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A PIV study of the laminar axisymmetric sudden expansion flow

K. J. Hammad, M. V. Ötügen, E. B. Arik

Abstract The laminar flow through an axisymmetric sudden expansion was investigated experimentally using real-time digital particle image velocimetry. An expansion ratio (downstream-to-upstream pipe diameter ratio) of 2 was selected for the study. The measurements covered the regions of separation, reattachment and re-development. Two dimensional velocity maps were obtained on the vertical center plane for six Reynolds numbers between 20 and 211, based on the upstream pipe diameter and bulk velocity. The stream function distributions are calculated and presented from the streamwise and radial velocity maps. The dependence of reattachment length, redevelopment length and recirculating flow strength on the Reynolds number are determined. Results show that not only the reattachment length but also the redevelopment length downstream of reattachment is a linear function of the Reynolds number. The recirculation eddy strength, on the other hand, has a non-linear dependence on the Reynolds number which becomes weaker as the Reynolds number is increased. The results indicate no instability- or buoyancy-driven flow asymmetry in the range $20 \leqslant Re \leqslant 211$.

List of symbols

D	diameter of downstream pipe
d	diameter of upstream pipe
g	gravitational acceleration
L_d	flow redevelopment length
L_r	reattachment length
n	index of refraction
r	radial coordinate
Re	Reynolds number, $Re=(\rho d U_i)/\mu$
U	streamwise velocity
u	normalized streamwise velocity, $u=U/U_i$
U_b	downstream streamwise bulk velocity
U_i	inlet streamwise bulk velocity
V	radial velocity
v	normalized radial velocity, $v=V/U_i$
V_s	radial component of buoyancy-driven secondary flow
x	streamwise coordinate from step

Greek symbols

β	thermal expansion coefficient
ΔT	temperature difference
μ	absolute viscosity
ρ	density
ψ	stream function

Subscripts:

cl	centerline value
min	minimum value
max	maximum value

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1 Introduction

Flows through sudden enlargements are of interest from the point of view of fundamental fluid mechanics as well as practical applications. There is keen interest in the understanding of such flows due to their widespread occurrence in many fluid applications including heat exchangers, combustors, nuclear reactors as well as in biological systems. On the fundamental fluid mechanics side, the flow through an axisymmetric sudden expansion has all the complexities of an internally separating and reattaching flow. However, it is relatively easy to study experimentally since the point of separation is fixed by the expansion step while the geometry affords a straightforward numerical scheme in the cylindrical coordinates. For this reason, the laminar axisymmetric sudden expansion flow has become a standard problem to test the performance of different computational schemes.

The gross features of the axisymmetric sudden expansion flow, both laminar and turbulent, are fairly well known through flow visualizations and some quantitative studies. However, the details of this flow structure such as velocity distributions, wall pressure and shear stress, redevelopment length and recirculation flow strength, are not well understood.

The turbulent axisymmetric sudden expansion flow has been investigated both experimentally and numerically by a number of researchers (Teyssandier and Wilson, 1974; Durret et al., 1988; Gould et al., 1990; Devenport and Sutton, 1993). The details of the flow structure, as well as the influence of inlet conditions, are fairly well understood in this flow regime. On the other hand, the number of studies involving the laminar axisymmetric sudden expansion flow is quite limited. The numerical studies of this flow include both finite element (Scott and Mirza, 1986) and finite difference approaches (Feuerstein et al., 1975; Bedakes and Knight, 1992) in order to obtain a solution to the problem. The literature on the experimental investigations of this flow is even more limited and quantitative results obtained in a systematic fashion covering a range of Reynolds numbers is currently lacking. The detailed experimental analysis of these flows is particularly challenging due to the limitation in the choice of measurement techniques that can be successfully used to gather quantitative data. For this reason, most of the previous studies of the laminar axisymmetric sudden expansion flows have essentially been limited to flow visualizations (Macagno and Hung, 1967; Back and Roschke, 1972; Iribarne et al., 1972; Monnet et al., 1982). These studies showed that the size of the recirculation region downstream of the expansion increases with increasing Reynolds numbers within the laminar flow regime.

In order to investigate spatially-resolved velocity, non-intrusive methods such as those based on lasers must be used to avoid flow disturbances. These methods require that the test section and the working fluid are optically transparent. Furthermore, it is generally difficult to obtain stable flow rates in the Reynolds number range where the flow is truly laminar. Recently, a laser Doppler velocimetry (LDV) study was carried out to provide some qualitative velocity distributions in a laminar flow through a 1 : 2.25 axisymmetric sudden expansion (Budwig and Tavoularis, 1995). Measurements were carried out for two Reynolds numbers ($Re = 131$ and 253) in this study. The results are interesting in that although the inlet velocity is axisymmetric, the separation and reattachment downstream of the step exhibit significant asymmetry. The measurements, apparently taken on the vertical center plane, indicate significantly shorter reattachment lengths occurring along the top wall of the expanded pipe as compared to that along the bottom wall. Since this study is primarily concerned with the corresponding pulsatile flow structure, the detailed behavior of the asymmetric steady flow is not available. Furthermore, no measurements were reported at lower Reynolds numbers to determine whether or not the asymmetry is due to circumferencial instabilities and if there exists a critical Reynolds number below which the expansion flow is truly axisymmetric.

A systematic study of the laminar sudden expansion flow was undertaken in the Reynolds number range of $Re = 20$ –211. The selected Reynolds numbers insured instability-free laminar flow in the complete range of study; the lowest critical Reynolds number reported in the literature for the onset of flow instabilities is $Re = 300$ (Back and Roschke, 1972). A digital PIV technique was used in order to provide quantitative data to form a baseline for future computational studies. It was also the aim of this study to investigate possible flow asymmetry and the effect of Reynolds number on this and other features of the flow.

2 Experimental details

2.1 Test facility

A schematic of the closed-loop experimental system is shown in Fig. 1. The system is composed of a 12.7 mm diameter inlet pipe, a 25.4 mm diameter test section, a return loop and a variable-speed dc motor-driven pump. The inlet pipe is 813 mm long which ensures a fully developed flow at the expansion step for all the cases studied. The test section is 965 mm long which allows the investigation of flow development downstream of reattachment. The material for the inlet pipe and the test section is vycor whose index of refraction ($n = 1.46$) closely matches that of the test fluid ($n = 1.45$). The test section is enclosed inside a 51 mm × 89 mm rectangular cross-section Plexiglas outer enclosure which extends into the inlet pipe as shown in Fig. 1. During the experiments, the enclosure is filled with the working fluid in order to prevent the distortion of the PIV image by the curved surface of the test section. The flow rate through the system is monitored by a coriolis-type mass flow meter and different flow rates are obtained by changing the rpm of the pump motor. The surface of the liquid in the inlet settling tank is 300 mm above the test section thus preventing the occurrence of any significant static pressure variations in the test section. The working fluid is diethylene glycol with an absolute viscosity of 0.038 Pa s at 20 °C temperature.

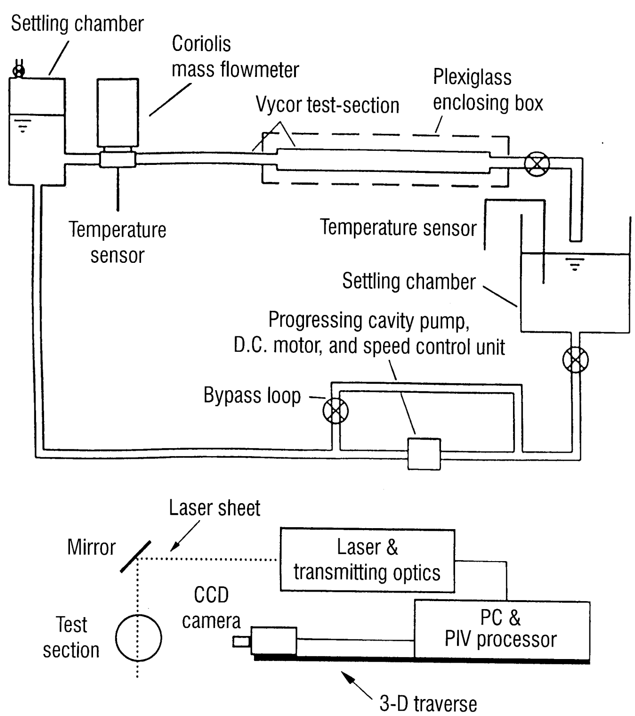

Fig. 1. Schematic of the test facility and PIV system

2.2 PIV system

A digital PIV system is used for the planar simultaneous measurements of axial and radial velocities in the vertical plane passing through the test section centerline. The optical system is powered by two Nd:YAG lasers each with approximately 10 mJ of pulse energy and a duration of 8 ns. The firing of the lasers are externally controlled and the repetition rates as well as the cross-pulse delays are adjustable. The laser outputs are frequency doubled to provide the 532 nm green line and the two beams are combined using a beam splitter. The combined beam is expanded into a sheet using a cylindrical lens. The 1 mm-thick laser sheet is directed through the test section via a mirror (Fig. 1). The sheet subsequently passes through a slot in the test section table and is terminated in a beam dump placed at the floor. Silicon carbide particles (16 μm nominal diameter) are used as light scatterers. The image of the scattering particles in the measurement plane is collected at right angle by a zoom lens and fed into a digital CCD camera. The digital particle images could be acquired at rates up to the video rate (30 Hz) using asynchronous illumination technique where the time difference between pairs of images are adjustable depending on velocity, giving a vector sampling rate of up to 15 Hz. The 768×484 pixel image plane of the camera is divided into 32×32 pixel size sub-regions with 50% overlap and the average particle displacement is calculated real-time for each of these sub-regions (interrogation areas) using a cross-correlation method. Thus, each of these interrogation areas provides a single velocity vector in the flow field and the spatial resolution of the measurements is determined by the size of the sub-regions and the thickness of the laser sheet. Based on these, the spatial resolution of the measured velocity is 0.36 and 0.45 mm in the radial and axial directions, respectively, and 1.0 mm along the third direction (depth). The data is stored on a personal computer for further analysis and graphing. The PIV system is placed on a three-dimensional traverse system so that different regions of the flow can be interrogated using the same alignment and optical settings. The positioning accuracy of the traverse system is determined to be 0.2 mm in the axial direction and 0.125 mm in the radial direction. Based on the predicted uncertainty in determining the particle displacement in each interrogation area and the accuracy in repositioning of the traverse system, the uncertainty in velocity is estimated to be better than 6% of the expected minimum velocity at each measurement plane. Estimates of the propagation of velocity uncertainty into the global flow characteristics such as reattachment length, redevelopment length and recirculating flow strength are presented in the appropriate graphs as error bars. Further details of the experiments and the uncertainty estimates can be found in Hammad (1997).

3 Discussion of results

The planar images of the flow field were obtained in small subregions of the domain of interest in order to achieve high spatial resolution. Each two-dimensional PIV image had a radial extent of approximately one step height and an axial extent of 1.6 step heights. Four rows of images were obtained at each vertical level to cover the full radial extent of the flow. The number of images in each vertical strip varied depending on the Reynolds number. For larger Reynolds numbers, longer axial distances were covered in order to capture the complete redevelopment of the flow past reattachment. In the end, for each Reynolds number case all the images were put together to form a composite picture of the flow field. The pulse delay of the lasers were varied in different regions of the flow in order to optimize the particle displacement for highest measurement accuracy.

The velocity vectors in the full vertical center plane of the test section are shown in Fig. 2 for a range of Reynolds numbers. Note that a different scale is chosen for the streamwise distance in each plot in order to capture the redevelopment region past reattachment. Even for the smallest Reynolds number of $Re=20.6$, the velocity field is symmetric about the pipe axis to within the measurement uncertainty. This confirms the planar laser sheet flow visualizations carried out along both the vertical and the horizontal center planes prior to the PIV measurements which indicate that the flow is axisymmetric in the Reynolds number range covered by the study. This finding contradicts that of Budwig and Tavoularis (1995) where a flow asymmetry is observed downstream of the expansion step at a Reynolds number as low as $Re=131$. In that study, while the measured velocity profiles remained nominally symmetric (about the pipe axis) on the horizontal plane, they were markedly skewed on the vertical plane with at least a 50% shorter reattachment length along the top wall. Axisymmetric wave instabilities were suggested by the authors as a possible cause of this strong asymmetry although in a follow up report (Budwig et al. 1997) they suggested that the observed asymmetry could also be the result of a thermal gradient. Other studies found that the threshold Reynolds number for the onset of flow instabilities was $Re>300$ (Back and Roschke, 1972). Another potential source for flow asymmetry on the vertical plane is the possible existence of a buoyancy-driven secondary flow structure caused by a small temperature difference between the fluid emerging from the smaller pipe and the wall of the larger pipe. This temperature difference can be caused by several mechanisms including possible viscous heating in the closed-loop test facility. For low Reynolds number flows, if the average streamwise velocity is sufficiently small, the secondary flow magnitude can become significant enough to skew the flow on the vertical plane leading to asymmetric reattachment. Indeed, recent flow visualizations by Wang and Lin (1997) clearly show that even when the temperature difference is less than 1 °C, at $Re=175$, the flow pattern exhibits significant asymmetry on the vertical plane. They suggested that Grashof number dictates the strength of this flow asymmetry. However, a simple inviscid analysis yields an estimate for the secondary flow magnitude as

$$V_s=\sqrt{\frac{gD\beta\Delta T}{2}} \tag{1}$$

where, ΔT is the temperature difference between the pipe wall and the fluid. Therefore, an appropriate non-dimensional parameter should be

$$\frac{V_s}{U_b}=\sqrt{\frac{gD\beta\Delta T}{2U_b^2}} \tag{2}$$

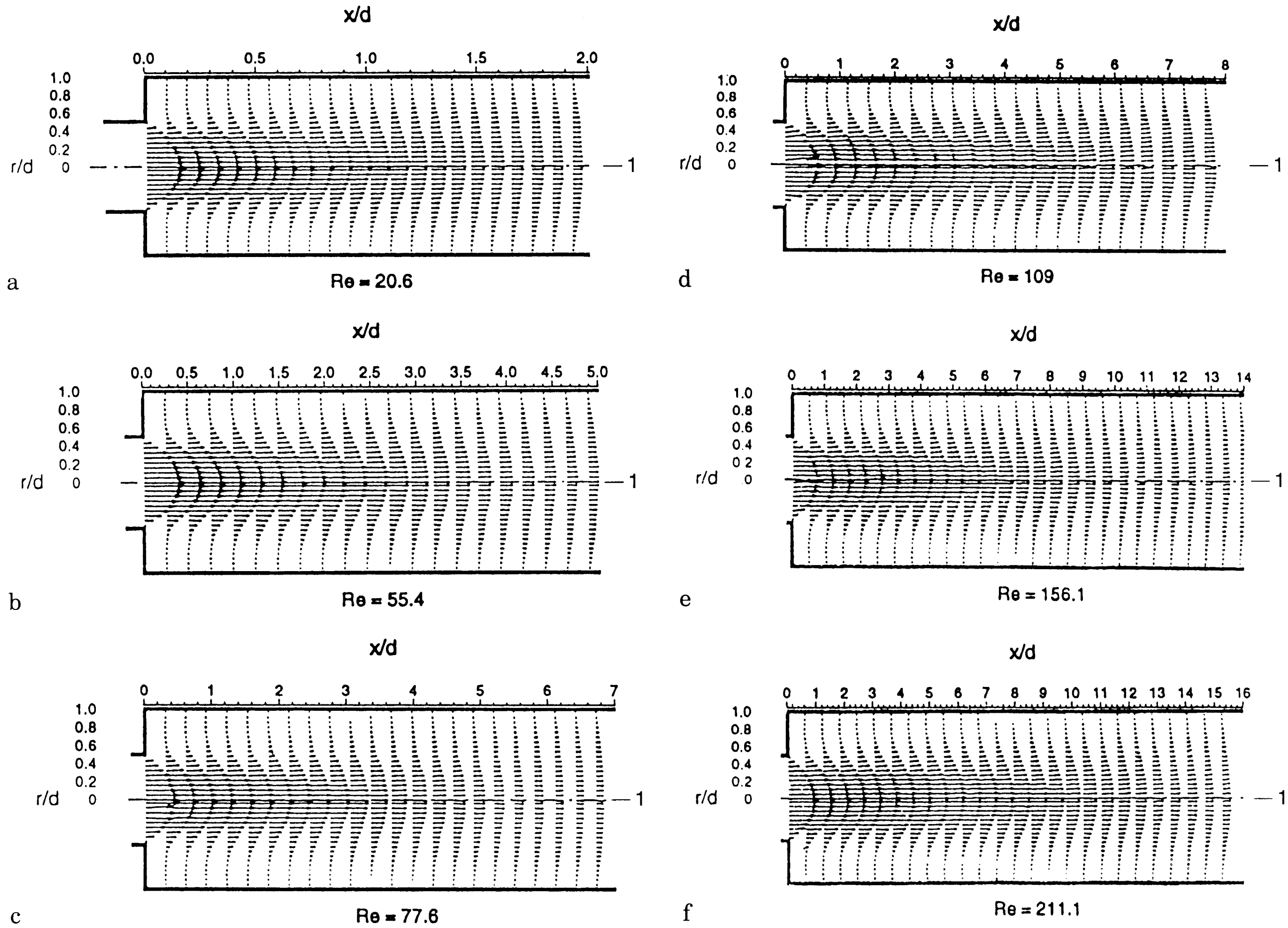

Fig. 2a–f. Velocity vectors in the vertical plane

As this equation indicates, smaller pipe diameters and larger average streamwise velocities lead to smaller flow asymmetry due to buoyancy. Therefore, for a fixed Reynolds number, the flow asymmetry can be minimized by using fluids with higher kinematic viscosities which in turn lead to larger average streamwise velocities. In the present study, diethylene glycol is used as the working fluid. While the viscosity of this fluid is about 38 times that of water, its thermal expansion coefficient is less three times that of water. Thus, it provides a secondary-to-bulk streamwise velocity ratio that is 22 times smaller than for water for the same Reynolds number. The relative significance of the thermal effects can be demonstrated in the following example. If water were used in our experiments for $Re = 20.6$ case, Eq. (2) would yield a velocity ratio in the order of one for a temperature difference as low as $\Delta T = 0.1$ °C. Since it is practically impossible to eliminate such minute temperature differences, that case would clearly have lead to a severe flow asymmetry.

In Fig. 2, the velocity vectors near the step still closely resemble the Poiseuille profile for all six cases. As described earlier, the fluid passes through a long pipe in order to ensure fully developed flow conditions upstream of the expansion. This allows for a meaningful comparison of the different Reynolds number cases. As the Reynolds number increases, the flow takes a longer axial distance to adjust to the sudden change in the pipe cross-sectional area. Fig. 2 also demonstrates that no flow instabilities exist even for $Re = 211$.

The stream functions, calculated from the velocity distributions, are shown in Fig. 3. These plots are also obtained on the vertical plane and they show a good level of flow symmetry. (The slight disparity in the reattachment length along the top and bottom walls as indicated by the stream function is in fact significantly smaller than the uncertainty in the calculated stream function values). As the Reynolds number is increased, the $\psi = 0$ line becomes straighter and it reaches the wall at increasing axial distances from the expansion step. Furthermore, higher Reynolds numbers result in stronger flow recirculation beneath the $\psi = 0$ line as indicated by the existence of streamlines with progressively larger negative values. The dependence of the recirculation flow strength on the Reynolds number is presented in Fig. 4. The recirculation flow strength is defined as the ratio of the minimum stream function value in the recirculation zone to the maximum stream function. Increasing Reynolds numbers lead to stronger corner eddies, however, this dependence is non-linear becoming weaker at higher Reynolds numbers. In fact $|\psi_{min}/\psi_{max}|$ ratio seems to approach an asymptotic value of approximately 0.14 at the high end of Reynolds numbers studied. The

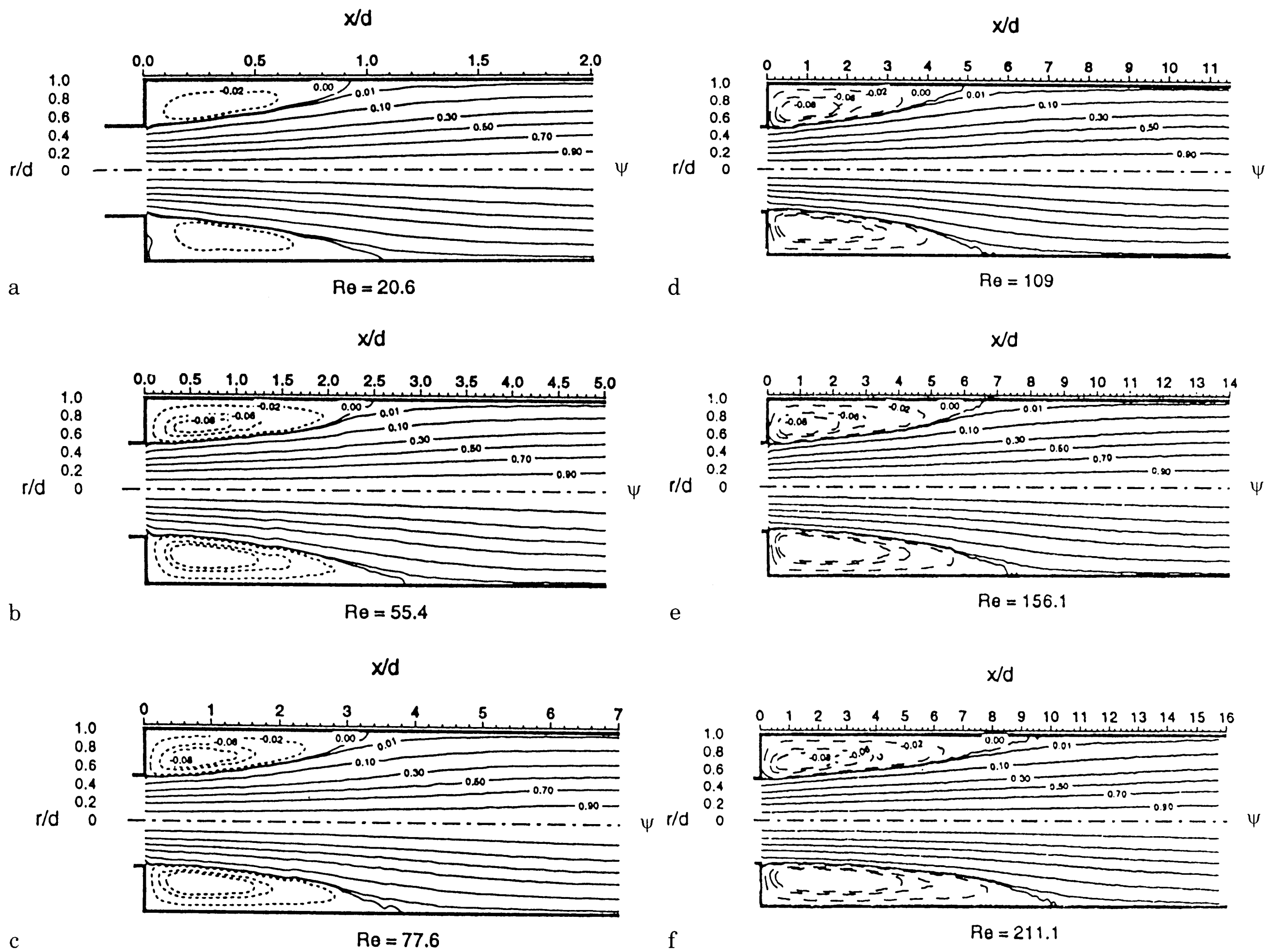

Fig. 3a–f. Distribution of the stream function in the vertical plane

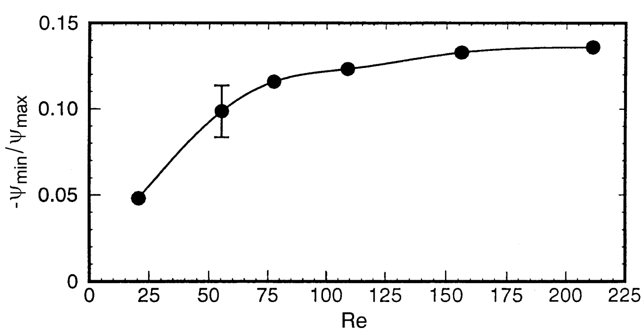

Fig. 4. Dependence of eddy strength on Reynolds number

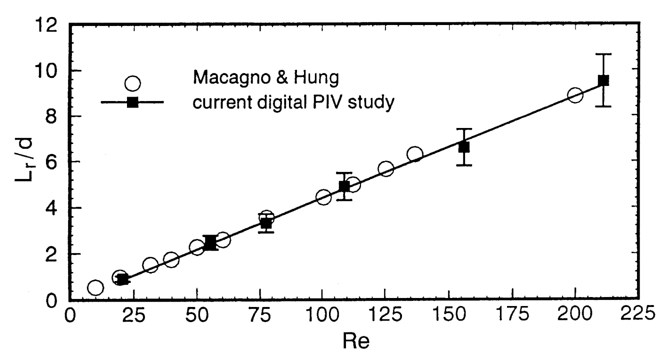

Fig. 5. Dependence of reattachment length on Reynolds number

reattachment length, on the other hand, is a linear function of the Reynolds number as shown in Fig. 5. In this figure, the present results are compared to the earlier flow visualization results of Macagno and Hung (1967). The agreement between those results and the present quantitative measurements is very strong confirming the earlier computations and visual observations which indicated that the reattachment length is roughly a linear function of the Reynolds number in this flow regime. It is also noted that, if the straight line representing this relationship (which is obtained through linear regression) were to be extended, it would pass through the origin. This seems to indicate the lack of any other flow regimes at lower Reynolds numbers where the reattachment has nonlinear Reynolds number dependence. This linear relationship in the laminar flow regime should in fact be expected considering that the average shear rate at any fixed axial distance normalized by the

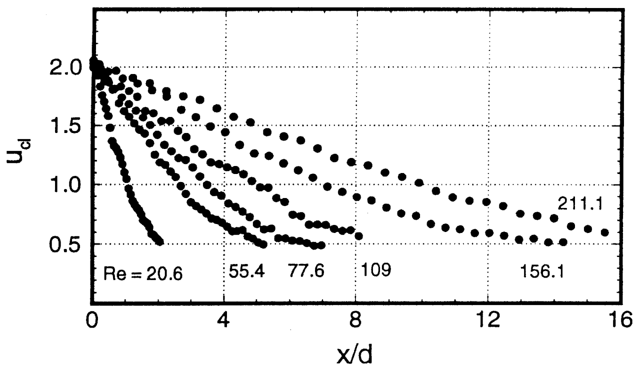

Fig. 6. Distribution of streamwise velocity on pipe axis

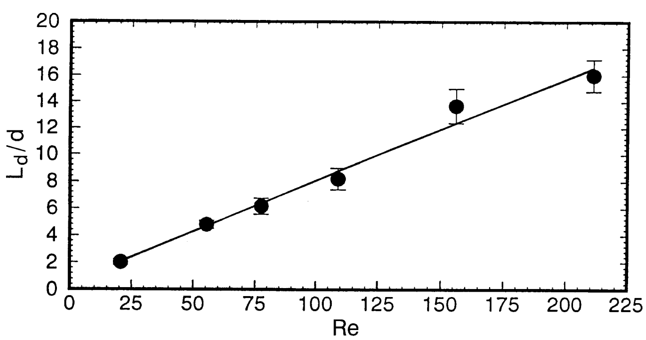

Fig. 7. Dependence of redevelopment length on Reynolds number

reattachment length, is nearly constant. Therefore, since the spreading rate of the separated shear layer in the radial direction is dictated by molecular diffusion only, for a given fluid, larger streamwise velocities lead to proportionately longer distances for the separated layer to reach the wall.

The slope of the straight line is calculated to be 0.044. Based on a normalization using the step height in place of the upstream diameter, this slope is 0.088. This value is slightly smaller than those previous results reported in Back and Roschke (1972) and Iribarne et al. (1972) which give values of 0.106 and 0.096, respectively. These small variations obtained in different studies can be attributed to several factors including the level of accuracy of the previous observations as well as the difference in the inlet conditions and the expansion ratios (which was 2.6 in the latter study).

The redevelopment of the flow past reattachment is investigated by tracking the evolution of the streamwise velocity. Figure 6 shows the axial distribution of the centerline streamwise velocity for six Reynolds numbers. For a fully developed laminar pipe flow the velocity distribution is parabolic and hence, the maximum (centerline) velocity is twice the bulk velocity. Therefore, normalized by upstream bulk velocity, the centerline velocity starts with a value of 2 at the expansion plane and should reach a value of 0.5 when the flow is fully developed at a sufficiently large distance further downstream. As expected, the centerline velocity approaches the value of 0.5 in an asymptotic fashion and at slower rates for larger Reynolds numbers. The dependence of the redevelopment length, L_d, on the Reynolds number is shown in Fig. 7. Here, L_d is defined as the axial distance, measured from the expansion plane, where the centerline velocity becomes $U_{cl} = 0.53$. Defined in this manner, the redevelopment length also grows linearly with Reynolds number. The slope of the straight line is approximately 0.08 indicating that the flow becomes fully developed at a distance roughly twice as long as the reattachment length.

4
Conclusions
In the Reynolds number range covered by the study, the velocity field does not show any asymmetry on the vertical plane; the radial velocity on pipe axis is zero throughout and the top and bottom reattachment lengths are approximately the same. Secondary flow structures caused by possible buoyancy effects are avoided by using diethylene glycol as the working fluid whose kinematic viscosity is 34 times that of water. Higher Reynolds numbers lead to longer reattachment and flow redevelopment lengths. Both the reattachment and the redevelopment lengths are linear functions of the Reynolds number. The slope of the reattachment plot is determined to be 0.044 confirming the earlier flow visualization results of Macagno and Hung (1967). The present study shows that the flow takes approximately twice as long as the reattachment length to reach a nearly fully developed state. The strength of the recirculating eddy increases with increasing Reynolds number in a nonlinear fashion tending towards an asymptotic value of $|\psi_{min}/\psi_{max}| \approx 0.14$.

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