for the measurement of poverty. Econ J 91:515–526

Cruces G (2005) Income fluctuations, poverty and well-being over time: theory and application to Argentina. London School of Economics, Discussion paper No. DARP 76

Davis P (2006) Poverty in time: exploring poverty dynamics from life history interviews in Bangladesh. CPRC Working Paper No. 69, Chronic Poverty Research Centre (CPRC), Manchester

Davis P, Baulch B (2009) Parallel realities: exploring poverty dynamics using mixed methods in rural Bangladesh. CPRC Working Paper No. 142, Chronic Poverty Research Centre (CPRC), Manchester

Dercon S, Hoddinott J (2003) Health, shocks and poverty persistence. Discussion Paper No. 2003/08, WIDER, Helsinki

Dercon S, Krishnan P (2000) In sickness and in health: risk-sharing within households in rural Ethiopia. J Polit Econ 108:688–727

Dutta I, Roope L, Zank H (2011) On intertemporal poverty: affluence-dependent measures. University of Manchester, Economics Discussion Paper, EDP 1112

Foster J (2009) A class of chronic poverty measures. In: Addison T, Hulme D, Kanbur R (eds) Poverty dynamics: interdisciplinary perspectives. Oxford University Press, Oxford, pp 59–76

Foster J, Santos ME (2012) Measuring chronic poverty. Oxford Poverty & Human Development Initiative (OPHI), Working Paper No. 52

Foster J, Greer J, Thorbecke E (1984) A class of decomposable poverty measures. Econometrica 52:761–766

Grab J, Grimm M (2007) Robust multiperiod poverty comparisons. Ibero-America Institute for Economic Research Working Paper No. 160

Gradín C, del Río C, Cantó O (2012) Measuring poverty accounting for time. Rev Income Wealth 58:330–354

Hoy M, Zheng B (2011) Measuring lifetime poverty. J Econ Theory 146:2544–2562

Hulme D (2003) Thinking 'small' and the understanding of poverty: Maymana and Mofizul's story. CPRC Working Paper No. 22, Chronic Poverty Research Centre (CPRC), Manchester

Hulme D, McKay A (2005) Identifying and measuring chronic poverty: beyond monetary measures. CPRC Working Paper No. 30, Chronic Poverty Research Centre (CPRC), Manchester

McKay A (2009) Assets and chronic poverty: background paper. CPRC Working Paper No. 100, Chronic Poverty Research Centre (CPRC), Manchester

Mendola D, Busetta A, Milito AM (2011) Combining the intensity and sequencing of the poverty experience: a class of longitudinal poverty indices. J R Stat Soc 174:1–21

Morduch J (1995) Income smoothing and consumption smoothing. J Econ Perspect 9:103–114

Moser C (1996) Confronting crisis. A comparative study of household responses to poverty and vulnerability in four poor urban communities. Environmentally Sustainable Development Studies and Monographs Series No. 8, World Bank, Washington

Narayan D, Chambers R, Shah MK, Petesch P (2000) Voices of the poor: crying out for change. World Bank, Oxford University Press, Oxford

Narayan D, Pritchett L, Kapoor S (2009) Moving out of poverty: success from the bottom up. Palgrave Macmillan and the World Bank, New York

Porter C, Quinn N (2008) Intertemporal poverty measurement: tradeoffs and policy options. CSAE WPS/2008–2021

Sen A (1976) Poverty: an ordinal approach to measurement. Econometrica 44:219–231
Sen A (1985) Commodities and capabilites. Oxford University Press, Oxford
Sen A (1987) The standard of living. Cambridge University Press, Cambridge
Sen B, Hulme D (2004) Chronic poverty in Bangladesh: tales of ascent, descent, marginality and persistence. CPRC Working Paper No. 43, Chronic Poverty Research Centre (CPRC), Manchester
UNDP Human Development Report Office (1990) Human development report 1990. Oxford University Press, New York
Watts H (1968) An economic definition of poverty. In: Moynihan DP (ed) On understanding poverty. Basic Books, New York
Woolcock M, Narayan D (2000) Social capital: implications for development theory, research and policy. World Bank Res Obs 15:225–249
Zheng B (1997) Aggregate poverty measures. J Econ Surv 11:123–162
Zheng B (2011) Measuring chronic poverty: a gravitational approach. University of Colorado, Denver