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Characterizing multidimensional inequality measures which fulfil the Pigou–Dalton bundle principle

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Abstract The *Pigou–Dalton bundle dominance* introduced by Fleurbaey and Trannoy (Social Choice and Welfare, 2003) captures the basic idea of the Pigou–Dalton transfer principle, demanding that, in the multidimensional context also, “a transfer from a richer person to a poorer one decreases inequality”. However, up to now, this principle has not been incorporated to derive multidimensional inequality measures. The aim of this article is to characterize measures which fulfil this property, and to identify sub-families of indices from a normative approach. The families we derive share their functional forms with others having already been obtained in the literature, the major difference being the restrictions upon the parameters.

1 Introduction

This article deals with the measurement of multidimensional inequality. Given different distributions, the concern of inequality measurement is to establish when one distribution is more unequal than another. As is well known, when only income is considered, the basic criterion for ordering distributions is the Pigou–Dalton transfer principle. Nevertheless, in recent years there has been considerable agreement that inequality is a multidimensional problem, and other attributes apart from income should also be taken into consideration to better measure the extent of inequality.

The straightforward generalization of the Pigou–Dalton transfer principle to any number of attributes may be established as follows: a transfer from a richer person to a poorer one, preserving the order, diminishes the inequality. Fleurbaey and Trannoy (2003) have formalized this extension as the *Pigou–Dalton bundle*

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dominance,[1] henceforth PDB. However this is not the only extension of the Pigou–Dalton transfer principle to the multi-attribute framework. Among the alternative proposals, the uniform majorization criterion, UM from now on, provided by Kolm (1977), is one of the most widely used.[2] A number of multidimensional inequality indices consistent with UM have already been obtained previously (Tsui 1995, 1999; Bourguignon 1999; List 1999; Gadjos and Weymark 2005). In contrast, up to now, PDB has not been included in deriving inequality measures.

Some difficulties arise when UM is used to order distributions, since inequality comparisons are allowed only when one distribution is obtained from another by a transfer of all the attributes in the same proportions. First, the reasons for transferring all the dimensions in the same proportions are not clear. Secondly, the idea of a transfer is not necessarily meaningful for all the attributes, for instance, for educational level or health status. Finally, if the transfers of all the attributes are made between any two people, where one is not necessarily richer than the other, the motivations for considering the new distribution more equal are not evident.

By contrast, PDB gets over the difficulties mentioned above. First of all, transfers only take place between two people, one unambiguously richer than the other. Second, it is not necessary to transfer all the attributes in the same proportions, and finally, the attributes considered as transferable can be selected. Although this appealing dominance criterion seems to lead to one of the coarsest inequality orderings not all the inequality indices, not even those fulfilling UM, as shown in Diez et al. (2007), are consistent with it.

In this article, we characterize classes of relative aggregative multidimensional inequality measures which are PDB consistent following two different approaches. In Sect. 2, we derive a multi-attribute extension of the generalized entropy family that fulfils PDB. In turn, in Sect. 3, following the normative approach pioneered by Atkinson (1970) and Kolm (1969, 1977) for constructing inequality indices, we derive social evaluation functions which satisfy PDB, and obtain relative inequality measures which are multidimensional generalizations of the Atkinson family. Similar exercises are carried out by Tsui (1999, 1995) assuming UM instead of PDB. In both cases, the functional forms of the measures obtained by Tsui and the indices derived in this article are the same, although there are two significant differences. On the one hand, there are restrictions upon the parameter values, which are far less complicated in our families. On the other hand, all the measures in our families are sensitive to the correlation between distributions of different attributes, and are consistent with a weaker version of the correlation increasing principle, henceforth WCIM, proposed by Tsui (1999). In contrast, Tsui's measures do not, in general, satisfy this property. In fact, the only aggregative measures that fulfil UM and take account of the statistical dependence between the attribute distributions are those that also fulfil PDB.

The implications of another interesting principle, the ALEP substitutability, which encompasses PDB and WCIM are examined as well, finding that this criterion and

[1] This principle is formally defined in Sect. 2. Fleurbaey and Trannoy (2003) show that this multidimensional version of the Pigou–Dalton transfer principle clashes with the Pareto principle in a society with heterogeneous individual preferences.

[2] This principle is defined in Sect. 2.

PDB place equivalent restrictions on a relative aggregative measure. We also show that invoking PDB is equivalent to assuming UM and WCIM in the case of the Atkinson family, but not in that of the generalized entropy.

Since most of the proofs follow that of Tsui's (1995, 1999) articles, we only present a scheme. The complete proofs can be provided by the authors upon request.

2 Multidimensional inequality measures which fulfil PDB

We consider a population consisting of $n \geq 2$ individuals endowed with a bundle of $k \geq 2$ attributes, such as income, health, education and so on. An $n \times k$ real matrix X represents a multidimensional distribution among the population. The *ij*th entry of X, denoted by x_{ij}, represents the *i*th individual's amount of the *j*th attribute. The *i*th row is denoted by x_i. For each attribute j, $\mu_j(X)$ represents the mean value of the *j*th attribute and $\mu(X) = (\mu_1(X), \ldots, \mu_k(X))$ is the vector of the means of the attributes. We denote $M(n, k)$ the class of $n \times k$ real matrices over the positive real elements, and D the set of all such matrices, that is, $D = \bigcup_{n \in \mathbb{N}_+} \bigcup_{k \in \mathbb{N}_+} M(n, k)$. Comparisons of the bundles of attributes are denoted as follows: $x_q \geq x_p$ if $x_{qj} \geq x_{pj}$ for all $j = 1, \ldots, k$, $x_q > x_p$ if $x_{qj} \geq x_{pj}$ and $x_p \neq x_q$.

In this article, a multidimensional inequality measure is a non-constant function $I : D \to \mathbb{R}$ satisfying the following four properties:

- *Continuity*: I is a continuous function in any individual's attribute.
- *Anonymity*: $I(X) = I(\Pi X)$ for any $X \in \mathrm{M}(n, k)$ and for any $n \times n$ permutation matrix Π.
- *Normalization*: $I(x) = 0$ if all the rows of matrix X are identical.
- *Replication Invariance*: $I(y) = I(x)$ if Y is obtained from X by a replication.

  As regards invariance properties, the following principle is used:
- *Scale Invariance principle, SI*: I satisfies SI if $I(X) = I(XC)$ for all $X \in M(n, k)$, where $C = \mathrm{diag}(c_1, \ldots, c_k)$, $c_j > 0 \quad j = 1, \ldots, k$.

Relative inequality indices are those that are scale invariant.

If the population in which we want to measure inequality is split into groups, then the aggregative principle allows us to relate inequality in each group to overall inequality:

- *Aggregative principle*: I It is *aggregative* if there exists a function A such that $I\left(\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}\right) = A(I(X_1), \mu(X_1), \mathrm{n}_1, I(X_2), \mu(X_2), \mathrm{n}_2)$ for all $X_1, X_2 \in D$ and A is a continuous and strictly increasing function in the index values $I(X_1)$ and $I(X_2)$.

Based on a well known result of Shorrocks (1984), Tsui (1999) shows that any aggregative index is ordinally equivalent to a measure that may be constructed using a two-stage procedure. In the first step, a function, denoted by ϕ, is used to aggregate the individual's bundle of attributes into a statistical summary. Then, these statistics are summed. This result is presented in the following lemma.

Lemma 1 *An inequality measure $I : D \to \mathbb{R}$ satisfies the aggregative principle if and only if there exist continuous functions ϕ and F such that, for every $X \in D$ with mean vector $\mu = \mu(X)$,*

$$F\left(I\left(X\right),\mu\right)=\frac{1}{n}\sum\nolimits_{1\leq i\leq n}\left(\phi\left(x_{i}\right)-\phi\left(\mu\right)\right) \tag{1}$$

where F is strictly increasing in $I(X)$ *and* $F(0,\mu)=0$.

None of the properties listed above allows the multidimensional measure to capture the essence of inequality. Multidimensional generalizations of the Pigou–Dalton transfer principle are proposed to make the index sensitive to the distributional inequality. The uniform majorization principle, introduced by Kolm (1977), is widely used.

- *Uniform majorization principle, UM:* I satisfies UM if $I(Y) < I(X)$ for any $X, Y \in \mathrm{M}(n,k)$ such that $Y = BX$ for some $n \times n$ bistochastic matrix B that is not a permutation matrix.

According to UM, inequality decreases when the individuals become closer in the attribute space by a transfer of all the dimensions in the same proportion. Thus, this axiom reflects an aversion towards the spread of the attribute distributions. However, UM does not take account of the statistical dependence between the distributions of the attributes, a crucial feature for an inequality index according to Atkinson and Bourguignon (1982). Tsui (1999) introduces the correlation increasing principle, one way in which this dependence is taken into account. A weak version of this principle will also play a role in this article.

Definition Let $X, Y \in M(n,k)$. Distribution X may be derived from distribution Y by a *correlation increasing transfer* if there exist two individuals p and q such that (i) $x_{pj} = \min\left\{y_{pj}, y_{qj}\right\}$ for all $j = 1,\ldots,k$, (ii) $x_{qj} = \max\left\{y_{pj}, y_{qj}\right\}$ for all $j = 1,\ldots,k$ and (iii) $x_m = y_m \quad \forall m \neq p, q$. A correlation increasing transfer is *strict* whenever $x_j \neq y_j$.

- *Correlation increasing principle, CIM*: I satisfies CIM if $I(Y) < I(X)$ for any $Y \in D$ and for all X matrices derived from Y by a permutation of rows and a finite sequence of correlation increasing transfers, at least one of which is strict.

As long as we replace a strict inequality sign with the inequality sign in the definition the *weak correlation increasing principle, WCIM,* is obtained.

Under a correlation increasing transfer, the bundles of two individuals are rearranged so that one of the individuals receives at least as much of every attribute as the other, and more of at least one attribute. Hence, CIM and WCIM ensure that inequality measures capture aversion to correlation between dimensions, and implicitly assume that all the attributes are substitutes.

Tsui (1999) examines the implications of being consistent with UM and CIM for an aggregative measure. Specifically, he shows that UM is equivalent to requiring that the function ϕ in Eq. 1 be strictly convex whereas WCIM is fulfilled if and only if ϕ is L-superaddition, that is, $\phi\left(x_p\right)+\phi\left(x_q\right) \geq \phi\left(y_p\right)+\phi\left(y_q\right)$ for all $y_p, y_q \in \mathbb{R}^k_{++}$ and for all x_p, x_q as specified in the previous definition. If ϕ has second partial derivatives, this latter condition is equivalent to $\phi''_{hl} \geq 0$ for all $h, l = 1,\ldots,k$ such that $h \neq l$ (Marshall and Olkin 1979).

As already mentioned in the introduction, some difficulties which arise with UM are got over by the PDB dominance proposed by Fleurbaey and Trannoy (2003). This criterion extends the proper idea behind the Pigou–Dalton transfer principle, that is, a transfer from a richer individual to a poorer one, which preserves the order, diminishes the inequality. This principle may be established as follows:

Definition Let $X, Y \in M(n, k)$. Distribution Y is derived from X by a *PDB transfer* if there exist two individuals p, q such that:

(i) $x_q > x_p$
(ii) $y_m = x_m \quad \forall m \neq p, q$
(iii) $y_q = x_q - \delta$ and $y_p = x_p + \delta$ where $\delta = (\delta_1, \ldots, \delta_k) \in \mathbb{R}_+^k$ with at least one $\delta_j > 0$
(iv) $y_q \geq y_p$

The first condition implies that, in the initial distribution, individual q is richer than individual p in all the attributes, whereas the fourth requirement is that this ranking is preserved by the transfer. The corresponding notion for the inequality measures is the following:

- *Pigou–Dalton bundle principle, PDB:* I satisfies PDB if $I(Y) \leq I(X)$ for any $X \in D$ and for all Y matrices derived from X by a finite sequence of PDB transfers.

It may be worth noting that PDB reflects an aversion towards the dispersion of the attributes, since closing the distance between two unambiguously ranked individuals, reduces inequality. We will also prove that, under particular assumptions, it reflects an aversion to attribute dependence as well.

A somewhat different way to capture aversion to both dispersion and attribute correlation is to introduce the notion of ALEP[3] substitutability in the inequality field, a concept well known in the literature. We first go over the definition of a comprehensive type of transfer.

Definition Let $X, Y \in M(n, k)$. Distribution Y is derived from X by a *compensating transfer* if there exist two individuals p, q such that:

(i) $x_q > x_p$
(ii) $y_m = x_m \quad \forall m \neq p, q$
(iii) $y_q = x_q - \delta$ and $y_p = x_p + \delta$ where $\delta = (\delta_1, \ldots, \delta_k) \in \mathbb{R}_+^k$ with at least one $\delta_j > 0$
(iv) $y_q \geq x_p$

There are two points worth noting about this definition. On the one hand, this kind of transfer encompasses the definition of a PDB transfer. Also, in this case, a prerequisite of the transfer is that individual q is richer in all the attributes than individual p. However, according to the fourth condition, this ranking may be reversed in the dimensions involved in the transfer. Whenever it is not reversed, we get a PDB transfer. On the other hand, this kind of transfers includes a correlation increasing transfer as well. This particular case is achieved when the amount of the attribute transferred is equal to the difference between the endowments of the individuals involved in the transfer.

[3] ALEP stands for Auspice–Lierne–Edgeways–Pareto.

In the inequality field, the redistributive principle corresponding to a compensating transfer can be formulated as follows.

- *ALEP principle, ALEP: I* satisfies ALEP if $I(Y) \leq I(X)$ for any $X \in D$ and for all Y matrices derived from X by a finite sequence of compensating transfers.

The primary consequence of this principle is that any measure fulfilling ALEP also satisfies PDB and WCIM.[4] Moreover, a direct implication of the ALEP principle for an aggregative measure is that function ϕ in Eq. 1 has non-decreasing increments, that is

$$\phi\left(x_p + \delta\right) - \phi\left(x_p\right) \leq \phi\left(x_q + \delta\right) - \phi\left(x_q\right) \tag{2}$$

for all $x_p \leq x_q$ and for all $\delta \in \mathbb{R}^k_+$.[5] When ϕ is twice continuously differentiable on $\mathbb{R}^k_{++}$, then Eq. (2) holds if and only if $\phi''_{hl} \geq 0$ for all $h, l = 1, \ldots, k$ (Chipman 1977).

Any aggregative measure consistent with PDB satisfies equation (2) whenever $x_p + \delta \leq x_q$.

The ensuing proposition shows that requiring UM and WCIM for an aggregative measure entails ALEP, and consequently PDB.

Proposition 2 *If an aggregative inequality measure $I : D \to \mathbb{R}$ satisfies UM and WCIM, then it fulfils ALEP.*

Proof Let x_p and x_q be the two individuals' bundles involved in a compensating transfer. Let us assume that they only transfer an amount δ_1 of the first attribute, and let $\delta = (\delta_1, 0, \ldots, 0)$. Given that I is an aggregative measure, it suffices to prove that $\phi\left(x_p\right) + \phi\left(x_q\right) \geq \phi\left(x_q - \delta\right) + \phi\left(x_p + \delta\right)$ where $x_q - \delta$ and $x_p + \delta$ are the two individuals' bundles after the transfer, and $x_p \leq x_q - \delta$. Let us consider $z = \left(x_{q1}, x_{p2}, \ldots, x_{pk}\right)$ the bundle of an additional individual. Then,

$$\begin{aligned}
&\phi\left(x_p\right) + \phi\left(x_q\right) + \phi(z) \\
&\geq \phi\left(x_q\right) + \phi\left(x_p + \delta\right) + \phi(z - \delta) \quad && \text{by the aggregative principle and UM,} \\
&\geq \phi\left(x_q - \delta\right) + \phi\left(x_p + \delta\right) + \phi(z) \quad && \text{by WCIM.}
\end{aligned}$$

Again by the aggregative principle, we get the result. □

Consequently, for an aggregative measure UM and WCIM are more demanding than ALEP, whereas this latter principle entails PDB. In the following discussion, we examine the relationships between these redistributive principles when the scale invariance condition is also assumed. First, the implications of PDB are derived.

Proposition 3 *A relative aggregative inequality measure I: $D \to \mathbb{R}$ satisfies PDB if and only if there exists a continuous increasing function, $F : \mathbb{R} \to \mathbb{R}_+ \, with F(0) = 0$ such that: either*

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[4] It should be pointed out that, in the multidimensional framework, PDB and Anonymity do not imply ALEP.

[5] Its counterpart for a utility function is defined in terms of non-increasing increments.

$$F(I(X)) = \frac{1}{n} \sum_{1 \le i \le n} \left[\prod_{1 \le j \le k} \left(x_{ij}/\mu_j \right)^{\alpha_j} - 1 \right] \tag{3}$$

where either $\alpha_j \ge 1$ *for all* j *or* $\alpha_j < 0$ *for all* j,
or

$$F(I(X)) = \frac{1}{n} \sum_{1 \le i \le n} \left[\sum_{1 \le j \le k} \beta_j \log \left(\mu_j / x_{ij} \right) \right] \tag{4}$$

where $\beta_j > 0$ *for all* j.

Proof Tsui (1999, theorem 3) derives the following functions ϕ in Eq. 1 for a relative aggregative measure: (i) $\phi(x_i) = \rho \prod_{1 \le j \le k} \left(x_{ij} \right)^{\alpha_j}$, (ii) $\phi(x_i) = \sum_{1 \le j \le k} -\beta_j \log \left(x_{ij} \right)$ and (iii) $\phi(x_i) = x_{ih} \sum_{1 \le j \le k} a_{hj} \log \left(x_{ij} \right)$.
Assuming PDB, we get the parameter restrictions of the proposition. □

Moreover, proposition 4 also shows that PDB and ALEP are equivalent for a relative aggregative measure, and hence all the measures in Eqs. 3 and 4 satisfy WCIM.

Proposition 4 *A relative aggregative inequality measure* $I \colon D \to \mathbb{R}$ *satisfies PDB if and only if it fulfils ALEP.*

Proof All the functions ϕ corresponding to measures in Eqs. 3 and 4 fulfil $\phi''_{hl} \ge 0$ for all $h, l = 1, \ldots, k$, and consequently these indices satisfy ALEP. □

Proposition 3 above, reveals that invoking PDB implies that function ϕ used to aggregate the individual's bundle of attributes is either increasing in all the attributes, (Eq. 3 with $\alpha_j \ge 1$ for all j), or decreasing in all the dimensions, (Eq. 3 with $\alpha_j < 0$ for all j or Eq. 4). As already mentioned, Tsui (1999) proves that assuming UM is equivalent to demanding that the aggregative function ϕ is strictly convex. However, the strictly convex functions are not necessarily monotone in the same direction in all the dimensions. A straightforward extension of Tsui's results shows that the implication of adding WCIM is that function ϕ should be decreasing in each component. Specifically, any relative aggregative inequality measure which fulfils both UM and WCIM coincides with the indices given in Eq. 3 with $\alpha_j < 0$ for all j or in Eq. 4. The rest of the relative aggregative measures fulfilling UM and not WCIM, do not satisfy PDB either.

Similar to the generalized entropy family in the unidimensional framework, we may interpret that the family obtained in proposition 3 has two tails depending on whether the indices are given by Eq. 3 with the parameter values greater than 1 or otherwise. As we are going to show in the next section, indices in the latter case are transformations of measures with underlying Pareto-consistent separable social evaluation functions.

3 Multidimensional inequality measures which fulfil PDB derived from social evaluation functions

The main goal of this section is to identify PDB-consistent normative inequality measures.

In the following, we assume that a multidimensional social evaluation function is a non-constant function $W : D \to \mathbb{R}$ that possesses the following three properties:

- *Continuity*: W is a continuous function in any individual's attributes.
- *Pareto principle*: W is strictly increasing in the elements of X.
- *Anonymity*: $W(X) = W(\Pi X)$ for any $X \in D$ and for any $n \times n$ permutation matrix Π.

The following invariance property is also assumed:

- *Homothermic principle, HP*: W is *homothermic* if for any two distributions $X, Y \in D$ such that $W(X) = W(Y)$ then $W(XC) = W(YC)$ for any $C = \operatorname{diag}(c_1, \ldots, c_k), c_j > 0, \quad j = 1, 2, \ldots, k$.

Tsui (1995) introduces the following separability axiom:

- *Separability:* W is separable if for all the subsets of individuals $S \subset \{1, 2, \ldots, n\}$, such that: $W(X) = W\left(\psi\left(X^S\right), X^C\right)$ where ψ is some continuous function, X^S is the submatrix of X including the vector of attributes of the individuals in S, and X^C is the complement of X^S.

When $n \geq 3$, Tsui (1995) proves that the separability axiom implies that the function W is ordinally equivalent to $\sum_{i=1}^{n} U(x_i)$, where $U : \mathbb{R}^k \to \mathbb{R}$ is an increasing function in each dimension.

We focus on the approach introduced by Kolm (1977) to derive the multidimensional generalization of the Atkinson–Kolm–Sen inequality indices. Kolm suggests the following multidimensional relative index:

$$I(X) = 1 - \delta(X)$$

where $\delta(X)$ is such that $W(X) = W\left(\delta(X) X_\mu\right)$ and the ith row of X_μ is equal to $\mu(X)$ for all i.

The inequality of a distribution as measured by this index can be interpreted as the fraction of the amount of each attribute that could be discarded if every attribute was equally redistributed, and the resulting distributions were indifferent to the original one according to W.

None of the properties above takes account of the distributional sensitivity of the social evaluation function. Counterparts of the principles proposed in the previous section can be assumed for a welfare function. We will say that W satisfies *PDB, WCIM or ALEP* if $W(Y) \geq W(X)$; and that W satisfies *UM* if $W(Y) > W(X)$, where the matrices X and Y are the same as those specified in the corresponding definitions.

The inequality index derived from a social evaluation function inherits structure and properties from W. It is a trivial exercise to prove that if W is separable then I is aggregative. Furthermore, I fulfils any of the redistributive properties if and only if the counterpart holds for the corresponding social evaluation function.

Tsui (1995) derives the homothetic separable evaluation functions and obtains a generalization of the Atkinson inequality indices consistent with UM. Based on his result, it is easy to come up with the restrictions of the parameters by invoking PDB.

Proposition 5 *Suppose that $n \geq 3$. A multidimensional social evaluation function $W : D \to \mathbb{R}$ satisfies Separability, PDB and HP if and only if W is ordinally equivalent to $\sum_{i=1}^{n} U(x_i)$ where $U : \mathbb{R}^k_{++} \to \mathbb{R}$ is an increasing function such that: either*

$$U(x_i) = a + b \prod_{1 \leq j \leq k} x_{ij}^{\alpha_j} \tag{5}$$

or

$$U(x_i) = a + \sum_{1 \leq j \leq k} \beta_j \log x_{ij} \tag{6}$$

where parameter a is an arbitrary constant, $b < 0$, $\alpha_j \leq 0$ and $\beta_j \geq 0$ or all j.

The corresponding inequality index is relative, aggregative, satisfies PDB, and has the forms

$$I(X) = 1 - \left[(1/n) \sum_{1 \leq i \leq n} \left[\prod_{1 \leq j \leq k} \left(x_{ij}/\mu_j \right)^{\alpha_j} \right] \right]^{1/\sum_{1 \leq j \leq k} \alpha_j} \tag{7}$$

or

$$I(X) = 1 - \prod_{1 \leq i \leq n} \left[\prod_{1 \leq j \leq k} \left(x_{ij}/\mu_j \right)^{\beta_j / \sum_{1 \leq j \leq k} \beta_j} \right]^{1/n} \tag{8}$$

Proof It accords with the proof of theorem 1 in Tsui (1995) and the results of proposition 3. □

The inequality measures derived in this proposition can be interpreted as a generalization of the Atkinson inequality indices. Since they are aggregative, they are monotonically related to a subfamily of the class obtained in proposition 3. This subfamily is the tail that not only fulfils PDB, but also UM and WCIM. In other words, from a normative point of view, assuming PDB is equivalent to requiring UM and WCIM for a relative aggregative measure.

Although this seems to be a bit intriguing at first glance, the close similarity between the utility functions displayed in Eqs. 5 and 6, and the aggregate individual functions ϕ for the corresponding relative measures shown in the proof of proposition 3, may shed light on this issue. Imposing Pareto-consistency implies that the utility function is to be increasing in all the components and, consequently, the corresponding ϕ function is to be decreasing in all the components. As mentioned in the previous section, measures in proposition 3 with ϕ decreasing in all the dimensions not only fulfils PDB, but UM and WCIM as well. The individual functions ϕ for the rest of PDB-consistent indices, for which UM does not hold, are increasing in all the components, and, consequently, they are not associated with Pareto-consistent separable welfare functions.

4 Conclusions

This article applies multidimensional generalizations of the Pigou–Dalton transfer principle, such as UM, ALEP substitutability and the PDB principle, to axiomatically design aggregative inequality indices. Each of these redistributive criteria imposes different ethical judgments, making it difficult to get full acceptance. We explore the different implications of imposing all these criteria along with CIM on aggregative measures.

The PDB principle is one of the simplest extensions of the Pigou–Dalton transfer principle that one can imagine. However, it is a useful and powerful criterion. This article reveals that this property leads to the same result as ALEP substitutability in the case of relative aggregative measures, and, consequently, under these conditions, encompasses sensitivity to the correlation between attributes. We also show that assuming UM and WCIM is in general too demanding, but they are equivalent to PDB for normative relative measures.

We have only focused on relative indices. A similar exercise is also possible invoking the translation invariance principle to derive absolute inequality indices.

Although recently, the Pigou–Dalton transfer principle is being reconsidered, it is the corner stone of the inequality measurement theory. In this sense, we hope that this article provides a greater understanding of this concept in the multidimensional framework.

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