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Does majoritarian approval matter in selecting a social choice rule? An exploratory panel study

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Abstract This study is an attempt to empirically detect the public opinion concerning majoritarian approval axiom. A social choice rule respects majoritarian approval iff it chooses only those alternatives which are regarded by a majority of "voters" to be among the "better half" of the candidates available. We focus on three social choice rules, the Majoritarian Compromise, Borda's Rule and Condorcet's Method, among which the Majoritarian Compromise is the only social choice rule always respecting majoritarian approval. We confronted each of our 288 subjects with four hypothetical preference profiles of a hypothetical electorate over some abstract set of four alternatives. At each hypothetical preference profile, two representing the preferences of five and two other of seven voters, the subject was asked to indicate, from an impartial viewpoint, which of the four alternatives should be chosen whose preference profile was presented, which if that is unavailable, then which if both of the above are unavailable, and finally which alternative should be avoided especially. In each of these profiles there is a Majoritarian Compromise-winner, a Borda-winner and a Condorcet-winner, and the Majoritarian Compromise-winner is always distinct from both the Borda-winner and the Condorcet-winner, while the Borda- and Condorcet-winners sometimes coincide. If the Borda- and Condorcet-winners coincide then there are two dummy candidates, otherwise only one, and dummies coincide with neither of the Majoritarian Compromise-, Borda- or Condorcet-winner. We presented our subjects with various types of hypothetical preference profiles, some where Borda

This is one of the last published works of Murat R. Sertel whom we lost on January 25, 2003. He was a genuine scholar and an excellent teacher.His brilliant approach toward unifying the diversified topics of economic theory has deeply influenced his colleagues and students.

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respecting majoritarian approval, some where it failed to do so, then again for Condorcet, some profiles it respected majoritarian approval and some where it did not. The main thing we wanted to see was whether subjects' support for Borda and Condorcet was higher when this social choice rule respected majoritarian approval than it did not. Our unambiguous overall empirical finding is that our subjects' support for Borda and Condorcet was significantly stronger as they respect majoritarian approval.

1 Introduction

Electoral systems pertain to democratic procedures according to which a social will is extracted out of individual wills. Each electoral system is a particular method to aggregate profiles of individual preferences into a social preference or to arrive at a choice based on such profiles. The electoral systems employed by a society reflect the democratic content of that society and thus should not left either to historical accident, or to narrow or shortsighted considerations of existing power groups.

In designing an electoral system, the first natural question is what the desirable properties are that a society wishes its electoral system to possess. This question can be answered empirically.

Although there are important studies on fairness and justice [Frohlich and Oppenheimer (1992); Amiel and Cowell (2001); Cowell and Schokkaert (2001)], these mainly focus on detecting a public understanding of distributive justice, not a public notion of fairness in the matter of how individual preferences should be mapped into a social choice. Sertel and Kara (2003) is the first extensive study in the literature which aims at eliciting public preferences concerning social choice rules in a completely hypothetical setting where, to a certain degree, a "veil of ignorance" [Rawls (1971)] is achieved.

The present study aims at detecting public support for 'majoritarian approval' axiom [Sertel (1986); Sertel and Yılmaz (1995, 1998)]. Unlike many other axioms in the social choice literature, majoritarian approval is based on a particular notion of 'fairness' in the sense that it defines the alternative(s) that should (and should not) 'deserve' to be chosen as (a) social choice given the individual preferences in the society.

A candidate receives "majoritarian approval" iff it is regarded by a majority of "voters" to be among the "better half" of the candidates available. More precisely, given m candidates and n "voters", a candidate receives majority approval iff at least $\lceil n/2 \rceil$ voters rank it among the best $\lceil m/2 \rceil$ candidates, where $\lceil x \rceil$ for any real number×denotes the smallest integer no less than x. The social choice rule μ which picks the set of all candidates, receiving majoritarian approval is called the majoritarian approval. A social choice rule (SCR) which refines μ is said to respect majoritarian approval.

The Majoritarian Compromise (M), defined below, and Borda's Rule (B) are two decisive SCRs of which M respects majoritarian approval while B fails to do so. Condorcet's Rule (C) fails even to be decisive, but neither is it a refinement of μ in any case.

In the panel study which we are reporting here, we presented each of our 288 subjects with four preference profiles of hypothetical societies confronted with a choice out of four alternatives. At each hypothetical preference profile, two representing the preferences of five and two other of seven voters, each subject was asked to indicate, from an impartial viewpoint, which of the four alternatives should be chosen for the society whose preference profile was presented, which if that is unavailable, then which if both of the above are unavailable, and finally which alternative should be avoided especially. (The subject's answers to this question reveal, naturally, the subject's total order of the candidates with respect to their worthiness to be elected for the hypothetical society being viewed.)

In each of these profiles, an example of which we display below, there is a *M*-winner, a *B*-winner and a *C*-winner, and the *M*-winner is always distinct from both the *B*-winner and the *C*-winner, while the *B*- and *C*-winners sometimes coincide. If the *B*- and *C*-winners coincide then there are two dummy candidates, *X* and *Y*, otherwise only one, *X*, and dummies coincide with neither of the *M*-, *B*- or *C*-winner.

The below table is a hypothetical preference profile representing $n=5$ voters' preferences over $m=4$ candidates. Each column displays a separate voter's ranking of the four candidates from best at the top toward worst at the bottom. Here we have $a=B=C$ so that the *B*- and *C*-winners coincide and $b=X$, $c=Y$ so that *b* and *c* are dummies, and *d* represents the *M*-winner.

Voter 1	*Voter 2*	*Voter 3*	*Voter 4*	*Voter 5*
a	a	b	b	c
d	d	d	c	d
b	b	a	a	a
c	c	c	d	b

In a previous series of studies (Sertel and Kara (2003)), we had presented subjects again with hypothetical preference profiles of seven voters over four alternatives where the four SCRs, Plurality (*P*), Plurality with a Run-off (*R*), *M* and *B* all disagree, each picking a distinct candidate among the four available. In those studies we asked our subjects which alternative should be chosen for the hypothetical society whose preference profile was presented. The overall results showed very strong support for *M* and *B*, but the *B*-winner always coincided with the Social Compromise (*SC*)-winner. In fact, when we eliminated the cases where the subjects were consciously choosing the SC-winner, of the remaining cases where the *B*-winner was being consciously chosen, the frequencies were significantly lower than those favoring *M*. It is the strong turnout in favor of *M* and *B* in these previous studies that encouraged us to include them in the present study. In those studies, however, we needed to use the preference profiles which separated the four SCRs, *P*, *R*, *M* and *B*, and this precluded the existence of a *C*-winner as we had to limit our profiles to the case of no more than seven voters. As *C* is a classical and salient SCR, we wanted to include it also in the present study.

A central question which has motivated the present study was whether the strong support for *M* as reported above was due to its respecting majoritarian approval. The pilot study pursuing this question in Kara (2001) obtained results encouraging and guiding the present study in this regard. Although, in that study, the preference profiles were manually generated for four alternatives and five voters, and some of the profile types which would be of interest (from the point of observing the change in the support for *B* and *C*) were not included in the study, the results obtained encouraged us to ask the same questions through a better design where the preference profiles would be computer generated guaranteeing randomization. In the present study we confronted each of our subjects with various types of computer generated hypothetical preference profiles, some where *B* respecting majoritarian approval, some where it failed to do so, then again for *C*, some profiles it respected majoritarian approval and some where it did not. The main thing we wanted to see was whether subjects' support for *B* and *C* was higher when this SCR respected majoritarian approval than it did not. Our unambiguous overall empirical finding is that our subjects' support for either of *B* and *C* was significantly stronger when the SCR respected majoritarian approval. However, the support for *M* which always respects majoritarian approval remains quite constant.

In Section 2, we present basic notions and definitions, and we introduce the "root profiles" for each four types of profile we used in our questionnaires. The preparation of menus and the execution of the study are explained in Section 3. In Section 4, we present our empirical findings. The final section features results on self-selectivity and majoritarian approval.

2 Basic notions and definitions

The present section has two parts in itself. The first (2.1) defines SCRs presented to the choice of our subjects, M, B and C. In our next sub-section (2.2) we assemble the root profiles and their types which we used in the present study.

2.1 Three social choice rules

By a *preference* on a set A we mean any function p: $A \rightarrow 2^A$ which assigns to every $x \in A$ a subset ("lower contour set") $p(A) \subset A$ such that, at all x, $y \in A$ we have

(1) $y \in p(x)$ or $x \in p(y)$ [completeness]
(2) $p(y) \subset p(x)$ whenever $y \in p(x)$ [transitivity]
(3) $y \in p(x)$ and $x \in p(y)$ only if $x=y$ [antisymmetry]

Such a preference clearly corresponds to a total (or "linear") order on A.

We denote $P(A)$ for the set of all preferences on a set A. For any positive integer n we write $N=\{1,..., n\}$,and by a *preference profile* for a society of n voters on a set A we mean any family $p = (p_i)_{i \in N} \in P(A)^N$ of preferences p_i on A indexed by "voters" $i \in N$.

A *social choice rule* (SCR) f is any function which assigns to every "preference profile" $\boldsymbol{p} \in P(A)^N$ a subset $f(\boldsymbol{p}) \subset A$ of "chosen alternatives".

To this end, given any preference profile $p = (p_i)_{i \in N}$ (with each p_i a total order on A), we define the function $g(\boldsymbol{p},s)$: $A \rightarrow N$ through

$$g(\boldsymbol{p},s)(x) = \#\{i \in N | \#p_i(x) = s\}$$

at each "score" $s \in \{0, ..., \#A\}$. Thus, $g(\boldsymbol{p},s)(x)$ is the number of voters $i \in N$ who find $x \in A$ to be of "score" s in their preferences p_i ($i \in N$) over A, where the "score" of an alternative $x \in A$ under a preference p is the cardinality of the "lower contour set" $p(x)$. We also define $\breve{g}(\boldsymbol{p},s)(x) = \#\{i \in N | \#p_i(x) \geq s\}$ to count the number of voters who find $x \in A$ to be of score s or greater according to a preference profile $\boldsymbol{p}$ on A. The top score for candidates in A is, of course, $\hat{s}(A) = \#A$. We abbreviate $g(\boldsymbol{p},\hat{s})$ to $\hat{g}(\boldsymbol{p})$.

It is now easy to make precise the definitions of our three functions (SCRs) f: $P(A)^N \rightarrow 2^A$ by use of the functions g and $\hat{g}$.

The Majoritarian Compromise (M) [Sertel 1986] In order to define the Majoritarian Compromise M, first we define the score $\check{s}(\boldsymbol{p})$ at any $\boldsymbol{p} \in P(A)^N$. This is the highest score attained by any $x \in A$ under $\boldsymbol{p}$ such that a majority, i.e. at least $n/2$ voters among n, give at least this score to x. Thus, writing

$$\check{s}(\boldsymbol{p}) = \max \left\{ s \in \{1, \ldots, \#A\} \middle| \exists \ \times \in A \text{ with } \breve{g}(\boldsymbol{p},s)(x) \geq n/2 \right\},$$

we are now able to define M as the SCR given, at any $\boldsymbol{p} \in P(A)^N$, by

$$M(\boldsymbol{p}) = \left\{ x \in A \middle| \breve{g}(\boldsymbol{p}, \check{s}(\boldsymbol{p}))(x) = \max{}_{y \in A} \ \breve{g}(\boldsymbol{p}, \check{s}(\boldsymbol{p}))(y) \right\}.$$

Thus, the M picks the candidates receiving at least the score $\check{s}$ from the largest number of voters, $\check{s}$ being the largest score which any majority has given (no less than) to any candidate. Since our profiles here preclude ties, M always picks a unique alternative at these profiles.

From Sertel and Yilmaz (1995, 1998) we know that $\check{s} \geq \lceil (\#A)/2 \rceil + 1$, where $\lceil . \rceil$ shows the integer part. Thus, M picks what, for some majority, is "relatively good", i.e. not among the worst $\lceil (\#A)/2 \rceil$ of the candidates.

Let us verbalize the formal definition above: If there is a first-best choice alternative in a preference profile that receives the majority of votes, then it is the M-winner. If there is no such alternative, then the alternatives that receive the support of majority as first or second-best choice are recorded and the one with maximal support is picked as the M-winner. If there is no such alternative, then the alternatives that receive the support of majority as first, second or third-best choice are recorded and so on. It is shown in Hurwicz and Sertel (1999) that there is always an alternative which gets the majority of votes in the "better half" of the alternatives. This guarantees the existence of a M-winner.

Borda's Rule (B) [Borda 1781] Borda's Rule B is defined, at each $\boldsymbol{p} \in P(A)^N$, by

$$B(\boldsymbol{p}) = \arg\max{}_A \sum\nolimits_{i \in N} \#p_i(.),$$

i.e.

$$B(p) = \left\{ x \in A | y \in A \Rightarrow \sum\nolimits_{i \in N} \#p_i(x) \geq \sum\nolimits_{i \in N} \#p_i(y) \right\}.$$

Thus, Borda's Rule B picks the candidates who maximize the total "Borda score" $\sum_{i \in N} \#p_i(.)$. Again, our profiles $\boldsymbol{p}$ here are such that the Borda winner is unique, i.e. $\#B(\boldsymbol{p})=1$.

Condorcet's Rule (C) [Condorcet 1785] Condorcet's Rule C is defined, at each $\boldsymbol{p} \in P(A)^N$, by

$$C(\boldsymbol{p}) = \{x \in A | \#\{i : y \in p_i(x)\} \geq n/2 \text{ for all } y \in A\}.$$

Thus, a candidate (alternative) is a C-winner iff it is preferred to each other candidate (alternative) by a majority (strict when n is odd) of voters.

2.2 Root profiles and their types

As in Sertel and Kara (2003), here again we represent $m \times n$ preference profile of n voters over m alternatives by $m \times n$ tables or matrixes as in our 4×5 example in Section 1. Again column j represents the preference (total ordering) of voter j in the sense that a candidate in a higher row is preferred to one in a lower. While we always have $m=4$ candidates, for voters we have $n=5$ in two types of hypothetical preference profile, and $n=7$ in the other two. It should be noted that we need n to be odd in order to avoid ties. In fact, in generating the profiles to be presented to our subjects we restricted ourselves to those profiles which gave a unique M-, a unique B- and a unique C-winner while M never agreed with B or C but in two types of profile, B and C agreed among themselves.

Observing that all three of the SCRs compared here are neutral and anonymous, many $m \times n$ profiles will be equivalent to other profiles with the same m and n, since neither the B- nor the C- nor the M-winners will change as we permute the names of the candidates (permute the set {a,b,c,d}) or the columns (the set $N=\{1,\ldots,n\}$ of voters). As in Sertel and Kara (2003), here again we call these equivalence classes "root profiles". While there were only three root profiles in our previous studies, (where we regarded only profiles which separated all four of P, R, M and B), in our present study we deal with a wealth of root profiles. Let us begin to acquaint ourselves with some of their types.

As long as we do not insist on separating the B- and C-winners, we can economize on n and make do with $m \times n = 4 \times 5$ profiles. There are just three root profiles with $(m,n)=(4,5)$ where the B- and the C-winners coincide and $B=C$ fails to respect majoritarian approval. We call these Type I profiles, denoted $M/B{\sim}C$, and we present them below. [$M/B{\sim}C$ is intended to communicate the characteristics that M respects majoritarian approval, falling above (to the left of) the "/" sign, while B and C fail majoritarian approval falling below the "/" sign.]

1.

B	B	X	X	Y
M	M	M	Y	M
X	X	B	B	B
Y	Y	Y	M	X

2.

B	B	X	Y	Y
M	M	M	M	X
X	X	B	B	B
Y	Y	Y	X	M

3.

B	B	X	X	Y
M	M	M	Y	M
X	Y	B	B	B
Y	X	Y	M	X

M/B~C: 4×5 profiles where *B*=*C* fails majoritarian approval. Here, *B* is the joint winner of *B* and *C*[1]

[1] The joint winner of *B* and *C* will sometimes be denoted as *B* for Type I and Type II profiles in the rest of the paper.

Our 4×5 *M*×*B*~*C*/profiles, classified as Type II, again have *B* and *C* coinciding with each other, but now also respecting majoritarian approval, although they are distinct from *M*. There are 38 such root profiles and the entire list of Type II profiles is presented in Appendix A. As they are quite numerous, we had to narrow down this class for our questionnaires, in fact selecting three representatives which we found to properly capture the spirit of Type II in a way which we detail as follows: We grouped the 38 profiles of Type II into three sub-groups according to the way the first row appears. Then, we randomly selected one profile out of each sub-group. We now present these three representatives of Type II profiles:

1.	*B*	B	M	X	X
	M	M	B	Y	M
	X	Y	X	B	B
	Y	X	Y	M	Y
2.	B	B	M	X	Y
	M	M	B	Y	M
	Y	X	X	B	B
	X	Y	Y	M	X
3.	B	B	X	X	Y
	M	M	M	B	M
	Y	Y	B	M	B
	X	X	Y	Y	X

M×*B*~*C*/: 4×5 profiles where *B*=*C* respects majoritarian approval

We should note at this point that there are no profiles of the type *M*/*B*×*C*, where *B* and *C* are distinct and fail majoritarian approval, and this is so not only when we restrict (*m*,*n*) to (4,5), but even for (*m*,*n*)=(4,7). As we had to abstain from presenting our subjects with profiles of more crowded hypothetical societies, we had to forego this type i×*B*×*C*×*M*. Similarly, we had to forego the type of profile where (*M* and) both *B* and *C* respect majoritarian approval while they all choose distinct winners. The reason was again that there are no profiles of this type for odd $n \leq 7$.

In the case of (*m*,*n*)=(4,7) we have two further types of profile: Type III which we denote as *M*×*B*/*C*, and Type IV, denoted as *M*×*C*/*B*. There are four root profiles of Type III, where *B* is distinct from *M* but respects majoritarian approval while *C* fails to do so, and there are eight root profiles of Type IV where instead, *C* is distinct from *M*, but respects majoritarian approval while *B* fails to do so. We present the root profiles of Type III below:

1.	*C*	C	C	M	B	B	B
	M	M	B	X	M	M	X
	B	B	M	C	C	X	C
	X	X	X	B	X	C	M
2.	*C*	C	C	M	B	B	B
	M	M	B	X	M	M	X
	B	B	X	C	C	X	C
	X	X	M	B	X	C	M
3.	*C*	C	C	X	B	B	B
	M	M	B	M	M	M	X
	B	B	M	C	C	X	C
	X	X	X	B	X	C	M
4.	*C*	C	C	X	B	B	B
	M	M	B	M	M	M	X
	B	B	X	C	C	X	C
	X	X	M	B	X	C	M

M×*B*/*C*: 4×7 profiles where *B*, but not *C* respects majoritarian approval

Below presented are the root profiles of Type IV:

1.	C	C	C	M	B	B	B	2.	C	C	C	M	B	B	B
	M	M	M	C	M	X	X		M	X	M	C	M	M	X
	B	B	B	B	X	M	C		B	B	B	B	X	X	C
	X	X	X	X	C	C	M		X	M	X	X	C	C	M
3.	C	C	M	M	B	B	B	4.	C	C	C	X	B	B	B
	M	M	C	C	M	X	X		M	M	M	C	M	M	M
	B	B	B	B	X	C	C		B	B	B	B	X	X	C
	X	X	X	X	C	M	M		X	X	X	M	C	C	X
5.	C	C	C	X	B	B	B	6.	C	C	C	X	B	B	B
	M	M	M	C	M	M	X		M	X	M	C	M	M	M
	B	B	B	B	X	C	M		B	B	B	B	X	C	X
	X	X	X	M	C	X	C		X	M	X	M	C	X	C
7.	C	C	X	X	B	B	B	8.	C	C	X	M	B	B	B
	M	M	C	C	M	M	M		M	M	C	C	M	M	X
	B	B	B	B	X	C	C		B	B	B	B	X	C	C
	X	X	M	M	C	X	X		X	X	M	X	C	X	M

M×*B*/*C*: 4×7 profiles where *B*, but not *C* respects majoritarian approval

As *M* always respects majoritarian approval we cannot test what happens to the support it receives when it fails to respect majoritarian approval. In fact, the support for *M* remains quite constant from one type of profile to another. Our central question therefore being whether *B* or *C* finds significantly stronger support when it respects majoritarian approval, we compared subjects' responses to four types of preference profile. Being restricted to $m=4$ and $n\leq 7$, our profiles did not permit *B* and *C* to disagree when they both failed to respect majoritarian approval. Thus, our comparison followed the pattern depicted in Fig. 1, below.

Our general question takes several precise forms here. For one, in passing from profiles of Type I (*M*/*B*~*C*) to Type II (*M*×*B*~*C*/), does the support for *B*=*C* increase significantly? The answer turned out to be "Yes". Next, in passing from Type I to Type III (*M*×*B*/*C*) profiles, does the support for *B* increase significantly? Again, the answer turned out to be in the affirmative. Third, does the support for *C* increase significantly when we pass from Type I to Type IV (*M*×*C*/*B*) profiles? In this case, the answer is negative. In fact, *C* receives less support in Type IV than *B*=*C* does in Type I profiles. Finally, there is the comparison of the support received by *B* and *C* at Type III and Type IV profiles. Here we find that either of *B* and *C* receives significantly stronger support when it respects majoritarian approval than when it does not, so that the support for *B* (resp. *C*) increases significantly as we pass from Type IV (resp. Type III) to Type III (resp. Type IV) profiles. Further comparisons (of Type III and IV with Type II) will be given in the Section 4 where we detail our empirical findings and present their analysis.

3 The menus and the subjects

In the present study, we presented each of our 288 subjects a "menu" of four profiles, one profile from each profile type. (See the exemplary menu in Appendix B) We had this sample size in order to be able to include all the possible combinations of the root profiles belonging to our four profile types. (3*3*4*8=288). That is, 96 batches of Type I profiles, 96 batches of Type II profiles, 72 batches of Type III profiles and 36 batches of Type IV profiles were presented to our subjects. The menus were prepared so that each subject was presented one root profile (with permuted columns and the names of the alternatives) from each type of profile. For each of the four profiles in the menu received by a subject, the subject was asked to state, from an impartial viewpoint, which of the four alternatives should be chosen for the society whose preference profile was presented, which if that is unavailable, then which if both of the above are unavailable, and finally which alternative should be avoided especially. The subjects were asked also to present their reasoning in giving their answers.

All the 288 subjects who participated in our study were volunteers responding to announcements posted in several hallways and other central locations at Boğaziçi University, İstanbul, İstanbul Bilgi University and Marmara University, İstanbul in officially assigned classrooms of which they were individually subjected to our menus during June, 2001.[1] The subjects were given no indication of the purpose of the study or of the team conducting it. They were asked to keep their responses void of any indication of their identities, so that they were guaranteed anonymity.

The subjects were each paid the equivalent of USD 5 in Turkish Liras, as promised in our announcement, for answering the questions in their menus. Unlike the other studies in the experimental literature, the subjects were not paid extra money depending on their answers since this would actually ruin the key features of the experimental setting, namely 'impartiality' and 'veil of ignorance'. The study was designed so as to prevent the subjects from identifying themselves with any of the members in the hypothetical society and to ensure that the subjects state their own 'judgments' independent of what other subjects might possibly state. Therefore, it would be against the aim of the experiment to make the subject payoff be conditional on a particular 'outcome'. Clearly, this setting does not give any incentive to the subjects to misrepresent their choices, but, on the other hand, it does not guarantee the subjects to take the choice task seriously. However, the mean and median time spent by the subjects for answering the questionnaires (17.1 min and 16.2 min, respectively) enables us to conclude that the subjects spent considerable time for the questionnaires instead of making their way out of the classroom as soon as possible.[2]

[1] This panel study obtains experimental data. That is, it is not a field research sampling from the "men in the street". Hence, here we are not concerned with the representativeness of the sample.
[2] It should be noted that the mean and median time spent by the subjects to answer the questionnaires is close to the ones in Sertel and Kara (2003). The extended time spent by the subjects can be explained by the observation that the students in Turkey are not used to get paid in return for answering questionnaires and when they are paid they feel obliged to seriously think before they answer them.

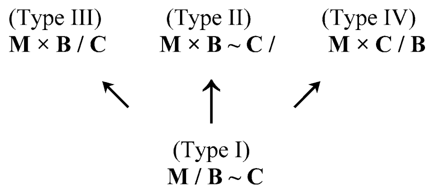

Fig. 1 Comparison of the support received by *B* and *C* with respect to the four types of preference profile

Our subjects were students, mostly undergraduates, from 11 departments[3] who had never taken a social choice course or participated in a similar experiment before.[4] Of the 288 subjects, 128 were female and 160 were male.

4 Empirical findings

4.1 Overall results

Of the 288 subjects, 72 (25%) and 24 (8.3%) consistently placed *B* and *M*, resp., at top of their rankings while no subject was consistently choosing *C*-winner as his/her top-ranked alternative. 68 (23.6%) subjects consciously applied *B*-scoring in all the profiles in their menus. The respective number of profiles at which the *M*-, the *B*- and the *C*- winner is top-ranked are 270, 700 and 480. On the other hand, the frequencies of the three to be bottom-ranked are 25, 3 and 8, respectively.

Our first notion of support for majoritarian approval simply has to do with how the subjects rank the "majoritarian approved" alternatives in their rankings.[5] We measure this support by use of a score MA which takes the values 0, 1, 2 and 3, respectively, accordingly as none of the majoritarian approved alternatives was either first or second-ranked, one of the majoritarian approved alternatives was top-ranked but the other majoritarian approved alternative was not second-ranked, none of the majoritarian alternatives was top-ranked, but one of them was second-ranked and both of the majoritarian approved alternatives were ranked in the first half of the subject ordering. The overall number of profiles with MA=0, MA=1, MA=2 and MA=3 are 50 (4.3%), 353 (30.7%), 315 (27.3%) and 434 (37.7%), resp. That is 95.7% of the profiles exhibit support for majoritarian approval at varying levels.

[3] Business Administration, Chemical Engineering, Civil Engineering, Computer Engineering, Economics, History, International Finance, Mechanical Engineering, Media and Broadcasting, Physics, Politics.

[4] During the pilot studies in Kara (2001), it is observed that the subjects who had taken a course of social choice theory fail to answer the questionnaires independent of what they had 'learned'. That is, they treat the task as if it is a test of what they 'know' about the matter of concern. Apparently, this creates a bias in terms of impartiality. The students who had previously participated in a similar experiment were prevented to be the subjects of the present study since the subjects usually have 'post-talks' and get exposed the way the other subjects approach to the matter which also leads to a bias.

[5] In all types of profile except for Type I, there are two "majoritarian approved" alternatives. One of these is always *M*-winner, the other is either *B*- or *C*- or *B*=*C*-winner.

The frequencies (number of profiles or subjects) of different levels of MA for each of our profile types are presented in Table 1.

The frequency of MA=0 for all types of profiles is significantly less than its random probability of occurrence. The frequencies for all values of MA are significantly different from the random probabilities of these values in Type III. On the other hand, only for MA=0 profiles the frequency of profiles in Type IV is different than the probability of having this outcome randomly.

4.2 Results by type

Table 2 presents the frequencies of different rankings concerning the *M*-, the *B*=*C* winner and the two dummy alternatives, *X* and *Y*, for Type I and Type II profiles.

The χ^2-test shows that *B*=*C* is top-ranked significantly more than *M* in Type I profiles. (Since χ^2=33.9 is greater than $\chi^2_{0.05,\ 1}$=3.84, we reject the null hypothesis that the frequencies for *B*=*C* and *M* being top-ranked are the same) On the other hand, the opposite statement is true for second-rank frequencies. (χ^2=39.6).

The χ^2-test also shows that *B*=*C* is top-ranked significantly more than M in Type II profiles. (Since χ^2=134.4 far exceeds $\chi^2_{0.05,\ 1}$=3.84, we reject the null hypothesis that the two frequencies are the same). The opposite statement is true for second-rank frequencies. (χ^2=72.4).

It should be noted that B is top-ranked in 54.2% of the Type I profiles, but this share increases to 72.2% at Type II profiles where B respects majoritarian approval.

Table 1 Frequencies of MA level

Profile type	MA level	% of profiles	Probability of randomness	χ^2	Significance check[a]
Type I: *M*/*B*~*C*	MA=0	17.4	1/2	14.70	+
	MA=1	30.2	1/4	0.29	−
	MA=2	52.4	1/4	6.68	+
Type II: *M*×*B*~*C*/	MA=0	0	1/6	27.49	+
	MA=1	19.5	1/3	2.72	−
	MA=2	3.8	1/3	34.30	+
	MA=3	76.7	1/6	39.60	−
Type III: *M*×*B*/*C*	MA=0	0	1/6	27.49	+
	MA=1	51.8	1/6	27.49	+
	MA=2	9.4	1/3	13.00	+
	MA=3	38.8	1/6	4.69	+
Type IV: *M*×*C*/*B*	MA=0	0	1/6	27.49	+
	MA=1	21.2	1/3	1.91	−
	MA=2	43.7	1/3	98	−
	MA=3	35.1	1/6	3.36	−

[a] Significance check sign is (+) when the χ^2 value exceeds $\chi^2_{0.05,\ 1}$=3.84 and is (−) otherwise

Table 2 Frequencies of rankings over *M*, *B*=*C*, *X* and *Y* in type I and type II profiles

Ranking	Type I		Type II	
	Number of profiles	Percent	Number of profiles	Percent
MBXY	58	20.1	27	9.4
MBYX	11	3.9	32	11.1
MXBY	11	3.9	7	2.4
MYBX	6	2.2	2	0.7
MXYB	0	0	0	0
MYXB	1	0.3	1	0.3
BMXY	86	30.0	61	21.2
BMYX	28	9.7	101	35.1
BXMY	34	11.8	27	9.4
BYMX	1	0.3	17	5.9
BXYM	4	1.3	2	0.7
BYXM	3	1.1	0	0
XBMY	4	1.3	6	2.1
YBMX	0	0	0	0
XMBY	37	12.8	4	1.4
YMBX	0	0	0	0
XMYB	0	0	0	0
YMXB	0	0	0	0
XBYM	4	1.3	1	0.3
YBXM	0	0	0	0
XYBM	0	0	0	0
YXBM	0	0	0	0
XYMB	0	0	0	0
YXMB	0	0	0	0

Table 3 presents the frequencies of different rankings concerning the *M*-, the *B*-, the *C* winner and the dummy alternative, *X*, for Type III and Type IV profiles.

The χ^2-test shows that *B* is top-ranked significantly more than both *M* and *C* in Type II profiles. (Since χ^2=177.3 far exceeds $\chi^2_{0.05,\ 1}$=3.84, we reject the null hypothesis that the top-rank frequencies for *B* and *M* are the same. χ^2=240.2 indicates the significant difference between the frequencies of *B* and *C* to be top-ranked.) On the other hand, there is also a significant difference between the top-rank frequencies of *M* and *C*. (Since χ^2=8.4 is greater than $\chi^2_{0.05,\ 1}$=3.84, we reject the null hypothesis that the top-rank frequencies for *M* and *C* are the same.) The χ^2-test also shows that *C* is second-ranked significantly more than both *M* and *B*. (Since χ^2=26.9 far exceeds $\chi^2_{0.05,\ 1}$=3.84, we reject the null hypothesis that the second-rank frequencies for *C* and *M* are the same. χ^2=64.6 indicates the significant difference between the frequencies of *B* and *C* to be second-ranked.) *M* is second-ranked significantly more than *B*. (Since χ^2=9.2 is greater than $\chi^2_{0.05,\ 1}$= 3.84, we reject the null hypothesis that the second-rank frequencies for *M* and *B* are the same.)

The χ^2-test shows that *B* is top-ranked significantly more than both *M* and *C* in Type IV profiles. (Since χ^2=31.3 exceeds $\chi^2_{0.05,\ 1}$=3.84, we reject the null hypothesis that the top-rank frequencies for *B* and *M* are the same. χ^2=5.3 indicates

Table 3 Frequencies of rankings over *M*, *B*, *C* and *X* in type III and type IV profiles

Ranking	Type III		Type IV	
	Number of profiles	Percent	Number of profiles	Percent
MCBX	12	4.3	32	11.1
MBCX	38	13.3	30	10.4
MBXC	0	0	1	0.3
MCXB	0	0	0	0
MXBC	0	0	0	0
MXCB	1	0.3	0	0
BMCX	72	25.0	56	19.5
BCMX	131	45.4	63	21.9
BCXM	3	1.0	3	1.0
BMXC	2	0.7	4	1.4
BXCM	1	0.3	0	0
BXMC	1	0.3	0	0
CBMX	16	5.5	26	9
CMBX	11	3.9	69	24.1
CMXB	0	0	0	0
CBXM	0	0	3	1.0
CXBM	0	0	1	0.3
CXBM	0	0	0	0
XCMB	0	0	0	0
XCBM	0	0	0	0
XBCM	0	0	0	0
XBMC	0	0	0	0
XBMC	0	0	0	0
XMCB	0	0	0	0

the significant difference between the frequencies of *B* and *C* to be top-ranked.) On the other hand, *M* is top-ranked significantly more than *C*. (Since χ^2=11.1 is greater than $\chi^2_{0.05,\ 1}$=3.84, we reject the null hypothesis that the top-rank frequencies for *M* and *C* are the same.) The χ^2-test also shows that M is second-ranked significantly more than both *B* and *C*. (Since χ^2=37.6 far exceeds $\chi^2_{0.05,\ 1}$=3.84, we reject the null hypothesis that the top-rank frequencies for *M* and *B* are the same. χ^2=7.0 indicates the significant difference between the frequencies of *M* and *C* to be second-ranked.) There is also a significant difference between the second-rank frequencies of *B* and *C*. (Since χ^2=12.8 is greater than $\chi^2_{0.05,\ 1}$=3.84, we reject the null hypothesis that the second-rank frequencies for *B* and *C* are the same.

The percentage of profiles where *B* is top-ranked is 72.9 in Type III profiles where *B* respects majoritarian approval, but this share is as low as 43.8% at Type IV profiles where if fails to respect majoritarian approval. On the other hand, *C* is top-ranked in only 9.4% of the Type III profiles where it does not satisfy majoritarian approval, but it is top-ranked in 34.4% of Type IV profiles where it respects this axiom.

4.3 Cross-type results

4.3.1 Aggregated changes in the rankings of B and C across types of profile

In this sub-section we aim to present how the subjects change the respective ranks of B and C as these SCRs individually or jointly respect majoritarian approval in the hypothetical profiles appeared in the subjects' menus compared to the cases where they fail to do so. In order to measure the difference in the subjects' rankings we present matrix $Q = \left(q_{ij}\right)_{i,j\in\{1,\ldots,4\}}$ where i denotes the rank of the SCR of concern (B or C) at any profile type where it does not respect majoritarian approval, j denotes the rank of the same SCR at another type where it does so and q_{ij} is the number of subjects who i-rank the SCR in the type of profile where it does not respect majoritarian approval, but j-rank it in the profile type where it respects the axiom. Note that $\sum_{i,j\in\{1,\ldots,4\}} q_{ij} = 288$ for every Q. That is, Q-matrix shows the number of subjects for each ranking combination concerning the SCR (B or C) in two different profiles whose characteristics are given above. Hence, q_{13} denotes the number of subjects who first ranked B (or C) in a profile type where B (or C) does not respect majoritarian approval and then third-ranked B (or C) in a profile type where B (or C) respects the axiom.

Let us now introduce the score Δ which gives the aggregated change in the subjects' rankings of the SCR of concern (B or C) as this SCR respects majoritarian approval, i.e.

$$\Delta = \sum_{i,j\in\{1,\ldots,4\}} (i-j)q_{ij}.$$

Aggregated changes in the rankings of B across types of profile

Below presented are $Q1$, $Q2$, $Q3$ and $Q4$ showing the number of subjects with each ranking combination of B in Type I and Type II, Type I and Type III, in Type IV and Type III, and Type IV and Type II, resp. Δ score for each Q is presented underneath the matrix.

* $Q1$:

Type I ($M/B{=}C$)	Type II ($M{\times}B{=}C/$)				
	B-1st Rank	B-2nd Rank	B-3rd Rank	B-4th Rank	Δ_i[6]
B-1st Rank	139	12	5	0	−22
B-2nd Rank	34	39	4	0	30
B-3rd Rank	34	15	4	1	82
B-4th Rank	1	0	0	0	3
Δ^j[7]	105	3	−14	−1	

Δ=93

[6] Let $\Delta_i = \sum_{,j\in\{1,\ldots,4\}} (i-j)q_{ij}$ for all i. Note that $\Delta = \sum_{i\in\{1,\ldots,4\}} \Delta_i$.
[7] Let $\Delta^j = \sum_{i\in\{1,\ldots,4\}} (i-j)q_{ij}$ for all j. Note that $\Delta = \sum_{j\in\{1,\ldots,4\}} \Delta^j$.

* *Q*2:

Type I ($M/B{=}C$)	Type III ($M{\times}B/C$)				
	B-1st Rank	B-2nd Rank	B-3rd Rank	B-4th Rank	Δ_I
B-1st Rank	132	17	6	1	−32
B-2nd Rank	40	27	10	0	30
B-3rd Rank	37	10	7	0	84
B-4th Rank	1	0	0	0	3
Δ^j	117	−7	−22	−3	

Δ=85

* *Q*3:

Type IV ($M{\times}C/B$)	Type III ($M{\times}B/C$)				
	B-1st Rank	B-2nd Rank	B-3rd Rank	B-4th Rank	Δ_i
B-1st Rank	104	12	9	1	−33
B-2nd Rank	42	14	4	0	38
B 3rd Rank	64	28	10	0	156
B 4th Rank	0	0	0	0	0
Δ^j	170	16	−22	−3	

Δ=161

* *Q*4:

Type IV ($M{\times}C/B$)	Type II ($M{\times}B{=}C/$)				
	B-1st Rank	B-2nd Rank	B-3rd Rank	B-4th Rank	Δ_I
B-1st Rank	105	16	5	0	−26
B-2nd Rank	31	24	5	0	26
B-3rd Rank	72	26	3	1	169
B-4th Rank	0	0	0	0	0
Δ^j	175	10	−15	−1	

Δ=169

The above results show that the support for *B*=*C* increases as it respects majoritarian approval. The support for *B* increases as we go from Type III to Type I since the support given to *B* when it respects majoritarian approval dominates the effect of its disagreeing with *C*. There is a strong support for *B* when its winner coincides with that of *C* and they together respect majoritarian approval. On the other hand, the support for *B* is remarkably high in Type III compared to Type IV. Table 4 summarizes the number of subjects who increased, decreased and did not change the rank of B in the pairwise comparisons stated.

Aggregated changes in the rankings of C across types of profile

Below presented are *Q*5, *Q*6, *Q*7 and *Q*8 showing the number of subjects with

Table 4 The number of subjects having changed and not having changed the rank of *B* across types

Profile types of comparison	# of subjects who increased *B*'s rank	# of subjects who decreased *B*'s rank	# of subjects who did not change *B*'s rank[a]
I and II	84 (29.2%)	22 (7.6%)	182 (63.2%)
I and III	88 (30.6%)	34 (11.8%)	166 (57.6%)
IV and III	134 (46.5%)	26 (9.0%)	128 (44.5%)
IV and II	129 (44.8%)	27 (9.4%)	132 (45.8%)

[a] Let us again note that there are 68 subjects who applied Borda-scoring for all the preference profiles that they were confronted with. Thus, they continuously picked *B*-winner in each of these profiles no matter whether *B* respects majoritarian approval or not

each ranking combination of *C* in Type I and Type II, Type I and Type IV, in Type III and Type IV, and Type III and Type II, resp. Δ score for each *Q* is presented underneath the matrix.

* *Q*5:

Type I (*M*/*B*=*C*)	Type II (*M*×*B*=*C*/)					
		C-1st Rank	C-2nd Rank	C-3rd Rank	C-4th Rank	Δ_I
	C-1st Rank	139	12	5	0	−22
	C-2nd Rank	34	39	4	0	30
	C-3rd Rank	34	15	4	1	82
	C-4th Rank	1	0	0	0	3
	Δ^j	105	3	−14	−1	

Δ=93

* *Q*6:

Type I (*M*/*B*=*C*)	Type IV (*M*×*C*/*B*)					
		C-1st Rank	C-2nd Rank	C-3rd Rank	C-4th Rank	Δ_i
	C-1st Rank	50	54	49	3	−161
	C-2nd Rank	21	33	23	0	−2
	C-3rd Rank	28	11	13	2	65
	C-4th Rank	0	0	1	0	1
	Δ^j	77	−43	−120	−11	

Δ=−97

We know that of the 72 subjects who consistently top-ranked the *B*-winner, 68 adopted *B*-scoring. Thus, these subjects ranked the alternatives according to their

B-scores. Hence, depending on the root profile of Type IV[8], 40 of them second-ranked and 28 of them third-ranked the *C*-winner in Type IV profiles. If we are to indicate the impact of the choices of subjects who adopted *B*-scoring on *Q*6, then we should note that Δ_1=−106 of Δ_1=−161 is a straightforward result of the answers by subjects who adopted *B*-scoring.

* *Q*7:

Type III ($M\times B/C$)	Type IV ($M\times C/B$)				
	C-1st Rank	C-2nd Rank	C-3rd Rank	C-4th Rank	Δ_i
C-1st Rank	10	8	8	1	−27
C-2nd Rank	32	65	48	1	−18
C-3rd Rank	57	24	30	1	137
C-4th Rank	0	1	0	2	2
Δ^j	146	18	−64	−6	

Δ=94

* *Q*8:

Type III ($M\times B/C$)	Type II ($M\times B{=}C/$)				
	C-1st Rank	C-2nd Rank	C-3rd Rank	C-4th Rank	Δ_i
C-1st Rank	22	5	0	0	−5
C-2nd Rank	107	30	8	1	97
C-3rd Rank	76	31	5	0	183
C-4th Rank	3	0	0	0	9
Δ^j	268	26	−8	−2	

Δ=284

[8] Below are the *B*-scores of the *M*, the *B*-, the *C*-winners and *X* at each root profile of Type IV:

Root profile	*M*	*B*	*C*	*X*
1	12	13	12	5
2	11	13	12	6
3	12	13	12	5
4	12	13	12	5
5	11	13	12	6
6	10	13	12	7
7	10	13	12	7
8	11	13	12	6

Of the 40 subjects second-ranking the *C*-winner, 24 were confronted with roots 2,5,6,7 and 8, and 26 were confronted with roots 1,3 and 4. The 24 subjects second-ranked the *C*-winner as a straightforward result of *B*-scoring. The 16 subjects, however, made their choice in favor of the *C*-winner at the profiles where the *M*- and the *C*-winners obtained the same *B*-score. Of the 28 subjects third-ranking the *C*-winner, 25 were confronted with root profiles of 1,3 and 4 and favored the *C*-winner and three others miscalculated the *B*-scores of the *C*-winner in roots 2,5 and 8

Of the 68 subjects who adopted *B*-scoring as the method of ranking the alternatives in the hypothetical preference profile, 49 second-ranked and 19 third-ranked the *C*-winner in Type II profiles depending of the root profile they were confronted with.[9] If we are to indicate the impact of the *B*-scorers on *Q*8, then we should note that Δ^1=87 of Δ^1=268 is a straightforward result of their answers.

The above results show that the support for *B*=*C* increases as it respects majoritarian approval. The results concerning *C* show that the increase in its support outstandingly increases as we move from Type III to Type II, i.e. when *C* picks the same unique winner as *B* and they together respect majoritarian approval instead of its disagreeing with *B* and not respecting majoritarian approval. However, the support for *C* declines as we go from Type I to Type IV. If we deduce the impact of all the choices made by the *B*-scorers, we see that no change in the support for *C* as it respects majoritarian approval. It is impossible, however, to distinguish the exact reason behind this result due to the experimental design. Still, we can state some of the possible reasons: First, the effect of *C*'s disagreeing with *B* might dominate the support of it's respecting majoritarian approval. Second reason, which is, in fact, an outcome of imposing *B* and *C* to have distinct winner is that, in Type IV profiles, the *B*-winner never shows up at the bottom row, whereas the *C*-winner appears once at the bottom row in three root profiles and twice at the bottom row in the remaining five root profiles. This construction of the profiles fulfilling our constraint in Type IV surely treats less favorable to *C* than it does to *B*. Hence, this effect should definitely be considered in interpreting the unchanged (or declined) support for *C* in Type IV profiles.

Table 5 summarizes the number of subjects who increased, decreased and did not change the rank of *C* in the pairwise comparisons stated.

4.3.2 The Wilcoxon signed rank test results

We use Wilcoxon Signed Ranks Test in order to be able to state whether the support for *B* and *C* increase as they respect majoritarian approval. We follow the same path of paired profile types as in Section 4.3.1.

Let $d_i = X_i - Y_i$ be the difference in the rank of *B* (or *C*) in Type X and Type Y stated by the subject i. Here Type X is a profile type where *B* (or *C*) fails to respect majoritarian approval and Type Y is a profile type where it respects the axiom.

[9] B-scores of the *M*-, the *B*-, the *C*-winners and *X* in the root profiles of Type II are as follows:

Root profile	*M*	*B*	*C*	*X*
1	12	13	12	5
2	11	13	12	6
3	11	13	12	6
4	10	13	12	7

Of the 49 subjects second-ranking the *C*-winner, 44 were confronted with roots 2,3, and 4, and 5 were confronted with root 1. The 44 subjects second-ranked the *C*-winner as a straightforward result of the ranking via *B*-scoring. The five subjects, however, made their choices in favor of the *C*-winner at root profile 1 where the *M*- and the *C*-winners obtain the same *B*-scores. Of the 19 subjects third-ranking the *C*-winner, eight were confronted with root 1 and favored the *M*-winner more than the *C*-winner and one subject was confronted with root 3, but miscalculated the *B*-scores and third-ranked the *C*-winner.

Table 5 The number of subjects having changed or not having changed the rank of B across types

Profile types of comparison	# of subjects who increased *C*'s rank	# of subjects who decreased *C*'s rank	# of subjects who did not change *C*'s rank
I and II	84 (29.2%)	22 (7.6%)	182 (63.2%)
I and IV	61 (21.2%)	131 (45.5%)	96 (3.3%)
III and IV	114 (39.6%)	67 (23.3%)	107 (37.1%)
III and II	217 (75.3%)	14 (4.9%)	57 (19.8%)

That is, if *B* is second-ranked in Type X profile and then top-ranked in Type Y profile by subject i, then d_i=1. If the contrary is true, then d_i=−1. The null hypothesis for any pair of comparison claims that there is no difference in the rankings of *B* (or *C*) as it respects majoritarian approval. We use α=0.01 as significance level and sample size N is the total number of subjects (288) minus the number of subjects who did not change the rank of *B* (or *C*) in the pair of profile types of concern. Let T^+ be the positive rank orders and z (for large samples) be calculated as follows:

$$z = \frac{T - N(N+1)/4}{\sqrt{N(N+1)(2N+1)/24}}$$

Since the direction of the difference is predicted, the region of rejection is one-tailed. The region of rejection consists of all z's obtained which are so extreme that the probability associated with their occurrence when the null hypothesis is true is equal to or less than α=0.01.

The Wilcoxon signed ranks test results for B

Type I and Type II:
N=288−82=106

	Number of subjects	Average rank assigned
d_{I}=+1	49	33.5
d_{I}=−1	17	33.5
d_{I}=+2	34	86
d_{I}=−2	5	86
d_{i}=+3	1	106
d_{i}=−3	0	–

Since T^+=4,671.5, T^-=999.5 and T^+>T^-, z=5.79 which has a probability far exceeding the probability associated with the occurrence when the null hypothesis (that there is no change in the support for *B*) is true, i.e. α=0.01. That is, there is a significant increase in the support for *B* for Type I to Type II.

Type I and Type III
N=288−166=122

	Number of subjects	Average rank assigned
d_{I}=+1	50	39
d_{I}=−1	27	39
d_{I}=+2	37	99
d_{I}=−2	6	99
d_{I}=+3	1	121.5
d_{I}=−3	1	121.5

Since T^{+}=5,734.5, T^{-}=1,768.5 and $T^{+}>T^{-}$, z=5.07 which has a probability far exceeding the probability associated with the occurrence when the null hypothesis (that there is no change in the support for B) is true, i.e. α=0.01. That is, there is a significant increase in the support for B for Type I to Type III.

Type IV and Type III
N=288−128=160

	Number of subjects	Average rank assigned[10]
d_{I}=+1	70	43.5
d_{I}=−1	16	43.5
d_{I}=+2	64	123
d_{I}=−2	9	123
d_{I}=+3	0	–
d_{I}=−3	1	160

Since T^{+}=10,917, T^{-}=1,963 and $T^{+}>T^{-}$, z=7.63 which has a probability far exceeding the probability associated with the occurrence when the null hypothesis (that there is no change in the support for B) is true, i.e. α=0.01. That is, there is a significant increase in the support for B for Type IV to Type III.

Type IV and Type II
N=288−132=156

	Number of subjects	Average rank assigned
d_{I}=+1	57	40
d_{I}=−1	22	40
d_{I}=+2	72	118
d_{I}=−2	5	118
d_{I}=+3	0	–
d_{I}=−3	0	–

Since T^{+}=10,776, T^{-}=1,470 and $T^{+}>T^{-}$, z=8.23 which has a probability far exceeding the probability associated with the occurrence when the null hypothesis

[10] Since there is more than one subject with the same d_{i}, an average rank is assigned to each of them.

(that there is no change in the support for B) is true, i.e. α=0.01. That is, there is a significant increase in the support for B for Type IV to Type II.

The wilcoxon signed ranks test results for **C**

Type I and Type II
N=288−82=106

	Number of subjects	Average rank assigned
d_I=+1	49	33.5
d_I=−1	17	33.5
d_I=+2	34	86
d_I=−2	5	86
d_I=+3	1	106
d_I=−3	0	–

Since T^+=4,671.5, T^-=999.5 and $T^+>T^-$, z=5.79 which has a probability far exceeding the probability associated with the occurrence when the null hypothesis (that there is no change in the support for C) is true, i.e. α=0.01. That is, there is a significant increase in the support for C for Type I to Type II.

Type I and Type IV
N=288−96=192

	Number of subjects	Average rank assigned
d_I=+1	33	56.5
d_I=−1	79	56.5
d_I=+2	28	151
d_I=−2	49	151
d_I=+3	0	–
d_I=−3	3	191

Since T^+=6,092.5, T^-=12,435.5 and $T^+<T^-$, z=4.11 which has a probability far exceeding the probability associated with the occurrence when the null hypothesis (that there is no change in the support for C) is true, i.e. α=0.01. That is, there is a significant fall in the support for C for Type I to Type IV.

Type III and Type IV
N=288−107=181

	Number of subjects	Average rank assigned
d_I=+1	56	57
d_I=−1	57	57
d_I=+2	58	147
d_I=−2	9	147
d_I=+3	0	–
d_I=−3	1	181

Since T^{+}=11,718, T^{-}=4,753 and $T^{+}>T^{-}$, z=4.93 which has a probability far exceeding the probability associated with the occurrence when the null hypothesis (that there is no change in the support for C) is true, i.e. α=0.01. That is, there is a significant increase in the support for C for Type III to Type IV.

Type III and Type II:
N=288−57=231

	Number of subjects	Average rank assigned
d_I=+1	138	76
d_I=−1	13	76
d_I=+2	76	190
d_I=−2	1	190
d_I=+3	3	230
d_I=−3	0	–

Since T^{+}=25,618, T^{-}=1,178 and $T^{+}>T^{-}$, z=12.02 which has a probability far exceeding the probability associated with the occurrence when the null hypothesis (that there is no change in the support for C) is true, i.e. α=0.01. That is, there is a significant increase in the support for C for Type III to Type II.

4.4 Results by root profiles

Our empirical findings for the three root profiles within Type I show that the top-rank frequencies in favor of M and B=C do not differ across the roots. Since χ^2=0.1 which is less than $\chi^2_{0.05,2}$=5.99, we fail to reject the null hypothesis that the top frequencies in favor of M and B=C do not change from one root to another.

Comparing our empirical findings for the three root profiles within Type II, we see that the top-rank frequencies in favor of M and B=C do not differ across the roots. Since χ^2=1.7 which is less than $\chi^2_{0.05,2}$=5.99, we fail to reject the null hypothesis that the top-rank frequencies in favor of M and B=C do not change from one root to another.

When we compare our empirical findings for the four root profiles within Type III, it is observed that the respective top-rank frequencies in favor of M, B and C do not differ across the roots. Since χ^2=2.8 which is less than $\chi^2_{0.05,6}$=12.59, we fail to reject the null hypothesis that the respective top-rank frequencies in favor of, B and C do not change from one root to another.

Finally, our empirical findings for the eight root profiles within Type IV indicate no difference for the respective top-rank frequencies in favor of M, B and C. Since χ^2=9.7 which is less than $\chi^2_{0.05,14}$=23.68, we fail to reject the null hypothesis that the respective top frequencies in favor of M, B and C do not change from one root to another.

5 Results of self-selectivity and majoritarian approval on refined data

Confronted with the hypothetical preference profiles in their menus, our 288 subjects gave us their rankings over, in fact, two different alternative sets according

Table 6 Frequencies of appearance of *M* and *B*=*C* in the effective half of subjects' rankings

	M	*B*=*C*
Type I	82.6	80.9
Type II	81.6	95.1
Type I and Type II united	82.1	87.5

to the extent of these alternatives' "deserving" to be adopted. One of these alternative sets consists of the *M*−, the *B*=*C*-winner and two dummy alternatives, ***X*** and ***Y***, and the other set features the *M*−, the *B*-, the *C*-winner and one dummy alternative, *X*. As a result, we obtained two profiles each of size 4×576 consisting of the subjects' rankings for Type I and Type II, and Type III and Type IV hypothetical profiles.

Self-selectivity was introduced to the social choice literature by Koray. (Koray 2000) A social choice function employed by a society to make a choice from a given alternative set it faces is "self-selective" if and only if it chooses itself from among other rival such functions when employed by the society to make this latter choice as well. That is, for each of the two large profiles defined above we applied *M*, *B* and *C* as the aggregation procedures in order to see if *M*, *B* and *C* were able to pick the *M*, the *B*- and the *C*-winner, respectively, *in* that profile.

Each of our SCRs', *M*'s, *B*'s and *C*'s, aggregation of the subjects' rankings for the united hypothetical profiles of Type I and Type II chooses *B*=*C* as the winner. That is, *B* and *C* are "self-selective" in the first of our two large profiles, but *M* fails to be so. (*M*, *B* and *C* all pick *B*=*C* as the winner in both Type I and Type II.)

On the other hand, only *B* achieves "self selectivity" in the profile consisting of the subjects' rankings for the united hypothetical profiles of Type III and Type IV. Both *M* and *C* fail to do so since they both pick *B* as the social outcome. In the typewise analysis, we see that *B* always picks itself while *C* chooses itself only in Type IV profiles. On the other hand, *M*, instead of picking itself, chooses *B* in Type III and *C* in Type IV. (See Appendix C for the self-selectivity results by the root profiles of within each profile type.)

The percentage of appearance in the better effective half of the profiles consisting of the subjects' rankings extracted from Type I and Type II, and Type III and Type IV are presented in Tables 6 and 7, resp.

Both *M* and *B*=*C* respect majoritarian approval in Type I and Type II and, as a result, in their united form. In Type I profiles, *M* is top or second-ranked by more people than *B*. The contrary is true for Type II. In the united form, however, *B* is the most frequently top or second-ranked SCR.

Only *M* fails to respect majoritarian approval in Type III while all *M* and *B* and *C* respect majoritarian approval in Type IV. In the united form of Type III and Type

Table 7 Frequencies of appearance of *M*, *B* and *C* in the effective half of subjects' rankings

	M	*B*	*C*
Type III	47.2	91.7	60.1
Type IV	66.7	64.6	68.4
Type III and Type IV united	56.95	78.50	64.25

IV profiles, again, all three of our SCRs satisfy majoritarian approval. (See Appendix D for the results by root profiles.)

6 Concluding remarks

Our unambiguous empirical findings indicate that there is a considerable public support for majoritarian approval. We should note, however, that the 68 subjects who adopted *B*-scoring top-ranked the *B*-winner no matter whether it respects majoritarian approval or not, and second-ranked the *M*- or the *C*-winner depending on the *B*-score they possess in the hypothetical preference profile of concern. That is, these subjects did not take majoritarian approval into consideration while they were ranking the alternatives. Keeping this effect in bay, however, we still come across a strong support for majoritarian approval.

In our cross-type analysis, we find strong evidence that the support for *B*=*C* increases as this joint winner becomes majoritarian approved. We can easily make the statement that the support for *B* increases as it respects majoritarian approval for all pairwise comparisons between profile types at which *B* respects majoritarian approval and the ones at which *B* fails to respect the axiom. The support for *C*, on the other hand, increases in all such pairwise comparisons—except for the case between Type I and Type IV profiles. There are certainly points to consider having to do with this result. The requirements imposed upon the preference profiles for each type causes the profiles to posses certain characteristics which might fail to treat each SCR winner equally. As a matter of fact, the relative positioning of the *C*-winner in Type I and in Type IV are very different, i.e. the *B*-winner never shows up at the bottom row, whereas the *C*-winner appears once at the bottom row in three root profiles and twice at the bottom row in the remaining five root profiles. This construction of the profiles fulfilling our constraint in Type IV surely treats less favorable to *C* than it does to *B*. This might have an impact on the choices of the subjects. However, it is hard to distinguish the extent of the impact on the result due to the design of the study.

One might question the difference between the respective public support for *M* and *B* in the study presented in Sertel and Kara (2003) and in the present study. Here, one should consider the constraints imposed to the preference profiles and examine the root profiles used in the two studies. In the first study, each of the *M*- and the *B*-winner appears once in the top row, and the *M*-winner appears twice at the bottom while the *B*-winner is never bottom-ranked in any of the root profiles. In the present study, however, the *M*-winner is never top-ranked, but bottom-ranked once in all the three root profiles of Type I. On the other hand, The *B*-winner is top-ranked twice and never bottom-ranked in these root profiles. In Type II, the *M*-winner appears once at the top and once at the bottom row in two out of three root profiles while the *B*-winner is at the top twice without appearing at the bottom row at all. In Type III, the *M*-winner is top-ranked only once in two out of four root profiles while it is bottom-ranked twice in two root profiles and once in the other two. Here, the *B*-winner appears three times and once at the top row and the bottom row, resp., in all four root profiles. In Type IV, The *M*-winner appears twice only in one of the root profiles and once in three profiles out of eight. It is bottom-ranked twice and once in five and three profiles, respectively. On the other hand, the *B*-

winner is top-ranked three times in all the root profiles while it is not bottom-ranked in any of them.

If we are to say a few words about the reasons underlying the popularity of *B*-scoring in this study, we should first note that the subjects were concerned about achieving consistency and scoring is an easy way of guaranteeing that. Second, scoring method enables the subject to achieve a full-ranking of the alternatives as we asked them to do so. Only when two alternatives (of course, which are distinct from the *B*-winner) obtained the same *B*-score, then the subject had to make a choice. Third, people are exposed to the idea of scoring in daily life through some TV shows and contests, such as "Biri Bizi Gözetliyor", Eurovision Song Contest, etc. Neither *M* nor *C* has such popular applications in daily life.

As a direction for future research, the study can be designed so as to include those cases where the requirements imposed on the preference profiles are relaxed, i.e. distinguishing the winner of two SCRs the most. We know that the more restrictions are imposed to the profiles, the more dramatically differ the positioning of the SCRs in concern, i.e. the number of times each SCR appears at the top and the bottom rows. That is, the study can be designed so as to enable the researcher to distinguish between the different impacts on the choices to be made by the subjects.

On the other hand, there surely is a need to invent certain statistical tests which can interpret the data to be obtained in this type of studies. As long as the design requires each subject to view more than one preference profile, an appropriate test is needed to deal with the interdependency problem associated with the data to be obtained.

As another direction for future research, the study can be extended over a greater variety of cultural settings, be this from society to society, or from student population to the "men in the street." Since majoritarian approval is an axiom of justice concerning the aggregation of individual preferences into a social outcome, a cross-cultural study could throw a light upon the existence question of a "universal" notion of justice in the above sense.

Finally, a computational study can be designed so as to detect the percentages for each SCR of concern to satisfy certain axioms. This study requires the extension of this detection for varying sizes of profiles. The results to be obtained from that study will be very useful to interpret the data obtained from such studies as the present one.

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Appendix A

Root profiles of type II

1.	*B*	B	X	X	Y	2.	*B*	B	X	X	Y
	M	M	B	M	M		*M*	M	B	M	M
	X	X	M	B	B		*X*	X	Y	B	B
	Y	Y	Y	Y	X		*Y*	Y	M	Y	X

3.	*B*	B	X	X	Y	4.	*B*	B	X	X	Y
	M	M	B	M	M		*M*	M	B	M	M
	X	Y	Y	B	B		*X*	Y	M	B	B
	Y	X	M	Y	X		*Y*	X	M	Y	X
5.	*B*	B	X	X	Y	6.	*B*	B	X	X	X
	M	M	B	M	M		*M*	M	B	M	M
	Y	Y	M	B	B		*Y*	Y	Y	B	B
	X	X	Y	Y	X		*X*	X	M	Y	X
7.	*B*	B	X	X	Y	8.	*B*	B	X	X	X
	M	M	B	M	M		*M*	M	B	M	M
	Y	Y	Y	Y	B		*Y*	Y	Y	Y	B
	X	X	M	B	X		*X*	X	M	B	X
9.	*B*	B	X	X	Y	10.	*B*	B	X	X	Y
	M	M	B	M	M		*M*	M	M	M	B
	X	X	Y	Y	B		*X*	Y	B	B	M
	Y	Y	M	B	X		*Y*	X	Y	Y	X
11.	*B*	B	X	X	Y	12.	*B*	B	X	X	Y
	M	M	M	M	B		*M*	M	M	M	B
	Y	Y	B	B	M		*X*	X	B	B	M
	X	X	Y	Y	X		*Y*	Y	Y	Y	X
13.	*B*	B	X	X	Y	14.	*B*	B	X	X	Y
	M	M	M	M	B		*M*	M	M	M	B
	X	X	B	B	X		*X*	Y	B	B	X
	Y	Y	Y	Y	M		*Y*	X	Y	Y	M
15.	*B*	B	X	X	Y	16.	*B*	B	X	X	Y
	M	M	M	M	B		*M*	M	M	M	B
	Y	Y	B	B	X		*Y*	Y	B	Y	X
	X	X	Y	Y	X		*X*	X	Y	B	M
17.	*B*	B	X	X	Y	18.	*B*	B	M	X	X
	M	M	M	M	B		*M*	M	B	M	Y
	Y	X	B	Y	X		*X*	X	X	B	B
	X	Y	Y	B	M		*Y*	Y	Y	Y	M
19.	*B*	B	M	X	X	20.	*B*	B	M	X	X
	M	M	B	M	Y		*M*	M	B	M	Y
	X	X	Y	B	B		*X*	Y	Y	B	B
	Y	Y	X	Y	M		*Y*	X	X	Y	M
21.	*B*	B	M	X	X	22.	*B*	B	M	X	X
	M	M	B	M	Y		*M*	M	B	M	Y
	X	Y	X	B	B		*Y*	Y	X	B	B
	Y	X	Y	Y	M		*X*	X	Y	Y	M

23.	*B*	B	M	X	X	24.	*B*	B	M	X	X
	M	M	B	M	Y		*M*	M	Y	B	M
	Y	Y	Y	B	B		*X*	X	B	Y	B
	X	X	X	Y	M		*Y*	Y	X	M	Y
25.	*B*	B	M	X	X	26.	*B*	B	M	X	X
	M	M	Y	B	M		*M*	M	Y	B	M
	X	Y	B	Y	B		*Y*	Y	B	Y	B
	Y	X	X	M	Y		*X*	X	X	M	Y
27.	*B*	B	M	X	Y	28.	*B*	B	M	X	Y
	M	M	B	M	X		*M*	M	B	M	X
	X	X	X	B	B		*X*	X	Y	B	B
	Y	Y	Y	Y	M		*Y*	Y	X	Y	M
29.	*B*	B	M	X	Y	30.	*B*	B	M	X	Y
	M	M	B	M	X		*M*	M	B	M	X
	X	Y	X	B	B		*X*	Y	Y	B	B
	Y	X	Y	Y	M		*Y*	X	X	Y	M
31.	*B*	B	M	X	Y	32.	*B*	B	M	X	Y
	M	M	B	M	X		*M*	M	B	M	X
	Y	Y	Y	B	B		*Y*	Y	X	B	B
	X	X	X	Y	M		*X*	X	Y	Y	M
33.	*B*	B	M	X	Y	34.	*B*	B	M	X	Y
	M	M	X	B	M		*M*	M	X	B	M
	X	X	B	Y	B		*X*	Y	B	Y	B
	Y	Y	Y	M	X		*Y*	X	Y	M	X
35.	*B*	B	M	X	Y	36.	*B*	B	M	X	Y
	M	M	X	B	M		*M*	M	Y	B	M
	Y	Y	B	Y	B		*X*	X	B	Y	B
	X	X	Y	M	X		*Y*	Y	X	M	X
37.	*B*	B	M	X	Y	38.	*B*	B	M	X	Y
	M	M	Y	B	M		*M*	M	Y	B	M
	X	Y	B	Y	B		*Y*	Y	B	Y	B
	Y	X	X	M	X		*X*	X	X	M	X

Appendix B

An exemplary questionnaire

University:
Department:
Year:

A group of citizens faces four alternatives. Exactly one of these is to be adopted.

Each citizen ranks the four alternatives according to his/her own preference. For example, a member ranking the alternatives as

a
b
c
d

has ranked "a" as his/her top choice, "b" as his/her second choice, "c" as his/her third choice and "d" as his/her last choice.

Below are presented four distinct groups whose members (citizens) exhibit various rankings of the alternatives according to their personal preferences. For each group, taking an impartial point of view, you are asked to indicate which alternative ("a" or "b" or "c" or "d") should be adopted, which should be adopted if this becomes unavailable, and which should be adopted that if, too, becomes unavailable, and which should especially be avoided. You are also encouraged to give a brief explanation concerning the reasoning on which your views rest for each of the four groups.

1.mbr.	2.mbr.	3.mbr.	4.mbr.	5.mbr.	6.mbr.	7.mbr.
a	b	c	a	c	a	c
b	d	b	c	d	b	b
c	a	a	b	a	c	d
d	c	d	d	b	d	a

1.mbr.	2.mbr.	3.mbr.	4.mbr.	5.mbr.	6.mbr.	7.mbr.
d	a	b	b	d	b	d
c	d	c	a	c	c	c
b	b	a	c	b	d	b
a	c	d	d	a	a	a

1.mbr.	2.mbr.	3.mbr.	4.mbr.	5.mbr.
a	c	b	b	a
d	d	d	c	d
b	a	a	a	b
c	b	c	d	c

1.mbr.	2.mbr.	3.mbr.	4.mbr.	5.mbr.
a	d	a	d	b
d	c	c	c	c
c	b	d	b	d
b	a	b	a	a

Appendix C

Self-selectivity results by root profiles

* Type I (*M*/*B*~*C*)

	Selecting SCR	Selected SCR
Root profile 1	M	B=*C*
	B	B=*C*
	C	B=*C*
Root profile 2	M	B=*C*
	B	B=C
	C	B=*C*
Root profile 3	M	B=*C*
	B	B=*C*
	C	B=*C*

* Type II (*M*×*B*~*C*/)

	Selecting SCR	Selected SCR
Root profile 1	M	B=*C*
	B	B=*C*
	C	B=*C*
Root profile 2	M	B=*C*
	B	B=*C*
	C	B=*C*
Root profile 3	M	B=*C*
	B	B=*C*
	C	B=*C*

* Type III (*M*×*B*/*C*)

	Selecting SCR	Selected SCR
Root profile 1	*M*	B
	B	B
	C	B
Root profile 2	M	B
	B	B
	C	B
Root profile 3	M	B
	B	B
	C	B
Root profile 4	M	B
	B	B
	C	B

* Type IV ($M \times C/B$)

	Selecting SCR	Selected SCR
Root profile 1	M	M
	B	M
	C	M
Root profile 2	M	C
	B	C
	C	C
Root profile 3	M	B
	B	B
	C	B
Root profile 4	M	M
	B	B and M
	C	No winner
Root profile 5	M	C
	B	C
	C	C
Root profile 6	M	C
	B	C
	C	No winner
Root profile 7	M	C
	B	B
	C	No winner
Root profile 8	M	B
	B	B
	C	B

Appendix D

Majoritarian approval results by root profiles

* Type I ($M/B{\sim}C$)

	M (%)	B=C (%)
Root profile 1	72.9	80.2
Root profile 2	89.6	80.2
Root profile 3	85.4	82.3

* Type II ($M \times B{\sim}C/$)

	M (%)	B=C (%)
Root profile 1	79.2	96.9
Root profile 2	99.0	98.0
Root profile 3	66.6	90.6

* Type III ($M \times B/C$)

	M (%)	B (%)	C (%)
Root profile 1	45.8	91.7	61.1
Root profile 2	45.8	94.4	58.3
Root profile 3	48.6	89.9	61.1
Root profile 4	48.6	91.7	59.8

* Type IV ($M \times C/B$)

	M (%)	B (%)	C (%)
Root profile 1	80.5	63.9	52.8
Root profile 2	50.0	61.1	88.9
Root profile 3	88.9	61.1	49.0
Root profile 4	80.6	63.9	55.6
Root profile 5	58.3	58.3	83.4
Root profile 6	61.1	63.9	75.0
Root profile 7	66.6	58.3	75.0
Root profile 8	47.2	86.1	66.7

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