Soc Choice Welfare (2003) 21: 347–384
DOI: 10.1007/s00355-003-0263-5

Social Choice and Welfare

On the informational basis of social choice

Marc Fleurbaey

Université de Pau et des Pays de l'Adour, UFR Droit, Economie, Gestion, BP 1633, 64016, Pau cedex, France (e-mail: marc.fleurbaey@univ-pau.fr) and CATT, THEMA, IDEP

Abstract. The notion of informational basis in social choice can be broadened so as to cover not only the standard notions related to interpersonal utility comparisons, but also information about utilities or preferences at (ir)relevant alternatives, non-utility features of alternatives, personal responsibility, unconcerned subpopulations, and feasibility constraints. This paper proposes a unified conceptual framework for all these notions, and analyzes the kind of information retained in each case. This new framework yields a deeper understanding of the difficulties and possibilities of social choice. New welfarism theorems are also obtained.

1 Introduction

The notion of informational basis has been coined by Sen (1970) and d'Aspremont and Gevers (1977), in order to describe the sets of data that are used in the determination of social preferences over alternatives. Although, in his work as a whole, Sen has studied the issue of information in social choice from many different angles, especially in the debate about welfarism, the notion of informational basis itself has often been conceived in a rather narrow way, that is, in terms of interpersonal comparisons of utility (or any similar notion of well-being). Moreover, a large consensus still exists in social choice about the fact that interpersonally comparable indices of individual well-being are a

I am indebted to W. Bossert, W. Gaertner, F. Gaspart, L. Gevers, N. Gravel, S. Kolm, M. Le Breton, Ph. Mongin, P. Hammond, K. Suzumura, K. Tadenuma, A. Trannoy and J. Weymark for many conversations on topics related to this paper. I have benefited from reactions of the audience at the conference in honor of Louis Gevers, Namur 1999, and at a seminar at the LSE, and from comments by two referees and the editor, F. Maniquet. None of them is responsible for the shortcomings of the paper.

necessary piece of information in order to obtain consistent social preferences and to avoid the difficulties so famously described in Arrow's impossibility theorem (Arrow 1951).[1] Such a consensus obviously reinforces the widespread impression that the question of the informational basis does indeed revolve around this key issue of interpersonal comparisons.

In this paper, I propose broadening the concept of informational basis, so as to make it possible to rigorously discuss, in a unified framework, the use of information not only about individual utilities but also about individual preferences, individual talents and handicaps, and about other features of the economy such as the set of feasible allocations.

The purpose of the paper is not just to provide a more suitable toolbox for the analysis of information in social choice. The broadened conceptual frame should make it easier to think about how to introduce more information than Arrow allowed in his famous impossibility theorem, and should make it more transparent that the consensus about the need for interpersonally comparable indices of well-being is questionable. In a variety of models, Samuelson (1977), Pazner (1979), Kaneko and Nakamura (1979), Dhillon and Mertens (1999), and Fleurbaey and Maniquet (1996, 2000, 2001) have articulated the dissident view that ordinal non-comparable preferences may be sufficient information for consistent and equitable social preferences. The concepts proposed in this paper will clarify how this outlying thesis fits in a general coherent picture.

The broader conceptual framework proposed in this paper will not only explain how social choice can be made possible in absence of interpersonally comparable indices of well-being. It will also relate the difficulties and possibilities of social choice to notions which are not usually thought to be central to such issues. Social preferences may be more or less hard to construct depending on the adoption of principles of responsibility, separability, and independence of feasibility constraints. In particular, it will be shown below that Arrow's theorem can be reinterpreted in terms of the following "trilemma". Reasonable social preferences cannot at the same time hold individuals responsible for their utility functions (as distinct from their preferences), disregard utilities at infeasible alternatives, and disregard feasibility constraints.

The paper is organized as follows. The next section presents the framework and the basic notions. The key concept is related to the information used in the *social ranking of two alternatives*. Then, Sect. 3 through 8 examine the use of information about, successively, utilities (interpersonal comparisons), relevant or irrelevant alternatives, non-utility features of alternatives (Paretianism), personal responsibility, unconcerned subpopulations (separability), and feasibility constraints. Since the classical notion of informational basis is limited to the first item of this list, the main achievement in these sections is the integration of all the other notions into a unified conceptual framework, and actually, one and the same family of axioms serves to cover all these informational issues. Every section is made up of a short formal analysis, followed by some comments

[1] For a very elegant presentation of this consensual view, see Sen (1999).

on the ethical foundations of the informational approaches under consideration. Then, sect. 9 studies some interesting combinations of informational requirements introduced separately in the previous sections. Section 10 examines the possibility of constructing consistent social preferences on the basis of various kinds of information and interpersonal comparisons, and proposes a distinction between different kinds of welfarism and non-welfarism. Section 11 summarizes the results and concludes.

2 Framework and basic notions

The standard social choice problem is the determination of a mapping which defines social preferences over a given set of alternatives as a function of the profile of preferences of a given population. This framework is too restrictive if one wants to study how social preferences could depend on other characteristics of the individuals (such as utility functions, or talents), on the population itself (demography), or on the set of alternatives.

A more general framework is the following. A *social choice entry* is $e = (\theta_N, X)$, where N is the set of individuals of the relevant population, $\theta_N = (\theta_i)_{i \in N}$ is the profile of individual characteristics, and X the set of alternatives. The *social choice problem* is to find a *social ordering function* (SOF) $\overline{R}$ such that, for every $e \in \mathscr{E}$, where $\mathscr{E}$ is a relevant domain of entries, $\overline{R}(e)$ is a complete preorder on X. Social preferences defined by the preorder $\overline{R}(e)$ will be denoted $x\overline{R}(e)y$, with related strict preferences $x\overline{P}(e)y$ and indifference $x\overline{I}(e)y$. Let $\overline{N} = \cup_{(\theta_N, X) \in \mathscr{E}} N$ denote the global population from which particular populations N are drawn.

The interest of this problem can be explained as follows. The set X may consist of all alternatives which are feasible in some general sense. But the actual decisions the population will have to make may be limited to a strict subset of X. For instance, X may be the set of all alternatives over which individual preferences are defined, and scarcity constraints may limit actual choices. Or X may be the set of technically feasible alternatives, and incentive compatibility constraints may make it impossible to achieve all alternatives which are technically feasible. Or the social decisions may always take the form of a piecemeal reform opposed to the status quo. There are a variety of possible constraints preventing society to choose directly from the whole set X. In all such contexts, a complete preorder over X is a very useful tool. Moreover, the theory would be quite useless if it were able to define social preferences for a very small class of social choice entries. It must be able to solve the problem for a sufficiently wide class, covering all relevant cases one may envision. This is why a function, not just one preorder for one entry, is sought.[2]

[2] As defined here, the social choice problem is still too narrow to study some important issues. In particular, it does not make it possible to study the issue of optimal population size, which requires a preorder over different alternatives for different populations. For a synthesis on this issue, see Blackorby et al. (1997).

Obviously, the nature of the social choice problem depends a lot on the domain of entries. Depending on $\mathscr{E}$, the social choice problem may consist in an abstract collective problem of choice, or in a more concrete allocation problem.[3]

It is assumed here that θ_i is a complete description of individual i's characteristics. For any notion of utility, one can then define a mapping U such that $U(\theta_i) : X_i \to \mathbb{R}$ is i's utility function, defined over some relevant domain X_i which contains X. We do not restrict the domain of definition of utility functions to X, because in some problems, X describes a feasible set and individual utilities are defined over a much broader set. As it will appear in the sequel, it is not an innocuous restriction in such cases to disregard utilities outside X.

The central topic of this paper will be the information used by a SOF $\overline{R}$ about an entry $e = (\theta_N, X)$ when ranking two alternatives $x, y \in X$. Typically, there is some information used by $\overline{R}$, and the rest is disregarded. This can be captured by introducing a function f, hereafter called a *data filter*, which screens out all irrelevant information and retains only what is considered relevant. In other words, the SOF $\overline{R}$ will satisfy the following axiom for some well chosen data filter f:

Independence of Non-f Information (INfI):

$$\forall e = (\theta_N, X), e' = (\theta'_{N'}, X') \in \mathscr{E}, \forall (x, y) \in X^2, \forall (x', y') \in (X')^2,$$
$$f(e, (x, y)) = f(e', (x', y')) \Rightarrow \left[x\overline{R}(e)y \Leftrightarrow x'\overline{R}(e')y' \right].$$

I claim that most if not all of the issues pertaining to the notion of informational basis in social choice revolve around particular features and properties of the data filter f. This claim will be illustrated, if not proved, in the following sections.

3 Utility transformations

The notion of informational basis has been traditionally associated to transformations of profiles of utility functions (Sen 1970; d'Aspremont and Gevers 1977), and transformations of vectors of utility levels (Gevers 1979). The purpose of this section is to relate such notions to INfI, and in the process to clarify a few points.

---

[3] The notion of social choice problem defined here is sufficiently general to cover the theory of fair allocation, which has mostly been confined to the search of coarse preorders distinguishing good allocations from bad ones. Such coarse preorders are nevertheless complete, and therefore the "allocation rules" of the theory of fair allocation can be described as SOFs for the social choice problem.

3.1 Formal analysis

Let U be a function deriving any individual i's utility function from her characteristics θ_i. That is, if i's utility function is u_i, then $u_i = U(\theta_i)$. With an abuse of notation, let $U(\theta_N)$ denote the profile of utility functions u_N such that $u_i = U(\theta_i)$ for every $i \in N$.

Let Φ_{ONC} denote the set of vectors of mappings $\varphi = (\varphi_i)_{i \in \overline{N}}$ such that for every $i \in \overline{N}, \varphi_i : \mathbb{R} \to \mathbb{R}$ is increasing. The subscript *ONC* refers to the "ordinal non-comparable" information setting that this set implies, as will be explained below. With a slight abuse of notation, for any $e = (\theta_N, X)$, let $\varphi(u_N)$ denote the vector of utility functions u'_N such that for every $i \in N$, and every $x \in X_i, u'_i(x) = \varphi_i(u_i(x))$. Similarly, let $\circ$ simultaneously denote the ordinary composition of functions, and also the composition operation applied to vectors of functions component-wise, so that $\varphi \circ \varphi' = (\varphi_i \circ \varphi'_i)_{i \in \overline{N}}$. With this convention, $(\Phi_{ONC}, \circ)$ is an algebraic group.[4]

In the literature, the sets of transformations have been used in the construction of invariance axioms such as the following one. Let $\Phi \subset \Phi_{ONC}$.

Invariance to Φ (*INV* Φ):

$$\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N),$$
$$\exists \varphi \in \Phi,\ u'_N = \varphi(u_N) \Rightarrow \overline{R}(e) = \overline{R}(e').$$

As an illustration, the axiom INVΦ_{ONC} means that any transformation of utility functions which does not alter individual ordinal preferences leaves social preferences unchanged. In other words, under this axiom, the only information about individual utilities that is used is contained in individual non-comparable preferences.

The question to be addressed here is how such an axiom can be translated into a requirement imposed on the data filter f of the INfI axiom. It is immediate that $\overline{R}$ satisfies INVΦ whenever it satisfies INfI for a data filter f such that:

$$\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N),$$

$$\exists \varphi \in \Phi,\ u'_N = \varphi(u_N) \Rightarrow f(e, .) = f(e', .).$$

But it is also useful to examine how a particular INfI axiom can serve to express the INVΦ condition, because this helps to understand the nature of the restrictions imposed by INVΦ. For any subset $\Phi \subset \Phi_{ONC}$, let $(\Phi_g, \circ)$ denote the subgroup of $(\Phi_{ONC}, \circ)$ generated by Φ (that is, the smallest superset of Φ which is a group). This set is unique, as recalled in the following lemma.

[4] The importance of algebraic groups in this area was emphasized by Roberts (1980).

Lemma 1. *Let* $\Phi \subset \Phi_{ONC}$, *and* Φ^{-1} *denote the set of inverses of elements of* Φ. *Then*

$$\Phi_g = \{\varphi \in \Phi_{ONC} | \exists \varphi_1, \ldots, \varphi_m \in \Phi \cup \Phi^{-1},\ \varphi = \varphi_1 \circ \ldots \circ \varphi_m\}.$$

The following proposition deciphers the consequences of INVΦ over the data filter f involved in INfI, when the two axioms are made equivalent. First, notice that the set Φ_g generates an equivalence relation $\sim_{\Phi_g}$ on the domain $\mathscr{E}$, which is defined by

$$(\theta_N, X) \sim_{\Phi_g} (\theta'_{N'}, X') \Leftrightarrow N = N' \text{ and } X = X' \text{ and } \exists \varphi \in \Phi_g, U(\theta'_N) = \varphi(U(\theta_N)).$$

For any $e \in \mathscr{E}$, let $\sim_{\Phi_g}(e)$ denote the equivalence class for $\sim_{\Phi_g}$ to which e belongs.

Proposition 1. *Consider any subset* $\Phi \subset \Phi_{ONC}$, *and assume that* $\mathscr{E}$ *is rich enough so that, for any* $e = (\theta_N, X) \in \mathscr{E}$, *and any* $\varphi \in \Phi_g$, *there exists* $(\theta'_N, X) \in \mathscr{E}$ *such that* $U(\theta'_N) = \varphi(U(\theta_N))$. *The axiom INV*$\Phi$ *is then equivalent to INfI for f defined by:*

$$f(e, (x, y)) = (\sim_{\Phi_g}(e), (x, y)) \quad .$$

Proof.[5] INVΦ implies INfI. Let $f(e, (x, y)) = f(e', (x', y'))$. Then, by definition of f, one has $x = x', y = y', N = N', X = X'$, and

$$\exists \varphi \in \Phi_g, U(\theta'_N) = \varphi(U(\theta_N)).$$

By the above lemma, there exist $\varphi_1, \ldots, \varphi_m \in \Phi \cup \Phi^{-1}$, such that $\varphi = \varphi_1 \circ \ldots \circ \varphi_m$. One then has

$$U(\theta'_N) = \varphi_1 \circ \ldots \circ \varphi_m(U(\theta_N)).$$

By the richness assumption on $\mathscr{E}$, and since $\varphi_m \in \Phi_g$, there exists $e^m = (\theta^m_N, X) \in \mathscr{E}$ such that $U(\theta^m_N) = \varphi_m(U(\theta_N))$, and therefore $U(\theta_N) = \varphi_m^{-1}(U(\theta^m_N))$. Since either $\varphi_m \in \Phi$ or $\varphi_m^{-1} \in \Phi$, then by INVΦ, one has $\overline{R}(e) = \overline{R}(e^m)$. Similarly, let $e^{m-1} = (\theta^{m-1}_N, X) \in \mathscr{E}$ be such that $U(\theta^m_N) = \varphi_{m-1}(U(\theta^m_N))$. By INV$\Phi$, one has $\overline{R}(e^m) = \overline{R}(e^{m-1})$. By iteration of this argument, one finally obtains $\overline{R}(e) = \overline{R}(e')$, and therefore $\overline{R}(e)$ and $\overline{R}(e')$ coincide on $\{x, y\}$.

INfI implies INVΦ. Let $e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, u_N = U(\theta_N)$, $u'_N = U(\theta'_N)$, be such that

$$\exists \varphi \in \Phi,\ u'_N = \varphi(u_N).$$

Take any $(x, y) \in X^2$. Since $\Phi \subset \Phi_g$, one has $f(e, (x, y)) = f(e', (x, y))$, so that, by IN$f$I, $\overline{R}(e)$ and $\overline{R}(e')$ coincide on $\{x, y\}$. Since this applies to any $(x, y) \in X^2, \overline{R}(e) = \overline{R}(e')$. ■

[5] Prop. 1 just gives the simplest example of f such that INVΦ and INfI are equivalent. There is a more general theorem saying that this equivalence is obtained for all f which are isomorphic to the one given here. This is true for all results below.

The formula $f(e,(x,y)) = (\sim_{\Phi_g}(e),(x,y))$ captures the substance of INVΦ. The only information that is retained about e is the equivalence class to which it belongs, and this implies lumping together profiles of utility functions which are related by transformations of Φ_g (not just Φ).

A corollary of this result, then, is that INVΦ is equivalent to INVΦ_g, which means that there is no obligation to restrict attention to sets of transformations Φ which are algebraic groups, and also no limitation in doing so. This corollary was proved in a particular case by d'Aspremont and Gevers (1977), when they showed (their Theorem 1) the equivalence of referring to the set of cardinal non-comparability

$$\Phi_{CNC} = \left\{\varphi \in \Phi_{ONC} | \exists \alpha \in \mathbb{R}^{\overline{N}}, \beta \in \mathbb{R}^{\overline{N}}_{++}, \forall i \in \overline{N}, \forall u \in \mathbb{R}, \varphi_i(u) = \alpha_i + \beta_i u\right\}$$

and to the set

$$\Phi_{AG} = \left\{\varphi \in \Phi_{ONC} | \exists \alpha \in \mathbb{R}, \beta \in \mathbb{R}^{\overline{N}}_{++}, \forall i \in \overline{N}, \forall u \in \mathbb{R}, \varphi_i(u) = \alpha + \beta_i u\right\}.$$

The equivalence between INVΦ_{AG} and INVΦ_{CNC} is simply the consequence of $(\Phi_{AG})_g = \Phi_{CNC}$.

For the rest of the discussion, it is useful to introduce the set of cardinal unit comparability

$$\Phi_{CUC} = \left\{\varphi \in \Phi_{ONC} | \exists \alpha \in \mathbb{R}^{\overline{N}}, \beta \in \mathbb{R}_{++}, \forall i \in \overline{N}, \forall u \in \mathbb{R}, \varphi_i(u) = \alpha_i + \beta u\right\}.$$

The axiom INVΦ_{CUC}, in particular, deletes any information about interpersonal comparisons of utility *levels* (because the α_i may change the zeros of individual utility functions independently), but preserves information about comparisons of utility *differences*. It is satisfied by the utilitarian SOF, which is defined here by: $\forall e = (\theta_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), \forall x, y \in X$,

$$x\overline{R}(e)y \Leftrightarrow \sum_{i \in N} u_i(x) \geq \sum_{i \in N} u_i(y).$$

And the set of ordinal measurability and full comparability

$$\Phi_{OFC} = \left\{\varphi \in \Phi_{ONC} | \forall i, j \in \overline{N}, \varphi_i = \varphi_j\right\}$$

yields an axiom INVΦ_{OFC} which preserves information about interpersonal comparisons of utility levels, but not more, and is satisfied in particular by the maximin SOF, defined by: $\forall e = (\theta_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), \forall x, y \in X$,

$$x\overline{R}(e)y \Leftrightarrow \min_{i \in N} u_i(x) \geq \min_{i \in N} u_i(y).$$

3.2 Ethical comments

An axiom like INVΦ restricts the kind of information that can be used by the SOF about the profile of individual utility functions. The literature[6] studying

[6] For two excellent surveys, see Bossert and Weymark (1998) and d'Aspremont and Gevers (2002).

such axioms has usefully clarified the informational content of various social welfare functions, most notably utilitarianism and the maximin criterion. But it usually left unclear why an axiom of the INVΦ sort should be viewed as appealing, in the normative perspective of the construction of good social preferences. A superficial reading might give the impression that it all has to do with the fact that some information about individual utilities may just happen not to be available, so that an axiom like INVΦ may help analyzing the consequences of this information shortage on the social objective. This idea is quite questionable. The construction of good social preferences should involve all ethically *relevant* information, independently of what is *available* or *possible*.[7] As an example, suppose a utilitarian social planner is told that the only information available is about individual levels of utility, without any clue about utility differences and intensities. Should this make the social planner accept the axiom INVΦ_{OFC}? This axiom is not satisfied by the utilitarian SOF. Should the planner abandon her utilitarian preferences and become an egalitarian, since the maximin SOF satisfies INVΦ_{OFC}? A more consistent attitude would be for her to keep her utilitarian social preferences and to conclude that the implementation of these preferences will be difficult. Trying to collect the relevant data might be the first way out to probe.

In conclusion, one should reserve the use of INV Φ for discarding irrelevant information, not for addressing information shortages. The various axioms INV Φ, for different sets Φ, need not appear equally justifiable, in this respect. For instance, it is not very hard to justify INVΦ_{ONC}, which excludes any information about utilities except that contained in ordinal non-comparable preferences, by invoking the individuals' responsibility for their subjective satisfaction. One may want to respect individual *preferences* and therefore take them into account, but disregard *utilities* as a purely private matter. This is just what INVΦ_{ONC} stipulates.[8] It seems harder to defend INVΦ_{CUC}. Truly enough, this axiom is logically weaker than INVΦ_{ONC}, so that, rigorously, it should be easier to justify. But it is hard to defend it without justifying INVΦ_{ONC} in the process. How could one argue that utility levels do not matter whereas utility differences may matter?[9] It is also quite hard to defend INVΦ_{OFC}, except through a direct defense of the absolute priority of the worst-off, as embodied in the maximin and leximin criteria, or in an equity axiom such as Hammond's (1976). But deriving a justification of INVΦ_{OFC} from the maximin criterion is

[7] This viewpoint is defended e.g., in Kolm (1996).

[8] This line of argument can be found in Rawls (1982), Dworkin (2000).

[9] One finds awkward arguments to this effect in Harsanyi (1976, p. 72 and pp. 75–76). One is that medical treatment should go to whom it benefits more, and not in priority to a poor (as opposed to a millionaire). A second example is about giving a small present to one of two boys, a happy one who derives great joy from presents, and an unhappy one who does not derive much pleasure from small presents. Harsanyi argues in favor of giving the present to the first one, on the ground that the giver is not responsible for the initial inequality. In both examples, the intuition seems to be driven by a matter of different spheres of justice.

unwelcome for a literature which goes the other way, deriving the maximin from $\mathrm{INV}\Phi_{OFC}$. In conclusion, the ethical foundations of the $\mathrm{INV}\phi$ approach are rather fragile, with at least one exception: $\mathrm{INV}\Phi_{ONC}$. The widespread belief that, in view of Arrow's impossibility, the $\mathrm{INV}\Phi_{ONC}$ axiom leads social choice to a dead end, has restricted its use. In view of the new possibilities discussed below, it may be rehabilitated.

This criticism of the INV Φ approach should not be understood as meaning that introducing interpersonally comparable indices of well-being is not sound. Quite to the contrary, the idea of constructing such indices, as defended for instance in Sen's broad theory of equality, is very respectable. The point here is that particular axioms such as $\mathrm{INV}\Phi_{CUC}$ or $\mathrm{INV}\Phi_{OFC}$ may not be very helpful. It is not on the basis of $\mathrm{INV}\Phi_{OFC}$ that Sen defends an egalitarian view, for instance, but because of the direct ethical value of equality.

4 Irrelevant alternatives

The INV Φ axioms (for various sets Φ) are the core of the traditional notion of informational basis. But they are just the starting point of our analysis. A first broadening of the notion of informational basis is now proposed, by encompassing the condition of "independence of irrelevant alternatives", a condition which has usually been used jointly with axioms bearing on transformations of utility functions.

4.1 Formal analysis

The following axiom says that a change in the population profile which does not alter the levels of utility at two alternatives should not modify social preferences on these two alternatives.

Independence of Other Alternatives (IOA):

$$\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N), \forall x, y \in X,$$

$$u_N\big|_{\{x,y\}} = u'_N\big|_{\{x,y\}} \Rightarrow \overline{R}(e)\big|_{\{x,y\}} = \overline{R}(e')\big|_{\{x,y\}}.$$

This axiom is usually called "independence of irrelevant alternatives". This name is, however, the source of two confusions. First, it gives the wrong impression that it is just an innocuous adaptation of Arrow's axiom of independence, which requires social preferences over two alternatives not to change when individual preferences on these two alternatives do not change. This axiom is actually a very substantial *weakening* of Arrow's axiom, as shown below.[10]

10 A formal definition of Arrow's axiom is given in Subsect. 9.2. The idea that IOA is a faithful translation of Arrow's axiom is tempting when one presents the introduction of utilities as a change of framework, rather than just a change of axioms in a more general framework. See e.g., Sen (1986, p. 1114): 'For a SWFL the Arrow conditions are readily redefined'. Hammond (1987) proposed the alternative name "Independence of Irrelevant Utilities".

Second, it makes a confusion between the uncontroversial requirement that irrelevant information should be disregarded, and the controversial definition of the irrelevant information. From the standpoint of deontology, it is probably preferable to use names expressing the content of an axiom in a neutral and transparent manner.

Now we can begin to see the power of the INfI axiom, since it can also express this new condition. What requirements on the data filter f does IOA imply? The SOF $\overline{R}$ satisfies IOA whenever it satisfies INfI for a data filter f such that: $\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N), \forall x, y \in X,$

$$u_N\big|_{\{x,y\}} = u'_N\big|_{\{x,y\}} \Rightarrow f(e,(x,y)) = f(e',(x,y)).$$

Again, it is useful to look for conditions under which the axioms IOA and INfI are actually equivalent.

Proposition 2. *A SOF $\overline{R}$ satisfies IOA if and only if it satisfies INfI for f defined by:*

$$f(e,(x,y)) = (u_N(x), u_N(y), X, (x,y)),$$

where $e = (\theta_N, X), u_N = U(\theta_N)$.

Proof. IOA implies INfI. Let $f(e,(x,y)) = f(e',(x',y'))$. This implies $x = x', y = y', N = N', X = X', u_N(x) = u'_N(x), u_N(y) = u'_N(y)$, and therefore, by IOA, $\overline{R}(e)\big|_{\{x,y\}} = \overline{R}(e')\big|_{\{x,y\}}$.

INfI implies IOA. Let $u_N\big|_{\{x,y\}} = u'_N\big|_{\{x,y\}}$. Then $f(e,(x,y)) = f(e',(x,y))$, so that by INfI, $\overline{R}(e)\big|_{\{x,y\}} = \overline{R}(e')\big|_{\{x,y\}}$. ■

As this proposition shows, IOA retains very little information about individual situations at x and y. And one may want to introduce some additional information, in particular about the profile of utility functions, which describes the kind of population concerned with the construction of social preferences. This can be done by taking account of utilities at other alternatives. There are many possible subsets of other alternatives which may be introduced. We limit our attention here to two typical examples.[11] The first one consists in taking account of utilities over the whole set X, and disregarding utilities outside X.

Independence of Non-Feasible Alternatives (INFA):

$$\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N),$$

$$u_N\big|_X = u'_N\big|_X \Rightarrow \overline{R}(e) = \overline{R}(e').$$

This axiom is equivalent to INfI for

$$f(e,(x,y)) = (u_N|_X, X, (x,y)).$$

[11] For a more extensive study along these lines, see Fleurbaey et al. (2002).

The second example introduces a condition stipulating that the upper and lower contour sets remain the same.[12] Define

$$UC_{u_i}(u) = \{z \in X_i | u_i(z) \geq u\} \quad LC_{u_i}(u) = \{z \in X_i | u_i(z) \leq u\}.$$

The following axiom allows social preferences to disregard the precise levels of utilities at all alternatives which are not indifferent to x and y, except for checking that they remain better, or worse, than x or y.

Independence of Non-Indifferent Alternatives (INIA):

$\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N), \forall x, y \in X,$

$$\forall i \in N, \left.\begin{array}{c} UC_{u_i}(u_i(x)) = UC_{u'_i}(u_i(x)) \\ UC_{u_i}(u_i(y)) = UC_{u'_i}(u_i(y)) \\ LC_{u_i}(u_i(x)) = LC_{u'_i}(u_i(x)) \\ LC_{u_i}(u_i(y)) = LC_{u'_i}(u_i(y)) \end{array}\right\} \Rightarrow \overline{R}(e)\big|_{\{x,y\}} = \overline{R}(e')\big|_{\{x,y\}}.$$

This axiom is equivalent to INfI for[13]

$$f(e,(x,y)) = \begin{pmatrix} u_N(x), u_N(y), (UC_{u_i}(u_i(x)))_{i\in N}, (UC_{u_i}(u_i(y)))_{i\in N}, \\ (LC_{u_i}(u_i(x)))_{i\in N}, (LC_{u_i}(u_i(y)))_{i\in N}, X, (x,y) \end{pmatrix}.$$

This axiom allows the social comparison of x and y to depend not only on their utility levels but also on how they are ranked by individuals with respect to other alternatives.

It may be useful at this point to provide examples of reasonable SOFs satisfying INFA or INIA but not IOA. In an abstract voting problem, the Borda rule, which applies the utilitarian criterion to individual Borda scores[14]

$$v_i(x) = \# u_i(LC_{u_i}(u_i(x)) \cap X),$$

(by standard convention, $u_i(A)$ denotes the range of u_i on any subset A) does satisfy INFA and INIA, but not IOA.

Less classical examples may be provided for the economic problem of dividing a total bundle $\Omega \in \mathbb{R}^{\ell}_{++}$ of ℓ goods among n individuals whose consumption set is $\mathbb{R}^{\ell}_{+}$, and whose preferences bear only on their personal consumption bundle x_i (for $i = 1, \ldots, n$). Pazner (1979) proposed to apply the maximin criterion to the vector of individual v_i defined by

$$v_i(x_i) = \min\{\lambda \geq 0 | u_i(\lambda\Omega) \geq u_i(x_i)\}.$$

---

[12] The idea of such a condition can be traced back to Hansson (1973).

[13] Notice that in INIA the upper and lower contour sets are defined for utility levels, so that for instance, $UC_{u_i}(u_i(x)) = UC_{u'_i}(u_i(x))$ and $LC_{u_i}(u_i(x)) = LC_{u'_i}(u_i(x))$ imply $u_i(x) = u'_i(x)$.

[14] There are several possible definitions of Borda scores when individuals may be indifferent between alternatives. The definition proposed here counts the number of indifference classes, in X, at and below x. For instance, if $X = \{x, y, z, t\}$, and if i has preferences defined by $u_i(x) > u_i(y) = u_i(z) > u_i(t)$, then $v_i(x) = 3 > v_i(y) = v_i(z) = 2 > v_i(t) = 1$.

This gives a SOF which does not satisfy IOA but satisfies INIA. It also satisfies INFA when the set X is defined as

$$X = \{x = (x_1, \ldots, x_n) \in \mathbb{R}_+^{n\ell} | x_1 + \ldots + x_n \leq \Omega\}.$$

Fleurbaey and Maniquet (1996) proposed another example of SOF, which computes the social value of an allocation as the smallest fraction of Ω which belongs to the convex hull of the union of individual upper contour sets in the consumption space:

$$\min\left\{ \lambda \geq 0 | \lambda\Omega \in co\left(\bigcup_{i \in N} \{z \in \mathbb{R}_+^{\ell} \mid u_i(z) \geq u_i(x_i)\}\right)\right\}.$$

This SOF is axiomatically justified in Fleurbaey and Maniquet (2001), and its main attractive feature is that, in economies with convex preferences, its first best subset of allocations, in X, is always exactly the subset of egalitarian Walrasian allocations (i.e. competitive equilibria with equal budgets for all individuals). This SOF satisfies INIA, but does not satisfy INFA nor IOA.

4.2 Ethical comments

The formula

$$f(e, (x, y)) = (u_N(x), u_N(y), X, (x, y))$$

exactly describes what information is retained under the operation of IOA. First, the content of the two alternatives x, y under consideration (and the set X) is fully registered. No relation is established by this axiom between different pairs of alternatives. For instance, the social ordering may be utilitarian for a particular pair (x, y) and egalitarian for another pair (x', y'). Second, for the contemplated pair (x, y), the only information that is retained about their consequences over individuals is contained in the levels of utility at these alternatives. The content of this information is not supplemented by any other individual characteristics, not even by a fuller description of the individual utility functions. This axiom therefore embodies a very restrictive kind of welfarism, which is utterly implausible a priori.

But two remarks may alleviate this negative impression. First, the mapping U which derives utility functions u_i from individual characteristics θ_i may be anything, and may take into account any feature of individual subjective or objective well-being. In other words, the kind of welfarism embodied in IOA is purely formal, and is compatible with any empirical content given to the notion of "utilities". Second, if one assumes U is well chosen so that any relevant information about individual situations at x and y is correctly summarized into the figures $u_N(x), u_N(y)$, then it is much less clear that IOA is restrictive. By tautology, if anything that counts *is* in those figures, the rest does not count, and can safely be disregarded.

If one follows this line of reasoning, the appeal of IOA is conditional on the availability of a good function U capturing all that counts about individual situations. This suggests two critical points. By assuming that, somewhat miraculously, we have a function U which gives the perfect measurement of individual well-being, the theory of social choice loses any grip on the substantial debate of how U should be defined. Since IOA somehow asserts the perfection of U, this leaves no space *within* the theory of social choice to examine alternative definitions of U. As soon as one adopts IOA for a particular U, all other measures of well-being are excluded. This drastic reduction of possible choices about U can be avoided only by treating IOA not as an (initial) axiom, but as a result of some anterior analysis.

A second critical point is that the construction of $u_i(x) = U(\theta_i)(x)$ means that individual well-being is defined by relying only on individual characteristics. This is rather appealing, but, nevertheless, a little restrictive. A priori, one may think of defining individual well-being as a function of the whole population profile, or of the situation of all individuals in x. In other words, a more general approach would authorize $u_i(x) = U(i, \theta_N)(x)$. IOA could be applied to this broader notion of well-being.

Let us now come back to the usual economist's world, in which u_i is an ordinary measure of subjective utility. In this context, IOA is quite unacceptable. Consider the following example. Two goods, 1 and 2, have to be distributed to two individuals, Ann and Brian. Allocation x gives Ann the bundle (4,6) and Brian the bundle (7,5). Allocation y gives them the bundles (5,7) and (6,4), respectively. We are told that Ann's utility is 20 in x, and 24 in y, while Brian's utility is 24 in x and 20 in y. According to IOA, this should be enough information to compare x and y. In summary, under IOA, the information which can be used is (ignoring the feasible set X):

	Allocation x	Allocation y
Ann's bundle (x_{A1}, x_{A2})	(4,6)	(5,7)
Brian's bundle (x_{B1}, x_{B2})	(7,5)	(6,4)
Ann's utility	20	24
Brian's utility	24	20

In view of the perfect symmetry of this table, social indifference is the unavoidable conclusion. But this is very unsatisfactory. Compare the following two utility profiles, which both yield the above utility figures at the two allocations:

$$\theta_N : \begin{cases} u_A = x_{A1} + 3x_{A2} - 2 \\ u_B = x_{B1} + 3x_{B2} + 2 \end{cases} \quad \theta'_N : \begin{cases} u'_A = 3x_{A1} + x_{A2} + 2 \\ u'_B = 3x_{B1} + x_{B2} - 2. \end{cases}$$

In profile θ_N, allocation x gives individuals bundles they both deem equivalent, since

$$u_A(4,6) = u_A(7,5) = 20 \quad u_B(4,6) = u_B(7,5) = 24,$$

and allocation x can actually be obtained as a Walrasian equilibrium in which the two agents have the same budget set. In contrast, allocation y gives Brian a bundle that both deem inferior to Ann's bundle, so that, in particular, Brian envies Ann (in the sense that he would rather have her bundle). This seems to give x a serious ethical advantage over y. Now, with profile θ'_N, the utility figures are the same at x and y, but the roles of x and y are inverted, since y is now an egalitarian Walrasian allocation, while in x Ann envies Brian.

If one believes that these considerations are relevant, then IOA is too restrictive and unduly eliminates relevant information. The additional information that has been used in this example would have been available under the weaker axioms INFA or INIA. Notice that we referred only to indifference curves at x and y, so that INIA seems to focus on the appropriate kind of information for many equity considerations. INFA retains information about all utilities over X, which is too much when other indifference curves are irrelevant, and is also too little for some equity notions. In particular, when X is an Edgeworth box, knowing whether an allocation is Walrasian or not, when it is not interior (more precisely, when one agent consumes all of one good), is not generally possible by looking only at indifference curves within the Edgeworth box. This explains why the Fleurbaey-Maniquet example of SOF defined above does not satisfy INFA.

5 Paretianism

The Pareto principle is usually presented in relation to democratic principles, the respect of unanimous preferences, but it is also well known that it limits the possibility to rely on non-utility information, and therefore pushes social choice in the direction of welfarism.

5.1 Formal analysis

The most relevant Pareto condition for the discussion of this informational feature is Pareto-Indifference:

Pareto-Indifference (PI):

$\forall e = (\theta_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), \forall x, y \in X,$

$$u_N(x) = u_N(y) \Rightarrow x\bar{I}(e)y.$$

This axiom is implied by INfI whenever f satisfies the following neutrality property: $\forall e = (\theta_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), \forall x, y, x', y' \in X,$

$$\left.\begin{array}{l} u_N(x) = u_N(x') \\ u_N(y) = u_N(y') \end{array}\right\} \Rightarrow f(e,(x,y)) = f(e,(x',y')).$$

Equivalence between PI and INfI is obtained as follows.

Proposition 3. *A SOF $\bar{R}$ satisfies PI if and only if it satisfies INfI for f defined by:*

$$f(e,(x,y)) = (e,(u_N(x),u_N(y))).$$

Proof. PI implies INfI. Assume $f(e,(x,y)) = f(e',(x',y'))$. This implies $e = e'$, and $(u_N(x), u_N(y)) = (u_N(x'), u_N(y'))$. By PI, this implies $x\bar{I}(e)x'$ and $y\bar{I}(e)y'$. And therefore $x\bar{R}(e)y \Leftrightarrow x'\bar{R}(e)y'$.

INfI implies PI. Assume $u_N(x) = u_N(y)$. Let $e' = e$ and $x' = y, y' = x$. Then $f(e,(x,y)) = f(e',(x',y'))$. By IN$f$I, one has $x\bar{R}(e)y \Leftrightarrow y\bar{R}(e)x$. This leaves $x\bar{I}(e)y$ as the only logical possibility. ∎

5.2 Ethical comments

From the formula

$$f(e,(x,y)) = (e,(u_N(x), u_N(y))),$$

it is clear that PI entails the impossibility for social preferences to take account of any non-utility or non-preference feature of individual situations at the alternatives considered. In this way, PI implies a good deal of welfarism. On the other hand, the fact that all information about e remains available for use in social preferences means that a lot can be done in relating $u_N(x)$ and $u_N(y)$ to corresponding features of individual situations. For instance, knowing that i is at $u_i(x)$ in x entails a full knowledge of the content of i's upper and lower contour sets. More generally, all characteristics of the population (and also of the set of alternatives X) can be used in order to take account of the nature of the problem and of the types of individuals involved.

It is instructive to compare the above formula with the similar formula for IOA:

$$f(e,(x,y)) = (u_N(x), u_N(y), X, (x,y)).$$

In the latter one sees that all information is lost about individual characteristics, so that it is impossible to put the levels of utility in perspective. On the other hand, knowledge of x and y makes it possible to have social preferences depend directly on features of x and y, which is not allowed by PI.

6 Responsibility

Individual responsibility has already been mentioned, when discussing INVΦ_{ONC} and responsibility for subjective satisfaction. When individuals are deemed responsible for some part of their situation in an alternative, this presumably means that social preferences may legitimately disregard that aspect of individual situations. Individual responsibility over something means that it belongs to the private sphere, and that social preferences need not bother about it. For instance, social preferences may focus on opportunity sets offered to individuals, and disregard the particular choices made in these sets by individuals, or they may focus on initial resources granted to individuals, and disregard what individuals make of these resources. Again, this is directly related to the distinction between relevant and irrelevant information.

6.1 Formal analysis

Assume that all features of individual i's situation at alternative x, for which individual i is *not* responsible, are summarized in some (possibly multi-dimensional) measure $c_i(x)$, and that the function c_i is itself obtained from individual characteristics θ_N through a mapping $c_i = C(i, \theta_N)$. The fact that C may depend on θ_N and not just on θ_i reflects the possibility that non-responsibility features of an individual may be jointly determined by characteristics of the whole population.

It is possible to express the fact that individual i is responsible for anything else by letting social preferences disregard all that is not recorded in c_i. Again, with an abuse of notation, $C(\theta_N)$ denotes $(C(i, \theta_N))_{i \in N}$.

Independence of Responsibility Features (IRF):

$\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall c_N = C(\theta_N), c'_N = C(\theta'_N),$

$c_N = c'_N \Rightarrow \overline{R}(e) = \overline{R}(e').$

The formal similarity between this axiom and INFA enables us to see immediately that the relevant formula for the data filter f, when IRF is equivalent to INfI, is:

$f(e, (x, y)) = (c_N, X, (x, y)).$

When alternatives x, y are so precisely described that they contain a description of features for which individuals are responsible (for instance, particular consumption bundles, or utility levels), $c_N(x), c_N(y)$ can erase the irrelevant data and focus on the relevant parameters of individual situations at x and y (e.g. budget sets).

An axiom like INVΦ_{ONC}, which conveys the idea that individuals are responsible for their utility functions (as distinct from their preferences), can also be reformulated as an IRF condition, simply by letting $C(i, \theta_N)$ record all individual characteristics except the utility function.

6.2 Ethical comments

Depending on the definition of C, IRF may be more or less restrictive. When $C(i, \theta_N)$ retains all information and, for instance, equals the constant function $c_i \equiv \theta_i$, IRF does not impose any restriction on social preferences.

One encounters axioms of the IRF sort in all branches of the literature on responsibility-sensitive egalitarianism.[15] For instance, there is a literature on fair allocation, dealing with the allocation of money to individuals whose utility function depends on money and a personal talent parameter. In this approach individuals are supposed to be responsible for their utility function, and INVΦ_{ONC} is retained as an expression of this responsibility. A stronger

[15] See the surveys by Fleurbaey (1998) and Fleurbaey and Maniquet (1999).

axiom requiring the allocation of money to be independent of individual preferences is even sometimes required. Another part of the literature studies social welfare functions which evaluate individual opportunity sets,[16] and an IRF axiom is satisfied in this approach since the social evaluation depends only on opportunity sets and disregards what exact option is chosen by individuals in their opportunity sets.

The literature does contain other conditions related to the idea of responsibility. For instance, some conditions request some preference for *equality* of budget sets between individuals who differ only in their responsibility characteristics. Such conditions express a neutrality requirement over responsibility characteristics, and prevent social preferences from expressing a bias in favor or against some particular exercise of responsibility by individuals. This idea of neutrality[17] bears on the desirable kind or degree of redistribution, and goes farther than the mere idea of responsibility. In a different vein, the literature dealing with social welfare functions computed on opportunity sets interprets the idea of responsibility as implying that social preferences should display no inequality aversion over individuals who have the same opportunities, and should therefore simply seek to maximize the *sum* of outcomes reached by such subpopulation. Again, this approach goes farther than the mere idea of responsibility, and indirectly advocates a particular (non-neutral) distribution of rewards.

7 Separability

Social preferences are separable when, in the comparison of two alternatives x and y, they disregard the fate of individuals who are not affected by the change from x to y or vice versa. The situation of subpopulations can then be studied separately.

7.1 Formal analysis

There are many ways to define the fact that an individual is not concerned by the choice between x and y. It all depends on how one should evaluate the situation of an individual. The traditional way refers to utility, and, as emphasized above,

[16] For instance, in Roemer (1998), an individual opportunity set is measured by the statistical distribution of outcomes in the subpopulation with identical non-responsible characteristics. This distribution depends on government policy and on the behavior of the population.

[17] Fleurbaey (1995, 1998) calls this kind of neutrality the principle of ‘‘natural reward’’ (don’t over-punish or over-reward those who exercise their responsibility in a particular way). When IRF is applied to functions c_i which are constant in x, and x describes the allocation of resources, then natural reward is entailed by IRF (and an anonymity requirement) since the optimal allocation of resources becomes independent of changes in individual responsibility characteristics. Individuals who differ only in their responsibility characteristics will then obtain similar resources.

this may cover any relevant notion of well-being. Now, when the fate of unconcerned individuals is measured by their utility, it seems consistent to use the same measure for the rest of the population. This justifies the following axiom, which says that the individuals who are indifferent between x and y can be disregarded (except for their mere existence), and that social preferences can then only look at the utility functions of the rest of the population.

Independence of Indifferent Individuals (III):

$$\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N), \forall M \subset N, \forall x, y \in X,$$

$$\left.\begin{array}{r} u_M(x) = u_M(y) \\ u'_M(x) = u'_M(y) \\ u_{N\backslash M} = u'_{N\backslash M} \end{array}\right\} \Rightarrow R(e)\big|_{\{x,y\}} = R(e')\big|_{\{x,y\}}.$$

Proposition 4. *A SOF $\overline{R}$ satisfies III if and only if it satisfies INfI for f defined by:*

$$f(e, (x, y)) = (u_{N\backslash M(e,x,y)}, N, X, (x, y)),$$

it where $M(e, x, y) = \{i \in N | u_i(x) = u_i(y)\}$.

Proof. III implies INfI. Let $f(e, (x, y)) = f(e', (x', y'))$. Then $N = N'$, $X = X'$, $x = x'$ and $y = y'$. In addition, for all $i \in M = M(e, x, y) = M(e', x, y), u_i(x) = u_i(y), u'_i(x) = u'_i(y)$, whereas $u_{N\backslash M} = u'_{N\backslash M}$. Therefore, by III, $R(e)\big|_{\{x,y\}} = R(e')\big|_{\{x,y\}}$.

INfI implies III. Let $u_M(x) = u_M(y), u'_M(x) = u'_M(y), u_{N\backslash M} = u'_{N\backslash M}$. One therefore has $M \subset M(e, x, y) \cap M(e', x, y)$, so that $N\backslash M(e, x, y) \subset N\backslash M$ and $N\backslash M(e', x, y) \subset N\backslash M$. As a consequence, $u_{N\backslash M(e,x,y)} = u'_{N\backslash M(e',x,y)}$. Then $f(e, (x, y)) = f(e', (x', y'))$, and by IN$f$I one gets $R(e)\big|_{\{x,y\}} = R(e')\big|_{\{x,y\}}$. ■

There are several variants of this axiom in the literature.[18] When dealing with an economic model, it is usually possible to describe an alternative x by individual bundles x_i for $i \in N$. Let $x_N = (x_i)_{i\in N}$ then denote the related alternative. One can then formulate an axiom saying that individuals whose *bundle* does not change can be disregarded and even removed from the population.[19]

Independence of Unconcerned Individuals (IUI):

$$\forall e = (\theta_N, X) \in \mathscr{E}, \forall M \subset N, \forall x_N, y_N \in X,$$

$$x_M = y_M \Rightarrow \left[x_N \overline{R}(e) y_N \Leftrightarrow x_{N\backslash M} \overline{R}(e^r) y_{N\backslash M}\right],$$

where $e^r = (\theta_{N\backslash M}, X - x_M)$ and $X - x_M = \{z_{N\backslash M} | (x_M, z_{N\backslash M}) \in X\}$.

[18] For instance, d'Aspremont and Gevers (1977) have an axiom which only disregards the utility of individuals who are totally indifferent over all alternatives of X.

[19] This axiom is closely linked to the consistency condition of the theory of fair allocation. On this condition, see e.g., Thomson (1996).

This axiom can be shown to be equivalent[20] to INfI for

$$f(e,(x_N,y_N)) = \left(\theta_{N\setminus M(x_N,y_N)}, X - x_{M(x_N,y_N)}, (x_{N\setminus M(x_N,y_N)}, y_{N\setminus M(x_N,y_N)})\right),$$

where $M(x_N,y_N) = \{i \in N | x_i = y_i\}$. Notice that, compared to the previous formula, N has disappeared, reflecting the fact that unconcerned individuals are dealt with as if they simply did not exist. More interestingly, this axiom is much less welfarist than III, since it minimally defines the fact of being unconcerned in terms of bundles, which leaves open many possibilities for the evaluation of individual well-being. This is why $\theta_{N\setminus M(x_N,y_N)}$ appears in the data filter f.

7.2 Ethical comments

Separability, and axioms like III and IUI, can be motivated in several ways. There is first an issue of informational parsimony and simplicity. Separable social preferences allow simple computations over subpopulations, and guarantee the consistency between separate studies at local levels and global studies for the whole population. A second idea is related to how democracy works. In any contest between two alternatives, the unconcerned individuals are likely not to express any preference in favor of any of them, and therefore their vote will not influence the final decision. A third, more normative viewpoint, is that the principle of subsidiarity requires decisions to be under the control of concerned individuals only. Unconcerned individuals may give advice and recommendations, but the ultimate decision power should remain entirely in the hands of concerned individuals.

Against these arguments, it is sometimes said that the evaluation of what is going on in a subpopulation may depend on how it fares with respect to the rest of the population. For instance, social preferences may be more egalitarian if the subpopulation under consideration is poor compared to the rest, and be less egalitarian if it is much richer than the rest.

Many axioms, in the literature of social choice and fair allocation, have some flavor of separability. For instance, the Strong Pareto principle adds to PI the statement that x is strictly better than y whenever part of the population is indifferent while the rest strictly prefers x. This can be derived from a version of III in which indifferent individuals can be removed from the population (as in IUI), combined with the Weak Pareto principle according to which unanimous strict preferences for x over y, in the population, entails the same for social preferences. Other examples are given by the Pigou-Dalton principle of transfer, and Hammond's (1976) equity axiom, which focus on two-individual subpopulations.

Separability axioms will generally be incompatible with non-individualistic definitions of utility $u_i = U(i, \theta_N)$ or non-responsibility features

[20] The proof is quite different from the previous one, and is given in the Appendix.

$c_i = C(i, \theta_N)$. This potential conflict will be illustrated below, concerning social preferences favoring Walrasian allocations.

8 Feasibility

As explained in Sect. 2, the set X may be the set of feasible alternatives, and feasibility may be conceived of in various ways. It may simply be the set of alternatives over which individual utilities are defined (e.g., individual bundles must belong to individual consumption sets), or the set of technically feasible alternatives, or the set of incentive-compatible alternatives, etc. Should social preferences over two alternatives depend on the general shape of the set X to which they belong? This question is again related to informational issues.

8.1 Formal analysis

When social preferences do not depend on the particular shape of X, they may satisfy the following axiom.

Independence of Feasible Set (IFS):

$\forall e = (\theta_N, X), e' = (\theta_N, X') \in \mathscr{E}, \forall x, y \in X \cap X',$

$$R(e)\big|_{\{x,y\}} = R(e')\big|_{\{x,y\}}.$$

There are variants of this condition for particular contexts. For instance, when individual characteristics θ_i comprise productive talents, the domain $\mathscr{E}$ may be such that there is only one set X for any given profile θ_N. In this case IFS is vacuously satisfied, but one may then want to modify IFS in order to say that social preferences should not depend on the profile of talents, but only on the profile of preferences, for instance.

Translating this axiom into the INfI language is done as follows:

Proposition 5. *A SOF $\overline{R}$ satisfies IFS if and only if it satisfies INfI for f defined by:*

$$f(e, (x, y)) = (\theta_N, (x, y)).$$

Proof. IFS implies INfI. Let $f(e, (x, y)) = f(e', (x', y'))$. This implies $x = x', y = y'$, and therefore $x, y \in X \cap X'$. In addition, $\theta_N = \theta'_N$. By IFS, one then has $R(e)|_{\{x,y\}} = R(e')|_{\{x,y\}}$.

INfI implies IFS. Let $\theta_N = \theta'_N$. Then $f(e, (x, y)) = f(e', (x, y))$. By IN$f$I, one has $R(e)|_{\{x,y\}} = R(e')|_{\{x,y\}}$. ■

8.2 Ethical comments

The appeal of IFS depends on the context and the notion of feasibility which determines the set X. The initial aim of the theory of social choice, as posited by Arrow (1951), was probably to construct social preferences over the whole set of alternatives X^* for which individual preferences are well-defined. The domain of social choice entries $\mathscr{E}$ then had a fixed set $X = X^*$, and changes of individual preferences were the only source of variability in the domain. In this way IFS was vacuously satisfied, but, more substantially, it was indeed the case that social preferences over two alternatives did not depend at all on feasibility constraints. In other words, from Arrovian social preferences over the global set of alternatives X^*, one can derive a SOF $\overline{R}$ on any domain of entries $e = (\theta_N, X)$ with $X \subset X^*$, by letting $\overline{R}(e)$ coincide with the Arrovian social preferences on X, and this SOF does indeed satisfy IFS in a non-trivial way.

The availability of general social preferences on a large set X^* is indeed an alluring perspective, but the constraints IFS imposes must not be neglected. Consider the problem of distributing bread and water to a given population. When there is no water, a particular ranking of allocations of bread will be formed. According to IFS, this ranking should be retained even if water became available. This is questionable, for the following reason. In absence of water, presumably some simple egalitarian ranking would seem reasonable for the allocation of bread. But when water is available, the allocation of bread could legitimately take account of how much individuals are willing to substitute water for bread.

The problem becomes acute under Pareto-Indifference. For simplicity, consider a population with two individuals, Ann and Brian. Assume for instance that, for the allocations of bread only, in absence of water, giving 10 to Ann and 8 to Brian is better than 12 and 6, respectively. Now suppose that Ann and Brian are indifferent between one-good bundles as described in the table:

	bread	water
Ann is indifferent between:	12	8
	10	6
Brian is indifferent between:	8	12
	6	10

By IFS, the above ranking of allocations of bread should be retained even when water is available. In an economy where both goods are available, PI entails that, if giving 10 of *bread* to Ann and 8 to Brian is better than 12 and 6, then giving 6 of *water* to Ann and 12 to Brian is better than 8 and 10. By IFS, this ranking of allocations of water should be retained even in the case when there is no bread.

This shows that IFS is very restrictive and questionable in such a context. It prevents social preferences from taking account of the relative scarcity of goods, and from focusing on the appropriate parts of individual preferences. For instance, if one thinks that an egalitarian Walrasian allocation is a good social objective, it makes little sense to look for social preferences that are independent of the relative scarcity of goods, since individual situations have to be evaluated in terms of budgets, and the relative prices of goods will depend on total supply.[21]

9 Combinations

In this section, we study how the combination of various informational axioms shapes the available information. It is impossible to examine all combinations here. But the translation of informational axioms into the INfI format makes it often quite easy to see the consequences of combining several such axioms. When a SOF satisfies INf_1I and INf_2I, for two filters f_1 and f_2, one may simply look at the information which is retained by both filters. For instance, combining PI and IFS means using the two filters

$$(e, (u_N(x), u_N(y)) \text{ and } (\theta_N, (x, y)),$$

which implies retaining only

$$(\theta_N, (u_N(x), u_N(y)).$$

9.1 IOA and PI

If one combines the filters for IOA and PI,

$$((u_N(x), u_N(y), X, (x, y)) \text{ and } (e, (u_N(x), u_N(y)),$$

in order to extract the common information, one gets

$$(u_N(x), u_N(y), X),$$

which expresses the welfarist approach according to which, in a given set X, only vectors of utilities are taken into account, irrespectively of any other information.

However, INfI for this third data filter is not in general equivalent to the combination of IOA and PI. This equivalence is obtained only on sufficiently rich domains. The classical "welfarism lemma" was obtained by d'Aspremont and Gevers (1977) under the assumption that all utility functions on X are admissible. This assumption of universal domain makes it impossible to apply the result to economic domains and raises the question of how general it is. Fortunately, a weaker richness assumption, which is satisfied on many economic domains, is sufficient.

[21] Except in the subdomain of homothetic preferences. See Eisenberg (1961), Fleurbaey and Maniquet (1996).

Let us assume that for every $e = (\theta_N, X) \in \mathscr{E}$, with $u_N = U(\theta_N)$, and any $x, y, x', y' \in X$, one can find $e^1 = (\theta_N^1, X), e^2 = (\theta_N^2, X) \in \mathscr{E}, u_N^1 = U(\theta_N^1)$, $u_N^2 = U(\theta_N^2)$, and $x'', y'' \in X$ such that

$$u_N^1(x) = u_N^1(x'') = u_N^2(x'') = u_N^2(x') = u_N(x),$$
$$u_N^1(y) = u_N^1(y'') = u_N^2(y'') = u_N^2(y') = u_N(y).$$

This assumption means that any pair of alternatives x, x' can be connected by indifference to a third alternative x'' for two profiles of preferences, and that this connection can be done for two pairs at the same time.[22]

Proposition 6. *Under the above richness assumption, a SOF $\overline{R}$ satisfies IOA and PI if and only if it satisfies INfI for f defined by:*

$$f(e, (x, y)) = (u_N(x), u_N(y), X).$$

Proof. IOA and PI jointly imply INfI. Let $f(e, (x, y)) = f(e', (x', y'))$. This means that $X = X', u_N(x) = u'_N(x'), u_N(y) = u'_N(y')$. By the richness assumption, one can find $e^1 = (\theta_N^1, X), e^2 = (\theta_N^2, X) \in \mathscr{E}, u_N^1 = U(\theta_N^1), u_N^2 = U(\theta_N^2)$, and $x'', y'' \in X$ such that

$$u_N^1(x) = u_N^1(x'') = u_N^2(x'') = u_N^2(x') = u_N(x),$$
$$u_N^1(y) = u_N^1(y'') = u_N^2(y'') = u_N^2(y') = u_N(y).$$

By PI, $x\overline{I}(e^1)x'', y\overline{I}(e^1)y'', x'\overline{I}(e^2)x'', y'\overline{I}(e^2)y''$. By IOA,

$$\overline{R}(e)\big|_{\{x,y\}} = \overline{R}(e^1)\big|_{\{x,y\}},$$
$$\overline{R}(e')\big|_{\{x',y'\}} = \overline{R}(e^2)\big|_{\{x',y'\}},$$
$$\overline{R}(e^1)\big|_{\{x'',y''\}} = \overline{R}(e^2)\big|_{\{x'',y''\}}.$$

Therefore one has

$$\begin{aligned} x\overline{R}(e)y &\Leftrightarrow x\overline{R}(e^1)y \quad \text{by IOA} \\ &\Leftrightarrow x''\overline{R}(e^1)y'' \quad \text{by PI} \\ &\Leftrightarrow x''\overline{R}(e^2)y'' \quad \text{by IOA} \\ &\Leftrightarrow x'\overline{R}(e^2)y' \quad \text{by PI} \\ &\Leftrightarrow x'\overline{R}(e')y' \quad \text{by IOA.} \end{aligned}$$

---

[22] It is not satisfied in domains where an alternative is strictly worse than all others for all admissible preferences, such as domains with strictly monotonic preferences and allocations containing a zero bundle for some agents. One can check that on such domains the welfarism result does not hold. For instance, the SOF which simply ranks all allocations in which no individual has a zero bundle over all other allocations does satisfy IOA and PI, but is not welfarist.

INfI implies PI. Let $u_N(x) = u_N(y)$. Then $f(e,(x,y)) = f(e,(y,x))$, implying $x\overline{R}(e)y \Leftrightarrow y\overline{R}(e)x$, and therefore $x\overline{I}(e)y$.

INfI implies IOA. Let $u_N(x) = u'_N(x), u_N(y) = u'_N(y)$. Then $f(e,(x,y)) = f(e',(x,y))$ and $f(e,(y,x)) = f(e',(y,x))$ so that by INfI, $\overline{R}(e)|_{\{x,y\}} = \overline{R}(e')|_{\{x,y\}}$. . ■

9.2 INVΦ and IOA

In the literature about social choice with interpersonal comparisons of utility, the axioms INVΦ and IOA are seldom used separately, and it is interesting to analyze the consequences of this association. This is rarely done because the usual axiomatic analysis first exploits the above welfarist result (combining IOA and PI) in order to obtain a preorder over utility vectors in $\mathbb{R}^N$, and then studies the consequences of INVΦ over this preorder. As it turns out, combining INVΦ and IOA has very clear consequences, independently of Pareto conditions.

First, for any $\Phi \subset \Phi_{ONC}$, any $\varphi \in \Phi$, any N and any vector $a_N \in \mathbb{R}^N$, let $\varphi(a_N)$ denote the vector $(\varphi_i(a_i))_{i\in N}$. The set Φ_g generates an equivalence relation $\approx_{\Phi_g}$ on $\cup_{N\subset\overline{N}}\mathbb{R}^N \times \mathbb{R}^N$, which is defined by

$$(a_N, b_N) \approx_{\Phi_g} (a'_{N'}, b'_{N'}) \Leftrightarrow N = N' \text{ and } \exists\varphi \in \Phi_g, a'_N = \varphi(a_N), b'_N = \varphi(b_N).$$

For any (a_N, b_N), let $\approx_{\Phi_g} (a_N, b_N)$ denote the equivalence class for $\approx_{\Phi_g}$ to which (a_N, b_N) belongs. Interestingly, different groups Φ_g may yield the same equivalence relation. A famous example, due to Sen (1970), is given by Φ_{ONC} and Φ_{CNC}.

Proposition 7. *Consider any subset $\Phi \subset \Phi_{ONC}$, and assume that $\mathscr{E}$ is rich enough so that, for any $e = (\theta_N, X) \in \mathscr{E}$, and any $\varphi \in \Phi_g$, there exists $(\theta'_N, X) \in \mathscr{E}$such that $U(\theta'_N) = \varphi(U(\theta_N))$. A SOF $\overline{R}$ satisfies INVΦ and IOA if and only if it satisfies INfI for f defined by:*

$$f(e,(x,y)) = (\approx_{\Phi_g} (u_N(x),\ u_N(y)), X, (x,y)),$$

where $e = (\theta_N, X), u_N = U(\theta_N)$.

Proof. INVΦ and IOA jointly imply INfI. Let $f(e,(x,y)) = f(e',(x',y'))$. This implies $x = x', y = y', N = N', X = X'$ and

$$\exists\varphi \in \Phi_g,\ U(\theta'_N)(x) = \varphi(U(\theta_N)(x)),\ U(\theta'_N)(y) = \varphi(U(\theta_N)(y)).$$

By the richness assumption, there exists $e'' = (\theta''_N, X) \in \mathscr{E}$ such that $U(\theta''_N) = \varphi(U(\theta_N))$. One therefore has

$$U(\theta'_N)(x) = U(\theta''_N)(x),\ U(\theta'_N)(y) = U(\theta''_N)(y).$$

By IOA, this implies $\overline{R}(e'')|_{\{x,y\}} = \overline{R}(e')|_{\{x,y\}}$. In addition, INVΦ, which is, as noted above, equivalent to INVΦ_g, entails that $\overline{R}(e'') = \overline{R}(e)$. Therefore $\overline{R}(e')|_{\{x,y\}} = \overline{R}(e)|_{\{x,y\}}$.

INfI implies INVΦ. This immediately follows from the fact that: $\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N)$,

$$\exists \varphi \in \Phi,\ u'_N = \varphi(u_N) \Rightarrow f(e,.) = f(e',.).$$

INfI implies IOA. This follows from the fact that: $\forall e = (\theta_N, X)$, $e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N), \forall x, y \in X$,

$$u_N|_{\{x,y\}} = u'_N|_{\{x,y\}} \Rightarrow f(e,(x,y)) = f(e',(x,y)). \qquad \blacksquare$$

The formula $f(e,(x,y)) = (\approx_{\Phi_g} (u_N(x), u_N(y)), (x,y))$ shows how the combination of INVΦ and IOA limits the information about the utility bivector $(u_N(x), u_N(y))$. It is also worth noting that, under the richness assumption of this proposition, INVΦ and IOA are, jointly, equivalent [23] to the following simple axiom:

Binary invariance to Φ (*BINV* Φ):

$\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall u_N = U(\theta_N), u'_N = U(\theta'_N), \forall x, y \in X$,

$$\exists \varphi \in \Phi, \begin{cases} u'_N(x) = \varphi(u_N(x)) \\ u'_N(y) = \varphi(u_N(y)) \end{cases} \Rightarrow R(e)|_{\{x,y\}} = R(e')|_{\{x,y\}}.$$

In view of the above proposition, BINVΦ is equivalent to BINVΦ' whenever $\approx_{\Phi_g} = \approx_{\Phi'_g}$. As an example, BINV$\Phi_{CNC}$ is equivalent to BINVΦ_{ONC}.[24]

The equivalence relation $\approx_{\Phi_{ONC}}$, restricted to a given population N for brevity of notations, may be described in the following, equivalent ways:

$$\begin{aligned}(a_N, b_N) \approx_{\Phi_{ONC}} (a'_N, b'_N) &\Leftrightarrow \exists \varphi \in \Phi_{ONC}, \forall i \in N, \begin{cases} a'_i = \varphi_i(a_i) \\ b'_i = \varphi_i(b_i) \end{cases} \\ &\Leftrightarrow \forall i \in N, \left(a'_i - b'_i\right)(a_i - b_i) > 0 \text{ or } a'_i - b'_i = a_i - b_i = 0. \\ &\Leftrightarrow a'_N - b'_N \text{ is in the same orthant(s) as } a_N - b_N.\end{aligned}$$

Recalling that the application of this equivalence relation is made on

$$\begin{aligned}(a_N, b_N) &= (u_N(x), u_N(y)) \\ (a'_N, b'_N) &= (u'_N(x), u'_N(y)),\end{aligned}$$

one sees that the axiom BINVΦ_{ONC} is actually Arrow's axiom of "Independence of Irrelevant Alternatives". Let $R(\theta_i)$ denote individual i's preferences on X_i, and let $R(\theta_N)$ denote $(R(\theta_i))_{i \in N}$.

Arrow Independence of Irrelevant Alternatives (Arrow IIA):

$$\forall e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, \forall R_N = R(\theta_N), R'_N = R(\theta'_N), \forall x, y \in X,$$

$$R_N|_{\{x,y\}} = R'_N|_{\{x,y\}} \Rightarrow \overline{R}(e)|_{\{x,y\}} = \overline{R}(e')|_{\{x,y\}}.$$

---

[23] The proof of this fact is given in the appendix.

[24] Bossert (1999) studies the related phenomenon that combining invariance conditions about utility profiles with Pareto indifference and IOA may entail much larger invariance conditions.

As another illustration, consider the case of cardinal unit comparability, related to the set of transformations Φ_{CUC} (defined above). $(\Phi_{CUC}, \circ)$ is an algebraic group, and the equivalence relation $\approx_{\Phi_{CUC}}$, restricted to a given population N, may be described in the following, equivalent ways:

$$(a_N, b_N) \approx_{\Phi_{CUC}} (a'_N, b'_N) \Leftrightarrow \exists \alpha \in \mathbb{R}^N, \beta \in \mathbb{R}_{++}, \forall i \in N, \begin{cases} a'_i = \alpha_i + \beta a_i \\ b'_i = \alpha_i + \beta b_i \end{cases}$$
$$\Leftrightarrow \forall i, j \in N, \frac{a'_i - b'_i}{a_i - b_i} = \frac{a'_j - b'_j}{a_j - b_j} \text{ or}$$
$$a'_i - b'_i = a_i - b_i = 0 \text{ or } a'_j - b'_j = a_j - b_j = 0$$
$$\Leftrightarrow \forall i, j \in N, \frac{a'_i - b'_i}{a'_j - b'_j} = \frac{a_i - b_i}{a_j - b_j} \text{ or } a'_j - b'_j = a_j - b_j = 0$$
$$\Leftrightarrow a'_N - b'_N \text{ is proportional to } a_N - b_N.$$

As a consequence, BINVΦ_{CUC} involves much more precise information than the mere comparisons of utility differences with which utilitarianism is commonly associated.[25] The ratios of differences have to remain unchanged under Φ_{CUC}, so that the direction of the vector of utility differences $u_N(x) - u_N(y)$ is unaltered.[26]

9.3 INFA, PI and IFS

There is an obvious tension between INFA and IFS. The former excludes information about individual utilities outside X, while the latter excludes any information about X. The combination of the two excludes a lot of information. This is illustrated by the following proposition.

Assume that:

1. The domain $\mathscr{E}$ is a Cartesian product $\Theta \times \Xi$, where Θ is the set of profiles θ_N and Ξ is the set of feasible sets X;
2. The set Ξ is such that for any $X, X' \in \Xi$, there is $X'' \in \Xi$ such that $X \cup X' \subset X''$;
3. There exist $\Xi^1, \Xi^2 \subset \Xi$ such that:
   (a) for any $X^1 \in \Xi^1, X^2 \in \Xi^2, X^1 \cap X^2 = \emptyset$;
   (b) for any $e = (\theta_N, X) \in \mathscr{E}, u_N = U(\theta_N)$, any $x \in X$, there exist $X^1 \in \Xi^1$, $x^1 \in X^1, X^2 \in \Xi^2, x^2 \in X^2, u_N(x) = u_N(x^1) = u_N(x^2)$;

[25] This point is emphasized in Bossert (1991) and Bossert and Weymark (1998). For instance, with three individuals, knowing that

$$u_1(x) - u_1(y) > u_2(y) - u_2(x) > u_3(y) - u_3(x) > 0$$

tells everything about individual preferences and comparisons of differences, but is insufficient for the utilitarian SOF to rank x and y.

[26] Although the association of INVΦ and IOA into BINVΦ is worth analyzing, without Pareto conditions, it does not yield a direct characterization of the traditional SOFs. BINVΦ alone does not prevent social preferences from being imposed or biased in favor of particular individuals.

(c) for any $X^1 \in \Xi^1, X^2 \in \Xi^2$, any $\theta_N, \theta'_N \in \Theta$, there exists $\theta''_N \in \Theta$ such that $U(\theta''_N)|_{X^1} = U(\theta_N)|_{X^1}$ and $U(\theta''_N)|_{X^2} = U(\theta'_N)|_{X^2}$.

In the third part[27] of this assumption, the idea is that the feasible sets X^1, X^2 are in the outskirts of the global domain, with an empty intersection and no constraint about connecting utility profiles. For instance, in a problem of division of unproduced commodities, X^1 may contain only allocations of good 1, and X^2 allocations of good 2.[28] The example with bread and water in Subsect. 8.2 may help in getting the intuition of the next result.

One indeed obtains the following new "welfarism" proposition.

Proposition 8. *Under the above assumption, a SOF satisfies INFA, PI and IFS if and only if it satisfies INfI for f defined by:*

$$f(e,(x,y)) = (u_N(x), u_N(y)).$$

Proof. INFA, PI and IFS jointly imply INfI. Let $f(e,(x,y)) = f(e',(x',y'))$, that is, $(u_N(x), u_N(y)) = (u'_N(x'), u'_N(y'))$. Let $e^1 = (\theta_N, X^1) \in \Theta \times \Xi^1, x^1, y^1 \in X^1$, and $e^2 = (\theta'_N, X^2) \in \Theta \times \Xi^2, x^2, y^2 \in X^2$, be such that

$$\left(u_N(x^1), u_N(y^1)\right) = (u_N(x), u_N(y))\left(u'_N(x^2), u'_N(y^2)\right) = \left(u'_N(x'), u'_N(y')\right).$$

Let $e^* = (\theta_N, X^*), e'^* = (\theta'_N, X^*) \in \mathscr{E}$ be such that

$$X \cup X' \cup X^1 \cup X^2 \subset X^*.$$

By PI, $x\overline{I}(e^*)x^1, y\overline{I}(e^*)y^1, x'\overline{I}(e'^*)x^2, y'\overline{I}(e'^*)y^2$. Therefore

$$x\overline{R}(e^*)y \Leftrightarrow x^1\overline{R}(e^*)y^1, x'\overline{R}(e'^*)y' \Leftrightarrow x^2\overline{R}(e'^*)y^2.$$

By IFS, one actually obtains

$$x\overline{R}(e)y \Leftrightarrow x^1\overline{R}(e^1)y^1, x'\overline{R}(e')y' \Leftrightarrow x^2\overline{R}(e^2)y^2.$$

Let $e^{**} = (\theta^*_N, X^*) \in \mathscr{E}, u^*_N = U(\theta^*_N)$, be such that $u^*_N|_{X^1} = u_N|_{X^1}, u^*_N|_{X^2} = u'_N|_{X^2}$. One then has

$$u^*_N(x^1) = u_N(x^1) = u_N(x) = u'_N(x') = u'_N(x^2) = u^*_N(x^2),$$

$$u^*_N(y^1) = u_N(y^1) = u_N(y) = u'_N(y') = u'_N(y^2) = u^*_N(y^2).$$

As a consequence, by PI $x^1\overline{I}(e^{**})x^2$ and $y^1\overline{I}(e^{**})y^2$. Besides, by INFA,

$$x^1\overline{R}(e^1)y^1 \Leftrightarrow x^1\overline{R}(e^{**})y^1, x^2\overline{R}(e^2)y^2 \Leftrightarrow x^2\overline{R}(e^{**})y^2.$$

---

[27] Notice that the third part implies the first, which is written down only for clarity's sake. Indeed, $U(\theta''_N)|_{X^1} = U(\theta_N)|_{X^1}$ and $U(\theta''_N)|_{X^2} = U(\theta'_N)|_{X^2}$ entail $U(\theta''_N)|_{X^1 \cap X^2} = U(\theta_N)|_{X^1 \cap X^2} = U(\theta'_N)|_{X^1 \cap X^2}$, which cannot hold for any θ_N, θ'_N if $X^1 \cap X^2 \neq \emptyset$.

[28] Part 3.b of the assumption is not satisfied in an abstract domain containing individual preferences without any indifference. One can obtain similar results on such domains by relying on different Pareto conditions.

By transitivity,

$$x^1\overline{R}(e^1)y^1 \Leftrightarrow x^2\overline{R}(e^2)y^2.$$

Finally,

$$x\overline{R}(e)y \Leftrightarrow x'\overline{R}(e')y'.$$

INfI implies INFA. When $u_N|_X = u'_N|_X$, $(u_N(x), u_N(y)) = (u'_N(x), u'_N(y))$ for any $x, y \in X$, so that $f(e,(x,y)) = f(e',(x,y))$ for any $x, y \in X$, and therefore $\overline{R}(e) = \overline{R}(e')$.

INfI implies PI. Same argument as in Proposition 6.

INfI implies IFS. This is obvious from the definition of f. ■

Combining this result with the welfarism result of Subsect. 9.1, one can deduce that, under PI and IFS, the axioms INFA and IOA are equivalent.[29]

10 Information and the possibility of social choice

10.1 Two routes

It is now well understood that Arrow's impossibility theorem comes from the fact that the axioms of the theorem delete too much information compared to what is needed for the construction of consistent social preferences. More precisely, Arrow IIA is the culprit. As explained above, Arrow IIA is equivalent to the combination of INVΦ_{ONC} and IOA. This decomposition indicates two routes for retrieving possibility results.

The most trodden route has consisted in weakening INVΦ_{ONC} and retaining information about interpersonal comparisons of utility (or any notion of well-being represented by U). It leads to traditional social welfare functions such as utilitarianism and the maximin criterion. The other route, suggested somewhat obscurely by Samuelson (1977)[30] and very clearly by Pazner (1979), consists in weakening IOA. Pazner actually proposed to weaken Arrow IIA into an axiom which is exactly the combination of INVΦ_{ONC} and INIA, and, in an economic domain (with monotonic preferences), requires the social ranking of two allocations to depend only on indifference curves at bundles obtained by individuals at the two allocations. While these authors gave examples of social preferences satisfying INVΦ_{ONC}, INIA, and PI, such as the Pazner example presented in Subsect. 4.1, more precise axiomatic derivations of social preferences from these axioms and

[29] Along these lines a strengthening of Arrow's impossibility theorem can be obtained See Fleurbaey et al. (2002).

[30] Samuelson (1977) focused on Kemp and Ng's (1976) single-profile independence axiom, and did not criticize Arrow IIA directly, under the dubious argument that multi-profile issues were largely irrelevant in welfare economics. A more explicit criticism of Arrow IIA appeared however in Samuelson (1987).

other equity principles were obtained much more recently, by Fleurbaey and Maniquet (2000, 2001).[31]

In an economic model (with monotonic preferences), where individual *i*'s indifference curve at allocation x may be denoted $I_i(x)$, and $I_N(x)$ may denote $(I_i(x))_{i \in N}$, the combination of INVΦ_{ONC}, INIA, and PI is equivalent to INfI for

$$f(e, (x, y)) = (I_N(x), I_N(y), X).$$

This can be compared to the combination of IOA and PI, which, in the same context, yields

$$f(e, (x, y)) = (u_N(x), u_N(y), X).$$

In other words, while traditional social welfare functions process and compare *utility* vectors, the new kind of social welfare functions obtained in the alternative approach would have vectors of *indifference curves* as their argument.

10.2 Welfarism and quasi-welfarism

The relationship of these approaches to welfarism deserves some scrutiny. As exemplified by the Borda rule or the Pazner example of SOF, some of the new kind of social welfare functions evaluate any given vector of indifference curves by first putting a real number on each of them, and then applying a traditional social welfare function to this vector of real numbers. Is that welfarist or not? Does it involve interpersonal utility comparisons just as the traditional approach? The confusion is reinforced by the fact that an approach may be formally similar to welfarism, but philosophically quite far away from it. I suggest distinguishing the following approaches, starting from the strongest form of "welfarism":

- **Real welfarism.** The "real thing", in matters of welfarism, consists in letting U measure subjective utility or satisfaction (there are several possibilities, and therefore several variants of real welfarism), and requiring the SOF to satisfy IOA and PI.
  A typical example is classical utilitarianism.
  Real welfarism involves interpersonal comparisons of subjective utilities.
- **Formal welfarism.** It adopts an exogenous definition of U, and requires the SOF to satisfy IOA and PI. But U may measure any objective or subjective notion of well-being, and it is provided by moral philosophy or the social decision-maker.

[31] Results based on different weakenings of IOA were obtained by Kaneko and Nakamura (1979) and Dhillon and Mertens (1999) in an abstract model with uncertainty.

An example, where moral philosophy provides U, is Sen's (1985, 1987) approach in terms of capabilities and functionings. Another example, involving the social decision-maker as the purveyor of U, is given by many publications in public economics.[32]

Formal welfarism involves interpersonal comparisons of whatever U measures.

- **Individualistic quasi-welfarism.** It lets U be any representation of individual preferences, and requires the SOF to satisfy INVΦ_{ONC} and PI. But the SOF which is finally obtained satisfies INfI for

  $$f(e,(x,y)) = (v_N(x), v_N(y), X),$$

  where v_i is a real-valued function derived from individual characteristics: $v_i = V(\theta_i)$. The satisfaction of PI forces $V(\theta_i)$ to be ordinally equivalent to $U(\theta_i)$. But the SOF does not satisfy IOA. It satisfies INIA when the value of $v_i(x)$ depends only on the upper and lower contour sets at x.

  The Borda rule is an example, where v_i is the individual Borda score. Bergson and Samuelson's concept of social welfare also belonged to this approach, although these authors did not venture to defend one particular function V. A more precise example is Pazner's (1979), which, as defined above, relies on

  $$v_i(x_i) = \min\{\lambda \geq 0 | u_i(\lambda\Omega) \geq u_i(x_i)\}.$$

  One sees that v_i is ordinally equivalent to u_i, but this SOF satisfies INVΦ_{ONC} because a change of u_i which does not alter ordinal preferences will leave the v_i unchanged.

  This approach involves interpersonal comparisons of v_i. But one should resist the temptation to view v_i as a measure of subjective utility. The only tint of welfarism here comes from PI, and the best interpretation of the v_i, in the Pazner example for instance, is not in terms of satisfaction, but in terms of resources. The quantity $v_i(x_i)$ does not measure i's satisfaction, but measures the *value* of bundle x_i, in terms of $\lambda\Omega$, according to i's opinion. Similarly, in the Borda rule, the Borda score is less a measure of subjective satisfaction than a measure of the *value* of an alternative, estimated, roughly, by the number of alternatives which are worse. This justifies replacing the word "welfarism" by "*quasi-welfarism*". For the Pazner example, one could even propose the word "*wealthfarism*".
- **Non-individualistic quasi-welfarism.** It lets U be any representation of individual preferences, and requires the SOF to satisfy INVΦ_{ONC} and PI. But the SOF which is finally obtained satisfies INfI for

[32] See e.g., Atkinson (1995), where U may in particular embody the social planner's aversion to inequality. A general discussion of formal welfarism in relation to real welfarism is made in Mongin and d'Aspremont (1998).

$$f(e,(x,y)) = (v_N(\theta_N,x), v_N(\theta_N,y), X),$$

where v_i is a real-valued function depending on the whole profile and the whole allocation.

An example is the Fleurbaey-Maniquet example of SOF, defined above, and which can be alternatively formulated with the maximin criterion applied to

$$v_i(\theta_N,x) = v_i^*(x_i, p(\theta_N,x)),$$

where

$$v_i^*(x_i,p) = \min\{\lambda \geq 0 | \exists z, pz \leq \lambda p\Omega, u_i(z) \geq u_i(x_i)\},$$
$$p(\theta_N,x) = \arg\max_p \min_i v_i^*(x_i,p).$$

That is, $v_i(\theta_N,x)$ is the money-metric utility function of i measured in terms of the initial endowment $\lambda\Omega$ which would give i the same satisfaction as x_i, at prices p computed so as to give allocation x the best maximin value.

This approach is quite far away from welfarism. First, like individualistic quasi-welfarism, it measures the value of resources rather than subjective satisfaction as such. Second, unlike individualistic quasi-welfarism, it does not evaluate the situation of an individual in isolation from the rest of the population.

One could moreover distinguish "real" and "formal" variants of quasi-welfarism, since the indifference curves which serve as the main input in the evaluation of an alternative may belong to the actual subjective indifference maps of the individuals, or instead represent any ordinal representation of individuals' objective interests.

10.3 Separability and Walrasian allocations

Interestingly, one may conjecture that a SOF from the last non-individualistic approach cannot in general satisfy the separability axiom III. This is easily checked for the Fleurbaey-Maniquet example. This is actually true for any SOF relying on the maximin criterion. But in general, a leximin refinement of the maximin may satisfy III, whereas this cannot be achieved with this particular example. As can be checked easily, no SOF for which the maximum value over the feasible set is obtained only by egalitarian Walrasian allocations can satisfy III or IUI.[33] There is a basic dilemma between separability (III or IUI) and orienting social preferences toward Walrasian allocations. This dilemma has been ignored in the theory of fairness because the allocation rule which selects the subset of egalitarian Walrasian allocations does satisfy the "consistency" requirement (which is logically weaker than IUI) that removing some individuals and their bundles still leaves the allocation (for the rest of the population) optimal.

10.4 A trilemma

The results of Subsect. 9.3 suggest a slightly more complex picture than the binary analysis of the possibility of social choice which has just been proposed. From these results, one can deduce that *no ordinal Paretian SOF can satisfy INFA and IFS without satisfying Arrow IIA*, and therefore running into the troubles of Arrow's theorem.

For instance, the Borda rule defined as above with individual Borda scores

$$v_i(x) = \# u_i(LC_{u_i}(u_i(x)) \cap X)$$

is ordinal (it satisfies INVΦ_{ONC}), and satisfies INFA, but not IFS. A variant of this rule would define Borda scores by

$$v_i(x) = \# u_i(LC_{u_i}(u_i(x))),$$

and obtain a modified Borda rule which satisfies IFS but not INFA. The results obtained in 9.3 show that there is no hope of finding another variant satisfying simultaneously INFA and IFS in general.

The Pazner example of SOF satisfies INFA but not IFS, when the feasible set X is defined as

$$X = \{x = (x_1, \ldots, x_n) \in \mathbb{R}_+^{n\ell} \mid x_1 + \cdots + x_n \leq \Omega\}.$$

The Fleurbaey-Maniquet example of SOF satisfies neither INFA nor IFS. An example of SOF satisfying IFS but not INFA is obtained by modifying Pazner's example, letting v_i be defined with respect to a fixed Ω_0, independent of available resources.

This suggests that Arrow's impossibility can be understood as reflecting not only a dilemma between ordinalism (INVΦ_{ONC}) and the restriction to local data (IOA), but also, and more deeply, a trilemma between ordinalism

[33] The core argument is given in Fleurbaey and Maniquet (2001). In a nutshell, suppose (x_1, x_2, x_3) is an egalitarian Walrasian allocation and (x_1', x_2', x_3) is not, with $x_3 = \Omega/3$, and that both (x_1, x_2) and (x_1', x_2') are egalitarian Walrasian allocations when total resources are $(2/3)\Omega$. This violates IUI, since in the reduced economy without the third individual and with resources $\Omega - x_3 = (2/3)\Omega$, social preferences are indifferent between (x_1, x_2) and (x_1', x_2'), whereas (x_1, x_2, x_3) is strictly preferred to (x_1', x_2', x_3) in the large economy. Besides, just by changing individual 3's preferences without removing him, one may reverse the situation so that (x_1', x_2', x_3) is an egalitarian Walrasian allocation and (x_1', x_2', x_3) is not. Which violates III.

($INV\Phi_{ONC}$), the restriction to local data (INFA) and disregarding feasibility constraints (IFS). Since IFS was somehow built in Arrow's framework, while IOA was an extreme kind of restriction to local data, this could not be apparent in the classical approach to social choice.

In more general terms, we can then say that, in a Paretian approach, there is a tension between three broad ethical principles:

- Individual well-being should be measured in an ordinal non-comparable way (e.g. individuals should be held responsible for their numerical level of well-being).
- Individual well-being at distant or infeasible alternatives is irrelevant.
- Social preferences should not be influenced by feasibility constraints or be limited to local alternatives.

As a consequence, there are not two but three different ways of obtaining possibility results in social choice: 1) introducing interpersonally comparable utilities; 2) introducing non-local information about individual preferences; 3) restricting social preferences to local or feasible alternatives. These three ways are not exclusive of each other, and can be combined in various degrees. For instance, the Pazner example of SOF satisfies INFA but not IOA, which means that it introduces some non-strictly local information, but without going beyond the feasible set, while it violates IFS.

11 Conclusion

In conclusion, the INfI axiom is a convenient instrument for the analysis of the informational content of various axioms. It disentangles the notion of informational basis from its historical origin tied to the perspective of interpersonal comparisons of utilities, and enabled us here to obtain some interesting insights in the general outlook and understanding of the problems and perspectives of social choice.

The following table summarizes the main informational dimensions and the related data filters (see the relevant sections for the notations).

Information	**Axiom**	**Data filter** $f(e,(x,y))$
Utility	$INV\Phi$	$\left(\sim_{\Phi_g}(e),(x,y)\right)$
Local alternatives	IOA	$(u_N(x),u_N(y),X,(x,y))$
	INFA	$\left(u_N\vert_X, X,(x,y)\right)$
Pareto	PI	$(e,(u_N(x),u_N(y)))$
Responsibility	IRF	$(c_N,X,(x,y))$
Separability	III	$\left(u_{N\setminus M(e,x,y)},N,X,(x,y)\right)$
Feasibility	IFS	$(\theta_N,(x,y))$

In this table, all axioms except PI restrict the information retained about e in $f(e,(x,y))$. The next table summarizes the compatibility patterns obtained when different informational restrictions are jointly imposed.

Information	**Axioms**	**Data filter** $f(e,(x,y))$
Loc. alt. & Pareto	IOA & PI	$(u_N(x),u_N(y),X)$
Loc. alt. & Utility	IOA & INVΦ	$(\approx_{\Phi_g}(u_N(x),u_N(y)),X,(x,y))$
e.g.: Arrow IIA	IOA & INVΦ_{ONC}	$\begin{pmatrix} \text{orthant}(u_N(x)-u_N(y)), \\ X,(x,y) \end{pmatrix}$
Loc. alt., Feas. & Par.	INFA, IFS & PI	$(u_N(x),u_N(y))$

As obvious from this table, the combination of informational restrictions about utilities, non-local alternatives, non-utility features of alternatives (Pareto), and feasibility is fateful for the possibility of social choice. Retaining information about utilities has been the most favored escape from the impossibility problem, including when utility in reinterpreted in a non-welfarist way, as in Sen's theory of capabilities. Another route consists in retaining information about individual characteristics at non-local alternatives, as proposed by Samuelson, Pazner and others. Along this route, retaining information about feasibility constraints is also often helpful, as exemplified by several SOFs presented above, which violate IFS by taking account of the relative scarcity of goods.

It is therefore important to broaden the concept of informational basis not only for a better conceptual foundation of the theory of social choice, but also to get a better picture of the dilemmas and possibilities of social choice in relation to informational restrictions. As emphasized in this paper, the selection of data to be retained must primarily be a question of ethical relevance. In the comparison of the two (or three) main possibility routes for social choice which have just been outlined, the relative relevance of utility information, non-local information and feasibility constraints has to be assessed in order to decide which route is the most fruitful. This paper has examined and provided a few arguments touching on these questions, and distinguished in particular various kinds of welfarism and quasi-welfarism, but its purpose is to point out the issue, not to settle it.

Appendix

Proposition 9. *A SOF $\overline{R}$ satisfies IUI if and only if it satisfies INfI for f defined by:*

$$f(e,(x_N,y_N)) = \left(\theta_{N\setminus M(x_N,y_N)}, X - x_{M(x_N,y_N)}, (x_{N\setminus M(x_N,y_N)}, y_{N\setminus M(x_N,y_N)})\right),$$

where $M(x_N,y_N) = \{i \in N | x_i = y_i\}$.

Proof. IUI implies INfI. Let $f(e,(x_N,y_N)) = f(e',(x'_N,y'_N))$. By IUI,

$$\left[x_N\overline{R}(e)y_N \Leftrightarrow x_{N\backslash M(x_N,y_N)}\overline{R}(e^r)y_{N\backslash M(x_N,y_N)}\right],$$

with $e^r = (\theta_{N\backslash M(x_N,y_N)}, X - x_{M(x_N,y_N)})$ and

$$X - x_{M(x_N,y_N)} = \{z_{N\backslash M(x_N,y_N)} | (x_{M(x_N,y_N)}, z_{N\backslash M(x_N,y_N)}) \in X\}.$$

Similarly,

$$\left[x'_N\overline{R}(e')y'_N \Leftrightarrow x'_{N'\backslash M(x'_N,y'_N)}\overline{R}(e'^r)y'_{N'\backslash M(x'_N,y'_N)}\right],$$

with $e'^r = (\theta'_{N'\backslash M(x'_N,y'_N)}, X' - x'_{M(x'_N,y'_N)})$ and

$$X - x'_{M(x'_N,y'_N)} = \{z_{N\backslash M(x_N,y_N)} | (x'_{M(x_N,y_N)}, z_{N\backslash M(x_N,y_N)}) \in X\}.$$

Now, since $e^r = e'^r$ and $x_{N\backslash M(x_N,y_N)} = x'_{N'\backslash M(x'_N,y'_N)}, y_{N\backslash M(x_N,y_N)} = y'_{N'\backslash M(x'_N,y'_N)}$, one has

$$x_{N\backslash M(x_N,y_N)}\overline{R}(e^r)y_{N\backslash M(x_N,y_N)} \Leftrightarrow x'_{N'\backslash M(x'_N,y'_N)}\overline{R}(e'^r)y'_{N'\backslash M(x'_N,y'_N)},$$

and as a result

$$x_N\overline{R}(e)y_N \Leftrightarrow x'_N\overline{R}(e')y'_N.$$

INfI implies IUI. Let $x_M = y_M$. One has

$$f(e,(x_N,y_N)) = \left(\theta_{N\backslash M(x_N,y_N)}, X - x_{M(x_N,y_N)}, (x_{N\backslash M(x_N,y_N)}, y_{N\backslash M(x_N,y_N)})\right),$$

and

$$f(e^r,(x_{N\backslash M},y_{N\backslash M})) = \begin{pmatrix} \theta_{(N\backslash M)\backslash M(x_{N\backslash M},y_{N\backslash M})}, (X - x_M) - x_{M(x_{N\backslash M},y_{N\backslash M})}, \\ (x_{(N\backslash M)\backslash M(x_{N\backslash M},y_{N\backslash M})}, y_{(N\backslash M)\backslash M(x_{N\backslash M},y_{N\backslash M})}) \end{pmatrix}.$$

Now, $(N\backslash M)\backslash M(x_{N\backslash M},y_{N\backslash M}) = N\backslash M(x_N,y_N)$, and

$$\begin{aligned} &(X - x_M) - x_{M(x_{N\backslash M},y_{N\backslash M})} \\ =&\{z_{N\backslash M(x_N,y_N)} | (x_{M(x_{N\backslash M},y_{N\backslash M})}, z_{N\backslash M(x_N,y_N)}) \in X - x_M\} \\ =&\{z_{N\backslash M(x_N,y_N)} | (x_M, x_{M(x_{N\backslash M},y_{N\backslash M})}, z_{N\backslash M(x_N,y_N)}) \in X\} \\ =&X - x_{M(x_N,y_N)}. \end{aligned}$$

Therefore $f(e,(x_N,y_N)) = f(e^r,(x_{N\backslash M},y_{N\backslash M}))$, so that by IN$f$I, $x_N\overline{R}(e)y_N \Leftrightarrow x_{N\backslash M}\overline{R}(e^r)y_{N\backslash M}$. ■

Proposition 10. *Consider any subset* $\Phi \subset \Phi_{ONC}$, *and assume that* $\mathscr{E}$ *is rich enough so that, for any* $e = (\theta_N, X) \in \mathscr{E}$, *and any* $\varphi \in \Phi$, *there exists* $(\theta'_N, X) \in \mathscr{E}$ *such that* $U(\theta'_N) = \varphi(U(\theta_N))$. *A SOF* $\overline{R}$ *satisfies* INVΦ *and IOA if and only if it satisfies BINV*Φ.

Proof. INVΦ and IOA jointly imply BINVϕ. Let $e = (\theta_N, X)$, $e' = (\theta'_N, X) \in \mathscr{E}, u_N = U(\theta_N), u'_N = U(\theta'_N), x, y \in X$, such that

$$\exists \varphi \in \Phi, \begin{cases} u'_N(x) = \varphi(u_N(x)) \\ u'_N(y) = \varphi(u_N(y)). \end{cases}$$

By the richness assumption, there exists $e'' = (\theta''_N, X) \in \mathscr{E}$ such that $U(\theta''_N) = \varphi(u_N)$. One therefore has

$$u'_N(x) = U(\theta''_N)(x), u'_N(y) = U(\theta''_N)(y).$$

By IOA, this implies $\overline{R}(e'')|_{\{x,y\}} = \overline{R}(e')|_{\{x,y\}}$. In addition, INVΦ entails that $\overline{R}(e'') = \overline{R}(e)$. Therefore $\overline{R}(e')|_{\{x,y\}} = \overline{R}(e)|_{\{x,y\}}$.

BINVΦ implies INVΦ. This immediately follows from the fact that:

$$\exists \varphi \in \Phi, u'_N = \varphi(u_N) \Rightarrow \forall x, y \in X, \exists \varphi \in \Phi, \begin{cases} u'_N(x) = \varphi(u_N(x)) \\ u'_N(y) = \varphi(u_N(y)). \end{cases}$$

BINVΦ_g implies IOA. Let $e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}, u_N = U(\theta_N)$, $u'_N = U(\theta'_N)$, and $x, y \in X$ be such that $u_N|_{\{x,y\}} = u'_N|_{\{x,y\}}$. The identity transformation belongs to Φ_g, and therefore

$$\exists \varphi \in \Phi_g, \begin{cases} u'_N(x) = \varphi(u_N(x)) \\ u'_N(y) = \varphi(u_N(y)), \end{cases}$$

so that, by BINV$\Phi_g, \overline{R}(e')|_{\{x,y\}} = \overline{R}(e)|_{\{x,y\}}$.

In order to prove that BINVΦ implies IOA, it is then sufficient to show that BINVΦ implies BINVΦ_g. Let $e = (\theta_N, X), e' = (\theta'_N, X) \in \mathscr{E}$, $u_N = U(\theta_N), u'_N = U(\theta'_N), x, y \in X$, such that

$$\exists \varphi \in \Phi_g, \begin{cases} u'_N(x) = \varphi(u_N(x)) \\ u'_N(y) = \varphi(u_N(y)). \end{cases}$$

By Lemma 1, there exist $\varphi_1, \ldots, \varphi_m \in \Phi \cup \Phi^{-1}$ such that $\varphi = \varphi_1 \circ \ldots \circ \varphi_m$. Let $e^m = (\theta^m_N, X) \in \mathscr{E}$ be such that $U(\theta^m_N) = \varphi^m(U(\theta_N))$. Such e^m exists, by the richness assumption, since $\varphi_m \in \Phi_g$. If $\varphi_m \in \Phi$, then, by BINVΦ,

$$\overline{R}(e)|_{\{x,y\}} = \overline{R}(e^m)|_{\{x,y\}}.$$

If $\varphi_m \in \Phi^{-1}$, then $\varphi_m^{-1} \in \Phi$, and

$$\begin{cases} u_N(x) = \varphi_m^{-1}(\varphi_m(u_N(x))) \\ u_N(y) = \varphi_m^{-1}(\varphi_m(u_N(y))), \end{cases}$$

so that, by BINVΦ, one again obtains

$$\overline{R}(e)|_{\{x,y\}} = \overline{R}(e^m)|_{\{x,y\}}.$$

If one takes $e^{m-1} = (\theta^{m-1}_N, X) \in \mathscr{E}$ such that $U(\theta^{m-1}_N) = \varphi^{m-1}\big(U(\theta^m_N)\big) = \varphi^{m-1}(\varphi^m(U(\theta_N)))$, a similar reasoning leads to

$$\overline{R}(e^m)|_{\{x,y\}} = \overline{R}(e^{m-1})|_{\{x,y\}}.$$

By iteration,

$$\overline{R}(e)|_{\{x,y\}} = \overline{R}(e')|_{\{x,y\}}.$$

References

Arrow KJ (1951) Social choice and individual values. Wiley, New York
Atkinson AB (1995) Public economics in action. The basic income/flat tax proposal. Clarendon Press, Oxford
Blackorby C, Bossert W, Donaldson D (1997) Critical-level utilitarianism and the population-ethics dilemma. Econo Philos 13: 197–230
Bossert W (1991) On intra- and interpersonal utility comparisons. Soc Choice Welfare 8: 207–219
Bossert W (1999) Welfarism and information invariance. University of Nottingham, mimeo
Bossert W, Weymark JA (1998) Utility theory in social choice. In: Barbera S, Hammond PJ, Seidl C (eds), Handbook of utility theory, vol.1. Kluwer, Dordrecht
d'Aspremont C, Gevers L (1977) Equity and the informational basis of collective choice. Rev Econ Stud 44: 199–209
d'Aspremont C, Gevers L (2002) Social welfare functionals and interpersonal comparability. In: Arrow KJ, Sen AK, Suzumura K (eds), Handbook of social choice and welfare, vol. 1. North-Holland, Amsterdam
Dhillon A, Mertens JF, (1999) Relative utilitarianism. Econometrica 67: 471–498
Dworkin R (2000) Sovereign virtue. The Theory and Practice of Equality. Harvard University Press, Cambridge, Mass
Eisenberg E (1961) Aggregation of utility functions. Manag Sci 7(4): 337–350
Fleurbaey M (1995) Equality and responsibility. Europe Econo Rev 39: 683–689
Fleurbaey M (1998) Equality among responsible individuals. In: Laslier JF, Fleurbaey M, Gravel N, Trannoy A (eds), Freedom in economics. Routledge, London
Fleurbaey M, Maniquet F (1996) Utilitarianism versus fairness in welfare economics. In: Salles M, Weymark JA (eds), Justice, political liberalism and utilitarianism: Themes from Harsanyi and Rawls. Cambridge University Press, Cambridge (forthcoming)
Fleurbaey M, Maniquet F (1999) Compensation and responsibility. In: Arrow KJ, Sen AK, Suzumura K (eds), Handbook of social choice and welfare, vol. 2. North-Holland, Amsterdam (forthcoming)
Fleurbaey M, Maniquet F (2000) Fair orderings with unequal production skills. THEMA 2000-17
Fleurbaey M, Maniquet F (2001) Fair social orderings. University of Pau and University of Namur, mimeo
Fleurbaey M, Suzumura K, Tadenuma K (2002) Informational requirements for preference aggregation in economic environments. University of Pau and Hitotsubashi University, mimeo
Gevers L (1979) On interpersonal comparability and social welfare orderings. Econometrica 47: 75–89
Hammond PJ (1976) Equity, Arrow's conditions and Rawls' difference principle. Econometrica 44: 793–804
Hammond PJ (1987) Social choice: the science of the impossible?. In: Feiwel GR (ed) Arrow and the foundations of the theory of economic policy. New York University Press, New York
Hansson B (1973) The independence condition in the theory of social choice. Theory Decision 4: 25–49

Harsanyi JC (1976) Essays on ethics, social behavior, and scientific explanation. Reidel, Dordrecht
Kaneko M, Nakamura K (1979) The Nash social welfare function. Econometrica 47: 423–435
Kemp MC, Ng YK (1976) On the existence of social welfare functions, social orderings and social decision functions. Economica 43: 59–66
Kolm SC (1996) Modern theories of justice. MIT Press
Mongin P, d'Aspremont C (1998) Utility theory and ethics. In: Barbera S, Hammond PJ, Seidl C (eds), Handbook of utility theory, vol. 1. Kluwer, Dordrecht
Pazner E (1979) Equity, nonfeasible alternatives and social choice: A reconsideration of the concept of social welfare. In: Laffont JJ (ed), Aggregation and revelation of preferences, North-Holland, Amsterdam
Rawls J (1982) Social unity and primary goods. In: Sen AK, Williams B (eds), Utilitarianism and beyond, Cambridge University Press, Cambridge
Roberts K (1980) Interpersonal comparability and social choice theory. Rev Econ Stud 47: 421–439
Roemer JE (1998) Equality of opportunity. Harvard University Press, Cambridge, Mass
Samuelson PA (1977) Reaffirming the existence of "reasonable" Bergson-Samuelson social welfare functions. Economica 44: 81–88
Samuelson PA (1987) Sparks from Arrow's anvil. In: Feiwel GR (ed), Arrow and the foundations of the theory of economic policy, New York University Press, New York
Sen AK (1970) Collective choice and social welfare. Holden-Day, San-Francisco
Sen AK (1985) Commodities and capabilities. North-Holland, Amsterdam
Sen AK, (1986) Social choice theory. In: Arrow KJ, Intriligator MD (eds), Handbook of mathematical economics, vol. 3. North-Holland, Amsterdam
Sen AK (1987) On ethics and economics. Basil Blackwell, Oxford
Sen AK (1999) The possibility of social choice. Am Econ Rev 89: 349–378
Thomson W (1996) Consistent allocation rules. University of Rochester, Mimeo