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Can preferences for catastrophe avoidance reconcile social discounting with intergenerational equity?

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Abstract A social welfare function treating all generations equally is derived from a set of axioms that allow for preferences for catastrophe avoidance or risk equity. Implications for the case where there is a risk of world extinction are studied. We show that substantial time discounting can arise from the planner's taste for catastrophe avoidance, even if the probability of the world ending is infinitesimally small.

1 Introduction

Many major policy decisions involve making trade-offs between the welfare of current and future generations. Policy guidance has then to rely on a given social objective. The most common approach involves assuming that the social planner aims at maximizing a social welfare function:

$$\mathrm{SW} = \sum_{t=0}^{\infty} \gamma^t U_t,$$

where t indicates time, U_t is the aggregate utility of cohort t and γ^t is the social discount factor (the discount rate being $1 - \gamma$).

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Such an approach has been extensively criticized by economists and philosophers. The main point is that generations are not treated equally, since they are assigned a weight that depends on when they were born. This is generally considered to be unfair, a point of view most clearly expressed by philosopher Henry Sidgwick who argued that "[...] the time at which a man exists cannot affect the value of his happiness from a universal point of view" (Sidgwick 1907, p. 414).

To our knowledge, there is only one argument that has been suggested to provide an ethical ground for using such a welfare function. Initially developed by Dasgupta and Heal (1974, 1979), it involves assuming that between any two dates there exists a positive probability $1 - \gamma$ that the world ends. An equitable utilitarian objective leads then to the above social welfare function.

Still, this argument is problematic when we turn to quantitative aspects. Social discount rates are usually taken between 1 and 5 % per year. But, for most people it would seem excessively pessimistic to assume a yearly probability of the word ending of 1 or 5%. With a 5% probability we would have more than a 50% chance of disappearing in the next 14 years. Even with a probability of the world ending of 1% per year, there would be the less than a 50% chance of seeing the world lasting more than 69 years. It is hard to believe that this should reflect the beliefs of a reasonable social planner.

The aim of this paper is to provide a theoretical foundation for a social welfare function displaying time discounting, but which is ethically acceptable, in the sense that it treats all generations equally. Our argument, just as that of Dasgupta and Heal, relies on the idea that at any time there is a positive probability that the world ends. However, the novelty is that we suggest different foundations that make it possible to distinguish the social discount rate from the probability of the world ending. Indeed we are able to break down social discount into two parts: one representing the risk of the extinction of the world, and the other related to aversion to correlated risks. It is thus possible to believe that the instantaneous probability of the world ending is very low, but that accounting for such a risk leads to introduce a discount rate that significantly differs from zero.

Theoretically speaking, our work relies on an axiomatic construction of the planner's preferences that largely resembles the one suggested by Harsanyi (1955). It extends Harsanyi's approach by considering a weaker version of the Pareto axiom, that allows us to consider preferences for "Catastrophe Avoidance", a notion that was initially suggested by Keeney (1980) and discussed further in Fishburn (1984).

The remainder of the paper is organized as follows. In Sect. 2, we introduce the notation. Section 3 will expose the axiomatic construction of the planner's preferences. Preferences for Catastrophe Avoidance are discussed in Sect. 4. In Sect. 5, we explore the consequences when there is a positive (but tiny) instantaneous probability of the world ending and see that it actually yields a rate of discount that is possibly far from zero. Last, in Sect. 6, we show how the additive representation can be recovered as a limit case of the more general representation that we derive.

2 The setting

We consider a society composed of consecutive generations. Time is discrete and infinite, and a period is denoted by $t \in \mathbb{N}$ (with $\mathbb{N}$ the set of natural numbers). Each potential generation is identified by the period at which it is supposed to be born. We use the adjective "potential" to emphasize the fact that those generations may never exist (for example because the end of the world occurs before their birth date). Generations do not overlap: people can live only one period. The situation of a generation is described by a unidimensional outcome x_t. This is an element of the set of possible outcomes $X = K \cup \{d\}$. K is an interval in the positive real line, $\mathbb{R}_+$. Elements of K can be seen as comparable measures of the standards of living of the generation. d indicates the non-existence status. X satisfies habitual topological properties.[1] In the sequel, we will assimilate generations to individuals. It is to be understood that we have in mind representative agents of the generations.

We wish to consider uncertain prospects. Let $\mathcal{P}$ be the set of all simple lotteries on X.[2] For simplicity, individuals are assumed to be identical and selfish (they have the same self-regarding preferences). Each individual's preferences are represented by a binary relation $\succeq^I$ defined on $\mathcal{P}$. We denote by $\succ^I$ the strict preference relation and by $\sim^I$ the indifference relation.

Like Harsanyi (1955) we restrict the study to the case where individual preferences admit an expected utility representation on $\mathcal{P}$. Stated differently, there exists a Bernoulli utility function $u(.)$ such that:

$$\forall p, \hat{p} \in \mathcal{P} : \ p \succeq^I \hat{p} \ \Leftrightarrow \ \sum_{x \in X} u(x) p(x) \geq \sum_{x \in X} u(x) \hat{p}(x).$$

By normalization, we can assume without loss of generality that $u(d) = 0$. We also need the following minimal sensitivity condition: there exist $\bar{x}$ and $\hat{x}$ in K such that $u(\bar{x}) > u(\hat{x}) > 0$. The differentiability of u will also be assumed in Sect. 3 in order to have a well-defined notion of the social discount rate. This assumption is not necessary to derive our representation result.

Let us now describe social outcomes and preferences. The literature on the evaluation of infinite utility streams initiated by Diamond (1965) makes no assumption as to the asymptotic properties of feasible utility streams.[3] In this paper, we consider the risk of the world ending. We are thus led to put some structure on possible intertemporal outcomes. We assume that the society will end in a finite time. The termination date is however unknown.

To be more specific, if we denote $X^{\mathbb{N}}$ the Cartesian product of individual outcome spaces, the social outcome space is

$$\chi = \left\{ \mathbf{x} \in X^{\mathbb{N}} \text{ such that } \exists T : \forall t \leq T \ x_t \neq d \text{ and } x_{t'} = d \ \forall t' > T \right\}$$

The social planner's preferences will be defined on $\mathcal{Q}$, the set of all simple lotteries on χ. Because the set χ is a strict subset of $X^{\mathbb{N}}$, we work with a smaller choice set

[1] In particular, X is a metric space using the following metric $m : X \times X \to \mathbb{R}^+$: $m(x, \hat{x}) = |x - \hat{x}|$, $\forall (x, \hat{x}) \in K \times K$; $m(x, d) = +\infty$, $\forall x \in K$; $m(d, d) = 0$.

[2] A simple lottery on X is a mapping $p : X \to [0, 1]$ with the property that (i) there exists a finite subset $J \subset X$ such that $p(x) > 0 \ \ \forall x \in J$ and $p(x) = 0 \ \ \forall x \in X \setminus J$; (ii) $\sum_{x \in J} p(x) = \sum_{x \in X} p(x) = 1$.

[3] See Fleurbaey and Michel (2003) for a recent paper containing an extensive review of this literature.

than the one considered by Epstein (1983). This is of particular importance: to restrict attention to outcomes in χ, rather than working on $X^{\mathbb{N}}$, enables to circumvent the necessity of impatience established by Koopmans (1960) and, in a framework involving risk, by Epstein (1983). The specifications that will be obtained in Proposition 1 are concrete examples of continuous social orderings that do not exhibit impatience or, in the context of intergenerational justice, preference for earlier generations.[4] Note that the restriction we impose bears some similarities with the one used by Ramsey (1928).[5]

The consideration of outcomes in χ seems appropriate if we think that the world has a finite but uncertain time horizon. In a comment on a paper by Koopmans, Fisher argued that the infinite horizon was primarily introduced as a convenience to escape the problems associated with an arbitrary terminal date (Fisher 1965). Our approach can achieve this task without making the assumption that the horizon is infinite.

We follow Harsanyi in assuming that the social planner's preferences admit an expected utility representation.[6] There exists a Bernoulli index U defined on χ such that:

$$\forall q, \hat{q} \in \mathcal{Q}: \; q \succeq^S \hat{q} \;\; \Leftrightarrow \; \sum_{\mathbf{x} \in \chi} U(\mathbf{x}) q(\mathbf{x}) \geq \sum_{\mathbf{x} \in \chi} U(\mathbf{x}) \hat{q}(\mathbf{x})$$

For any subset of the set of individuals, $I \subset \mathbb{N}$, and for any $q \in \mathcal{Q}$, we denote q_I the marginal distribution of q on the outcomes of individuals in I.[7] In particular, we denote q_t the marginal distribution on individual t's outcomes. Remark that $q_t \in \mathcal{P}$ for any $q \in \mathcal{Q}$ and any $t \in \mathbb{N}$.

Last we introduce a subset of $\mathcal{Q}$ that will be of particular importance. Let $\prod_{t \in \mathbb{N}} \mathcal{P}$ be the set of product lotteries on $X^{\mathbb{N}}$.[8] We define the set of "independent distributions", $\mathcal{R}$, by:

$$\mathcal{R} = \mathcal{Q} \cap \prod_{t \in \mathbb{N}} \mathcal{P}.$$

An element of $\mathcal{R}$ thus describes a societal risk composed of independent individual risks. An experiment where each individual would be asked to flip a coin could be represented by an element of $\mathcal{R}$. The case of a social planner who flips a coin to determine all individual outcomes cannot be represented by an element of $\mathcal{R}$. This is also the case for any aggregate risk, as with the risk of an ecological catastrophe that

4 Thus our conclusions also contrast with the results obtained in the literature on the aggregation of infinite utility streams. The structure of χ explains the different conclusions.

5 Ramsey (1928) considered an infinite horizon but he restricted attention to paths converging to a constant tail at the "bliss"level. We assume that all paths have a constant tail composed of outcomes d.

6 Harsanyi's approach is not free from controversy. In particular, it has been argued that the expected utility axioms might not be appropriate for the social planner (Diamond (1967), Epstein and Segal (1992)). This contention has however also been criticized, for instance in Deschamps and Gevers (1977), Broome (1984) or Fleurbaey (2007).

7 Let $I = \{i_1, i_2, \ldots\}$. q_I is a a mapping $q_I : X^I \to [0, 1]$ such that, for all $(\bar{x}_{i_1}, \bar{x}_{i_2}, \ldots)$ in X^I, $q_I(\bar{x}_{i_1}, \bar{x}_{i_2}, \ldots) = \sum_{t \in \mathbb{N} \setminus I} \sum_{\bar{x}_t \in X} q\,(\mathbf{x} \in \chi : x_i = \bar{x}_i \; \forall i \in I; x_t = \bar{x}_t)$.

8 A simple lottery q belongs to $\prod_{t \in \mathbb{N}} \mathcal{P}$ if, for any t and t' in $\mathbb{N}$ and any x_t and $x_{t'}$ in X, $q_{\{t,t'\}}(x_t, x_{t'}) = q_t(x_t) q_{t'}(x_{t'})$.

would deteriorate the welfare of several generations, or that associated with the world ending.

3 Planner's preferences

In this section, we state the assumptions made on the planner's preferences and then provide a representation result.

Our first axiom is a restricted Pareto axiom:

Axiom 1 Restricted Pareto (RP):

$$\forall q, \hat{q} \in \mathcal{R}, \text{ if } \forall t \in \mathbb{N}, \; q_t \succeq^I \hat{q}_t \text{ then } q \succeq^S \hat{q}$$
$$\text{If, furthermore, } \exists t' \,/\, q_{t'} \succ^I \hat{q}_{t'} \text{ then } q \succ^S \hat{q}.$$

The above axiom is called Restricted Pareto, because we apply Pareto's principle to independent distributions only. This axiom is weaker than the standard strong Pareto axiom, which would be obtained by replacing $\mathcal{R}$ by $\mathcal{Q}$ in the above definition.

The reason for using a restricted version of the Pareto axiom, instead of the standard one, is that we want the planner's preferences to reflect individual preferences when independent risks are concerned, but possibly to deviate from individual preferences when collective risks come at play. This possibility was first considered by Keeney (1980) and Fishburn (1984). More recently we also find it in Manski and Tetenov (2007). Such a restricted axiom is necessary if we want to allow for social judgments on how individual risks are combined.

This seems reasonable if we consider that the social planner should not only care for individuals' happiness but also implement some coordination between individuals' behaviors in order to avoid undesirable social outcomes. For instance, the social planner might want to avoid major social catastrophes. Fishburn argued that social risk diversification would be particularly "appealing if the fate of the human race were at stake"(Fishburn 1984, p. 904). This is precisely the issue that is discussed in the present paper.

But the restricted axiom also enables to consider the converse of catastrophe avoidance, namely "Risk Equity"(the notion was introduced in Keeney (1980)). Risk Equity may also seem appropriate because it guarantees that people do not face too different fates. Our axiomatic makes it possible to consider this principle as well and to study its consequences on the social discount rate.

Restricted Pareto is however not innocuous. Like Pareto's principle it may be criticized on the ground that it bears only on ex-ante evaluations.[9] The social planner may have concerns about the ex-post distribution of welfare that would be incompatible with the acceptance of the restricted Pareto axiom. The present paper does not tackle these issues. Restricted Pareto only permits to consider the role of correlated generations' fate on social welfare.

[9] See Hammond (1981), Myerson (1981) or Fleurbaey (2007) on this problem. See also Rabinowicz (2002) on the use of Pareto's principle restricted to ex-post situations.

The second major axiom we use is an independence axiom called Independence of the Utilities of the Dead.[10] It stipulates that, at any given date, the choice between two socially risky prospects does not depend on the situation of individuals who trespassed. The principle is usually justified by the fact that we cannot change the fate of those already dead, so that their welfare should not matter for present choices.

Denote δ_x the one-point measure on $x \in X$.[11] The Independence of the Utilities of the Dead axiom is as follows:

Axiom 2 Independence of the utilities of the dead (IUD): Let q, q', $\hat{q}$ and $\hat{q}'$ be four simple lotteries in $\mathcal{Q}$ for which there exists $T \in \mathbb{N}$ such that:

- $q_t = q'_t = \delta_{x_t}$ and $\hat{q}_t = \hat{q}'_t = \delta_{\hat{x}_t}$ for all $t \leq T$.
- $q_{\mathbb{N}\setminus\{1,\ldots,T\}} = \hat{q}_{\mathbb{N}\setminus\{1,\ldots,T\}}$ and $q'_{\mathbb{N}\setminus\{1,\ldots,T\}} = \hat{q}'_{\mathbb{N}\setminus\{1,\ldots,T\}}$.

Social preferences satisfy the (IUD) axiom if, for any such q, q', $\hat{q}$ and $\hat{q}'$, the following equivalence holds:

$$q \succeq^S q' \;\Leftrightarrow\; \hat{q} \succeq^S \hat{q}'$$

Last, we would like the social planner to treat all generations impartially. In the literature, the concept of intergenerational equity has often been represented using finite permutations of individual outcomes.[12] A finite permutation π is a bijection $\pi : \mathbb{N} \to \mathbb{N}$ such that, for some $T \in \mathbb{N}$, $\pi(t) = t$ for all $t \geq T$. Let Π denote the set of all finite permutations. For any $\mathbf{x} \in \chi$ and any $\pi \in \Pi$, $\mathbf{x}(\pi)$ denotes the sequence $(x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(n)}, \ldots)$. Our anonymity axiom is as follows:

Axiom 3 Anonymity (A): $\forall \mathbf{x} \in \chi$, $\forall \pi \in \Pi$ such that $\mathbf{x}(\pi) \in \chi$:

$$\delta_{\mathbf{x}} \sim^S \delta_{\mathbf{x}(\pi)}.$$

The axiom holds on sure prospects. This is enough to obtain our result. Its counterpart in terms of expected utility would be stronger but is more difficult to write. We prefer this weaker axiom which is simpler and sufficient for our purpose. Remark also that the anonymity axiom rules out any pure time preference of the social planner.

We are now able to state a representation result.

Proposition 1 *Assume that individual preferences admit and expected utility representation on $\mathcal{P}$ and that social preferences admit an expected utility representation on $\mathcal{Q}$. Then social preferences satisfy (RP), (IUD) and (A), if and only if they can be represented by an expected utility using the Bernoulli utility function:*

$$U(\mathbf{x}) = \frac{1}{\varepsilon} \times \left(1 - \prod_{t \in \mathbb{N}} (1 - \varepsilon u(x_t))\right) \tag{1}$$

[10] The axiom was proposed by Blackorby, Bossert and Donaldson in a series of papers on intertemporal social ethics. See for instance Blackorby et al. (2002).

[11] A one-point measure is a simple lottery $\delta_x \in \mathcal{P}$ such that $\delta_x(x) = 1$ and $\delta_x(y) = 0$, $\forall y \in X \setminus \{x\}$. The definition of one-point measures on $\mathbf{x} \in \chi$ is similar.

[12] For some discussion of permutations as expressing impartiality, and the presentation of different permutation conditions, see Fleurbaey and Michel (2003).

with $\varepsilon \neq 0$ *such that* $\varepsilon u(x_t) < 1$ *for all* $x_t \in X$*, or the Bernoulli utility function:*

$$U(\mathbf{x}) = \sum_{t\in\mathbb{N}} u(x_t). \tag{2}$$

Proof See Appendix 1. □

Note that the above representation of the social planner's preferences is contingent on the normalization assumption that has been made on u, namely $u(d) = 0$. In particular, because $u(d) = 0$, the infinite product and sum that appear in (1) and (2) are well defined for any $\mathbf{x} \in \chi$. Note also that $\varepsilon > 0$ is possible only if u is bounded from above. Conversely, $\varepsilon < 0$ is possible only if u is bounded from below. We want to consider these two cases and therefore assume that the function u is bounded.

The additive welfare function (Eq. 2) corresponds to the limit of the multiplicative representation (Eq. 1) when ε tends to zero. We will therefore consider the multiplicative representation as the general one, remembering that the additive one is obtained by taking $\varepsilon = 0$. Only the additive case would be obtained if we were to replace (RP) by the standard strong Pareto axiom.[13]

The representation of preferences given by Proposition 1 resembles the one found in Eqs. 9.16 and 9.17 of Meyer (1976). Our proof indeed makes use of a standard recursivity argument, similar to the one used by Meyer (1976) and Epstein (1983). Their axioms were adjusted to fit the intertemporal social choice framework. The proof also had to be adapted to the choice set we consider, which is larger than the one studied by Meyer (1976) but smaller than the one studied by Epstein (1983) because of the structure of χ. Proposition 1 should not be considered as the main achievement of the paper. Our contribution is of a different nature: it involves highlighting the judiciousness of the multiplicative representation in social choice and exploring its consequences when there is an exogenous risk of the world ending.

First, we explain in the next section how the parameter ε that enters Eq. (1) is related to catastrophe avoidance.

4 Catastrophe avoidance

Keeney (1980), who discusses the social evaluation of fatality risks, defines catastrophe avoidance as follows: social preferences are said to exhibit a preference for catastrophe avoidance if the probability π_1 of having n_1 fatalities is preferred to a probability π_2 of having n_2 fatalities when $n_1 < n_2$ and $\pi_1 n_1 = \pi_2 n_2$. Stated otherwise, under the assumption of a preference for Catastrophe Avoidance, for a given number of expected fatalities, the social planner prefers the case of an accident that kills few people to a less likely accident that kills more people.

In our setting, we are not concerned by fatality risks, but by the mere existence of individuals. Still, the transposition of Keeney's definition to our framework is trivial. The planner's utility exhibits preferences for catastrophe avoidance if and only if, in the case where all existing individuals are provided with the same positive utility level,

[13] Proof available upon request.

the planner prefers the lottery where the number of individuals that will ever exist is M with probability $(1-\pi_1)$ and $M-n_1$ with probability π_1 , to the lottery where the number of individuals is M with probability $(1-\pi_2)$ and $M-n_2$ with probability π_2, when $\pi_1 n_1 = \pi_2 n_2$ and $n_1 < n_2$.

With the representation in Eq. (1), a preference for catastrophe avoidance occurs if for all $u > 0$ and all n_1, n_2, π_1, π_2 such that $\pi_1 n_1 = \pi_2 n_2$ and $n_1 < n_2$ we have:

$$(1-\pi_1)(1-\varepsilon u)^M + \pi_1(1-\varepsilon u)^{M-n_1} > (1-\pi_2)(1-\varepsilon u)^M + \pi_2(1-\varepsilon u)^{M-n_2}.$$

Simplifying the above inequality by $(1-\varepsilon u)^M$ and using $\pi_1 n_1 = \pi_2 n_2$, this leads to:

$$\frac{(1-\varepsilon u)^{-n_1} - 1}{n_1} > \frac{(1-\varepsilon u)^{-n_2} - 1}{n_2}.$$

Thus preferences for catastrophe avoidance exist if and only if $f(n) = \frac{(1-\varepsilon u)^{-n}-1}{n}$ is a decreasing function. But $f'(n) = \frac{(1-\varepsilon u)^{-n}}{n}\left[\log(1-\varepsilon u) + \frac{(1-\varepsilon u)^{n}-1}{n}\right]$ which is negative for all $n > 0$ if and only $\varepsilon > 0$. Assuming preference for catastrophe avoidance is therefore equivalent to taking $\varepsilon > 0$.

Indeed, as shown by the second theorem of Keeney (1980), preference for catastrophe avoidance is equivalent to risk aversion with respect to the number of existing individuals (when all existing individuals are provided with the same utility level). To measure the strength of the preference for catastrophe avoidance, we can therefore use an Arrow–Pratt coefficient of risk aversion with respect to the number of existing individuals. More precisely, assume that all individuals that ever exist are provided with the same amount of commodity x, which yields the same utility level $u(x)$. For any N denote

$$\mathcal{U}(N, x) = U(\underbrace{x, \ldots, x}_{N \text{ times}}, d, \ldots)$$

the social utility when the number of individuals that will ever exist is N . Also denote $\Delta\mathcal{U}(N, x) = \mathcal{U}(N+1, x) - \mathcal{U}(N, x)$ and $\Delta^2\mathcal{U}(N, x) = \Delta\mathcal{U}(N+1, x) - \Delta\mathcal{U}(N, x)$. We can define the index of catastrophe avoidance as the following Arrow–Pratt coefficient:

$$\mathrm{ICA}(x) = -\frac{\Delta^2\mathcal{U}(N, x)}{\Delta\mathcal{U}(N, x)}$$

It is a matter of simple computation to show that, when the social welfare function takes the form shown in (1), we have $\mathcal{U}(N, x) = \frac{1}{\epsilon} - \frac{1}{\epsilon}(1-\varepsilon u(x))^N$ and $\mathrm{ICA}(x) = \varepsilon u(x)$. In particular the index of catastrophe avoidance equals zero when $\varepsilon = 0$. This means that utilitarian preferences (which fulfill the unrestricted Pareto axiom) cannot exhibit preferences for catastrophe avoidance.

The appeal of the catastrophe avoidance principle has been disputed. For instance, Fleurbaey (2007) notices that catastrophe avoidance implies a preference for ex-post inequalities. Catastrophe avoidance also means that we prefer to sacrifice the last generation for sure than to give the last two generations half a chance of existing.[14] One might rather want social preferences to satisfy risk equity. Risk equity corresponds to the idea that equalizing independent risks is socially desirable (see Keeney 1980). In our framework, this is possible only if $\text{ICA}(x) < 0$. Accordingly, the criterion obtained in Proposition 1 can accommodate risk equity if we choose $\varepsilon < 0$.

There are evidences (quoted in Keeney (1980)) that many people would endorse catastrophe avoidance in the case of large risks such as the end of the world. On the other hand, risk equity seems also ethically appealing. Without taking side for any of these principles, we will now see that preferences (or distaste) for catastrophe avoidance as expressed in the ICA play a key role when looking at time discounting arising from the risk of the world ending.

5 The risk of the world ending and the social discount rate

For the sake of simplicity we consider the case where uncertainty only bears on the timing of the world's disappearance. The planner's problem involves ranking infinitely long consumption plans, knowing that for a reason that is independent of his behavior, the world will stop existing at a finite date (consumption becoming then impossible). This problem is similar to the one considered by Dasgupta and Heal (1979). To simplify matters further, we assume that at each period there is a probability p that the end of the world occurs and a probability $(1 - p)$ that the world survives. We also consider infinitely long consumption plans $\mathbf{x} \in K^{\mathbb{N}}$ such that $u(x_t) > 0$ for all $t \in \mathbb{N}$.

The situation that we consider involves a countably infinite number of events, one for each possible ending date. It can be described by a discrete lottery but not by a simple lottery. It is however possible to extend the representations in Eqs. (1) and (2) from the set of all simple lotteries to a convex set of discrete lotteries. We encounter one problem though: the Bernoulli utility functions in (1) and (2) are not bounded, except when $\varepsilon > 0$ and $u(x_t) > 0$ for all t. When the Bernoulli utility function is not bounded, we may have a problem with the convergence of the expected utility for some discrete lotteries, so that the choice criterion is not well-defined.[15] Social preferences using Bernoulli functions in Eqs. (1) and (2) therefore admit an expected utility representation only on the set of discrete lotteries for which the expected utility is finite. For any ε we denote $\mathcal{Q}_\varepsilon$ denote this set. The set $\mathcal{Q}_\varepsilon$ necessarily contains all simple lotteries on χ.

Consider a consumption plan $\mathbf{x} = (x_t) \in K^{\mathbb{N}}$. For any $T \geq 0$ there is a probability $p(1 - p)^T$ that the world will last exactly T periods. In such a case the consumption plan $\mathbf{x}$ yields a social utility $\frac{1}{\varepsilon} \times \left(1 - \prod_{t=1}^{T} (1 - \varepsilon u(x_t))\right)$ if $T > 0$ and zero if $T = 0$. The expected utility associated with $\mathbf{x}$ is therefore:

$$W(\mathbf{x}) = \frac{1}{\varepsilon} \sum_{T=1}^{\infty} p(1-p)^T \left(1 - \prod_{t=1}^{T} (1 - \varepsilon u(x_t))\right). \tag{3}$$

[14] We thank the associate editor for suggesting this example.

[15] On this issue, see Hammond (1998, p. 187).

Since the social planner aims at maximizing expected utility, $W(\mathbf{x})$ is a social welfare function representing the planner's preferences over consumption plans. A consumption plan $\mathbf{x}$ will be preferred to an alternative plan $\hat{\mathbf{x}}$ if and only if $W(\mathbf{x}) > W(\hat{\mathbf{x}})$.

The infinite sum in Eq. (3) converges whatever $p > 0$ for all admissible $\varepsilon \geq 0$. The lottery that we consider therefore belongs to $\mathcal{Q}_\varepsilon$ for these values of ε. In the case of risk-equity preferences ($\varepsilon < 0$) however, the sum may not converge when p is sufficiently low. In that case, we must restrict attention to values p large enough so that the discrete lottery belong to $\mathcal{Q}_\varepsilon$. All the results in this section are given for values p satisfying this condition.

As it emerges from Eq. (13) in Appendix 2, the social welfare function in Eq. (3) can be rewritten:[16]

$$W(\mathbf{x}) = (1-p) \sum_{T=1}^{+\infty} u(x_T) \prod_{t=1}^{T-1} \left[(1 - \varepsilon u(x_t))\,(1-p) \right] \tag{4}$$

We recognize here stationary recursive preferences. Indeed, if we denote ${}_t\mathbf{x} = (x_t, x_{t+1}, \ldots)$ the infinite sequence of generations' consumptions from period t on, we have the recursive relation:

$$W({}_t\mathbf{x}) = u(x_t) + \left[(1 - \varepsilon u(x_t))\,(1-p) \right] W({}_{t+1}\mathbf{x}). \tag{5}$$

Stationary recursive preferences have been studied at length in the economic literature ever since Koopmans (1960). The particular form obtained here was originally introduced by Uzawa (1968) in continuous time and Epstein (1983) in discrete time. As is well-known, recursive preferences generally display endogenous discounting.

Like Koopmans (1960) or Epstein (1983), we define the rate of time discounting along a constant consumption path by looking at how the marginal utility of consumption changes with time. Formally, if we denote ${}_{\text{con}}x$ the constant consumption program for which $x_t = x \in K$ for any $t \in \mathbb{N}$, the rate of time discounting is defined by:

$$\rho(x) = 1 - \left. \frac{\frac{\partial W}{\partial x_2}}{\frac{\partial W}{\partial x_1}} \right|_{\mathbf{x} = {}_{\text{con}}x}. \tag{6}$$

When preferences are given by the social welfare function W shown in Eq. (3) we have the following result:

Proposition 2 *The social rate of discount is*

$$\rho(x) = p + (1-p)\varepsilon u(x) = p + (1-p) ICA(x)$$

Proof See Appendix 2. □

Note that when $\varepsilon = 0$, that is in the standard utilitarian case, we find that the rate of discount equals the instantaneous probability of the world ending, which is consistent

[16] We use the habitual convention that $\prod_{t=1}^{0} g(x_t) = 1$ for all functions g.

with the results of Dasgupta and Heal. From Proposition 2 we also have the immediate consequences:

Corollary 1 *1) The rate of discount is greater than the hazard rate of the world disappearing whenever social preferences exhibit preferences for catastrophe avoidance (i.e. $\rho > p$ whenever $\varepsilon > 0$).*

2) The rate of discount is lower than the hazard rate of the world disappearing whenever social preferences exhibit preferences for risk equity (i.e. $\rho < p$ whenever $\varepsilon < 0$).

3) In the case $\varepsilon > 0$, when the probability of the world ending is infinitesimally small, the rate of discount approximately equals $ICA(x) = \varepsilon u(x)$.

These three points deserve some comments. First we see that preference for catastrophe avoidance makes the social planner discount the future more heavily. The reason is that the planner wants to avoid the worst catastrophe in which: (1) only few individuals ever come to life; (2) those few individuals sacrifice most of their resources for the sake of future generations that will actually never exist.

Conversely and for symmetrical reasons, preference for risk equity lowers the social discount rate. This is true only for sufficiently large value of p, for, when $\varepsilon < 0$, the lottery we consider does not belong to $\mathcal{Q}_\varepsilon$ when p tends to zero.

Lastly, even if the instantaneous probability of the world ending is very small, the rate of discount may be quite large. This may seem counterintuitive, as one might expect that when p is infinitesimally small, the planner would not exhibit pure time preference, as a consequence of the anonymity axiom. There is however a simple intuition that explains why the rate of discount does not tend to zero when p tends to zero. The point is that when p tends to zero the expected number of individuals that will ever exist (which equals $\frac{1}{p}$) tends to infinity. As a consequence, the smaller p, the greater the loss when the world ends. Thus, when p tends to zero, the probability of a catastrophe occurring does tend to zero, but the magnitude of the catastrophe tends to infinity. Both factors compensate and lead the social planner to use a non-negligible social discount even if p is very small. At the limit $p \to 0$, the discount rate is equal to the index of catastrophe avoidance.

Recursive preferences have been increasingly used in optimal growth theory with seminal contributions by Uzawa (1968), in continuous time, and Beals and Koopmans (1969) in discrete time. Several papers have provided interesting developments on the existence and convergence of optimal paths in many settings, for instance Epstein (1983), Lucas and Stockey (1984), Epstein (1987) or Palivos et al. (1997). They obtain results similar to the standard neoclassical growth model with exogenous discounting when social preferences display 'increasing marginal time preference'; that is when the discount rate $\rho(x)$ is increasing in x. With preferences displayed in Eq. (4), this is the case whenever $u(x)$ is increasing in x, a conventional assumption.

6 Recovering the additive social welfare function

An indisputable drawback of the multiplicative representation is that the expected utility is not additive. For most applications, this is a source of substantial increase in complexity. Still, following a strategy similar to that of Bommier (2006), additivity can be recovered by considering the limit case where the difference in welfare between existing or not existing is assumed to be much greater than the difference in welfare between having a low or a high level of consumption. More precisely, assume that the x_t (when different from d) remain in a bounded domain $[x_{\min}, x_{\max}]$ and that the function u is such that:

$$u(x_t) = 0 \quad \text{when } x_t = d,$$
$$u(x_t) = 1 + \lambda v(x_t) \quad \text{when } x_t \neq d,$$

where λ is very small and v is a bounded function over $[x_{\min}, x_{\max}]$. In such a case the difference in welfare between existence and non-existence is approximately equal to 1, while the difference in welfare between having $x_{\min}$ or $x_{\max}$ equals $\lambda(v(x_{\max}) - v(x_{\min}))$, which is assumed to be much smaller than 1.

Consider now one of the lotteries we have studied in Sect. 5. Assume that the lottery belongs to Q_ε and consider the limit case where λ is infinitesimally small. Note that in such a case, the index of catastrophe avoidance is independent of x and equals ε.

We obtain following result:

Proposition 3 *In the limit where $\lambda \to 0$ the social planner's preferences are represented by the social welfare function*

$$W_\varepsilon(\mathbf{x}) = \sum_{t=0}^{\infty} \gamma^t v(x_t)$$

where $\gamma = (1 - p)(1 - \varepsilon)$.

Proof See Appendix 3. □

We therefore obtain the standard additive representation, with a discount rate that equals $p + (1 - p)\,\varepsilon$. Thus, we end up with a formulation that is the same as that of Dasgupta and Heal, though with the fundamental difference that the rate of discount is now augmented by a factor that depends on the index of catastrophe avoidance. Thus, there is no contradiction between assuming that the probability that the world disappears is very low, and that the rate of discount is significantly greater than zero. Nor is there any inequitable bias in favor of present generations.

Despite its simplicity, the additive approximation may be controversial, for we have to assume that the difference in welfare between existence and non-existence is much larger than the difference in welfare between possible lives. This is of course disputable. Blackorby et al. (1995) argue for example there are states of extreme poverty that are, from the planner's point of view, worse than non-existence. If one is reluctant to use the additive approximation, there is no other solution than to rely on the recursive formula given by Eq. (4).

Preferences for catastrophe avoidance permits to reconcile intergenerational equity with *endogenous* time discounting. The simplifying assumption of *exogenous* discounting can be seen as a limit case that corresponds to the additive approximation detailed above. Whether or not this limit case may be considered as relevant depends on how wide we think the welfare gap is between existence and non-existence.

7 Conclusion

We have extended the standard utilitarian aggregation of preferences *à la* Harsanyi to account for a possible planner's taste for catastrophe avoidance or for risk equity. This was done by replacing the usual Pareto axiom by a weaker axiom. This axiom stipulates that there is no divergence between the social planner's and the individuals' preferences as long as uncorrelated risks are considered, but that some divergence may occur when correlated risks are at play. The axiom allows planners to express their own views on collective consequences.

Preferences for catastrophe avoidance were found to play a key role when accounting for the probability that the world may end. We showed that an equitable social planner who has no pure time preference, but preferences for catastrophe avoidance, discounts the welfare of future generations with a rate that is greater than the instantaneous probability of the world coming to an end. More importantly this rate does not vanish when the instantaneous probability of the world ending tends towards zero. In other words, substantial time discounting does not necessarily reflect the planner's lack of equity, or the planner's belief that the world will soon end. There is a third source of social discounting that results from the combination of preference for catastrophe avoidance with the belief that there is indeed a positive (but, possibly very small) probability that the world will end.

The end of the world is a very stylized representation of an event with durable consequences. Its key characteristics, for our analysis of time discounting, is that it durably and negatively impacts individuals' utilities and marginal utilities.[17] In fact, it can be shown that, when the planner exhibits preferences for Catastrophe Avoidance, the planner's discount rate increases with the likelihood of an event having these characteristics occurring. Natural extensions of this paper therefore involve considering less caricatured risks, such as the possibility of an ecological catastrophe, and see how they may affect time discounting.

Appendices

In all the proofs we use the convention that $\prod_{t=1}^{0} g(x_t) = 1$ for all functions g.

Appendix 1: Proof of Proposition 1

It is straightforward to check that if the planner's preferences are represented by the multiplicative or additive utility function shown in (1) and (2) they satisfy (RP), (IUD) and (A). We will therefore focus on showing that (RP), (IUD) and (A) imply these particular specifications.

[17] When the world ends, the utilities and marginal utilities of future individuals are irreversibly set to zero.

Consider a Bernoulli utility function that represents the planner's preferences:

$$\tilde{U}(\mathbf{x}) = \tilde{U}(x_1, x_2, x_3, \ldots).$$

Let q and $\hat{q}$ be two lotteries in $\mathcal{Q}$ such that $q_1 \neq \hat{q}_1$ and $q_t = \hat{q}_t = \delta_d$ for all $t > 1$. The (RP) axiom implies that, for any such q and $\hat{q}$:

$$\sum_{x\in X} \tilde{U}(x, d, d, \ldots)q_1(x) \geq \sum_{x\in X} \tilde{U}(x, d, d, \ldots)\hat{q}_1(x) \Leftrightarrow \sum_{x\in X} u(x)q_1(x) \geq \sum_{x\in X} u(x)\hat{q}_1(x)$$

Thus $\tilde{U}(x, d, d, \ldots)$ must be an individual Bernoulli utility function. There exist real numbers $a > 0$ and b such that $\tilde{U}(x, d, \ldots) = au(x) + b$ for all $x \in X$. Denote U the function on $X^{\mathbb{N}}$ such that $U(\mathbf{x}) = \frac{\tilde{U}(\mathbf{x})-b}{a}$. U is a Bernoulli utility functions for the planner that satisfies the condition:

$$U(x, d, d, \ldots) = u(x) \quad \text{for all } x \in X \tag{7}$$

Our sensitivity condition implies that there exists $\bar{x} \in K$ such that $u(\bar{x}) > 0$. For any $T \in \mathbb{N} \setminus \{1\}$, define $V_T(x_T, x_{T+1}, \ldots) \equiv U(\bar{x}, \ldots, \bar{x}, x_T, x_{T+1}, \ldots)$. Consider q and $\hat{q}$ in $\mathcal{Q}$ such that $q_T \neq \hat{q}_T$, $q_t = \hat{q}_t = \delta_{\bar{x}}$ for all $t < T$, and $q_t = \hat{q}_t = \delta_d$ for all $t > T$. The (RP) axiom implies that:

$$\sum_{x\in X} V_T(x, d, d, \ldots)q_T(x) \geq \sum_{x\in X} V_T(x, d, d, \ldots)\hat{q}_T(x) \Leftrightarrow \sum_{x\in X} u(x)q_T(x) \geq \sum_{x\in X} u(x)\hat{q}_T(x).$$

Thus $V_T(x, d, d, \ldots)$ must be a Bernoulli utility function for generation T, which means that there exist real numbers $a_T > 0$ and b_T such that $V_T(x, d, \ldots) = a_T u(x) + b_T$ for all $x \in X$. Denote $U_T(x_T, x_{T+1}, \ldots) \equiv \frac{V_T(x_T, x_{T+1}, \ldots)-b_T}{a_T}$. U_T is such that:

$$U_T(x, d, d, \ldots) = u(x) \quad \text{for all } x \in X \tag{8}$$

We also take $U_1(x_1, x_2, \ldots) \equiv U(x_1, x_2, \ldots)$.

Consider now $q, q', \hat{q}$ and $\hat{q}'$, four probability measures in $\mathcal{Q}$ satisfying the following conditions for some $T \in \mathbb{N}$:[18]

- $q_t = q'_t = \delta_{\bar{x}}$ for all $t \leq T - 1$, $q_T = q'_T = \delta_{x_T}$, and $\hat{q}_t = \hat{q}'_t = \delta_{\bar{x}}$ for all $t \leq T$.
- $q_{\mathbb{N}\setminus\{1,\ldots,T\}} = \hat{q}_{\mathbb{N}\setminus\{1,\ldots,T\}}$ and $q'_{\mathbb{N}\setminus\{1,\ldots,T\}} = \hat{q}'_{\mathbb{N}\setminus\{1,\ldots,T\}}$.

[18] In the case $T = 1$, the first condition can be written $q_1 = q'_1 = \delta_{x_1}$, and $\hat{q}_1 = \hat{q}'_1 = \delta_{\bar{x}}$.

According to the (IUD) axiom, $q \succeq^S q' \Leftrightarrow \hat{q} \succeq^S \hat{q}'$. Using the definition of U_T we obtain:

$$\sum_{{}_{T+1}\mathbf{x}\in\chi} U_T(x_T, x_{T+1}, \ldots) q_{\mathbb{N}\setminus\{1,\ldots,T\}}(x_{T+1}, \ldots) \geq \sum_{{}_{T+1}\mathbf{x}\in\chi} U_T(x_T, x_{T+1}, \ldots) q'_{\mathbb{N}\setminus\{1,\ldots,T\}}(x_{T+1}, \ldots)$$
$$\Longleftrightarrow$$
$$\sum_{{}_{T+1}\mathbf{x}\in\chi} U_{T+1}(x_{T+1}, \ldots) q_{\mathbb{N}\setminus\{1,\ldots,T\}}(x_{T+1}, \ldots) \geq \sum_{{}_{T+1}\mathbf{x}\in\chi} U_{T+1}(x_{T+1}, \ldots) q'_{\mathbb{N}\setminus\{1,\ldots,T\}}(x_{T+1}, \ldots)$$

The equivalence is true for any $x_T \in K$ and for any $q_{\mathbb{N}\setminus\{1,\ldots,T\}}$ and $q'_{\mathbb{N}\setminus\{1,\ldots,T\}}$. Denote $U_{T,x_T}(x_{T+1}, \ldots) \equiv U_T(x_T, x_{T+1}, \ldots)$. The equivalence implies that U_{T,x_T} and U_{T+1} are two Bernoulli utility functions representing the same preference ordering on the uncertain future after period T. The function U_{T,x_T} must therefore be obtained from U_{T+1} by a positive affine transformation. In other words, for any $x_T \in K$ there exist $v_T(x_T)$ and $w_T(x_T) > 0$ such that

$$U_T(x_T, x_{T+1}, \ldots) = v_T(x_T) + w_T(x_T) \times U_{T+1}(x_{T+1}, \ldots). \tag{9}$$

The normalization condition (8) implies that:

$$v_T(x_T) = u(x_T), \quad \forall x_T \in X, \ \forall T \in \mathbb{N}. \tag{10}$$

Remark that the (A) axiom imposes that $U_T(x, y, d, \ldots) = U_T(y, x, d, \ldots)$. Using (9) and (10), this implies that for any $x, y \in K$ we must have:

$$u(x) + w_T(x)u(y) = u(y) + w_T(y)u(x)$$

or equivalently:

$$\frac{1 - w_T(x)}{u(x)} = \frac{1 - w_T(y)}{u(y)}$$

The ratio $\frac{1-w_T(x)}{u(x)}$ is therefore constant for any $x \in K$. Let denote it by ε_T. We have

$$w_T(x) = 1 - \varepsilon_T u(x) \quad \forall x \in K. \tag{11}$$

Since $w_T(x) > 0$ we must have $\varepsilon_T u(x) < 1$.

Gathering the results in (9), (10) and (11) for $t = 1, \cdots, T$, we obtain that:

$$\begin{aligned} &U(x_1, \ldots, x_T, d, d, \ldots) \\ &= \sum_{t=1}^{T} u(x_t) \prod_{\tau=1}^{t-1} (1 - \varepsilon_\tau u(x_\tau)) + U_{T+1}(d, d, \ldots) \prod_{\tau=1}^{T} (1 - \varepsilon_\tau u(x_\tau)) \end{aligned}$$

We also know that $U_{T+1}(d, d, \ldots) = 0$, so that:

$$U(x_1, \ldots, x_T, d, d, \ldots) = \sum_{t\in\mathbb{N}} u(x_t) \prod_{\tau=1}^{t-1} (1 - \varepsilon_\tau u(x_\tau)). \tag{12}$$

We need to prove that $\varepsilon_T = \varepsilon$ for all $T \in \mathbb{N}$. First consider the outcomes $(x_1, x_2, x_3, d, d, \ldots)$ and $(x_3, x_2, x_1, d, d, \ldots)$. Axiom (A) requires that $U(x_1, x_2, x_3, d, d, \ldots) = U(x_3, x_2, x_1, d, d, \ldots)$. Using Eq. (12) and after some simplifications, we get: $\varepsilon_1 (u(x_1) - u(x_3)) = \varepsilon_2 (u(x_1) - u(x_3))$. Recall that our sensitivity requirement imposes that there exist $\bar{x}$ and $\hat{x}$ in K such that $u(\bar{x}) > u(\hat{x}) > 0$. Taking $x_1 = \bar{x}$ and $x_3 = \hat{x}$, we end up with $\varepsilon_1 \left(u(\bar{x}) - u(\hat{x})\right) = \varepsilon_2 \left(u(\bar{x}) - u(\hat{x})\right)$. This implies $\varepsilon_1 = \varepsilon_2$.

Consider $(\bar{x}, \ldots, \bar{x}, x_t, x_{t+1}, x_{t+2}, d, d, \ldots)$ and $(\bar{x}, \ldots, \bar{x}, x_{t+2}, x_{t+1}, x_t, d, d, \ldots)$, for any $t \in \mathbb{N} \setminus \{1\}$. The same argument as above, using $x_t = \bar{x}$ and $x_{t+2} = \hat{x}$, proves that $\varepsilon_t = \varepsilon_{t+1}$. This completes the proof that $\varepsilon_T = \varepsilon$ for all $T \in \mathbb{N}$ so that Eq. (12) becomes:

$$U(x_1, \ldots, x_T, d, d, \ldots) = \sum_{t \in \mathbb{N}} u(x_t) \prod_{\tau=1}^{t-1} (1 - \varepsilon u(x_\tau))$$

If $\varepsilon = 0$, we get the additive Bernoulli function:

$$U(\mathbf{x}) = \sum_{t \in \mathbb{N}} u(x_t).$$

When $\varepsilon \neq 0$ we compute:

$$\begin{aligned}
&1 - \varepsilon U(x_1, \ldots, x_T, d, d, \ldots) \\
&\quad = 1 - \sum_{t=1}^{T} \varepsilon u(x_t) \prod_{\tau=1}^{t-1} (1 - \varepsilon u(x_\tau)) \\
&\quad = 1 + \sum_{t=1}^{T} (1 - \varepsilon u(x_t)) \prod_{\tau=1}^{t-1} (1 - \varepsilon u(x_\tau)) - \sum_{t=1}^{T} \prod_{\tau=1}^{t-1} (1 - \varepsilon u(x_\tau)) \\
&\quad = 1 + \sum_{t=1}^{T} \prod_{\tau=1}^{t} (1 - \varepsilon u(x_\tau)) - \sum_{t=1}^{T} \prod_{\tau=1}^{t-1} (1 - \varepsilon u(x_\tau)) \\
&\quad = \prod_{t=1}^{T} (1 - \varepsilon u(x_t)) = \prod_{t \in \mathbb{N}} (1 - \varepsilon u(x_t)) .
\end{aligned}$$

We eventually get the multiplicative specification:

$$U(\mathbf{x}) = \frac{1}{\varepsilon} \left(1 - \prod_{t \in \mathbb{N}} (1 - \varepsilon u(x_t)) \right).$$

Appendix 2: Proof of Proposition 2

For any $\mathbf{x} \in K^{\mathbb{N}}$ and $T > 0$ denote

$$U(\mathbf{x}, T) = \frac{1}{\varepsilon} \times \left(1 - \prod_{t=1}^{T} [1 - \varepsilon u(x_t)]\right)$$

and, for $T = 0$,

$$U(\mathbf{x}, 0) = 0.$$

We have, for any $T \geq 0$:

$$\begin{aligned} U(\mathbf{x}, T+1) - U(\mathbf{x}, T) &= \frac{1}{\varepsilon} \times \left(1 - \left[1 - \varepsilon u(x_{T+1})\right]\right) \times \prod_{t=1}^{T} [1 - \varepsilon u(x_t)] \\ &= u(x_{T+1}) \times \prod_{t=1}^{T} [1 - \varepsilon u(x_t)]. \end{aligned}$$

When the lottery belongs to $\mathcal{Q}_\varepsilon$, we can write:

$$\begin{aligned} W(\mathbf{x}) = \sum_{T=0}^{+\infty} p(1-p)^T U(\mathbf{x}, T) &= \sum_{T=0}^{+\infty} \left[(1-p)^T - (1-p)(1-p)^T\right] U(\mathbf{x}, T) \\ &= \sum_{T=0}^{+\infty} (1-p)^T U(\mathbf{x}, T) - \sum_{T=0}^{+\infty} (1-p)^{T+1} U(\mathbf{x}, T) \\ &= \sum_{T=0}^{+\infty} (1-p)^T U(\mathbf{x}, T) - \sum_{T=0}^{+\infty} (1-p)^{T+1} U(\mathbf{x}, T+1) \\ &\quad + \sum_{T=0}^{+\infty} (1-p)^{T+1} [U(\mathbf{x}, T+1) - U(\mathbf{x}, T)] \\ &= \sum_{T=1}^{+\infty} (1-p)^T u(x_T) \prod_{t=1}^{T-1} [1 - \varepsilon u(x_t)]. \end{aligned} \tag{13}$$

Now, we can easily compute the partial derivative of $W(\mathbf{x})$ with respect to x_t:

$$\begin{aligned} \frac{\partial W(\mathbf{x})}{\partial x_t} &= u'(x_t)(1-p)^t \prod_{\tau=1}^{t-1} [1 - \varepsilon u(x_\tau)] \\ &\quad - \varepsilon u'(x_t) [1 - \varepsilon u(x_t)]^{-1} \sum_{T=t}^{+\infty} (1-p)^{T+1} u(x_{T+1}) \prod_{\tau=1}^{T} [1 - \varepsilon u(x_\tau)]. \end{aligned}$$

Along a constant consumption path $\mathbf{x} =_{\text{con}} x$, denoting $u(x)$ and $u'(x)$ by u and u', the above expression reduces to:

$$\begin{aligned}\frac{\partial W(\mathbf{x})}{\partial x_t} &= u'(1-p)^t(1-\varepsilon u)^{t-1} \\ &\quad -\varepsilon u'\,[1-\varepsilon u]^{-1}\sum_{T=t}^{+\infty}(1-p)^{T+1}u(1-\varepsilon u)^T \\ &= (1-p)^t(1-\varepsilon u)^{t-1} \\ &\quad \times\left(u' - \varepsilon u u'\,[1-\varepsilon u]^{-1}\sum_{T=t}^{+\infty}(1-p)^{T-t+1}\,[1-\varepsilon u]^{T-t+1}\right) \\ &= (1-p)^t(1-\varepsilon u)^{t-1}\left(u' - \varepsilon u u'\,[1-\varepsilon u]^{-1}\sum_{T=0}^{+\infty}((1-p)\,(1-\varepsilon u))^T\right).\end{aligned}$$

We eventually obtain that:

$$\rho(x) = 1 - \left.\frac{\frac{\partial W(\mathbf{x})}{\partial x_2}}{\frac{\partial W(\mathbf{x})}{\partial x_1}}\right|_{\mathbf{x}=_{\text{con}}x} = 1-(1-p)(1-\varepsilon u) = p+(1-p)\varepsilon u.$$

Appendix 3: Proof of Proposition 3

Consider a lottery that belongs to Q_ε. The planner's preferences over consumption plans are represented by:

$$W(\mathbf{x}) = \frac{1}{\varepsilon}\sum_{T=1}^{+\infty} p(1-p)^T\left(1-\prod_{t=1}^{T}[1-\varepsilon u(x_t)]\right)$$

Substitute $u(x) = 1+\lambda v(x)$ in the above formula and write that:

$$W(\mathbf{x}) \simeq W(\mathbf{x})|_{\lambda=0} + \lambda\frac{\partial W}{\partial \lambda}|_{\lambda=0}$$

to obtain

$$\begin{aligned}W(\mathbf{x}) &\simeq \frac{1}{\varepsilon}\sum_{T=1}^{+\infty} p(1-p)^T\left(1-[1-\varepsilon]^T\right) \\ &\quad +\lambda\sum_{T=1}^{+\infty} p(1-p)^T\,[1-\varepsilon]^{T-1}\left(\sum_{t=1}^{T} v(x_t)\right).\end{aligned}$$

The first term is a constant and does not affect preferences. Switching the summation signs, the second term equals

$$\begin{aligned}
&\lambda \sum_{t=1}^{+\infty} v(x_t) \sum_{T=t}^{+\infty} p(1-p)^T \, [1-\varepsilon]^{T-1} \\
&= \lambda p \, [1-\varepsilon]^{-1} \sum_{t=1}^{+\infty} v(x_t) \sum_{T=t}^{+\infty} [(1-p)(1-\varepsilon)]^T \\
&= \frac{\lambda p \, [1-\varepsilon]^{-1}}{1-(1-p)(1-\varepsilon)} \sum_{t=1}^{+\infty} \gamma^t v(x_t)
\end{aligned}$$

where $\gamma = (1-p)(1-\varepsilon)$. The term $\frac{\lambda p[1-\varepsilon]^{-1}}{1-(1-p)(1-\varepsilon)}$ is a positive multiplicative factor which does not affect preferences. We therefore see that the planner's preferences can be represented by

$$\sum_{t=1}^{+\infty} \gamma^t v(x_t).$$

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