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ORIGINAL PAPER

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A general equilibrium model of multi-party competition

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Abstract The paper introduces a generalized spatial model that is motivated by the frequent changes in party identity and electoral laws that characterize transitional party systems. In this model, parties may (1) change their platforms, (2) their identities through coalitions and splits and (3) if they form a winning coalition, the electoral law. The equilibrium is defined as a state such that no party or coalition can strictly benefit from changing the electoral law, its platform, or from splitting or coalescing. The results show that while there are games with no institutional or coalitional-split equilibria, such equilibria do exist under relatively undemanding conditions. The main finding is that once an institutional and identity equilibrium is achieved, it is generically robust against small trembles in party platforms or voter preferences. This robustness facilitates greater stability in terms of institutions and party identities in mature party systems where such trembles are smaller than in transitional systems.

1 Introduction

Spatial approaches dominate the formal modeling of party politics. In such models, parties or electoral candidates usually modify their spatial platforms to increase their chances of election (Downs 1957).[1] However, modifying a platform is only one mechanism among many to improve one's electoral prospects in multi-party systems. A party might also institute electoral reform or change its identity via electoral splits, mergers, or temporary electoral coalitions. All such activities take

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[1] Surveys of spatial and other formal voting models include Austen-Smith and Banks (1999), Calvert (1986), Coughlin (1992), Enelow and Hinich (1984, 1990), Hinich and Munger (1997), Miller (1995), Ordeshook (1986), Schofield (1985) and Shepsle (1991).

place before elections and may affect a party's electoral prospects significantly. Although empirically-oriented scholars analyze the frequent changes to electoral laws and party identities, formal studies of party competition are dominated by the spatial paradigm.

The abundance of models based on party platforms, as opposed to electoral law and party identity activities, is rooted in the clear predominance of issue-politics in Western democracies. This predominance of issue politics is not a universal phenomenon in the empirical world, though. Parties in emerging democracies operate differently than those in their more-established counterparts. Platform changes are usually less prominent in the buzz of transitional politics. Instead, party politics revolves around the emergence of new parties, electoral splits and coalitions, and the perpetual modification of electoral laws. The institutional or identity changes that actually occur are only the tip of an iceberg. There are numerous other threats of splits, rumors of coalescing, and electoral reform projects that never come to fruition.[2] Thus, the puzzling question emerges: Why are there so many identity and electoral law changes in transitional democracies compared to the lack of similar activity in established party systems?

A good proxy for the relative importance of electoral law and identity changes in emerging democracies may be the overwhelming number of such events. Figures 1 and 2 show the major attempts to change electoral institutions and the modifications to party identities that occurred in Poland between 1989 and 2000. Pictures drawn for other transitional democracies would be quite similar to the Polish one. These types of events are less common in more established party systems. For example, a similar picture drawn for the American party system would consist of empty lines, with an exception made for the Reform party that emerged in 1993.

Since the fall of communism in 1989, the Polish parliamentary electoral law has been changed nine times. The changes were deep and often had profound political consequences. In addition, 12 major attempts to change the law were unsuccessful (Fig. 1). The total number of all electoral law-related projects debated by the electoral law committee was much greater than the number of actual reforms: 16 such projects were considered between October 14, 1999 and April 30, 2001 alone. The identity transformations were even more spectacular (Fig. 2). The total number of parties skyrocketed to over 200 in the early post-communist years and the effective number of parliamentary parties after the first free elections in 1991 came close to 11. Over the entire period, there were about 70 major entries, legalizations, coalitions, mergers, splits, defections, and withdrawals of major political players. While the frequency of both kinds of change clearly decreased over time, politics motivated by institutional and identity change is far from over.

I provide a game-theoretic model that incorporates issue politics, electoral heresthetics, and identity transformations into a single framework. While many important questions remain unsettled, the results of this paper are compatible with a vision of party politics where (1) parties may adjust their platforms; (2) they may merge, coalesce or split; (3) a winning coalition may change the electoral law. These changes to platforms, identities and/or electoral laws are motivated by the

[2] A sample of papers documenting identity and institutional fever in transitional democracies includes Waller (1995) on Bulgaria, Kopecky (1995) on the Czech Republic, Grofman et al. (1999, 2000) on Estonia, Lomax (1995) on Hungary, Kaminski (2001, 2002) on Poland, and Filippov et al. (1999) on Russia.

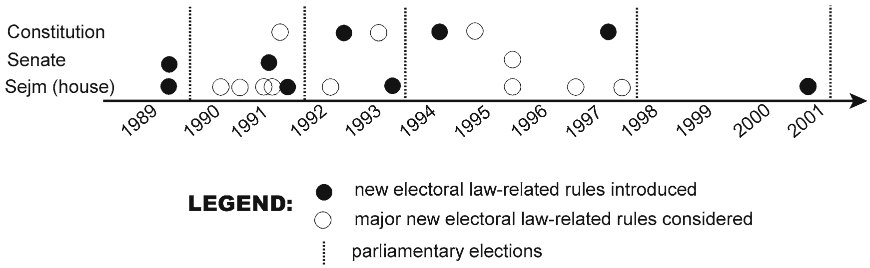

Fig. 1 Institutional changes in Poland affecting the allocation of parliamentary seats, 1989–2001. *Note.* Only major changes or attempted changes by political parties are represented. Changes in the electoral laws for presidential elections and local government elections are not shown

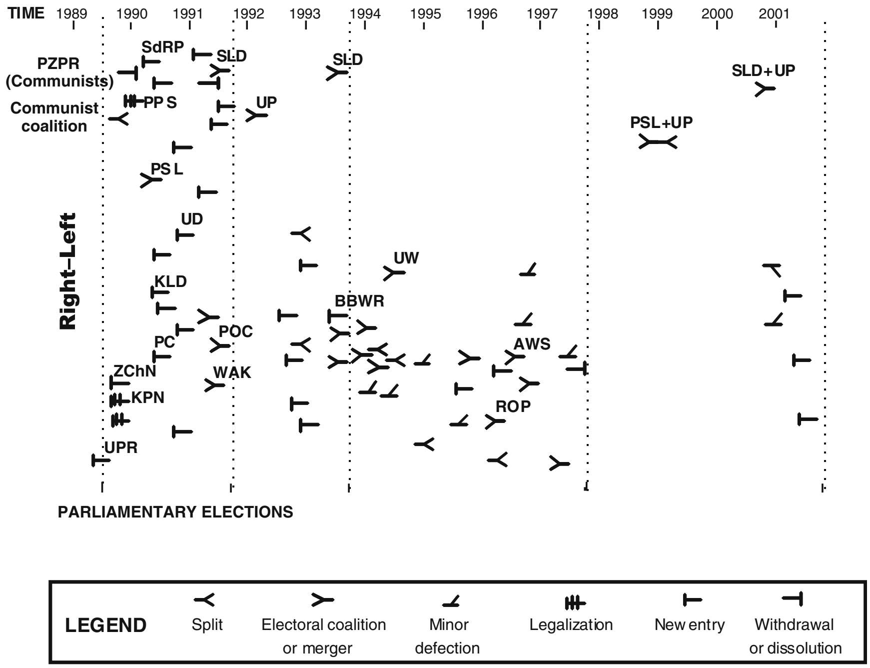

Fig. 2 Changes of political identities in Poland, 1989–2001. Major splits, mergers, coalitions, entries of new parties, legalizations, defections, dissolutions, and withdrawals from electoral race. *Note.* Every graphical symbol represents an identity change of a political player or a set of players. Minor players, minor changes of players' identities, and players other than political parties are omitted. Dates are approximate. Since the "Left–Right" dimension represents combined social and economic liberalism, spatial distances between parties are approximate. In some cases, such as PSL and UW, the combination of social and economic dimensions distorts the spatial distance between the players significantly

parties' expected payoffs—usually an increase in their vote or seat share (see Fig. 3). In emerging democracies, sudden switches in voter preferences often change the payoffs of the electoral game. This frequently creates opportunities for increasing seat share via electoral reforms and identity changes. Amidst a poorly informed

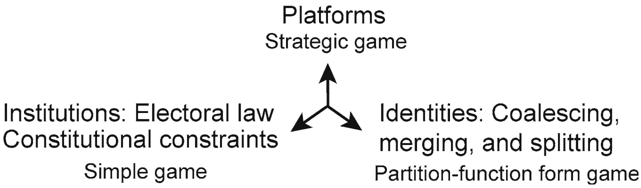

Fig. 3 Three main components of party politics in multi-party systems and corresponding models

electorate, the fine-tuning of platforms may not promise such opportunities. As a result, its relative importance is lower.

Figure 3 depicts all three kinds of party activities and formal models used to represent similar phenomena. The mechanism behind such activities is illustrated by the examples of consolidation of rightist parties in Poland between 1993 and 1997, and by the 2001 Polish electoral reform.

1.1 Example: consolidation of the Polish rightist parties between 1993 and 1997

The parties that entered the Polish 1993 parliamentary elections were highly fragmented. Between 1993 and 1997, the right side of the political scene witnessed complex coalitional changes. After various attempts to create a single large coalition or party, a big coalition, called the AWS, emerged finally as the main focal point for rightist voters.

The AWS' leaders emphasized their desire to maximize house seat shares and worked hard to resolve personal animosities and ignore minor programmatic differences. The electoral law was friendly towards larger parties and provided ample incentives for coalescing. However, the rightist leaders recognized that some potential coalitions were expected to be harmful. When negotiating with the small radical rightist ROP, the AWS' leader Krzaklewski announced that it was important to "examine first whether our electorates are additive" (Zdort 1997). The polls showed a 2% loss for a potential coalition AWS–ROP. It is likely that this would have resulted in a loss of seat share. As a result, the coalition did not materialize.

Consider an example illustrating hypothetical opportunities from coalescing from Table 1. A leftist party L and two rightist parties R_1 and R_2 compete in elections. When the two rightist parties compete separately (coalitional structure $L-R_1-R_2$), they win jointly 60% of the vote. The electoral law rewards large parties and this support is translated into only 48% of seats. Now assume that party analysts calculate that when R_1 and R_2 coalesce (coalitional structure L–R_1R_2), such a grand coalition of the rightist parties will lose some supporters and receive 58% of the vote. Although the percentage of the vote received by the right falls, the mechanical properties of the electoral law would translate this support into 65% of the seats. Thus, by coalescing, the two rightist parties can gain 17% of seats.

The rightist parties in Poland faced similar incentives for coalescing after 1993. First, the parties' programmatic similarity meant that the parties did not lose too many votes through coalescing. In fact, their votes were approximately *additive*. This means that the total support was about equal the total support for separate

Table 1 Hypothetical vote and seat shares of three parties with the rightist parties fragmented and a grand rightist coalition

Coalitional structure	Party or coalition	Vote share	Seat share
$L–R_1–R_2$	L	40	52
	R_1	28	20
	R_2	32	28
$L–R_1R_2$	L	42	35
	R_1R_2	58	65

$L–R_1–R_2$ denotes a party system with the rightist parties fragmented and $L–R_1R_2$ denotes a party system with a grand coalition of the rightist parties formed

parties. Second, the electoral law was friendly towards larger entities. Under the combined effect of both factors, the coalition of rightist parties could expect to win a larger seat share than the total seat share won by the coalition's members.[3] Similarly, an electoral law favoring smaller parties and electorates disliking larger coalitions would exercise a pressure on a coalition towards splitting into smaller components.

The possibility of increasing the total seat share by a set of parties through coalescing, or alternatively, by splitting a party, will motivate the part of our model representing incentives for splits and mergers.

1.2 Example: Polish electoral reform of 2001

Immediately after the 1997 elections, the main post-solidarity party AWS supported a majoritarian electoral reform against the moderately proportional status quo. The AWS was soon joined by the main post-communist party SLD against the minor post-solidarity party UW and the minor post-communist peasant party PSL. Over a few years, the AWS gradually changed its position and finally joined the UW and PSL in an effort to make the electoral law even more proportional.

The electoral reform was completed at the very last moment, 6 months before the elections. In March 2001, the AWS, UW, and PSL introduced the new electoral law despite strong protests from SLD. The votes-to-seats properties of the new law were more friendly for smaller and medium parties. The main changes included an increase in the average district magnitude from 7.5 to 11.2 and the substitution of d'Hondt apportionment formula with modified Sainte Laguë.

The reason for the electoral reform and the AWS' turnabout was perfectly clear to the vast majority of political commentators in Poland. After the 1997 elections, the support of both SLD and AWS oscillated around 30% against about 10% support for the smaller parties. However, the support for the SLD gradually increased to 45–50% over the following years. The support for the AWS followed an opposite trend, approaching single digits. This put the AWS on a par with the PSL and slightly above the UW. At a time when the reform could be implemented, the analysts of the AWS, PSL, and UW reached the same conclusion that electoral reform would be beneficial for them (see Table 2).

[3] Simulations show that had the rightist parties united before the 1993 elections, they would have won about 35% of house seats against the actual 3.5% (Kaminski et al. 1998).

Two factors were important. First, the three parties supporting the electoral reform could expect to make substantial gains at the expense of the SLD. The SLD led the polls with support close to 50%. The other major parties had similar and pretty stable scores around 10%. Under this distribution, even substantial changes of a smaller party's support would not reverse the sign of estimated gains. Second, the three parties had enough voting power and were sufficiently disciplined to implement the electoral reform.

Thus, the coalition of reformers had both the necessary incentives and voting power to change the law. The existence of gains for all members of a coalition with a winning power will define the partial equilibrium in the model dealing with electoral reform.

1.3 Plan of paper

I have two objectives in writing this paper. First, I attempt to establish a general framework for modeling various aspects of multiparty competition that cannot be represented within the boundaries of the traditional spatial approach. Second, I investigate certain basic properties of equilibria that emerge in such a context.

The next section introduces notation, the main concepts, and defines games in the effectiveness function form (Rosenthal 1972). The model is presented in Sect. 2 as a special case of such games. The equilibrium concept employed in the model represents various decision-making processes that lead to changes in party platforms, an electoral law, or players' identities. The results of Sect. 3 aim at providing a formal justification for the higher frequency of institutional and identity changes observed in transitional versus established democracies. Section 4 concludes.

2 Preliminaries

A game in the effectiveness function form (e-game) represents a set of states of the world, the ways players can alter such states, and the players' incentives for action. Formally, an e-game consists of Rosenthal (1972) and Chwe (1994):

Table 2 Estimated seat shares of major parties at the time of 2001 electoral reform under the old and new electoral laws

Party	Est. seat share (under old law)[a]	Est. seat share (under new law)[a]	Voting power[b]	Support [b]		
				New	Old	Indiff
AWS	12.0	14.6	38.4	36.8	0.9	0.7
SLD	57.8	50.2	36.8	0	36.8	0
UW	0.4	2.6	9.3	9.3	0	0
PSL	11.7	14.3	5.7	5	0.2	0.5

Seat shares are in percentages. Voting power was recorded on 03/07/2001, the day the new electoral law was introduced. Four parties with the greatest voting power are listed. "Support" lists those party MP's who supported the new law, those who supported the old law, and abstainers as the percentage of all MP's taking part in the voting session

[a] Wyniki symulacji (1998)

[b] Wyniki glosowania nr 75 - posiedzenie 103 (2001)

(1) A non-empty set of *players* N;
(2) A non-empty set of *states* Q;
(3) An *effectiveness function*, or an e-*function*, ε: $2^N \times Q \to 2^Q$ such that $q \in \varepsilon(K, q)$ for all $K \subset N$, $K \neq \emptyset$;
(4) A *payoff function* ν: $N \times Q \to \mathbb{R}$.

For any state q and a non-empty coalition K, the value of the e-function $\varepsilon(K, q)$ or $\varepsilon^q(K)$ is interpreted as the set of states that K can achieve, reach, or enforce starting from an initial state q. If $\varepsilon^q(K) = \{q\}$, i.e., if K can only retain the status quo q, then K is called a *dummy* at q. The set of all states that can be reached by all coalitions from q is denoted by $\varepsilon(q) = \cup_{K \subset N} \varepsilon^q(K)$. The payoff of a player i under the state q is denoted as $\nu(i, q)$ or $\nu^q(i)$. Any "e-game with payoffs removed," i.e., a triple $\langle N, Q, \varepsilon\rangle$ is called a *game form*. A game form represents a strategic backbone of a class of games.

One of the advantages of e-games is that they are general enough to represent various cooperative and non-cooperative models. Informally, a standard characteristic function form TU game may be represented as an e-game with the states parametrized by specific coalitions and with the e-function allowing a set of parties K to move from any state to the state representing coalition K.[4] The dynamics motivating such an e-game can be interpreted as "changing the present coalition." However, an e-game allows for other kinds of moves, such as changing an individual's strategy. The model introduced in this paper will allow for both cooperative and non-cooperative moves. Every strategic game can be represented as an e-game with a transformation outlined in Example 1 below.

Example 1 Consider a typical 2×2 strategic game $\langle N, S_1, S_2, \nu_1, \nu_2\rangle$, where $N = \{P_1, P_2\}$, $S_1 = \{s_1, s_2\}$, $S_2 = \{t_1, t_2\}$, and ν_1, ν_2: $S_1 \times S_2 \to \mathbb{R}$ denote players, strategy sets, and payoffs respectively. A corresponding e-game is defined as follows:

Players: N;
States: $Q = \{s_1t_1, s_1t_2, s_2t_1, s_2t_2\}$;
Payoffs: ν_1, ν_2;
e-function: $\varepsilon(P_1, s_it_j) = \{s_it_j, s_kt_j\}$, where $i, j, k \in \{1, 2\}$ and $i \neq k$; $\varepsilon(P_2, s_it_j) = \{s_it_j, s_it_k\}$, where $i, j, k \in \{1, 2\}$ and $j \neq k$; $\varepsilon(P_1 \cup P_2, s_it_j) = \{s_it_j\}$ for $i, j \in \{1, 2\}$.

The e-function represents the players' options for deviating from a given state. A single player may change the state by changing his strategy. However, there are no actions in Example 1 available exclusively to coalitions to change states, i.e., all actions available to P_1 and P_2 as a coalition are available to them as single players. This is shown by $\varepsilon(P_1 \cup P_2, s_it_j) = \{s_it_j\}$. Thus, every non-singleton coalition in a strategic game is a dummy.[5]

[4] The effectiveness function is a generalization of the effectivity function e^*: $2^N \to 2^Q$ which, combined with payoffs, provides a direct generalization of a TU characteristic function $e^\#$: $2^N \to \mathbb{R}$.

[5] The e-function can be made superadditive, i.e., such that for any two coalitions $K_1, K_2 \subset N$, $K_1 \subset K_2$ implies $\varepsilon^q(K_1) \subset \varepsilon^q(K_2)$ for all $q \in Q$. The interpretation of such an e-function would be different, but both definitions can be made equivalent with respect to equilibrium selection with appropriate definitions of equilibria. The present definition offers a more convenient notation.

A *simple game* is a pair $\langle N, \mathcal{W} \rangle$, where N is a nonempty set and $\mathcal{W}$ is a function defined for every coalition (subset) of N, i.e., $\mathcal{W}$: $2^N \rightarrow \{-1, 0, 1\}$ such that the following conditions are satisfied: (1) For all $K \subset N$, $\mathcal{W}(K) + \mathcal{W}(N - K) = 0$; (2) For all $L, K \subset N$, if $L \subset K$, then $\mathcal{W}(L) \leqq \mathcal{W}(K)$; (3) $\mathcal{W}(N) = 1$. If the set of players N is obvious from the context, a simple game will be denoted by $\mathcal{W}$. If for $K \subset N$, $\mathcal{W}(K) = -1, 0, 1$, then we call K a *losing*, *blocking*, and *winning* coalition respectively. The three conditions have an intuitive interpretation: (1) if a coalition is winning, losing, or blocking, then the coalition of all others must be losing, winning, or blocking, respectively; (2) if a coalition expands, then its voting power does not decrease; (3) the grand coalition is winning.

A *coalitional structure* $\mathcal{C}$ is any partition of N into subsets excluding the partition whose sole element is the grand coalition N . Excluding the grand coalition-based structure is a helpful convention that represents the empirically obvious fact that grand coalitions of all parties do not form under usual circumstances. A coalitional structure $\mathcal{C}_1 = \{K_1, \ldots, K_k\}$ is a *superstructure* of $\mathcal{C}_2 = \{L_1, \ldots, L_m\}$ if (1) $\mathcal{C}_1 = \mathcal{C}_2$ or (2) $\mathcal{C}_1 \neq \mathcal{C}_2$ and for all $i = 1, \ldots, m$, there exists $t \in \{1, \ldots, k\}$ such that either $L_i = K_j$ for some $j = 1, \ldots, k$, or $L_i \subsetneq K_t$. Intuitively, a superstructure emerges when some members of an old structure merge into one bigger coalition. For example, when the rightist parties merged into the AWS in 1996 in Poland, the resulting coalitional structure was a superstructure of the initial structure with the rightist parties fragmented. If $\mathcal{C}_1$ is a superstructure of $\mathcal{C}_2$ then $\mathcal{C}_2$ is called a *substructure* of $\mathcal{C}_1$. By convention, $\mathcal{C}_1$ is a superstructure and a substructure of itself. For any non-empty set X, $X^{\mathcal{C}_1}$ denotes the Cartesian product of X taken $|\mathcal{C}_1|$ times with the dimensions indexed by members of $\mathcal{C}_1$, where $|\mathcal{C}_1|$ denotes the number of elements of the set $\mathcal{C}_1$.

3 Model

The model is a subclass of e-games. States are parametrized by three essential pieces of information about the party system: the coalitional structure of parties, the vector of platforms, and the electoral law. Thus, in this model states are not primitives, but rather are defined with other concepts. Similarly, other parameters define the e-function. The full list of parameters is as follows:

S is any non-empty set and $|S| \geq 2$;
$\mathcal{I}$ is any finite non-empty set;
$\mathcal{W}$ is a simple game defined over N, the set of players;
f is a function such that for all pairs of coalitional structures $\mathcal{C}_1, \mathcal{C}_2$ such that $\mathcal{C}_1$ is a superstructure or a substructure of $\mathcal{C}_2$, and an issue space S , $f^{\mathcal{C}_1\mathcal{C}_2}: S^{\mathcal{C}_1} \rightarrow S^{\mathcal{C}_2}$. When $\mathcal{C}_1 = \mathcal{C}_2$, f is the identity function: $f^{\mathcal{C}_1\mathcal{C}_1} \equiv Id$.

The *issue space* S is interpreted as the set of potential party platforms. The *institutional space* $\mathcal{I}$ is interpreted as the set of available electoral laws. An obvious technical assumption is that $S \cap \mathcal{I} = \emptyset$. $\mathcal{W}$ says who can change the electoral law. Function f simply says how party platforms respond to identity changes. In effect it describes what happens with the platform(s) when parties or coalitions merge or split.

Definition 1 enlists the ways in which a party system can be modified by a party or a coalition of parties. All parts of this definition are explained in the comments below.

Definition 1 *A generalized spatial model is an e-game parametrized with a tuple $\langle N, S, \mathcal{I}, \mathcal{W}, f, \nu \rangle$ such that:*

(1) N is a set of players, $|N| \geq 3$;

(2) Q includes all tuples $q = \langle \mathcal{C}^q, s_1^q, \ldots, s_{|\mathcal{C}^q|}^q, EL^q \rangle$ s. t. (i) $\mathcal{C}^q = \{P_1, \ldots, P_{|\mathcal{C}^q|}\}$ is a coalitional structure whose members are called parties; (ii) $s^q = (s_1^q, \ldots, s_{|\mathcal{C}^q|}^q) \in S^{\mathcal{C}^q}$ are parties' platforms; (iii) $EL^q \in \mathcal{I}$ is an electoral law;

(3) For any $q \in Q$ and any $K \subset N$, $\varepsilon^q(K) \subset Q$ is the smallest set such that:

(i) Platform changes. For any $K = P_i \in \mathcal{C}^q$, let $q^{t,s\_i} = \langle \mathcal{C}^q, s_1^q, \ldots s_{i-1}^q, t, s_{i+1}^q, \ldots, s_{|\mathcal{C}^q|}^q, EL^q \rangle$. Then for all $t \in S$, $q^{t,s\_i} \in \varepsilon^q(K)$.

(ii) Coalescing. For any $K \in \mathcal{C}^1$, where $\mathcal{C}^1$ is a superstructure of $\mathcal{C}^q$ resulting from the merger of some parties into K, let $q\text{-}^K = \langle \mathcal{C}^1, f^{\mathcal{C}^q\mathcal{C}^1}(s^q)_1, \ldots, f^{\mathcal{C}^q\mathcal{C}^1}(s^q)_{|\mathcal{C}^1|}, EL^q \rangle$. Then $q\text{-}^K \in \varepsilon^q(K)$.

(iii) Splitting. For any $K = P_i \in \mathcal{C}^q$ and for any $\mathcal{C}^1$, a substructure of $\mathcal{C}^q$ resulting from a split of K into its partition $L_1, \ldots, L_k$, let $q\text{-}^{L_1,\ldots,L_k} = \langle \mathcal{C}^1, f^{\mathcal{C}^q\mathcal{C}^1}(s^q)_1, \ldots, f^{\mathcal{C}^q\mathcal{C}^1}(s^q)_{|\mathcal{C}^1|}, EL^q \rangle$. Then $q\text{-}^{L_1,\ldots,L_k} \in \varepsilon^q(K)$.

(iv) Institutional change. For any $K = \cup_{k=1}^m P_{i_k}$ such that $P_{i_k} \in \mathcal{C}^q$ for $k = 1, \ldots, m$ and $\mathcal{W}(K) = 1$, and any $EL \in \mathcal{I}$, let $q\text{-}^{EL} = \langle \mathcal{C}^q, s_1^q, \ldots, s_{|\mathcal{C}^q|}^q, EL \rangle$. Then $q\text{-}^{EL} \in \varepsilon^q(K)$ for all $EL \in \mathcal{I}$.

(4) For each $q \in Q$, $\nu^q \colon \mathcal{C}^q \to \mathbb{R}_+$ s.t. $\sum_{i=1}^{|\mathcal{C}^q|} \nu^q(P_i) = 100$.

Comments: Any generalized spatial model will hereafter be called a *game*.

Ad. (1): *Players* in this model are atomic components of potentially larger decision-making entities rather than sole decision-makers. We assume that there are at least three players.

Ad. (2): Each possible *state* of a party system is fully described by three pieces of information: the coalitional structure of players, the platforms of all parties (i.e., the members of the coalitional structure), and the electoral law.

Ad. (3): The *effectiveness* function represents four possible ways that the players can modify the state, directly affecting exactly one of the three pieces of information. Coalescing and splitting are alternative ways of modifying the coalitional structure and, since the total number of parties changes, such modifications affect indirectly the parties' platforms. The e-function for any specific coalition and state is fully determined by the parameters f and $\mathcal{W}$.

First, each party can unilaterally select any platform. The coalitional structure, other platforms, and the electoral law remain constant. This is the classical component of spatial models.

Second, any set of parties that is smaller than the grand coalition can coalesce. By convention, any party may (trivially) coalesce with itself. The platforms of coalescing parties are merged, and the platforms of other parties are possibly modified, according to the function f. While one can argue that the platforms of non-merging parties should remain constant, the present formulation allows for more generality.

Third, any single party can split into smaller components. By convention again, a (trivial) split of a party into itself is allowed. The platforms of parties resulting from a split are modified according to the function f. Note that while a set of parties can merge in exactly one way, a party consisting of two or more players can split in more ways (including the trivial split). One can require that the platforms of non-splitting parties remain unchanged, or that splits be linked with mergers. For instance, when a party splits and its components later re-merge, one can require its platform to remain unchanged. Noting these possibilities, I resist the temptation to modify the general formulation adopted here. After all, the specific formula for f remains an interesting empirical question.

Finally, any winning coalition of parties from the coalitional structure can select any electoral law.

The four types of moves are the only possible ones. Other coalitions of players, i.e., proper subset of parties or coalitions that cross-cut at least one party's boundaries, are dummies and cannot change anything.

Ad. (4): Payoffs are interpreted here as the percentages of house seats. A slight departure from the e-game model is that the payoffs are defined for parties, i.e., for coalitions of players rather than players.[6] Finally, let us introduce the following useful convention and define a *total payoff* for a set of parties. For every coalition $K \subset N$ s.t. $K = \cup_{k=1}^{m} P_{i_k}$ where $P_{i_k} \in \mathcal{C}^q$ for $k = 1, \ldots, m$, let $v^q(K) = \sum_{k=1}^{m} v^q(P_{i_k})$. Note that the total payoff of parties is not assumed to be equal to the payoff of the coalition of parties when such a coalition forms. The latter number is indexed by a different state that results from q by the coalescing of P_{i_k} into K. In fact, parties in the present model only have incentives to coalesce when their total payoff *before* coalescing can be strictly increased by coalescing.

There are no voters in the model. All voter preferences are fully represented by the properties of the payoff function.

The main novelty of this model of party competition is the use of the e-function. Various modifications of the e-function can be considered. While the present article studies a particular form of the e-function, Sect. 3.5 discusses some possible extensions, such as farsightedness or allowing for defections from one party to another.

3.1 Evaluating the e-function: an example

In the following example, I evaluate a game form for a specific combination of parameters at a specific state. There are three parties with three potential platforms and two electoral laws. The task is to compute $\varepsilon(q) = \cup_{K \subset N} \varepsilon^q(K)$.

Example 2 Players: $N = \{P_1, P_2, P_3\}$;

Parameters: $S = \{l, c, r\}$, where party positions can be interpreted as leftist, centrist, and rightist; $\mathcal{I} = \{FPP, PR\}$; $\mathcal{W}(\{P_i, P_j\}) = \mathcal{W}(\{P_1, P_2, P_3\}) = 1$ for all $i, j \in \{1, 2, 3\}$, $i \neq j$; f says that in mergers, the party with a lower index imposes its platform; in splits, parties retain the former platform of the pre-split party; and the platform of the non-concerned party remains intact.

Consider the state $q = \langle P_1 - P_2 - P_3, lcr, PR \rangle$. $P_1 - P_2 - P_3$ denotes the coalitional structure of all three parties competing separately; lcr means that

[6] This minor modification of the standard e-game can be avoided by appropriate changes in the payoffs and subsequent definitions of the equilibria. The price paid in complexity is definitely not worth the effect.

parties P_1, P_2, and P_3 have platforms l, c, and r, respectively, and that the electoral law is PR. The e-function at q can be evaluated as follows:

Party P_1 can choose any of the platforms l, c, or r: $\varepsilon^q(P_1) = \{\langle P_1 - P_2 - P_3, lcr, PR >, \langle P_1 - P_2 - P_3, ccr, PR\rangle, \langle P_1 - P_2 - P_3, rcr, PR\rangle\}$. By permuting names, we can similarly evaluate $\varepsilon^q(P_2)$ and $\varepsilon^q(P_3)$.

The two parties P_1 and P_2 can change the electoral law, coalesce or do nothing. Thus, $\varepsilon^q(P_1 \cup P_2) = \{\langle P_1 - P_2 - P_3, lcr, PR\rangle, \langle P_1 - P_2 - P_3, lcr, FPP\rangle, \langle P_1 P_2 - P_3, lr, PR\rangle\}$. Note that the l and c platforms of parties P_1 and P_2 are merged into a new platform l for $P_1 P_2$ since party P_1 dictates the choice of platform in the coalition $P_1 P_2$. Again, by permuting parties' names, one can evaluate $\varepsilon^q(P_1 \cup P_3)$ and $\varepsilon^q(P_2 \cup P_3)$.

By our convention in the definition of coalitional structure in Section 2, the three parties P_1, P_2, and P_3 cannot form a grand coalition but they can change the electoral law. Thus, $\varepsilon^q(P_1 \cup P_2 \cup P_3) = \{\langle P_1 - P_2 - P_3, lcr, PR\rangle, \langle P_1 - P_2 - P_3, lcr, FPP\rangle\}$.

This evaluation reconstructs a part of the game form for one particular state $q = \langle P_1 - P_2 - P_3, lcr, PR\rangle$. All possible modifications of q are shown in Fig. 4.

3.2 Equilibrium

In spatial models, the key assumption about system dynamics is that a party has an incentive to change its platform s_i if such a change increases its payoff. In the general equilibrium concept employed below, the Nash principle of "no player has incentives to deviate from the present outcome of a game by changing his strategy" is extended to parties, their various coalitions, and various modifications of states. The incentives to change a state are represented as strategic opportunities of changing a platform, coalescing, splitting, or changing the electoral law.

The global equilibrium in the model requires that the conjunction of four partial-equilibrium conditions holds.

Definition 2 *For any game* Γ *and any state* $q = \langle \mathcal{C}^q, s_1^q, \ldots, s_{|\mathcal{C}^q|}^q, EL^q\rangle$, *$q$ is a spatial-coalitional-split-institutional equilibrium, or simply an equilibrium, if the following conditions are satisfied:*

1. *Platform stability (Nash equilibrium): For all* $K = P_i \in \mathcal{C}^q$, $sup_{t \in S} \nu^{q^{t,s\_i}}(K) = \nu^q(K)$;

2a. *Coalitional stability: For all* $K = \cup_{k=1}^m P_{i_k}$, *where all* $P_{i_k} \in \mathcal{C}^q$, $\nu^{q^{-K}}(K) \leqq \nu^q(K)$;

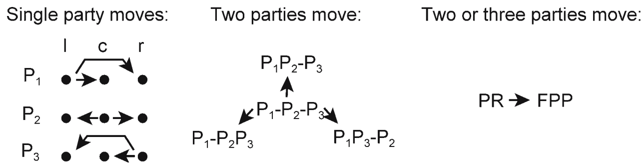

Fig. 4 Graphical representation of $\gamma(q)$ for $q = \langle P_1 - P_2 - P_3$ lcr PR$\rangle$

2b. *Split stability: For all* $K = P_i \in \mathcal{C}^q$, $max\{\nu^{q-^{L_1,\ldots,L_k}}(K) : L_1, \ldots, L_k$ *is a partition of* $K\} = \nu^q(K)$*;*

3. *Institutional stability: For all* $K = \cup_{k=1}^{m} P_{i_k}$*, where all* $P_{i_k} \in \mathcal{C}^q$ *and* $\mathcal{W}(K) = 1$*,* $max_{EL \in I} min_{k=1,\ldots,m}[\nu^{q-^{EL}}(P_{i_k})] = \nu^q(P_{i_k})$*.*

A state q is a partial equilibrium if it satisfies a subset of conditions 1–3.

Comments: The four different conditions represent the four different kinds of incentives that might induce parties or coalitions of parties to modify the status quo.

First, a party may be motivated to modify its platform and look for a best reply to the existing configuration of other parties' platforms. The relevant partial equilibrium condition is simply the Nash equilibrium in the spatial game, defined by holding players' identities and the electoral law constant.

Second, parties may be enticed to coalesce if the electoral law or voter preferences reward larger entities through appropriate payoffs. Similarly, a coalition may find it beneficial to dissolve into smaller units. Coalitional stability stipulates that no parties (not constituting together a grand coalition) would benefit from coalescing. Split stability requires that no coalition would benefit from a split.

Finally, every party may evaluate possible gains from modifying the electoral law. When a coalition of parties considers whether to support a particular electoral law, then unanimity is required for the move to take place. Thus, the coalition's incentives to move can be summarized by the possible gain or loss to the member who benefits least. (Of course, there may still be a winning proper subset in K that might be unanimously interested in the change.) Institutional stability demands that no winning coalition of parties K can change the electoral law to the strict benefit of each of its members.

Note that there are three cases in which parties can take actions that do not affect the status quo, i.e., a party can hold its platform, a trivial one-party split can occur, or a winning coalition can retain the existing electoral law. In all such cases the gain is null and the "no incentives to deviate" principle says that the maximum gain from these activities is zero. In the case of coalition-making, the status quo outcome is not an option for two or more parties and the "no incentives to deviate" principle says that parties do not gain anything or that they may strictly lose by coalescing.

Definition 2 can be modified in order to incorporate other incentives or equilibrium concepts. I note here some interesting possibilities. The Nash equilibrium can be weakened, following various authors, to the *uncovered set* (Miller 1980), the *yolk* (Ferejohn et al. 1984), or the *heart* (Schofield 1993). All these solution concepts were motivated by a desire to return a non-empty equilibrium set for all spatial two-party games. Split and coalitional stability can be supplemented by a requirement that no profitable *defections* are possible, i.e., that no player or group of players can increase the initial parties' total payoff by leaving one party and joining another party. While splits and coalitions are clearly the most important identity changes, major defections may occur from time to time. Finally, the institutional equilibrium can be made more inclusive by requiring that no member of a winning coalition strictly lose from the change of an electoral law and that at least one member strictly gains. It is clear that variants of the general equilibrium defined above can be produced more or less mechanically by combining variants of partial equilibria.

3.3 Subgames and partial equilibria

Assume that for some reason one is exclusively interested in evaluating the prospects of various electoral reforms and wants to hold other aspects of the party system constant. Thus, one needs to fix some coalitional structure and spatial position of parties and then evaluate the payoffs for all electoral laws. Such an operation leads to the concept of a *subgame* and can also be used to derive games that focus only on specific aspects of party activity.

A subgame represents a particular aspect of the larger game. For instance, in a spatial subgame, parties are not allowed to change their identities or the electoral laws, but they can change their platforms. In such a case, the general model is "folded" to the underlying spatial model. Note however that such an operation is not independent of the parameters that are held constant since parties' payoffs in a spatial model may differ under various electoral laws and certainly will differ under different coalitional structures. It is clear that every configuration of electoral law and coalitional structure may produce a specific spatial subgame. Thus, it is necessary to index such a subgame with the electoral law and coalitional structure that are being held fixed.

Formally, a subgame can be specified in the most general way by a subset of all states that parties must stay within. It is possible interpret this restriction in a natural way: states within the subset are precisely those states that can be reached in this particular subgame. One can also assume that the set of players and their payoffs remain unchanged and that the e-function is restricted to the set of permissible states. This leads to the following definition of a subgame of an e-game.

Definition 3 *An e-game* $\Gamma_1 = \langle N_1, Q_1, \varepsilon_1, \nu_1\rangle$ *is a subgame of an e-game* $\Gamma_2 = \langle N_2, Q_2, \varepsilon_2, \nu_2\rangle$ *if (i)* $N_1 = N_2$*; (ii)* $Q_1 \subset Q_2$, $Q_1 \neq \emptyset$*; and (iii) for all* $q \in Q_1$ *and all* $K \subset N_1$, $K \neq \emptyset$, $\varepsilon_1(K, q) = \varepsilon_2(K, q) \cap Q_1$ *and* $\nu_1(K, q) = \nu_2(K, q)$.

In other words, a subgame corresponds to exactly one non-empty subset Q_1 of Q_2 and the e-function is confined to states from Q_1. Such a subgame must be played within the boundaries of Q_1, where Q_1 represents the restrictions imposed on parties' moves.

Every partition of Q_2 into k subsets will create exactly k subgames. In our model, partitions of Q_2 that result from fixing some state parameters are especially interesting. For instance, Γ can be partitioned into spatial subgames that correspond to every coalitional structure and every electoral law. Thus, we have $|\mathcal{I}|$ n-player spatial subgames (one subgame for every electoral law and with all players competing separately), $|\mathcal{I}| \times \frac{n(n-1)}{2}$ $(n-1)$-player spatial subgames (one subgame for every electoral law and every coalitional structure with exactly one two-player coalition), etc. In every such subgame, $\varepsilon(q)$ is restricted only to spatial moves. Similarly, every vector of platforms and every coalitional structure defines exactly one institutional game.

There are six kinds of subgames in the model that result from restricting possible modifications to one or two of the three basic aspects of a party system. Since the results are formulated for the case where party platforms are held constant, I will define formally only three of these subgames. Definition 4 introduces an *institutional* subgame.

Definition 4 *Let* $\Gamma = \langle N, Q, \varepsilon, \nu \rangle$ *be a game with parameters* S, $\mathcal{I}$, $\mathcal{W}$, f. *Let* $\Gamma_1 = \langle N, Q_1, \varepsilon_1, \nu_1 \rangle$ *be a subgame of* Γ *and let* $q = \langle \mathcal{C}^q, s_1^q, \ldots, s_{|\mathcal{C}^q|}^q, EL^q \rangle \in Q_1$. *Then,* Γ_1 *is an institutional subgame if* $Q_1 = \{p \in Q : \mathcal{C}^p = \mathcal{C}^q$ *and* $s_i^p = s_i^q$ *for* $i = 1, \ldots, |\mathcal{C}^p|\}$.

Comments: Fixing the coalitional structure and party platforms produces a family of institutional subgames. There is exactly one subgame for every coalitional structure/vector of platforms combination. In each such subgame, parties can only change electoral laws. Such a subgame could be used to model electoral reforms, such as the 2001 Polish reform.

The remaining two cases that include identity subgames and institutional-identity subgames must be handled differently. Since the total number of parties changes, a change in coalitional structure necessarily implies a change in the vector of platforms as well. Thus, party platforms cannot be fixed when the coalitional structure is changed. This property may bring a troublesome consequence. It may happen that two parties first coalesce and next split, thereby changing their spatial positions as well as their payoffs. This means that, after completing an identity cycle, party platforms may change.[7] Even worse, there might also be different payoffs. Clearly, this is not a desirable property. However, such pathologies are empirically unmotivated. Hereafter I will restrict my attention to subgames where payoffs are independent of cyclical transformations of identity or to possible interactions of such transformations with the changes in electoral laws.

Definition 5 *Let* $\Gamma = \langle N, Q, \varepsilon, \nu \rangle$ *be a game.* Γ *is called "regular" if for any sequence of states* $q_1, \ldots, q_m \in Q$ *such that* q_{i+1} *results from a split of a party or coalition of parties in* q_i *or from a change of the electoral law in* q_i, *for* $i = 1, \ldots, m-1$ *and* $\mathcal{C}^{q_1} = \mathcal{C}^{q_m}$, $EL^{q_1} = EL^{q_m}$, *it holds that* $\nu(P, q_1) = \nu(P, q_m)$ *for all* $P \in \mathcal{C}^{q_1}$.

Comment: Definition 5 says that if states change only through coalitions, splits, or modifications of electoral laws and that if a sequence of such identity and institutional changes eventually produces the initial coalitional structure and the same electoral law, then such a cyclical change does not affect the parties' payoffs in a regular game. A simple example of a regular game is when payoffs depend only on coalitional structures and electoral laws, and not on party platforms. This means that an institutional-identity and identity subgame can be defined as follows:

Definition 6 *Let* $\Gamma = \langle N, Q, \varepsilon, \nu \rangle$ *be a regular game with parameters* S, $\mathcal{I}$, $\mathcal{W}$, f, *let* $\Gamma_1 = \langle N, Q_1, \varepsilon_1, \nu_1 \rangle$ *be a subgame of* Γ, *and let* $q = \langle \mathcal{C}^q, s_1^q, \ldots, s_{|\mathcal{C}^q|}^q, EL^q \rangle \in Q_1$. *Then:*

[7] With a finite policy space some party platforms must change after a cycle of splits and coalitions. Consider any game with $|\mathcal{I}| = 1$, $|S| = 2$, and $|N| = 3$, and a simple merger-split cycle. For the three-party structure, the total number of states is $|S|^3 = 8$. When parties 1 and 2 merge, the total number of states for the new structure decreases to $|S|^2 = 4$. Next, when the coalition 12 dissolves, it can dissolve into at most four three-party states. Thus, none of the (at least four) remaining three-party states can result from a split of coalition 12. A coalition-split cycle starting from such a state will produce a state with different platforms.

(i) Γ_1 *is an institutional-identity subgame if* $Q_1 = \{p \in Q$: *there is a sequence of states in* Q $p = q_1, \ldots, q_m = q$ *such that* q_{i+1} *results from a split of a party or coalition of parties in* q_i, *or from the electoral law change in* q_i, *for* $i = 1, \ldots, m-1$;

(ii) Γ_1 *is an identity subgame if* $Q_1 = \{p \in Q : EL^p = EL^q$ *and there is a sequence of states in* Q $p = q_1, \ldots, q_m = q$ *such that* q_{i+1} *results from a split of a party or coalition of parties in* q_i *for* $i = 1, \ldots, m-1\}$.

Some of the partial equilibrium results of the next section are formulated for specific categories of subgames. Let Γ be a game at some state q. Γ is subgame-institutionally stable at q if the institutional subgame defined by q has an institutional equilibrium. Otherwise, Γ is institutionally cyclical at q. Similar terminology applies to the remaining two categories of subgames. By definition, any identity, institutional or other equilibrium in the identity, institutional, etc., subgame is a partial identity, institutional, etc., equilibrium in the entire game Γ.

Note that every specific subgame is well described by a subset of parameters describing the larger game. For instance, for an identity-institutional subgame one does not need S and f. I will often refer to a subgame as an "identity-institutional" or other game.

3.4 Farsightedness versus myopia

Thus far I have assumed that parties can make only one move at a time. However, more "farsighted" parties could anticipate that their move will produce some opportunities for them from a different activity or induce a reaction from other parties. The objective of this section is to show some intuition behind the problem of farsightedness and present an argument suggesting that parties have tremendous difficulties with collecting the information necessary for farsighted actions (on farsightedness see Chwe 1994). The discussion of the problem is informal and by no means exhaustive.

In the example considered below, parties cannot instantaneously gain by coalescing or reforming the electoral law, but they could gain by making both moves.

Example 3 Let Γ be an identity-institutional game parametrized with $N = \{P_1, P_2, P_3\}$, $\mathcal{I} = \{PR, FPP\}$, $\mathcal{W}(\{P_i, P_j\}) = \mathcal{W}(\{P_1, P_2, P_3\}) = 1$ for all $i, j \in \{1, 2, 3\}, i \neq j$.

In order to define the payoffs concisely, let's denote $q = \langle P_1 - P_2 - P_3, PR\rangle$, $p = \langle P_1 - P_2 - P_3, FPP\rangle$, $r = \langle P_1P_2 - P_3, PR\rangle$, and $s = \langle P_1P_2 - P_3, FPP\rangle$. Payoffs are as follows: $\nu^q(P_1) = \nu^q(P_2) = \nu^q(P_3) = 33\frac{1}{3}$; $\nu^p(P_1) = \nu^p(P_2) = 10$ and $\nu^p(P_3) = 80$; $\nu^r(P_1P_2) = 60$ and $\nu^r(P_3) = 40$; $\nu^s(P_1P_2) = 80$ and $\nu^s(P_3) = 20$. For coalitions P_2P_3 or P_1P_3, the payoffs are obtained by permuting the names of parties.

Note: in state q, parties P_1 and P_2 have no incentive to change the electoral law (i.e., to go from q to p) since only P_3 benefits from such a change. Moreover, all two-party coalitions lose some seats (state r and its permutations). Thus, state q is an identity-institutional equilibrium. However, parties P_1 and P_2 could both coalesce and change the electoral law and benefit from the resulting payoff in state s.

When one allows parties to evaluate consequences of two or more moves, then the set of states available from a given initial state expands. Thus, a modification of the equilibrium concept could require that a coalition of parties have no incentives to change the state via making any number of such intermediate moves. An obvious consequence would be that the set of equilibrium states in a game would shrink or, for some games, remain unchanged. Another modification could result from letting the parties compare the current and anticipated state, i.e., taking into account not only immediate but rather all anticipated consequences of a move.

The extent of the parties' farsightedness is an empirical question. The fundamental constraint on farsightedness comes from the tremendous methodological problems associated with estimating potential seat shares. Although estimating the seat-share consequences of an electoral reform or altering a coalitional structure is not a simple task, relevant survey-based simulation methodologies are available. In fact, parties in Poland increasingly used simulations to make informed decisions about changes to electoral laws or coalitional structures. However, anticipating the consequences of both kinds of moves is a much more complex a task. No reliable methodology exists for this.

Moreover, anecdotal evidence suggests that parties usually act myopically and are sometimes hurt by unanticipated after-effects of their actions. In the 2001 electoral reform case, the consequences of the reform estimated with survey data for when the coalitional structure was held fixed were common knowledge. Among the four largest parties, three estimated gainers supported the reform and the estimated loser opposed it. However, the new electoral law, which was more friendly towards smaller parties, encouraged splits and entries. In fact, the three parties supporting reform actually lost some votes. In the case of the AWS, the apparent secondary effects of electoral reform were especially strong and lethal. Most of its electorate was captured by its two former small factions that entered the elections as separate parties. As a result, the AWS obtained about a fourth of the total support for all three parties and did not pass the minimal threshold for seat distribution.

While the problems associated with farsightedness are interesting and deserve further study, this paper does not intend to provide "a theory of everything." Thus, I will examine formally only the equilibrium concept defined in Section 3.2, hoping that it balances relative simplicity with solid descriptive power.

3.5 Possible modifications of the model: summary

The model can be modified in a number of ways. This paper intends to provide a concise description of the main ideas: (1) that it is possible to think about a party system as a set of states characterized by various parameters and (2) that incentives for changing a state may be represented in many fashions. Many of the possible modifications to the model were already discussed earlier in the text. Here is a short catalog of the most natural modifications:

(a) Farsightedness: Explicit consideration of the extent to which players include further consequences of their actions in their calculations;
(b) Modifications of equilibria without farsightedness: Application of partial stability concepts such as more or less restrictive versions of split-coalition equilibrium or the uncovered set;

(c) Other changes to coalitional structures: Accounting for defections (e.g., when a faction of a party splits and immediately joins another party), withdrawal of a party from the electoral race and possibly other changes to the coalitional structure;
(d) Other institutions: Parametrization of states with other components of political systems, such as the upper house, constitutions changed by a qualified majority or the timing of elections;
(e) Other interpretations of payoffs: Accounting for policy-motivated players with non-constant sum payoffs;
(f) Non-transferrable payoffs from coalescing: Accounting for presidential elections.

4 Existence and generic properties of equilibria

The present section investigates the existence of institutional and coalition-split equilibria and their robustness with respect to changes in payoffs. First, examples of games with top cycles are presented. Since the number of electoral laws and coalitional structures is finite, the existence of a top cycle in a game is equivalent to the non-existence of an equilibrium. However, the second set of results shows that equilibria do exist under rather mild conditions, and that the non-existence of an equilibrium seems to be a rare event. The final result says that if an equilibrium exists, then generically (or "almost always") the equilibrium will not be de-stabilized by a small perturbation in payoffs. Thus, institutional and identity equilibria are robust with respect to small "trembles" in payoffs, perception, or party platforms, given that payoffs are continuous functions of the platforms.

We assume below that $\mathcal{W}$ represents a simple majority rule, i.e., $\mathcal{W}(P) = 1$ if and only if a party or a coalition of parties P command more than $\pi \geq \frac{1}{2}$ of the (lower house) vote. We note here that this restriction can be relaxed in some cases, or the results can be strengthened to account for a wider class of simple games.

The central concept for the next sections is a game form. A game form $\gamma = \langle N, S, \mathcal{I}, \mathcal{W}, f \rangle$ lists players and parameters for describing states and the e-function (see Example 2); the only parameter distinguishing game forms from games is the lack of payoffs. Thus, one can think of a game form as the concise description of a given strategic environment for an entire class of games that would result from adding all possible payoff functions. Finally, note that since only identity-institutional subgames of the entire game are examined, the two spatial characteristics of the game, issue space S and function f, play no role in the results. The spatial aspects of the game are not examined since they are obviously prone to the instability predicted by Plott (1967).

4.1 Existence of top cycles

The results of this section examine the existence of institutional or identity top cycles.[8]

Let's fix a game form $\gamma = \langle N, S, \mathcal{I}, \mathcal{W}, f \rangle$ and $q \in Q$.

[8] The proofs are in the appendix.

Proposition 1 *If* $|\mathcal{I}| \geq 3$ *and* $|C^q| \geqslant 3$*, then there exists* $\mathcal{W}'$ *and* ν *such that the game* $\langle N, S, \mathcal{I}, \mathcal{W}', f, \nu\rangle$ *is subgame-institutionally cyclical at* q*.*

Proposition 2 *If* $n \geqslant 4$*, then there exists* ν *such that the game* $\langle N, S, \mathcal{I}, \mathcal{W}, f, \nu\rangle$ *is subgame-identity cyclical at* q*.*

Comment: Proposition 1 shows that, if there are at least three parties in $\mathcal{C}^q$ and at least three electoral laws in $\mathcal{I}$, then for any game form it is possible to construct a game, possibly with changing the voting power of parties, that has no institutional equilibrium at any state. Essentially, the top institutional cycle in the proof represents the voting paradox. In Proposition 2, an example of a subgame with no identity equilibrium is constructed when N includes at least four players. The latter result generalizes Proposition 2 in Kaminski (2001) and simplifies the construction of the top cycle. Both results mean that there exists a possibility–at least theoretically–that in some electoral games there exist incentives for perpetual splitting and coalescing, or never-ending electoral heresthetics.

4.2 Existence of equilibria

While there is some possibility that a top institutional or identity cycle exists at a given state, such cycles seem to be rather unlikely empirically. The next two results examine conditions that guarantee the existence of an equilibrium. I start with two sufficient conditions for the existence of an institutional equilibrium. Let's fix a game $\Gamma = \langle N, S, \mathcal{I}, \mathcal{W}, f, \nu\rangle$ and a state $q \in Q$.[9]

Proposition 3 *The following conditions are sufficient for the existence of a subgame-institutional equilibrium at* q*:*

(i) For some blocking coalition of parties $K = \cup_{k=1}^{m} P_{i_k}$*, where* $P_{i_k} \in \mathcal{C}^q$ *for* $k = 1, \ldots, m$*, there exists an electoral law that simultaneously maximizes the payoffs of all* P_{i_k}*;*
(ii) Parties from $\mathcal{C}^q$ *have at* q *single-peaked preferences over* $\mathcal{I}$*.*

Comment: Proposition 3 (i) guarantees the existence of an institutional equilibrium for party systems with a blocking party or a blocking coalition of parties sharing their top choice. Condition (ii) is often satisfied when parties are relatively small. In electoral law bargaining, the set of available alternatives is usually small. Parties typically evaluate electoral laws according to their "degree of proportionality." The smallest party, or parties, prefer more proportionality to less, whereas the largest party, or parties, prefer less proportionality to more. The ideal choice of medium-sized parties is somewhere between these extremes. The assumption of single-peakedness is well-justified in such environments.

Proposition 4 now examines the three sufficient conditions for the existence of a subgame identity equilibrium. For this result, I need the following definition:

[9] Single-peakedness is defined in the usual way, i.e., that it is possible to order the set of alternatives in such a way that each payoff function ν_i is 1-1 and has exactly one local maximum. Formally, let X be any non-empty set and $\nu_1, \ldots, \nu_n : X \to \mathbb{R}$ s.t. each ν_i is 1-1. The family $\{\nu_1, \ldots, \nu_n\}$ is *single-peaked* if there exists $\prec$, a strong ordering of X, such that for all $x_1, x_2, x_3 \in X$, and all ν_i, if $x_1 \prec x_2 \prec x_3$, then it is not true that $\nu_i(x_1) > \nu_i(x_2)$ and $\nu_i(x_3) > \nu_i(x_2)$.

Definition 7 *A regular game Γ is bi-superadditive at q if N can be partitioned into subsets P_1, P_2 such that for any states p, r in the identity subgame defined by q such that $r = p^{-P}$ and $P \subset P_1$ or $P \subset P_2$, $\nu^r(P) \geq \nu^p(P)$.*

Assume that a coalition P forms that consists exclusively of players from one of the two subsets P_1 or P_2. Bi-superadditivity simply means that no matter what the other details of the coalitional structure, the payoff of P does not decrease, comparing with the sum of payoffs of P_1 and P_2. This condition is empirically plausible in highly polarized party systems, with a single meaningful left-right dimension, and with an electoral law promoting larger players. Sets P_1 and P_2 can be interpreted as sets of "rightist" and "leftist" parties divided into two clusters.

Proposition 4 *Let Γ be a regular game. Then:*

(i) Γ is always subgame-coalitionally stable and subgame-split stable at q;
(ii) If $n = 3$, then Γ is subgame-identity stable at q;
(iii) If Γ is bi-superadditive at q, then it is subgame-identity stable at q.

Comment: Proposition 4 (i) guarantees the existence of (possibly separate) coalitional equilibrium and split equilibrium at any state. Part (ii), a more general version of Proposition 1 in Kaminski (2001), guarantees that all three-player games have a joint coalitional-split equilibrium at any state. Finally, bi-superadditivity at q is shown to guarantee the existence of an identity equilibrium at q. Note that superadditivity and subadditivity, defined in the usual way, are also sufficient for the existence of identity equilibrium but such conditions are less plausible empirically, in comparison to bi-superadditivity.

A more detailed characterization of sufficient conditions for joint identity-institutional equilibria is an open question. Note that in some cases such conditions can be obtained by combining sufficient conditions from Propositions 3 and 4. A related problem is the characterization of equilibria under various assumptions regarding players' farsightedness. Finally, the existence of a regular identity-institutional subgame with both institutional and identity equilibrium, but with no joint identity-institutional equilibrium is another open question.

4.3 Generic properties of equilibria

The implications of Propositions 3 and 4 are similar: sufficient conditions for institutional and identity equilibria are less restrictive than the famous Plott (1967) pairing conditions for the spatial two-party equilibrium. Indeed, the Plott conditions essentially imply the generic non-existence of a spatial equilibrium for two parties in the usual spatial setting. Such a claim is not true for identity or institutional equilibria. Proposition 5 below formalizes this statement.

First, I need a few clarifying definitions. Let M be any m-dimensional subset of a Cartesian space. A subset F of M is called *generic* if it is open and dense in M. If F is generic in M, then for any continuous probability distribution defined over M, the probability of drawing an element of M that belongs to F is 1. (One can think of F as including "almost all" elements of M.)

Let's fix a game form $\gamma = \langle N, S, \mathcal{I}, \mathcal{W}, f\rangle$ and a state q, and let's denote $m = |\mathcal{C}^q|$ and $k = |\mathcal{I}|$. We can represent every regular identity subgame at q

by specifying its payoffs, i.e., as a point in the Cartesian space of an appropriate dimension. Denote the set of all such games as $M^{CS}_{\gamma,q}$. Similarly, the set of all institutional games at q is denoted as $M^{I}_{\gamma,q}$ and the set of all identity-institutional games at q is denoted as $M^{CSI}_{\gamma,q}$. (See the Appendix for further details of the construction.)

Now, let $E^{CS}_{\gamma,q} \subset M^{CS}_{\gamma,q}$ denote the set of all regular stable identity subgames. Furthermore, let $\bar{E}^{CS}_{\gamma,q} \subset E^{CS}_{\gamma,q}$ be a subset of subgames such that for every $G \in \bar{E}^{CS}_{\gamma,q}$ there exists an open neighborhood of G in $M^{CS}_{\gamma,q}$ that consists exclusively of points from $E^{CS}_{\gamma,q}$. In words, $\bar{E}^{CS}_{\gamma,q}$ consists of all regular stable identity subgames such that a sufficiently small change of payoffs does not de-stabilize the game.

Similarly, $E^{I}_{\gamma,q} \subset M^{I}_{\gamma,q}$ is the set of all stable institutional subgames and $\bar{E}^{I}_{\gamma,q} \subset E^{I}_{\gamma,q}$ consists of all subgames such that for every $G \in \bar{E}^{I}_{\gamma,q}$ there exists an open neighborhood of G in $M^{I}_{\gamma,q}$ that consists exclusively of points from $E^{I}_{\gamma,q}$.

Finally, let $M^{CSI}_{\gamma,q}$, $E^{CSI}_{\gamma,q}$, and $\bar{E}^{CSI}_{\gamma,q}$ represent all regular identity-institutional subgames, all such subgames that are identity-institutionally stable, and the subset of $E^{CSI}_{\gamma,q}$ with equilibria robust against small changes of payoffs, respectively.

Proposition 5 *For all γ and $q \in Q$,*

(i) $M^{CS}_{\gamma,q} - E^{CS}_{\gamma,q}$ *is not generic in* $M^{CS}_{\gamma,q}$*;*
(ii) $M^{I}_{\gamma,q} - E^{I}_{\gamma,q}$ *is not generic in* $M^{I}_{\gamma,q}$*;*
(iii) $M^{CSI}_{\gamma,q} - E^{CSI}_{\gamma,q}$ *is not generic in* $M^{CSI}_{\gamma,q}$*.*

The main point to be taken from Proposition 5 is that, at least in some cases, identity and institutional equilibria are robust against small changes in payoffs. Remember that in a typical multi-dimensional spatial equilibrium, for any $\varepsilon > 0$, however small, one can change voter positions by less than ε and destabilize the equilibrium. In a one-dimensional Black model, a small move of the median voter(s) does not destabilize the equilibrium bit it changes its location. In contrast, sufficiently small trembles do not de-stabilize some institutional and identity equilibria. In fact, the next result shows that this property holds generically in the set of all games with equilibria.

Theorem 1 *[Generic tremble-stability of institutional and identity equilibria] For all γ and $q \in Q$,*

(i) $\bar{E}^{CS}_{\gamma,q}$ *is generic in* $E^{CS}_{\gamma,q}$*;*
(ii) $\bar{E}^{I}_{\gamma,q}$ *is generic in* $E^{I}_{\gamma,q}$*;*
(iii) $\bar{E}^{CSI}_{\gamma,q}$ *is generic in* $E^{CSI}_{\gamma,q}$*.*

Theorem 1 says that stability with respect to identities and/or institutions is generically robust against sufficiently small changes in payoffs. This result can be interpreted in the context of transitional versus mature party systems: whereas the former experience large external shocks to voter preferences (the payoff function) that can destabilize equilibria, the latter experience much smaller shocks that may fit within the "limits of tolerance" of a given equilibrium. Since disruptions provide parties with incentives to adjust their identities and change electoral institutions, one can expect less frequent identity and institutional changes in mature democracies.

An immediate consequence of Theorem 1 is the following Corollary:

Corollary 1 *Let S^E be a Euclidean manifold. For any game form $\gamma = \langle N, S^E, \mathcal{I}, \mathcal{W}, f \rangle$ and $q \in Q$, if $\Gamma = \langle N, S^E, \mathcal{I}, \mathcal{W}, f, v \rangle$ is a regular game that is identity-institutionally stable at q with the payoff function $v(., s, .)$ continuous at q, then the set of all identity-institutional subgames at q that are stable in some spatial neighborhood of q is equal to $\bar{E}^{CSI}_{\gamma,q}$ and it is generic in $E^{CSI}_{\gamma,q}$.*

The Corollary says that, if some state q is identity and institutionally stable and if the payoff function at this state is continuous with respect to platform changes by parties, then the equilibrium generically has some tolerance for platform trembles, i.e., sufficiently small platform changes do not disturb the equilibrium. Thus, once identity and institutional stability is reached in a party system, parties have some degree of freedom in adjusting their platforms without upsetting identity and institutional stability. The fact that platform trembles in mature democracies are of a smaller magnitude than those in transitional ones provides another reason to expect that identity and institutional equilibria will last longer. This is because, ceteris paribus, parties in mature democracies will face lower incentives to change their identities and electoral institutions.

4.4 Interpretation of results: summary

The results of this paper provide a preliminary catalog of the model's basic properties. The main focus has been to explain the high frequency of identity changes and electoral reforms that are observed under conditions of high voter volatility.

High voter volatility tends to be a phenomenon specific to transitional democracies.[10] For example, spectacular changes in support often occur in the final weeks of electoral campaigns and attract the attention of the media in these countries. In Russia, the "phenomenal rise of Vladimir Putin from obscurity to the position of the unquestioned frontrunner in the presidential contest [mirrored] Yeltsin's rise in opinion polls during the 1996 campaign" (Moser, 2001, p 155). High voter volatility may be translated into even higher seat share volatility. For the 1997 and 2001 parliamentary elections in Poland, Pedersen's seat volatility index was equal to an astronomical number of 53.3. Such a value in a two party system corresponds to the case when, after winning a close election in period 1, the former winner loses all seats in period 2 to the competitor![11]

Under high voter volatility, payoffs change abruptly. If shocks in payoffs are the results of changing voter preferences, payoff trembles are exogenous to the present model. It is possible to think about such changes as switching from one game to another: The larger the shock, the less similar the new game is to the original one. Alternatively, the source of a shock disturbing the gains from coalescing or electoral reform may be endogenous and related to platform adjustments.

[10] See Geddes (1996, p 19), Grofman et al. (1999, p 241), and Moraski and Loewenberg (1999, p 18). Remington and Smith (1996, p 483) note that the "unsettled partisan and policy preferences of the Russian electorate make it difficult for political strategists to adjust electoral and institutional arrangements to their interests."

[11] The index is given by the formula $\frac{1}{2}\sum_{i=1}^{n}|v_i^1 - v_i^2|$, where the summation is over all n parties winning seats in at least one of two consecutive elections and v_i^k is party i's seat share in election k, for $k = 1, 2$. The index assumes values between 0 (no change in the distribution of seats) and 100 (no party winning seats at period 1 wins seats at period 2).

Consider an equilibrium in a party system experiencing an exogenous or endogenous payoff tremble in the light of this paper's findings. Although the results indicate that the game might involve institutional and/or identity cycles (Propositions 1 and 2), they also suggest that institutional and identity equilibria exist under relatively unrestrictive assumptions (Propositions 3, 4, and 5). Thus, it is likely that the game does have an institutional and/or identity equilibrium.

Assuming that such equilibria exist, assume further that the party system, possibly after various adjustments, has reached a state of institutional and identity equilibrium. Suppose that an endogenous or exogenous shock now disturbs the equilibrium payoffs and the opportunity payoffs related to splitting, coalescing, or implementing electoral reform. What happens with the corresponding partial equilibria depends on the magnitude of the shock. Since institutional and identity equilibria are generically robust against small trembles in payoffs (Theorem 1 and Corollary 1), a small shock–such as those in a mature party system–is likely not to upset the existing equilibrium. Only occasionally will the combination of various external shocks and platform adjustments create incentives for splits, coalescing, or electoral reform in a mature democracy. In contrast, the expected magnitude of a shock in a transitional democracy is greater. As a result, it is more likely that the equilibrium will be upset and that incentives for adjusting party identities or electoral laws will arise.[12]

Thus, the specific property of institutional and identity equilibria that can be called a *generic robustness against small trembles in payoffs* facilitates relatively greater institutional and identity stability in mature versus transitional democracies. This is simply because mature systems are less likely to experience large trembles.

Note that a typical spatial model predicts no difference between party systems with greater versus smaller shocks for platform-based politics. In such a model, equilibria are generically non-existent and incentives for adjusting platforms are generically present regardless of the magnitude of shocks in the party system. This prediction seems to fit empirical observations.

5 Conclusion

In this paper, I attempted to provide a unified approach to platform, identity, and institutional aspects of party politics and to set a clear direction for possible modifications. Formally, this unification is possible thanks to the flexibility of e-games. Such games are not confined to the Cartesian-product based structure of player interactions and they allow for various kinds of "changes" or "moves." In the model presented in this paper, parties can modify the state of the party system by choosing their platforms, by splitting/coalescing or, if they form a winning coalition, by selecting a different electoral law. The model can easily be adjusted to incorporate additional institutional parameters of the party system or more comprehensive identity changes.

[12] Another factor facilitating greater coalitional fluidity in new democracies was suggested by a referee. Since voters do not have firm expectations about the meaning of party labels, the empirical e-games in such systems are more "forgiving." In contrast, in mature democracies voters punish parties for mergers and are reluctant to vote for resulting new parties. The associated e-games have more equilibria and such equilibria are more robust against preference changes.

The formalization of non-Cartesian product based interactions allows for a general extension of the classical spatial framework, letting strategic and cooperative components co-exist in a single framework. One might claim that the presence of an entangled cooperative/non-cooperative decision-making knot is the most fundamental property of interactions that are essentially political rather than economic.[13]

The results show that institutional and identity-related equilibria are easier to reach and preserve than equilibria resulting from issue politics. The general conditions for the existence of a two-party spatial equilibrium were shown to be very restrictive (Plott 1967; Davis et al. 1972; Kramer 1973; Cox 1987b; McKelvey and Schofield 1987). While the case of three or more parties was not researched equally thoroughly, spatial equilibria in many multiparty models do not exist at all (Eaton and Lipsey 1975; Shaked 1975) or do not exist generically.[14] This conclusion was somewhat softened by a crop of contributions showing (1) that equilibrium concepts such as the uncovered set, the yolk, or the heart, founded on different ideas of stability, generate non-empty solution sets for each game (Miller 1980; Ferejohn et al. 1984; Schofield 1993); (2) that "structure-induced" equilibrium may be facilitated by various institutional constraints (Shepsle and Weingast 1981; Shepsle 1979); (3) that probabilistic models of voting generate equilibria more easily than deterministic ones (Coughlin and Nitzan 1981); or (4) that equilibria may emerge under various voting systems and party objectives (e.g., Cox 1987a, 1990). Nevertheless, equilibria in typical spatial models are fragile. This theoretically established fragility is supported by the constant modifications of party platforms in all political campaigns and elections. Such activity is in striking contrast with the relative stability of institutional and identity-related aspects of mature party systems. However, parties modify electoral laws, coalesce, or split as often as they announce new platforms in transitional democracies. This activity may be explained by larger shocks that result from changes in preferences and party platforms in such democracies. Identity and institutional equilibria are generically robust to sufficiently small shocks but may be de-stabilized by larger shocks.

A Appendix: Proofs

Proof of Proposition 1 For any q such that $|C^q| \geqslant 3$, we will construct a top cycle at q. Let $P_1, P_2, P_3 \in \mathcal{C}^q$ and $EL_1, EL_2, EL_3 \in \mathcal{I}$. For all $K = \cup_{k=1}^{m} P_{i_k}$, where $P_{i_k} \in \mathcal{C}^q$ for $k = 1, \ldots, m$, let $\mathcal{W}'(K) = 1$ iff $P_1 \cup P_2 \subset K$ or $P_2 \cup P_3 \subset K$ or $P_3 \cup P_1 \subset K$. Otherwise, $\mathcal{W}'(K) = -1$. Define $\nu(P_1, q^{-EL_1}) = \nu(P_2, q^{-EL_2}) = \nu(P_3, q^{-EL_3}) = 70$; $\nu(P_1, q^{-EL_2}) = \nu(P_2, q^{-EL_3}) = \nu(P_3, q^{-EL_1})$

13 The opposite point of view, best expressed by the so-called Nash program, states that it is desirable to translate every cooperative game into a non-cooperative game with a good match between players, outcomes, and equilibria. While economics provides support for such a program, some descriptive cooperative models in politics (e.g., of coalition formation) are critically more succint than their non-cooperative counterparts. For cooperative models with an intentionally normative interpretation, like cost allocation or bankruptcy models, the "Nash transformation" rarely makes any sense. See Schofield and Sened (1997) for similar arguments in favor of a cooperative approach.

14 Cox (1990) conjectures that, for the "canonical" spatial model and any continuous distribution of voters "If…there exists a multicandidate equilibrium, then an arbitrarily small change in the voter distribution can always be made such that no multicandidate equilibrium exists for the changed distribution."

$= 30$; $\nu(P_1, q^{-EL_3}) = \nu(P_2, q^{-EL_1}) = \nu(P_3, q^{-EL_2}) = 0$; and $\nu(P_i, q^{-EL_j}) = 0$ for $j = 1, 2, 3$ if $i > 3$ in case of $|\mathcal{C}^q| > 3$; $\nu(P_i, q^{-EL_j}) = \nu(P_i, q^{-EL_1})$ for all i if $j > 3$ in case of $|\mathcal{I}| > 3$.

Denote by $EL_i \succ^q_K EL_j$ the binary relation of strict preferences over $\mathcal{I}$ representing the fact that all parties in the coalition K derive strictly higher payoffs from EL_i than EL_j while holding the coalitional structure and platforms constant at q. Let $\sim^q_K$ be the indifference relation associated with $\succ^q_K$. By construction, we have that $EL_1 \succ^q_{P_1 \cup P_3} EL_2 \succ^q_{P_2 \cup P_1} EL_3 \succ^q_{P_2 \cup P_3} EL_1$. In addition, $EL_3 \succ^q_{P_2 \cup P_3} EL_j$ for all $j > 3$. This means that for any electoral law $EL_j \in \mathcal{I}$, there is at least one winning coalition such that all its members strictly prefer EL_i to EL_j for $i = 1, 2,$ or 3. Thus, Γ is institutionally cyclical at q. □

Proof of Proposition 2 Since $n \geqslant 4$, $\{1, 2, 3, 4\} \subset N$. The game's payoffs will depend only on coalitional structures, i.e., the payoffs will be independent of S and $\mathcal{I}$. Set Q is partitioned into five subsets with different coalitional patterns among players 1, 2, 3, and 4. Notation $ij(q)$ means that players i and j are in the same coalition in state q and $i - j(q)$ means that i and j are in separate coalitions in q; $K(ij)$ denotes the coalition including i and j, etc.

(1) $123 - 4(q)$ or $134 - 2(q)$ or $124 - 3(q)$ or $1 - 234(q)$ or $1234(q)$, i.e., at least three players from $\{1, 2, 3, 4\}$ are in the same coalition K. Then we define $\nu^q(K) = 0$ and $\nu^q(L) = \frac{100|L|}{|N|-|K|}$ for all $L \in \mathcal{C}^q$, $L \neq K$. Note that $|N| - |K| > 0$ since $\mathcal{C}^q$ include at least two structures.

In the remaining cases, the payoff of a coalition is the total weight of its members, i.e., $\nu^q(K) = \sum_{i \in K} w(i)$, where (i) for $i \in N - \{1, 2, 3, 4\}$, $w(i) = \frac{100}{|N|}$ and (ii) for $i \in \{1, 2, 3, 4\}$ we have:

(2) $1i - j - k(q)$ and (3) $1 - i - jk(q)$: $w(1) = w(i) = \frac{101}{|N|}$, $w(j) = w(k) = \frac{99}{|N|}$;

(4) $1i - jk(q)$ and (5) $1 - i - j - k(q)$: $w(1) = w(i) = w(j) = w(k) = \frac{100}{|N|}$;

where $i, j, k \in \{2, 3, 4\}$ denote different players, i.e., $i \neq j \neq k \neq i$. Obviously, for all q, $\nu^q(K) \geqslant 0$ for all $K \in \mathcal{C}^q$ and $\sum_{K \in \mathcal{C}^q} \nu^q(K) = 100$.

Every state is dominated by a state in a different subset via split or a coalition. The pattern of cycles is as follows: (1): K splits into single players which gives (5). In other cases, payoff changes following splits or coalitions depend only on the changes in weights among 1, 2, 3, and 4. In case (2), $K(j)$ and $K(k)$ coalesce $\to$ (4); then, $K(1i)$ splits $\to$ (3); then, $K(jk)$ splits $\to$ (5); then, $K(1)$ and $K(i)$ coalesce $\to$ (2). □

Proof of Proposition 3 Ad. (i): Let $K = \cup_{k=1}^m P_{i_k}$, where all $P_{i_k} \in \mathcal{C}^q$, be a blocking coalition and let EL^* simultaneously maximize the payoff of all members of K at q, i.e., $EL^* \in ArgMax_{EL \in \mathcal{I}} \nu(P_{i_k}, q^{-EL})$ for $k = 1, \ldots, m$. Let L be any winning coalition of parties from $\mathcal{C}^q$. Since K has at least blocking power, at least one member of K, say P_{i_1}, must be a member of L. Otherwise $K \subset N - L$ and since $N - L$ is losing, K must be losing as well, contrary to our assumption. Consider the state $p = q^{-EL^*}$. Since $EL^* \in ArgMax_{EL \in \mathcal{I}} \nu(P_{i_1}, q^{-EL})$, then for all $EL \in \mathcal{I}$, $\nu^{q^{-EL}}(P_{i_1}) \leqslant \nu^q(P_{i_1})$, i.e., at least one player in L has no incentive to change the electoral law when the state is p. Thus, p is an institutional equilibrium.

Ad. (ii): Let EL be a median electoral law with respect to the ordering that facilitates single-peakedness and the parties' voting power and let $p = q^{-EL}$. Then, for any alternative electoral law EL^*, the parties that strictly prefer EL to EL^* have at least half of the voting power. This means that there is no winning coalition such that for every member of this coalition P_i, $\nu^{q-EL}(P_i) \leqslant \nu^q(P_i)$. Thus, p is an institutional equilibrium. □

Proof of Proposition 4 Ad (i): Any structure that consists of two parties only is coalitionally stable while any structure that consists of single players is split stable.

Ad (ii): See Kaminski (2001), Proposition 1.

Ad (iii): Consider a state r in the identity subgame of q such that $\mathcal{C}^r = \{P_1, P_2\}$. Since $\mathcal{C}^r$ includes only two coalitions, r is coalitionally stable. Assume that r is split unstable, e.g., that for some partition $R_1, \ldots R_k$ of P_1, we have that $r = p^{-P_1}$ and $\nu^p(P_1) > \nu^r(P_1)$. But since all R_i are subsets of P_1, bi-superadditivity implies that $\nu^p(P_1) \leqq \nu^r(P_1)$, a contradiction. □

Identity and institutional subgames as subsets of Cartesian space: Let's fix a game form $\gamma =< N, S, \mathcal{I}, \mathcal{W}, f >$ and $q \in Q$. Let $|\mathcal{I}| = k$ and $|\mathcal{C}| = m$. In order to represent a regular identity subgame at q as a point in a Cartesian space, assign to each coalition under each possible coalitional structure exactly one dimension. For $n = 3$, we have three two-party structures (six dimensions) and one three-party structure (three dimensions) and the relevant Cartesian space is $\mathbb{R}^9$. In general, for n players, denote the dimension of the corresponding space by $\dim(n)$ and the number of different coalitional structures by $coal(n)$. It is straightforward that $\dim(n)$ and $coal(n)$ do not depend on γ and q.

Since, by assumption, payoffs are non-negative and the sum of payoffs of all coalitions from a given structure is 100, the set of points in $\mathbb{R}^{\dim(n)}$ that represents all regular identity subgames is the Cartesian product of simplices of dimension $\dim(n) - coal(n)$. Denote this set by $M^{CS}_{\gamma,q}$. For $n = 3$, $M^{CS}_{\gamma,q} = 100 \times \Delta^2_1 \times \Delta^2_2 \times \Delta^2_3 \times \Delta^3_1$ is of dimension 5, where the first three simplices Δ^2_i encode the payoffs for three two-party structures and Δ^3_1 encodes the payoffs for the unique three-party structure. Also, for notational convenience, index parties corresponding to different dimensions by $K_i, i = 1, \ldots, \dim(n)$. This indexing identifies each party in each structure uniquely and allows us to skip the state in the payoff function. For any two games $G, G' \in M^{CS}_{\gamma,q}$, the metrics is introduced as follows:

$$\rho_{CS}(G, G') = \sum_{i=1}^{\dim(n)} |\nu_G(K_i) - \nu_{G'}(K_i)|. \tag{1}$$

Distance ρ_{CS} is more convenient to handle than the usual distance in $\mathbb{R}^n$. Note that the families of open sets are identical in both cases.

Similarly, the set of all institutional subgames at q can be interpreted as the product of k m-dimensional simplices and a subset of $\mathbb{R}^{k\times m}$. Denote this set by $M^I_{\gamma,q}$. The dimension of $M^I_{\gamma,q}$ is equal to $k \times m - k$, where every dimension represents a payoff of a specific party under a specific electoral law. Index parties corresponding to different dimensions by $K_i, i = 1, \ldots, k \times m$. For any two games

G, $G' \in M^{I}_{\gamma,q}$, the metrics is introduced as follows:

$$\rho_I(G, G') = \sum_{i=1}^{k\times m} |\nu_G(K_i) - \nu_{G'}(K_i)|. \tag{2}$$

Finally, the set $M^{CSI}_{\gamma,q}$, the set of all institutional-identity subgames at q, can be represented as taking k sets $M^{CS}_{\gamma,q}$, one for each electoral law. Thus, $M^{CSI}_{\gamma,q} \subset \mathbb{R}^{k\times \dim(n)}$ and it is of dimension $k \times (\dim(n) - coal(n))$. For any two games G, $G' \in M^{CSI}_{\gamma,q}$, the metrics is introduced in a similar way as in the preceding cases:

$$\rho_{CSI}(G, G') = \sum_{i=1}^{k\times \dim(n)} |\nu_G(K_i) - \nu_{G'}(K_i)|. \tag{3}$$

In all three cases, the δ-neighborhood of a game G is defined as the set of all G' such that $\rho_t(G, G') < \delta$, for $t = CS, I, CSI$, and it is denoted as $U(G, \delta)$.

Proposition 5 and Theorem 1 will be proved for the most general case (iii) of identity-institutional subgames. It is straightforward to notice that sets $M^{CS}_{\gamma,q}$ can be interpreted as a $M^{CSI}_{\gamma,q}$ for $|\mathcal{I}| = 1$, and $M^{I}_{\gamma,q}$ can be interpreted as a projection of $M^{CSI}_{\gamma,q}$ by fixing a coalitional structure with the same metrics; the sets of equilibria can be interpreted in a similar way. Thus, the proof for case (iii) given below can be used to prove cases (i) and (ii).

Definition 8 *For an identity-institutional subgame G, $q \in Q$ is a strong equilibrium if for all $K = \cup_{k=1}^{m} P_{i_k}$, where all $P_{i_k} \in \mathcal{C}^q$, (i) if $m \geq 2$, then $\nu^{q^{-K}}(K) < \nu^q(K)$; (ii) if $m = 1$, $L_1, \ldots, L_t$ is a partition of K and $t \geq 2$, then $\nu^{q^{-L_1,\ldots,L_k}}(K) < \nu^q(K)$; (iii) if $\mathcal{W}(K) = 1$, then for every $EL \in \mathcal{I}$, $EL \neq EL^q$ there is some k such that $\nu^{q^{-EL}}(P_{i_k}) < \nu^q(P_{i_k})$. In such a case we call G strongly stable.*

An equilibrium is strong if every proper coalition or split produces strict losses, and if for every winning coalition K, every proper change in the electoral law produces strict losses for at least one member of K.

Lemma 1 *If G is strongly stable, then $G \in \bar{E}^{CSI}_{\gamma,q}$.*

Proof Let $G^* \in M^{CSI}_{\gamma,q}$ and $q \in Q$ be a strong equilibrium. Denote by δ^q the minimum of all numbers $\nu^q(K) - \nu^{q^{-K}}(K)$, $\nu^q(K) - \nu^{q^{-L_1,\ldots,L_k}}(K)$, $\nu^q(P_{i_0}) - \nu^{q^{-EL}}(P_{i_0})$, for all $K = \cup_{k=1}^{m} P_{i_k}$, for all $P_{i_k} \in \mathcal{C}^q$ such that $m \geqslant 2$, in the case of coalescing; $m = 1$ and $t \geqslant 2$ for splitting, and if $\mathcal{W}(K) = 1$, then for every $EL \in \mathcal{I}$, $EL \neq EL^q$, $P_{i_0} \in ArgMax_{P_i \in K}[\nu^q(P_{i_0}) - \nu^{q^{-EL}}(P_{i_0})]$. The number δ^q denotes the minimal loss suffered by coalescing or splitting parties, or by the least fortunate member of a winning coalition changing the electoral law. The facts that q is strong, and that there are a finite number of cases considered, imply that

$$\delta^q > 0. \tag{4}$$

Consider $U(G^*, \delta)$ for $\delta = \delta^q$. We will show that q is an equilibrium in $U(G^*, \delta)$.

Assume that q is not an equilibrium for some $G \in U(G^*, \delta)$, i.e., that the following inequality holds:

$$\nu_G(K_1, q) - \nu_G(K_1, p) < 0, \tag{5}$$

for at least one of three cases:

(i) K_1 is a coalition of at least two parties from q and $p = q^{-K_1}$;
(ii) $K_1 \in \mathcal{C}^q$ and for some partition $L_1, \ldots, L_t$ of K_1, $p = q^{-L_1 \ldots L_t}$;
(iii) $K_1 \in K$, where $\mathcal{W}(K) = 1$, $p = q^{-EL}$ and K_1 is the member of K that suffers greatest loss in G^* when EL^q is changed to EL;

Since $G \in U(G^*, \delta)$, we know that

$$|\nu_{G^*}(K_1, q) - \nu_G(K_1, q)| + |\nu_{G^*}(K_1, p) - \nu_G(K_1, p)| \leqq \rho_{CSI}(G, G^*) < \delta. \tag{6}$$

Inequality (6) implies that

$$\nu_{G^*}(K_1, q) - \nu_G(K_1, q) - \nu_{G^*}(K_1, p) + \nu_G(K_1, p) < \delta \tag{7}$$

and, after adding to the respective sides of inequality (5):

$$\nu_{G^*}(K_1, q) - \nu_{G^*}(K_1, p) < \delta. \tag{8}$$

In all cases (i)–(iii), given the definition of δ, inequality (8) contradicts inequality (4). □

The next two Lemmas complete the main part of the proof.

Lemma 2 $\bar{E}^{CSI}_{\gamma,q}$ *is open in* $E^{CSI}_{\gamma,q}$.

Proof Let $G^* \in \bar{E}^{CSI}_{\gamma,q}$. By definition of $\bar{E}^{CSI}_{\gamma,q}$, for some $U(G^*, \delta)$, every $G \in U(G^*, \delta)$ is stable. Let $G' \in U(G^*, \frac{\delta}{2})$; thus $U(G', \frac{\delta}{2}) \subset U(G^*, \delta)$. This means that every $G \in U(G', \frac{\delta}{2})$ is stable and that $G' \in \bar{E}^{CSI}_{\gamma,q}$. Thus, $\bar{E}^{CSI}_{\gamma,q}$ is open in $M^{CSI}_{\gamma,q}$ and, consequently, it is open in $E^{CSI}_{\gamma,q}$. □

Lemma 3 $\bar{E}^{CSI}_{\gamma,q}$ *is dense in* $E^{CSI}_{\gamma,q}$.

Proof Let $G^* \in E^{CSI}_{\gamma,q}$. For any $\delta > 0$, we will find a strongly stable game $G \in U(G^*, \delta)$. Since Lemma 1 implies that $G \in \bar{E}^{CSI}_{\gamma,q}$, this will conclude the proof.

First, we will find a game $G' \in U(G^*, \frac{\delta}{2})$ such that q is an equilibrium in G' and $\nu^p(K) > 0$ for all $p \in Q$ and all $K \in \mathcal{C}^p$. Consider the game $G^\# : \nu^q(K) = \frac{100|K|}{n}$, i.e., such that the payoff of a coalition is equal to the percentage of players of this coalition in N. Define G' as follows:

$$G' = \frac{\delta}{200 \times \mathrm{coal}(n)} G^\# + \frac{200 \times \mathrm{coal}(n) - \delta}{200 \times \mathrm{coal}(n)} G^*. \tag{9}$$

We will leave it to the reader to check that G' is a game, $G' \in U(G^*, \frac{\delta}{2})$, that q is an equilibrium in G', and that $\nu^q(K) \geqslant \frac{\delta|K|}{2n \times \mathrm{coal}(n)}$ for all $p \in Q$ and all $K \in \mathcal{C}^p$. Game G is now defined as a game G' with slight perturbations of payoffs for $p \in Q$ resulting from coalescing or splitting of a party/parties from q or from changing an electoral law in q that will make sure that coalescing, splitting, or changing the electoral law (for at least one member of a winning coalition), brings strict losses:

(i) For $K \in \mathcal{C}^q$ and for a partition of K, $L_1, \ldots, L_m$, where $m \geqslant 2$, denote $p = q\text{-}^{L_1,\ldots,L_m}$. Then $\nu_G(L_i, p) = \nu_{G'}(L_i, p) - \frac{\delta|L_i|}{2n\times\mathrm{coal}(n)}$. For all parties $R_j \in \mathcal{C}^p$, $R_j \neq L_i$ for $i = 1, \ldots, m$, $\nu_G(R_j, p) = \nu_{G'}(R_j, p) + \frac{\delta|K|\times|R_j|}{2n\times(n-|K|)\times\mathrm{coal}(n)}$;

(ii) For $K = \cup_{j=1}^m P_{i_j}$, where $P_{i_j} \in \mathcal{C}^q$ for $j = 1, \ldots, m \geqslant 2$, denote $p = q\text{-}^K$. Then $\nu_G(K, p) = \nu_{G'}(K, p) - \frac{\delta|K|}{2n\times\mathrm{coal}(n)}$. For all parties $R_j \in \mathcal{C}^p$, $R_j \neq K$, $\nu_G(R_j, p) = \nu_{G'}(R_j, p) + \frac{\delta|K|\times|R_j|}{2n\times(n-|K|)\times\mathrm{coal}(n)}$;

(iii) For any $EL \in \mathcal{I}$, $EL \neq EL^q$, let $K_1 \in \mathcal{C}^q$ be any losing or blocking party from q. Define $\nu_G(K_1, q\text{-}^{EL}) = \nu_{G'}(K_1, q\text{-}^{EL}) + \frac{\delta|K_1|}{2n\times\mathrm{coal}(n)}$. For all parties $R_j \in \mathcal{C}^q$, $R_j \neq K$, $\nu_G(R_j, q\text{-}^{EL}) = \nu_{G'}(R_j, q\text{-}^{EL}) - \frac{\delta|K|\times|R_j|}{2n\times(n-|K|)\times\mathrm{coal}(n)}$.

Again, we leave it to the reader to check that G is a game, $G \in U(G', \frac{\delta}{2})$ and that q is a strong equilibrium in G . Since $G' \in U(G^*, \frac{\delta}{2})$ and $G \in U(G', \frac{\delta}{2})$, $G \in U(G^*, \delta)$. □

Proof of Proposition 5 (iii) Immediately from Lemma 2. □

Proof of Theorem 1 (iii) Immediately from Lemmas 2 and 3. □

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References

Austen-Smith D, Banks JS (1999) Positive political theory 1. Collective preference. The University of Michigan Press, Ann Arbor

Calvert RL (1986) Models of imperfect information in politics. Harwood Academic Publishers, Chur

Chwe M (1994) Farsighted coalitional stability. J Econ Theory 63:299–325

Coughlin P, Nitzan S (1981) Directional and local electoral equilibria with probabilistic voting. J Econ Theory 24:226–240

Coughlin P (1992) Probabilistic voting theory. Cambridge University Press, New York

Cox G (1987a) Electoral equilibrium under alternative voting institutions. Am J Polit Sci 31:82–108

Cox G (1987b) The uncovered set and the core. Am J Polit Sci 31:408–422

Cox GW (1990) Multicandidate spatial competition in advances in the spatial theory of voting. In: Enelow JM, Hinich MJ (eds) Cambridge University Press, Cambridge

Davis OA, DeGroot MH, Hinich MJ (1972) Social preference orderings and majority rule. Econometrica 40:147–157

Downs A (1957) An economic theory of democracy. Harper, New York

Eaton BC, Lipsey R (1975) The principle of minimum differentiation reconsidered: some new developments in the theory of spatial competition. Rev Econ Stud 42:27–50

Enelow JM, Hinich MJ (1984) The spatial theory of voting: an introduction. Cambridge University Press, Cambridge

Enelow JM, Hinich MJ (eds) (1990) Advances in the spatial theory of voting. Cambridge University Press: Cambridge

Ferejohn JA, McKelvey RD, Packel EW (1984) Limiting distributions for continuous state markov voting models. Social Choice Welf 1:45–67

Filippov MG, Ordeshook PC, Shvetsova OV (1999) Party fragmentation and presidential elections in post-communist democracies. Const Polit Econ 3–26
Geddes B (1996) Initiation of new democratic institutions in Eastern Europe and Latin America in Institutional Design in New Democracies. Eastern Europe and Latin America. In: Lijphart A, Waisman CH (eds) Westview Press, Boulder
Grofman B, Mikkel E, and Taagepera R (1999) Electoral systems change in Estonia, 1989–1993. J Baltic Stud 30:227–49
Grofman B, Mikkel E, Taagepera R (2000) Fission and fusion of parties in Estonia, 1987–1999. J Baltic Stud 31:329–357
Hinich MJ, Munger MC (1997) Analytical politics. Cambridge University Press, Cambridge
Kaminski MM (2001) Coalitional stability of multi-party systems: evidence from Poland. Am J Polit Sci 45:294–312
Kaminski MM (2002) Do parties benefit from electoral manipulation? Electoral laws and Heresthetics in Poland, 1989–1993. J Theoret Polit 14:325–359
Kaminski MM, Lissowski G, Swistak P (1998) The "revival of communism" or the effect of institutions? The 1993 Polish parliamentary elections. Public Choice 97:429–449
Kopecky P (1995) Factionalism in parliamentary parties in the Czech Republic: a concept and some empirical findings. Democratization 2:138–151
Kramer G (1973) On a class of equilibrium conditions for majority rule. Econometrica 41: 285–297
Lomax B (1995) Factions and factionalism in Hungary′s new party system. Democratization 2:125–137
McKelvey RD, Schofield NJ (1987) Generalized symmetry conditions at a core point. Econometrica 55:923–934
Miller N (1980) A new solution set for tournaments and majority voting. Am J Polit Sci 24:68–96
Miller NR (1995) Committees, agendas, and voting. Harwood Academic Publishers, Chur
Moraski B, Loewenberg G (1999) The effect of legal thresholds on the revival of former communist parties in East-Central Europe. J Polit 61: 151–170
Moser RG (2001) Unexpected outcomes. Electoral systems, political parties, and representation in Russia. University of Pittsburgh Press, Pittsburgh
Ordeshook PC (1986) Game theory and political theory. Cambridge University Press, Cambridge
Plott CW (1967) A notion of equilibrium and its possibility under majority rule. Am Econ Rev 57:787–806
Remington TF, Smith SS (1996) Political goals, institutional context, and the choice of an electoral system: the Russian parliamentary election law. Am J Polit Sci 40:1253–1279
Rosenthal RW (1972) Cooperative games in effectiveness form. J Econ Theory 5:88–101
Schofield NJ (1985) Social choice and democracy. Springer, Berlin Heidelberg Newyork
Schofield N (1993) Political competition and multiparty coalition governments. Eur J Polit Res 23:1–33
Schofield N, Sened I (1997) A general equilibrium approach to the study of multiparty electoral systems. Typesrcipt, Washington University in St. Louis
Shaked A (1975) Non-existence of equilibrium for the two-dimensional three-firms location problem. Rev Econ Stud 42:51–56
Shepsle KA (1979) Institutional arrangements and equilibrium in multidimensional voting models. Am J Polit Sci 23:27–60
Shepsle KA (1991) Models of multiparty electoral competition. Harwood Academic Publishers, London
Shepsle K, Weingast B (1981) Structure induced equilibrium and legislative choice. Public Choice 37:503–519
Waller M (1995) Making and breaking: factions in the process of party formation in Bulgaria. Democratization 2:152–167
Wyniki glosowania nr 75 - posiedzenie 103 (2001) Sejm: Warsaw: http://orka.sejm.pl
Wyniki symulacji (2001) in Rzeczpospolita 05/02: Warsaw
Zdort MD (1997) ROP z AWS o wyborach in Rzeczpospolita 06/11: Warszawa