Experiments in Fluids [Suppl.] S61–S69

The accuracy of remapping irregularly spaced velocity data onto a regular grid and the computation of vorticity

R. K. Cohn, M. M. Koochesfahani

Abstract The velocity data obtained from molecular tagging velocimetry (MTV) are typically located on an irregularly spaced measurement grid. To take advantage of many standard data processing techniques, the MTV data need to be remapped onto a grid with a uniform spacing. In this work, accuracy and noise issues related to the use of a least-squares-fit to various low order polynomials for the remapping of these data onto a uniformly spaced grid and the subsequent computation of vorticity from these data are examined. This information has relevance to PIV data processing as well. It has been previously noted that the best estimate of the velocity vector acquired through the use of tracer techniques such as PIV, is at the midpoint of the displacement vector. Thus, unless special care is taken, PIV data are also initially obtained on an irregular grid. The error in the remapped velocity and the calculated vorticity field is divided into a mean bias error and a random error. In the majority of cases, the mean bias error is a more significant source of error than the more often quoted random error. Results of the simulation show that the best choice for remapping is the use of a least-squares fit to a 2nd order polynomial and the best choice for vorticity calculation is to use a 4th order accurate, central, finite difference applied to uniformly sampled data. The actual value of the error depends upon the data density and the radius used for the selection of velocity measurements to be included in the remapping process. Increasing the data density and reducing the fit radius improve the accuracy.

1 Introduction

In recent years, many researchers have made use of full-field, two-component optical velocity measurement techniques, such as particle image velocimetry (PIV), to derive flow quantities such as the out-of-plane vorticity from velocity data. The velocity field acquired from PIV is normally thought to be gathered on a uniformly spaced grid that allows for a variety of standard post-processing methods to be utilized. The development of molecular tagging velocimetry (MTV) has placed an additional complication on the calculation of flow variables in that the data are not normally collected on a uniformly spaced grid. This paper deals with the questions related to remapping MTV data onto a regularly spaced grid and the methods used to compute the out-of-plane vorticity component from these remapped data sets.

Molecular tagging velocimetry is a full-field optical diagnostic which allows for the non-intrusive measurement of a fluid velocity field. This technique takes advantage of molecules that have long-lived excited states when tagged by a photon source. This technique can be thought of as the molecular equivalent of PIV. Rather than tracking particles placed in the flowing medium, the luminescence of regions of the flow containing the tracer molecules is tracked. A more complete description of the implementation of the molecular tagging technique, its applications, and the parameters necessary for an optimal experiment can be found in Gendrich and Koochesfahani (1996), Gendrich et al. (1997), and Koochesfahani (1999). The accuracy of velocity measurements made using MTV is comparable to the digital version of PIV (DPIV).

In the implementation of MTV, a series of laser-lines is used to generate a two-dimensional spatial distribution in the intensity field within the flowing medium. Velocity vectors are calculated at the intersection of these laser-lines. Generally, the measurement locations are not uniformly spaced. Thus, it is necessary to place the velocity data onto a regular grid before flow variables, such as vorticity, can be computed via standard finite difference techniques. It should be noted that even though it is possible to generate a series of regularly spaced laser-lines in the flow, it is still necessary to remap the data. Both an unpublished study conducted at Michigan State University and Spedding and Rignot (1993) have reported that the best estimate of the location of the velocity vector determined by a measurement technique which tracks a tracer in a flow is located at the midpoint of the displacement vector. Thus, unless special care is taken in the selection of the measurement windows, data collected from PIV measurements is also not on an uniformly spaced grid.

Few studies have examined the effect of remapping randomly spaced velocity data onto a regular grid. Agui and Jimenez (1987) reported that low order polynomial fits and "kriging" techniques produced the most accurate

R. K. Cohn (✉)
AFRL/PRSA, 10 E. Saturn Blvd, Edwards Air Force Base
CA 93524, USA

M. M. Koochesfahani
Department of Mechanical Engineering
Michigan State University
East Lansing, MI 48825, USA

This work made use of shared facilities of the MRSEC Program of the National Science Foundation, Award Number DMR-9400417. It was supported by the Palace Knight Program of the United States Air Force Research Laboratory.

representation of the actual velocity field. However, the advantage was small with respect to other methods and no quantitative information on the performance of the polynomial and kriging methods was given. Spedding and Rignot (1993) compared an inverse distance approach with the use of a "global basis function" and found that the global basis function produced generally more accurate results; however, the results were highly dependent upon the measurement density.

Several authors have examined the accuracy of various means to compute vorticity from velocity data already on a regular grid. Spedding and Rignot (1993) used a 2nd order accurate finite difference technique for the inverse distance method and directly differentiated the global basis function to compute vorticity. It was found that direct differentiation of the global basis function produced generally superior results. However, as with the velocity results, the accuracy was highly dependent upon the ratio of a characteristic length scale of the flow, L, to the mean spacing between measurements, δ. Abrahamson and Lonnes (1995) found that calculating vorticity by computing the local circulation around a point resulted in slightly more accurate vorticity results than differentiating a least-squares fit to a model velocity field. Luff et al. (1999) compared the 2nd and 4th order accurate finite difference methods and an eight-point circulation method in the calculation of vorticity in the presence of both noise and missing data points. In terms of only the computed vorticity rms, the 2nd order accurate finite difference technique produced the best results.

One shortcoming of the above mentioned studies is that only the random component of the error field is examined. Fouras and Soria (1998) found that the error in the vorticity field could be better represented if it is divided into two portions: a mean bias error due to spatial filtering, and a random error resulting from the propagation of error in the velocity measurements into the vorticity calculation. In some cases, the mean bias error can be significantly larger than the random error. This study recommends differentiating a 2nd order polynomial least-squares fit to the velocity data for the calculation of vorticity based on the 21 closest points. However, at low data densities, this produces larger bias errors than the use of a finite difference method. The results based on differentiating the fit were sensitive to the number of points used in the fit. This work is based entirely on regularly sampled velocity data; issues connected to remapping an irregular data set were not considered.

The aforementioned investigations suggest different optimum methods for vorticity computation depending upon the criterion used to assess the error. In our work, we directly compare several of the different vorticity calculation methods that were determined in the previous studies to produce the best results. In addition, the effect of the remapping of the velocity field on the estimation of the vorticity is also considered.

2 Comparison method

The present study makes use of a simulation of an Oseen vortex in order to study the effect of remapping an irregularly sampled velocity field onto a regular grid and the calculation of the out-of-plane vorticity component. This flowfield has also been used in the works of Spedding and Rignot (1993), Fouras and Soria (1998), and Luff et al. (1999). The azimuthal velocity, u_θ, and out-of-plane vorticity, ω of this flow field are described by:

$$u_\theta = \frac{\omega_{\max} r_{\mathrm{core}}^2}{2r}\left(1 - \mathrm{e}^{-(r^2/r_{\mathrm{core}}^2)}\right)$$
$$\omega = \omega_{\max}\mathrm{e}^{-(r^2/r_{\mathrm{core}}^2)} \quad . \tag{1}$$

An example of the velocity and vorticity field generated can be seen in Fig. 1. Note that even though the mean data density is the same in both the irregularly sampled velocity field (Fig. 1a) and the remapped velocity field (Fig. 1b), it is visually easier to discern the vortical structure in the regular velocity measurement.

In order to simulate the irregular sampling found in the original velocity field measurements, the simulation data are irregularly spaced. The irregular spacing is generated by sub-dividing the measurement field into $\delta \times \delta$-sized regions, where δ is the mean spacing between velocity measurement points. A random number generator is then used to determine a location for the simulated velocity within each $\delta \times \delta$-sized region. Equation (1) is then used to establish the velocity at this location. In this manner, the mean spacing between measurement points remains equal to δ; however the actual location of the measurement varies.

The random error inherent in MTV and PIV measurements is simulated by the addition of noise to the velocity field. The method used is similar to that in Luff et al. (1999). A random number generator is used to add a random percentage of noise, with a maximum value of n% to each component of the Cartesian components, i.e., u and v, of the velocity field. Using this formulation, the velocity at each point in the simulation has a value of:

$$u = u_{\mathrm{act}}(1 + n_{\mathrm{random}}) \tag{2a}$$
$$v = v_{\mathrm{act}}(1 + n_{\mathrm{random}}) \tag{2b}$$

where n_{random} is a random number with a value $-n < n_{\mathrm{random}} < +n$. The quantities u_{act} and v_{act} represent the actual Cartesian velocities determined from u_θ in Eq. (1). Although error values ranging from $0\% < n < 10\%$ were examined in the simulations, only 0% and 6% values are presented as they are representative of the other noise values.

The velocity data in the present study are remapped onto a regular grid by means of a local least-squares fit to a two-dimensional 2nd, 3rd, or 4th order polynomial. The u and v velocity fields are fit separately. Only the velocity measurements located within the fit radius, R, from the regular grid point are used in the fitting procedure. In this study, this radius is normalized by the mean spacing between velocity measurements, δ. For all cases, the number of points used in the fit is such that the least-squares fit is over-determined. That is, the number of points used in the fitting process exceeds the minimum number necessary for a successful fit, as determined by the number of coefficients in the polynomial. For the 2nd, 3rd, and 4th order polynomials, this minimum number of

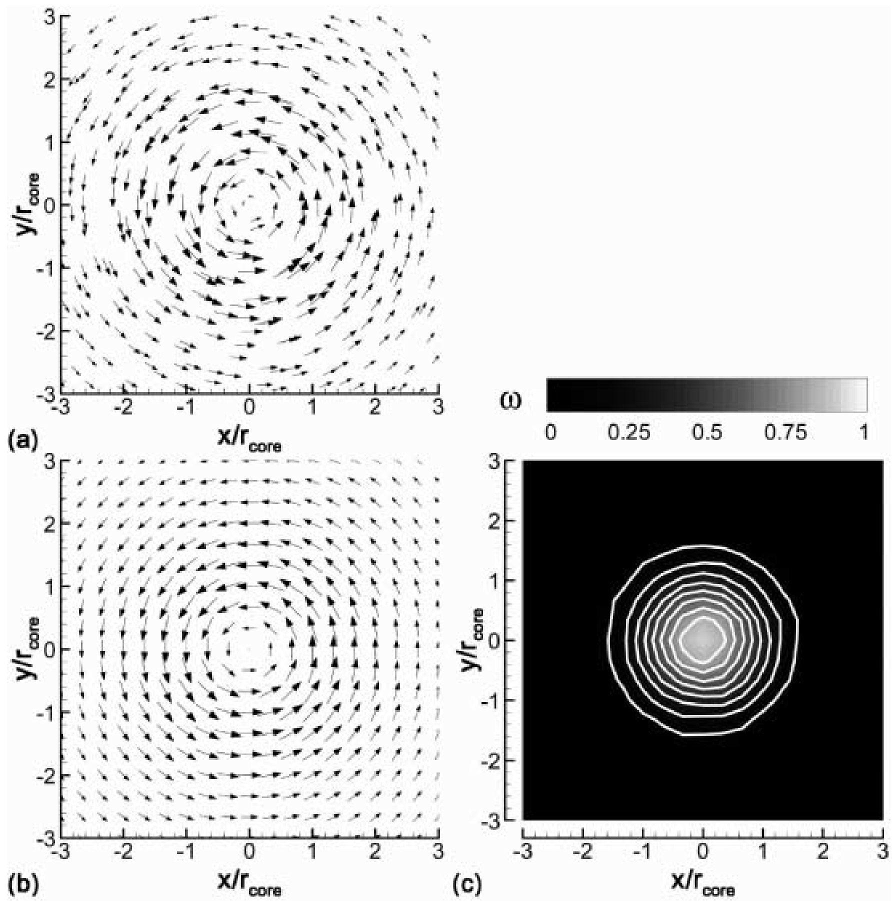

Fig. 1a–c. Sample normalized velocity and vorticity fields of a Gaussian core vortex. **a** Original velocity vector field on an irregular grid; **b** velocity vector field placed upon a uniformly spaced grid; **c** flooded contour plot of normalized calculated vorticity field. Contour lines are placed at 0.125, 0.25, ...,1

points are 6, 10, and 15, respectively. Note, therefore, that the minimum value of R that can be used for the 2nd order fit is smaller than that for the 4th order fit.

After the fits for the two velocity components are generated, each of the local fits is evaluated at the coordinate of the regular grid point in order to determine the velocity at that location. Clearly choosing a value of R that is too large will result in a considerable amount of spatial filtering of the data, while a small value of R will limit the ability of the fit to reduce the random noise present in the original data. Note that the order of the polynomial places a limit on the minimum size of R that can be used, as described earlier. For all of the studies conducted, the density of the remapped, uniformly spaced grid remains the same as the initial irregularly spaced measurement grid.

Four methods are used to estimate the out-of-plane vorticity,

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad . \tag{3}$$

The first two methods estimate the two derivatives in the definition of vorticity by means of a 2nd or 4th order accurate central finite difference technique (also referred to as 1st and 2nd order finite difference, respectively, in several other works). The third method performs a direct differentiation of the polynomial least-squares fit used in the remapping of the velocity field. This method has the advantage that it can be used to estimate the vorticity at any point within measurement region. The final vorticity calculation method computes the circulation of the 8 points in the rectangular region extending one regular grid point in each direction around the point to be examined. The calculated circulation value is then divided by the area in order to determine the vorticity. This method has been shown in Raffel et al. (1998) to be identical to a filtered version of the 2nd order accurate central finite difference technique. Figure 2 illustrates the data points used for the calculation of the vorticity in these various methods.

This study examines the effect of varying the normalized mean data density, L/δ and the normalized fit radius, R/δ on the accuracy of the remapped velocity field and the calculated vorticity field. The characteristic flow scale, L, used in this study is the vortex core radius, r_{core} defined as the distance from the peak vorticity to the location where the vorticity has dropped by a factor of e^{-1}. Simulations are conducted for values of L/δ ranging from 2 to 10 and

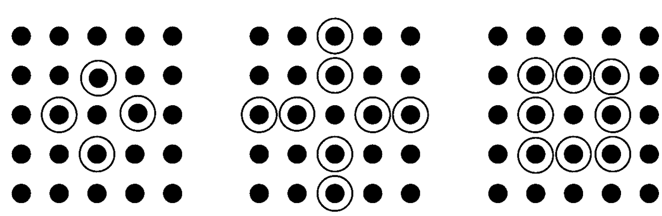

Fig. 2a–c. Velocity measurement locations used in the estimation of the spatial derivatives. **a** 2nd order accurate finite difference **b** 4th order accurate finite difference **c** 8-pt circulation method

for R/δ ranging from 2 to 6. Note that in order for the fit to be over-determined, not all of these values can be used for all polynomial orders of the least-squares fit.

As in the results of Fouras and Soria (1998), the accuracy of both the velocity and vorticity calculations methods are assessed in terms of the mean bias error caused by spatial filtering and the random error. Both the propagation of the error in the original measurements to the remapped field and the placement of the randomly spaced points onto the regular grid generate the random error. For each parameter condition investigated, 5000 independent simulations are conducted. This number of samples was found to be sufficient for the convergence of the mean statistical quantities, such as the mean bias error in this simulation, and results in only a small difference in the random error in a small number of cases. These cases are generally ones with larger errors that would not be recommended for use. The mean bias error will be denoted by the subscript "bias" and refers to difference between the mean value of these 5000 velocity (or computed vorticity) measurements and the exact value at each point in the flowfield determined from Eq. (1). The random error is quantified by the rms of the velocity (or computed vorticity) data in the sample set. For example, for the x-component of velocity, these are defined as:

$$u^*_{\text{bias}} = u_{\text{act}} - \bar{u}, \quad u^*_{\text{rms}} = \left(\frac{1}{n} \sum_{i=1}^{n} (u_i - \bar{u})^2 \right)^{1/2}, \quad \bar{u} = \frac{1}{n} \sum_{i=1}^{n} u_i, \quad n = 5000 \ . \tag{4}$$

In these expressions, u_i is the velocity at a particular point of an individual realization in the simulation. All of the velocity and vorticity error values reported here are normalized by their respective maximum values determined from Eq. (1). The normalized values are shown without the asterisk.

3 Remapping results

Figure 3a displays the mean velocity bias error for the 2nd, 3rd, and 4th order polynomial fits. The results presented in this figure are for the case of 0% added noise because the addition of noise has no effect on the bias error, as it is a mean quantity. For all three fits, the mean velocity data density is kept fixed at $L/\delta = 3.0$. That is, there are nominally seven velocity vectors along the vortex core diameter. For the 2nd and 3rd order polynomial fits, three different values of R/δ are examined. However, only the two larger values are used for the 4th order polynomial fit to ensure there are enough data points available for the fit. In terms of the mean bias error, reducing this radius results in a significant decrease in the bias error. For the 2nd and 3rd order polynomial fits, reducing from $R/\delta = 4$ to $R/\delta = 2$ results in a decrease of the peak mean bias error from 8% to less than 1%. This effect is present, although less dramatic, in the results for the 4th order polynomial fit. Note that the values specified are the maximum bias error. The bias errors at other locations is significantly smaller.

In terms of the mean bias error, the most accurate results are obtained using the least-squares fit to a 4th order polynomial. However, the difference in the bias error between the 4th order fit using $R/\delta = 3$, and the 2nd order fit with $R/\delta = 2$ is approximately 0.6%. It is interesting to note that the results for the 2nd and 3rd order polynomial fits are nearly identical. For all three fit orders, the peak bias error occurs at approximately $0.6r_{\text{core}}$. In the region $r/r_{\text{core}} > 1.5$, the velocity values tend to be overestimated, rather than underestimated. Since the area of the region $r/r_{\text{core}} > 1.5$ is significantly larger than the region where the velocity values are underestimated, the net average of the bias error becomes very small. Thus, one should be cautious about the use of an accuracy measure that is averaged over the entire vortical structure as this does not represent the actual error seen at any individual measurement location.

Figures 3b and c show the random error found in the remapped velocity field for cases of 0% and 6% added noise respectively. From Figs. 3b and c it can be seen that, generally, the value of the rms error is less than 2% at all locations. This value can only be reduced by a maximum of 1.5% by the optimal choice of fit order and R/δ, whereas a reduction of 8% is seen in the bias error. It is also interesting to note that for the case of 0% added noise, reducing R/δ results in a decrease in the random error. However, for the case of 6% added noise, reducing R/δ results in an increase in the random error.

Unless otherwise stated, the remainder of the results presented in this paper will use the 2nd order polynomial for the remapping process. This choice is made because of this condition has a very small bias error, nearly identical to the other fit orders, and it is not as computationally intensive as the 3rd and 4th order polynomials. Figure 4 shows the effect of grid density on the accuracy of the remapping of the 2nd order polynomial fit for $R/\delta = 2$. As shown in Fig. 4a, increasing the grid density can reduce the bias error in the remapping. For $L/\delta = 2$, the bias error is nearly 3% of the maximum velocity. Increasing L/δ to 3 results in a bias error of less than 1%. It should be noted that these values are only valid for $R/\delta = 2$. In order to achieve a bias error of less than 1% for a larger value of R/δ, such as $R/\delta = 4$, L/δ must be greater than 6.

Figures 4b and c show the effect of increasing the grid density on the random error. For the 0% added error cases, increasing the grid density also results in a noticeable decrease in the random error. The addition of noise, shown in Fig. 4c, generally increases the random error. The 6% noise added to the data tends to dominate the random error results and leads to the random error profiles being nearly identical for the values of L/δ examined in this study.

4 Vorticity calculation results

In this section, the error generated by the four methods for calculating the out-of-plane vorticity field will be examined. First, we will discuss the results from directly differentiating the various polynomial orders used in the remapping procedure in order to determine the vorticity value. Then, the results from this method will be compared

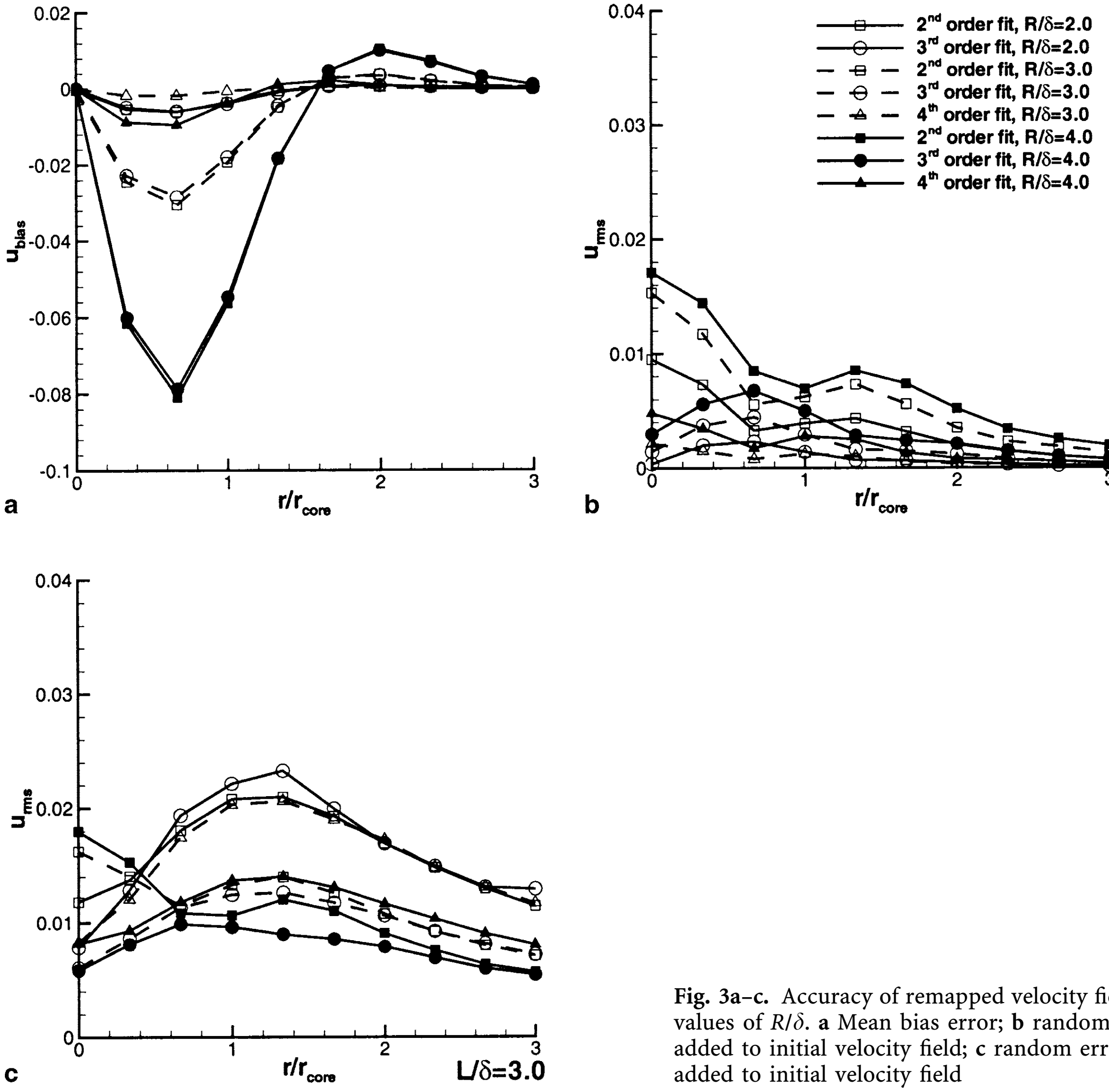

Fig. 3a–c. Accuracy of remapped velocity field for three different values of R/δ. **a** Mean bias error; **b** random error with 0% noise added to initial velocity field; **c** random error with 6% noise added to initial velocity field

with those from the finite difference and circulation methods applied to the remapped data using the 2nd order polynomial.

Figure 5a shows the mean bias error for differentiating the 2nd, 3rd, and 4th order polynomials for several values of R/δ. Similar to the velocity bias error results, decreasing R/δ decreases the bias error. The smallest values for the mean bias error are found using the smallest values of R/δ and the 3rd and 4th order polynomials. These selections result in a bias error of less than 4%. It is interesting to note that although the error in the remapping of the velocity field through the use of the 2nd and 3rd order polynomials are nearly identical, the vorticity estimates by differentiating these polynomials differ. Further, the vorticity estimates that result from differentiating the 3rd and 4th order polynomials (for the same value of R/δ) are very similar.

Figures 5b and c show the effect of the order of the polynomial fit on the random component of the error. For the case in which no noise is added to the velocity data, the random error is less than 1.5%. The addition of 6% random noise results in only a small increase in the random error for the majority of cases. The 3rd order fit and $R/\delta = 2$, are atypical in that a considerably larger increase, including a spike in the data, is seen in the random error. It is believed that the larger error is generated because the number of points utilized for the fit is only slightly larger than the minimum number of points required. With the exception of that case, there is generally little difference in the random error among the various polynomial fits. In the remainder of this paper, only the vorticity estimated from differentiating the 4th order fit with $R/\delta = 3$ will be compared with those calculated using the finite difference and circulation methods. The results generated by direct differentiation of the 3rd order fit were not selected for further comparison because for $R/\delta = 2$, where the bias error is noticeably less than that of the 4th order differentiation, the random error is significantly larger when noise is present in the original data. Furthermore, the 4th order fit results in significantly better velocity remapping results compared to the 3rd order fit. Thus, the 4th order fit seems the more suitable selection for performing the remapping and vorticity calculation.

Figure 6a compares ω_{bias} found by the four different methods considered here. Once again, the effect of adding

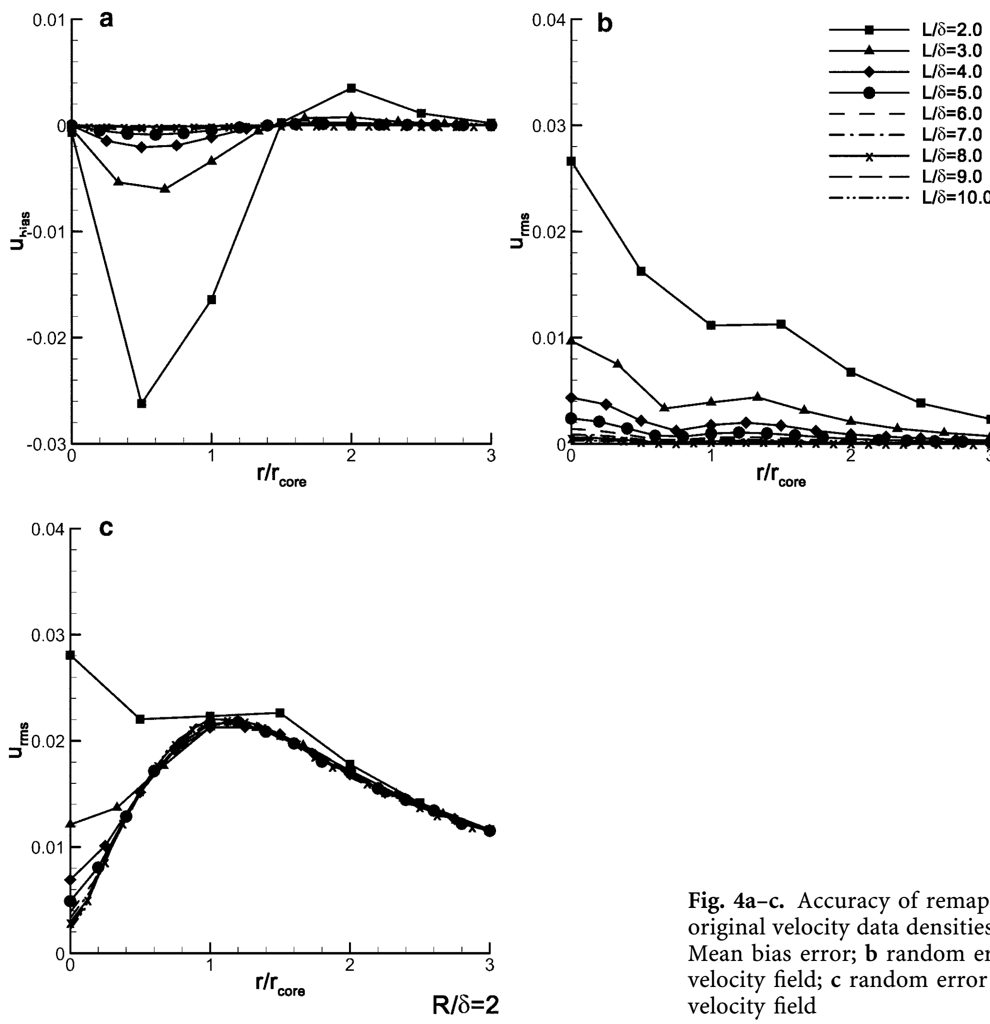

Fig. 4a–c. Accuracy of remapped velocity field for different original velocity data densities for the 2nd order polynomial fit. **a** Mean bias error; **b** random error with 0% noise added to initial velocity field; **c** random error with 6% noise added to initial velocity field

noise to the initial velocity field on the bias error is negligible, therefore, only the case of 0% added noise is shown. The qualitative features of the four methods are very similar. The maximum ω_{bias} occurs at $r/r_{\text{core}} = 0$ which is the location of the peak vorticity. For $r/r_{\text{core}} > 1.5$, there is a small overshoot where the vorticity value is overestimated. Note that although the numerical amount of the overshoot is small relative to that of the undershoot, the area occupied by the region of overshoot is roughly three times larger than the region of undershoot. Thus, the overall area-averaged vorticity bias error is very small. As a result, the estimate of the overall circulation of the vortex computed by integrating the vorticity field from any of these methods is accurate to better than 0.1% even though the peak bias error can be as large as 20%.

In terms of the mean bias error, it is apparent that differentiating the 4th order polynomial fit, and the use of the 4th order accurate finite difference technique produce the most accurate vorticity field information. For comparison purposes, the results from $R/\delta = 4$ are also shown. As expected, reducing the value of R tends to improve the accuracy of the vorticity calculation as well as decreasing the difference between the accuracy of the two techniques. The circulation method and the 2nd order accurate finite difference method produce results with a significantly larger bias error than the other two methods.

Figures 6b and c show the random error for the vorticity calculation methods for the cases of 0% and 6% added noise respectively. Generally, differentiating the 4th order polynomial produces the smallest random error while the 4th order accurate finite difference method produces the largest. As with the velocity field, the improvement which can be realized through the use of the optimal method to minimize the random error is much smaller than that which can be realized by minimizing the bias error. However, the difference in the random error between these two techniques is approximately 1%.

As expected, increasing the density of the original data also dramatically reduces the mean bias error as seen in Fig. 7a. This figure only shows results for the 4th order accurate finite difference technique and $R/\delta = 2$, however, the qualitative features of all of the methods are identical. Increasing the mean data density, L/δ, from 2 to 4 results in a decrease of the mean bias error from approximately 7% to less than 1%. Further increases result in only a small decrease in the bias error. As in the results presented for the remapping uncertainty, increasing the density of the data also results in a decrease of the random error for the

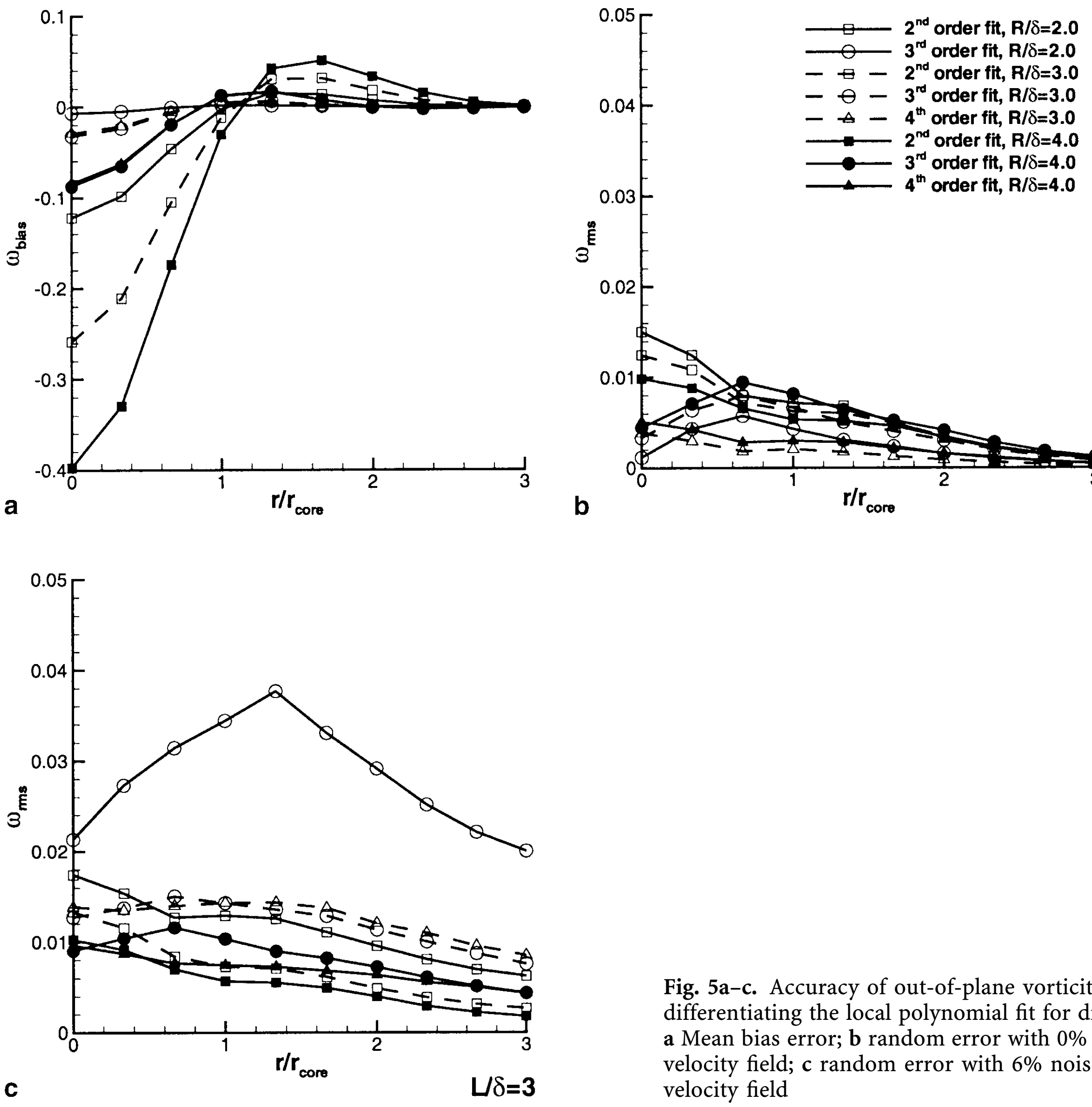

Fig. 5a–c. Accuracy of out-of-plane vorticity field computed by differentiating the local polynomial fit for different values of R/δ. **a** Mean bias error; **b** random error with 0% noise added to initial velocity field; **c** random error with 6% noise added to initial velocity field

case where no noise is added to the original data shown in Fig. 7b. However, when noise is added to the original velocity field, as shown in Fig. 7c, the random error in vorticity is found to increase with the increase in the grid density. For $L/\delta = 10$, the random error is nearly 6% of the peak vorticity value, which is only slightly smaller than the peak bias error introduced in the $L/\delta = 2$ case. For moderate values of L/δ, such as $L/\delta = 3$, the peak random error is about 2% of ω_{max}.

It should be noted that applying the uncertainty analysis to the finite difference calculation for vorticity shows that the error in the vorticity is connected to the error in the velocity in the form $\omega_{rms} \sim u_{rms}/\delta$. Therefore, as the grid density increases, the vorticity error is expected to increase as well; a trend that is not seen in Fig. 7b. The reason is that the source of the noise itself, i.e., u_{rms}, is not fixed and varies depending on the grid density during the remapping process (e.g., see Fig. 4b). The combined effect is illustrated in Fig. 7b. However, for the case where noise has been added to the initial data (Fig. 7c), ω_{rms} shows the expected increase. In this case, u_{rms} (see Fig. 4c) remains nearly constant as grid density varies.

5
Conclusions

The effect of remapping irregularly spaced velocity measurements onto a uniformly spaced grid and the accuracy of the out-of-plane vorticity computed from this information are studied through the use of a Gaussian core vortex simulation. The effect of varying the normalized grid density L/δ (ratio of the flow characteristic length to the mean spacing in the initial velocity measurement) and the normalized maximum radius from which points are used in the remapping process, R/δ, are examined. The error resulting from the remapping and the calculation of the out-of-plane vorticity is divided into a mean bias error due to spatial filtering and a random error due to the remapping process itself and the propagation of the error in the original velocity data.

It is found that in general the errors resulting from the bias error are significantly larger than the random error. In terms of the mean velocity bias error, it is necessary for the grid density to be suitably high and for the fit to be local, i.e., small values of R/δ, to generate an accurate remapping. In this study, the least-squares fit to a 4th order

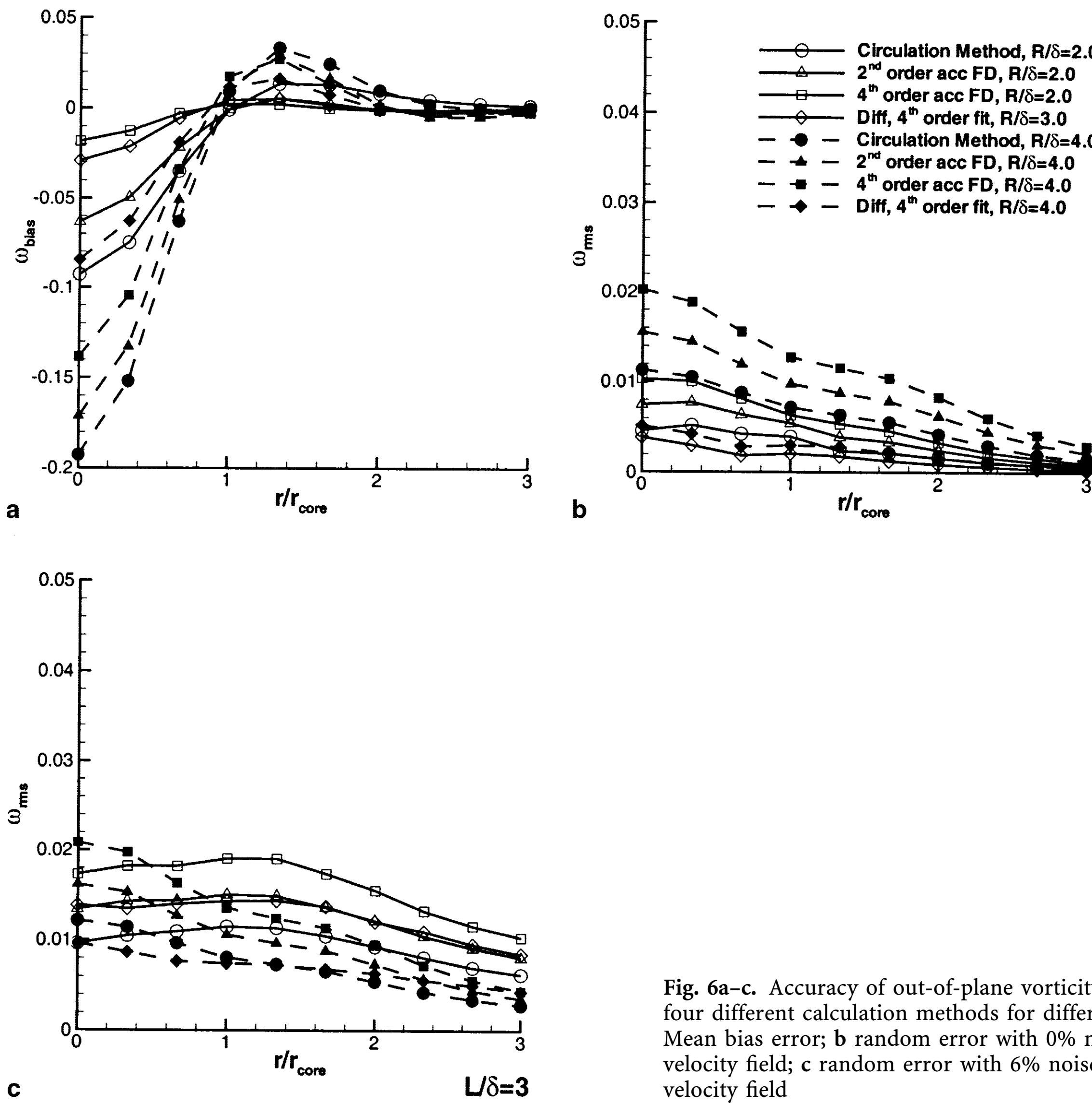

Fig. 6a–c. Accuracy of out-of-plane vorticity field computed by four different calculation methods for different values of R/δ. **a** Mean bias error; **b** random error with 0% noise added to initial velocity field; **c** random error with 6% noise added to initial velocity field

polynomial produced the most accurate remapping; however, due to the large number of points needed for the fit to be determined, it is not possible to use the same small values of R/δ as can be used with the lower order polynomials. The difference in the bias error between the 4th order polynomial with $R/\delta = 3$ and the 2nd order polynomial with $R/\delta = 2$ is very small. Thus, it is felt that the use of a 2nd order polynomial for the remapping is appropriate. In order to obtain results with a maximum mean velocity bias error of approximately 1% or less for the 2nd order polynomial, the normalized data density L/δ should be greater than or equal to 3, and R/δ should be less than or equal to 2. The difference in the random component of the error between the various fit orders is relatively small compared to the changes seen in the bias error. The rms values only varied by approximately 2% of the peak value when comparing different methods.

The most accurate results for the vorticity calculation were obtained by differentiating the 4th order polynomial fit and by use of the 4th order accurate finite difference method on data remapped by a 2nd order polynomial fit. Although the accuracy of these two techniques are comparable, the 2nd order polynomial fit combined with the 4th order accurate finite difference technique is better suited for the calculation of vorticity values from the PIV or MTV data since this method is less computationally intensive. In the calculation of vorticity, the noise present in the original data influences the selection of suitable values for R/δ and L/δ. In the absence of noise in the initial velocity data, L/δ should be made as large as possible for lowest vorticity random and bias errors. However, if the measured velocity field contains significant sources of noise, a large value of L/δ is not desirable as it results in a considerable increase in the random error, offsetting any potential decrease in the bias error. A choice of $L/\delta = 3$ and $R/\delta = 2$ yields vorticity values with both a mean bias and random error of less than 2% of the peak vorticity value for both the noise-free and the noisy initial velocity field results. It should be noted that the error values quoted are the maximum error across the vortex core. The error at most points is considerably less than the quoted 2%.

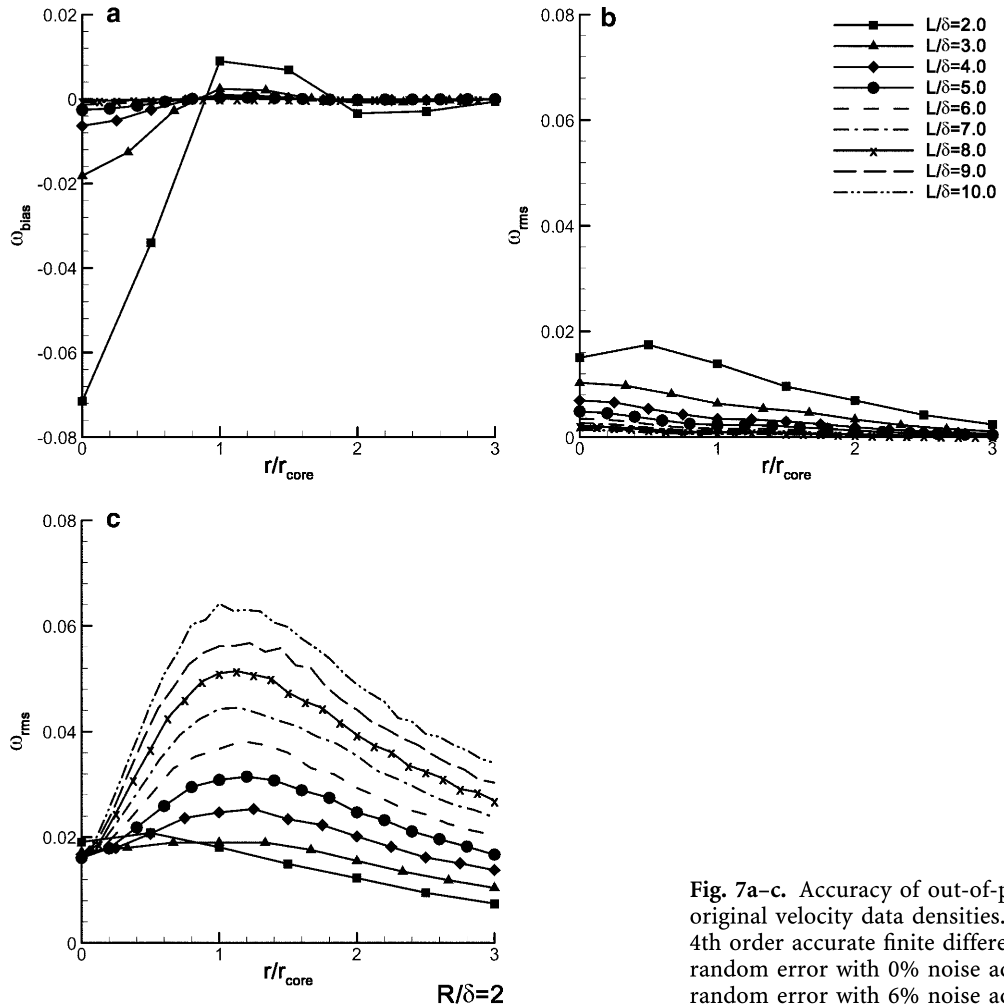

Fig. 7a–c. Accuracy of out-of-plane vorticity field for different original velocity data densities. Vorticity is calculated using the 4th order accurate finite difference method. **a** Mean bias error; **b** random error with 0% noise added to initial velocity field; **c** random error with 6% noise added to initial velocity field

It is noted that if the actual values of R/δ and L/δ are known in an experiment, it is possible to use the results of these simulations to extrapolate and estimate the actual vorticity value from under-resolved data.

References

Abrahamson S; Lonnes S (1995) Uncertainty in calculating vorticity from 2D velocity fields using circulation and least-squares approaches. Exp Fluids 20: 10–20

Agui J; Jimenez J (1987) On the performance of particle tracking. J Fluid Mech 186: 447–468

Fouras A; Soria J (1998) Accuracy of out-of-plane vorticity measurements from in-plane velocity field data. Exp Fluids 22: 409–430

Gendrich CP; Koochesfahani MM (1996) A spatial correlation technique for estimating velocity fields using molecular tagging velocimetry. Exp Fluids 22: 67–77

Gendrich CP; Koochesfahani MM; Nocera DG (1997) Molecular tagging velocimetry and other novel applications of a new phosphorescent supramolecule. Exp Fluids 23: 361–372

Koochesfahani MM (1999) Molecular tagging velocimetry (MTV): progress and applications. AIAA Paper 99-3786

Luff JD; Drouillard T; Rompage T; Linne MA; Hertberg JR (1999) Experimental uncertainties associated with particle image velocimetry (PIV) based vorticity algorithms. Exp Fluids 26: 36–54

Raffel M; Willert C; Kompenhans J (1998) Particle image velocimetry: a practical guide. Springer, Berlin Heidelberg New York

Spedding GR; Rignot EJM (1993) Performance analysis and application of grid interpolation techniques for fluid flows. Exp Fluids 15: 417–430