



# Assessing Klebanoff's data

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## Abstract

In 1955, Klebanoff published the first full set of turbulence stress measurements in a zero-pressure-gradient boundary layer (Klebanoff characteristics of turbulence in a boundary layer with zero-pressure gradient. NACA Report 1247, 1955). These results have achieved landmark status, and they are still widely used for comparisons with measurements and computations. The purpose of this paper is to show that these data are inaccurate in a number of ways, and that future comparisons should avoid using these results.

## 1 Introduction

When reporting new computations or measurements of turbulence, there is often a need to validate the new data against existing, high-quality data. The critical assessments of experimental work by Coles and Hirst (1968); Fernholz and Finley (1996); Chauhan et al. (2009); Pirozzoli and Smits (2023), and the critique of direct numerical simulations by Schlatter and Örlü (2010), have helped to identify such comparison data sets. In addition, there are a number of data bases freely available for this purpose, like those maintained by (ERCOFTAC 2024; NASA 2024; Johns Hopkins University 2024), the (KTH Royal Institute of Technology 2024) and (Universidad Politecnica de Madrid 2024).

Despite this multiplicity of resources, Klebanoff's 1955 measurements of the turbulent stresses in a zero-pressure-gradient boundary layer (Klebanoff (1955), hereafter K55) continue to be widely used, either as a reference data set, even though they are not part of any of the data collections listed here, or for a variety of other objectives. In fact, K55 is still routinely cited, about as often per year as it has been since its original publication, and it continues to make its appearance in widely-quoted (and recent) publications (Piquet (2013); Schlichting and Gersten (2016); Nakagawa (2017); Bose and Park (2018), for example).

Here, we re-analyze the K55 data and compare them with more contemporary experiments and computations. We find

that the measurements are inaccurate in a number of ways, and, despite the popularity of K55, it is strongly recommended that more recent data drawn from experiments and DNS should be used instead for future comparisons.

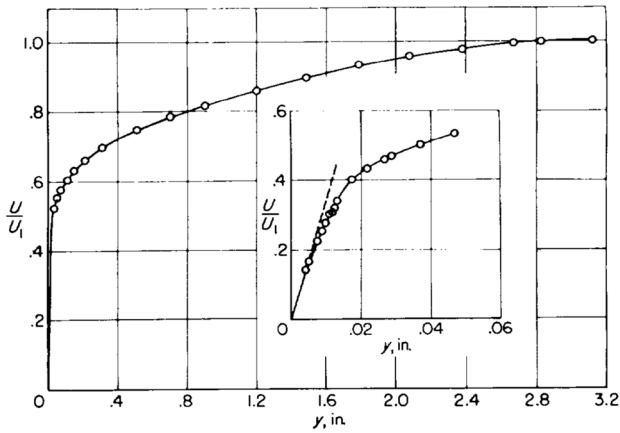
When Klebanoff published his results, they were the first full set of turbulence stress measurements made in a zero-pressure-gradient boundary layer, which explains their landmark status. Figures 1 and 2 show the data in their original presentation, where  $u'$ ,  $v'$  and  $w'$  are the rms velocity fluctuations in the streamwise, wall-normal and spanwise directions ( $x$ ,  $y$  and  $z$ ), respectively, and  $\overline{uv}$  is the (negative) shear stress.

The experiment was performed on a flat plate mounted on the centerline of the tunnel at a station located 10.5 ft (3.20 m) from the plate leading edge. The first 2 ft (0.61 m) was covered with #16 floor-sanding paper to trip the flow and artificially thicken the boundary layer. The measurements were obtained using a constant current anemometer with a compensation network, with a reported flat frequency response over the bandwidth of the amplifier (2 to 70,000 Hz). Some (unspecified) filtering was done to reduce noise. The wire sensors for all probes had a diameter  $d = 2.5\mu\text{m}$  with a length  $\ell = 0.5\text{ mm}$  ( $\ell/d = 197$ , with an estimated  $\ell^+ \approx 18$ ). In some instances (unspecified), the diameter was reduced to  $1.3\mu\text{m}$ . Given this information, it seems unlikely that the measurements were subject to any significant spatial or temporal filtering, except in the near-wall region where some spatial filtering is to be expected.

Table 1 lists the flow parameters pertinent to this experiment. Only the freestream velocity  $U_1 = 50\text{ ft/s}$  (15.24 m/s), the boundary layer thickness  $\delta = 3\text{ in.}$  (76.2 mm) and the Reynolds number based on the distance to the virtual origin

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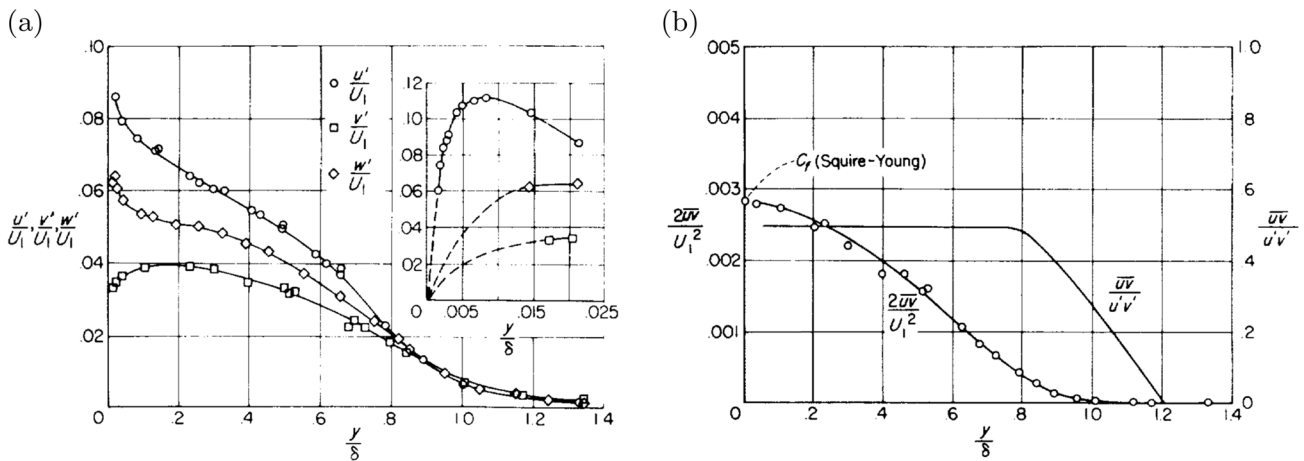


**Fig. 1** Original Klebanoff mean velocity profile (Klebanoff 1955), reproduced with permission

$x_V$ , that is,  $Re_x = x_V U_1 / \nu = 4.2 \times 10^6$ , were given in the text, with  $x_V = 14.5$  ft (4.42 m).

## 2 Data Analysis

To examine the data in more detail, we need additional information such as the skin friction coefficient,  $C_f$ , the Reynolds number based on the momentum thickness,  $Re_\theta$  and the friction Reynolds number,  $Re_\tau = \delta u_\tau / \nu$ . Here, as in the rest of this paper,  $\delta$  is understood to be the 99% boundary layer thickness, that is, the distance from the wall where  $U = 0.99U_1$ . In addition,  $\theta$  is the momentum thickness,  $C_f = 2\tau_w / (\rho U_1^2)$  is the skin friction coefficient,  $u_\tau = \sqrt{\tau_w / \rho}$  is the friction velocity,  $\tau_w$  is the shear stress at the wall and  $\rho$  is the fluid density. Since the original records are lost, we used DataThief<sup>1</sup> to reconstitute the data.

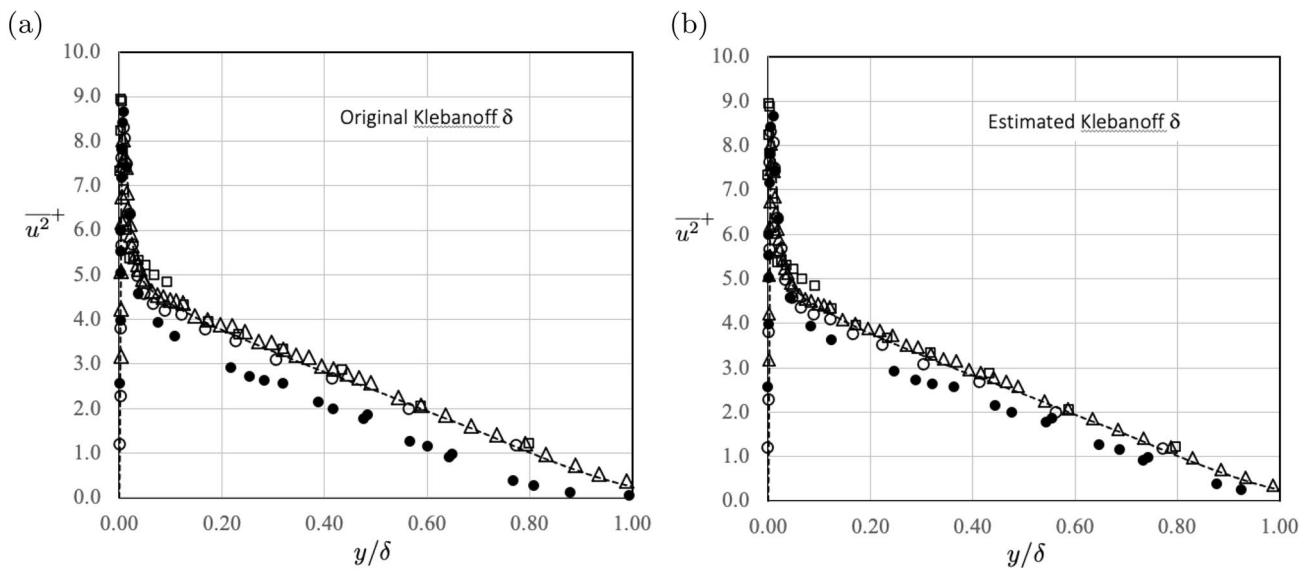


**Fig. 2** Original Klebanoff turbulence profiles (Klebanoff 1955), reproduced with permission

**Table 1** Boundary layer parameters (<sup>1</sup>denotes value estimated from original data)

	$U_\infty$ (m/s)	$Re_x$	$Re_\theta$	$\delta$ (mm)	$\delta^*$ (mm)	$\theta$ (mm)	$C_f$	$Re_\tau$
Klebanoff (1955)								
From text	15.24	$4.2 \times 10^6$	6394 <sup>1</sup>	76.2 (66.2 <sup>1</sup> )			0.00283 <sup>1</sup>	2406 <sup>1</sup>
From 1/7th power law			7360	75.8	9.47	7.37	0.00280	2755
Klebanoff (1952)								
From text	16.76	$4.66 \times 10^6$	7820	78.7 (63.9 <sup>1</sup> )	10.1	7.16		2551 <sup>1</sup>
From 1/7th power law							0.00275	
Adjusted to 15.24 m/s	15.24	$4.2 \times 10^6$	7200	81.5 (66.2 <sup>1</sup> )	10.45	7.41	0.00280	2404 <sup>1</sup>
DeGraaff and Eaton (2000)								1692
DeGraaff and Eaton (2000)								4336
Osaka, Kameda and Mochizuki (1998)								1750 <sup>1</sup>
Sillero et al. (2013) DNS								1848

<sup>1</sup> B. Tummers, DataThief III (2006) <https://datathief.org/>.



**Fig. 3** Comparison in outer scaling for  $\overline{u^2}^+$ . •, Klebanoff (1955)  $Re_\tau = 2406$ ; ○, DeGraaff and Eaton (2000)  $Re_\tau = 1692$ ; □, DeGraaff and Eaton (2000)  $Re_\tau = 4336$ ; △, Osaka, Kameda and Mochizuki (1998)  $Re_\tau = 1750$ ; - - - -, Sillero et al. (2013)  $Re_\tau = 1848$

The skin friction coefficient was found from the data point in Fig. 2b at  $y = 0$  marked “Squire–Young,” which gives  $C_f = 0.00283$ . Presumably, it was not measured directly but inferred from that correlation. There is some historical and circumstantial evidence that 1/7th power laws were used in this investigation (see, for example, the calculation of the boundary layer thickness—K55 page 16). Using the 1/7th power law,  $C_f = 0.0592/Re_x^{0.2} = 0.00280$ , in good agreement with the Squire–Young value.

As to the boundary layer thickness, the 1/7th power law relationship gives  $\delta = 0.37x_V/Re_x^{0.2} = 2.99$  in (75.8 mm), in good agreement with the value of 3 in (76.2 mm) reported by K55. It would follow then that  $\delta^* = \delta/8 = 9.47$  mm, and  $\theta = 7\delta/72 = 7.37$  mm. In the absence of other information, we then get  $Re_\theta = 7360$  and  $Re_\tau = 2755$ . As to the value of  $\delta$  given by K55, however, it seems incompatible with the velocity distribution shown in figure 1, where we estimate that the 99% thickness is closer to 2.61 in. (66.2 mm), which then yields  $Re_\theta = 6394$  and  $Re_\tau = 2406$ .

Additional support for our K55 estimates is provided by the earlier results obtained by Klebanoff (1952) using the same experimental configuration as in K55, but at a 10% higher freestream velocity (see Table 1). The 1/7th power laws were used to scale these data to the lower velocity, and we found good agreement with the K55 values inferred here, as shown in Table 1. As for K55 the 99% thickness for Klebanoff & Diehl was found directly from the velocity profile.

Therefore, our best estimates for K55 are  $\delta = 66.2$  mm,  $C_f = 0.00283$ ,  $Re_\theta = 6394$  and  $Re_\tau = 2406$ . Surprisingly, these essential parameters have not been reported previously for this iconic experiment.

### 3 Data comparisons

Figures 3, 5, 6 and 7 show how the K55 data compare with the experiments of DeGraaff and Eaton (2000) and Osaka, Kameda and Mochizuki (1998) and the DNS of Sillero et al. (2013) (see table 1). In our notation,  $\overline{u^2}^+ = \overline{u^2}/u_\tau^2$ , and the overbar denotes time-averaging. Similarly,  $\overline{v^2}^+ = \overline{v^2}/u_\tau^2$ ,  $\overline{w^2}^+ = \overline{w^2}/u_\tau^2$  and  $-\overline{uv}^+ = -\overline{uv}/u_\tau^2$ . These particular data sets were chosen because they were taken at broadly similar Reynolds numbers to K55, and because they are among the very few high-quality sets that report all components of the Reynolds stress tensor. It should be noted that Osaka, Kameda and Mochizuki (1998) used the 99.5% thickness, which is about 4% larger than the 99% thickness. The value of  $Re_\tau = 1750$  given in Table 1 for this data set uses the 99% thickness estimated here.

#### 3.1 Streamwise turbulence distribution

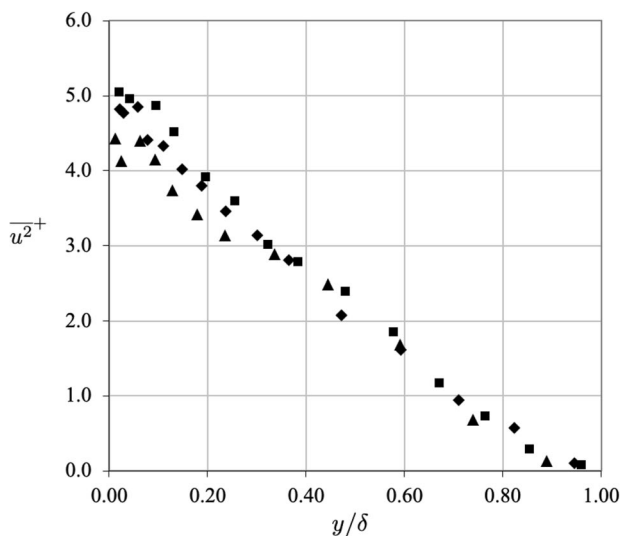
Figure 3 shows the comparisons in outer scaling for  $\overline{u^2}^+$ . Figure 3a uses the original boundary layer thickness (76.2 mm), and we see that in the middle of the layer the K55 values are about 25% lower than the other results. In figure 3b we show the same data using the 99% thickness found here (66.2 mm). It is clear that changing the boundary layer thickness cannot explain all of the observed discrepancies.

Instead, we note that Klebanoff’s experiment used an artificially thickened boundary layer. From Klebanoff (1952), we estimate that in K55 the boundary layer thickness at the end of the sandpaper was about  $\delta_i = 38$  mm, so that the

measuring station was approximately  $65\delta_i$  downstream of the rough to smooth transition. In terms of the mean flow, we would therefore expect the flow to be fully recovered from the step change (Antonia and Luxton 1972), but this may not hold for the turbulence. For example, Van Buren et al. (2020) found that in a pipe flow downstream of a similar step in roughness the turbulent stresses were exceedingly slow to adjust to the new wall condition ( $> 120$  radii), and they first fell below their equilibrium values before seemingly asymptoting to the fully recovered state. The sensitivity of the boundary layer development downstream of various tripping devices has been well documented (Marusic et al. 2015; Vila et al. 2017), especially at lower Reynolds numbers.

The measurements by Klebanoff (1952) support a similar conclusion here. In that experiment, at  $U_1 = 108$  ft/s (32.9 m/s), the authors found that the  $u'/U_1$  profiles at 3, 5.5 and 8.5 ft downstream of the roughness (0.91, 1.68 and 2.59 m, respectively) collapsed onto a single curve. We would expect, however, that the profiles ought to collapse in friction velocity scaling, not in freestream scaling. This is illustrated by the collapse of the DeGraaff & Eaton data at  $Re_\tau = 1692$  and 4336, as shown in Fig. 3. Yet the Klebanoff & Diehl profiles in friction velocity scaling are clearly still evolving with downstream distance, particularly for  $y/\delta < 0.4$ , as shown in Fig. 4. It seems likely, therefore, that the turbulence in K55 is still recovering from the step change in roughness.

The discrepancies seen in outer scaling are less obvious in inner scaling (Fig. 5). We see that the inner peak maximum for K55 agrees well with the other data, although its position is closer to  $y^+ = 25$  than the commonly accepted value of 15.



**Fig. 4** Comparison in outer scaling for  $\overline{u^2}^+$  at  $U_1 = 108$  ft/s (32.9 m/s). Distance from the leading edge: ▲, 5 ft (1.52 m); ◆,  $x = 7.5$  ft (2.29 m); ■,  $x = 10.5$  ft (3.20 m). These locations correspond to distances of 0.91, 1.68 and 2.59 m downstream of the step change in roughness. Data from Klebanoff (1952)

### 3.2 Wall-normal turbulence distribution

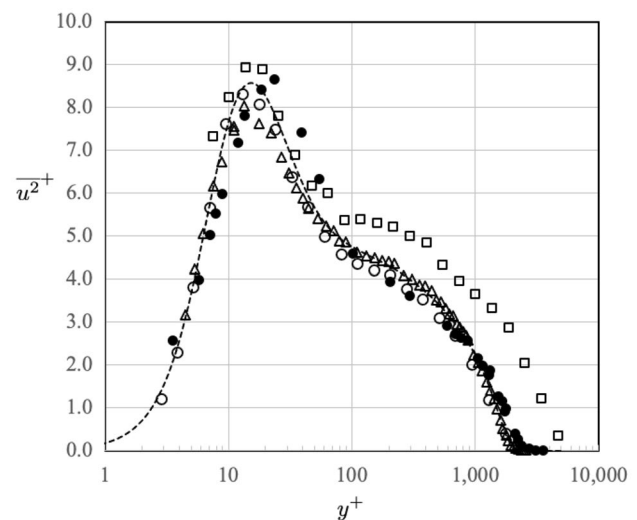
Figure 6a indicates that at about  $y/\delta = 0.4$ , the K55 value of  $\overline{v^2}^+$  is approximately 30% too low, using the original boundary layer thickness given by K55. This discrepancy reduces to about 20% when using the 99% thickness estimated here (Fig. 6b).

### 3.3 Spanwise turbulence distribution

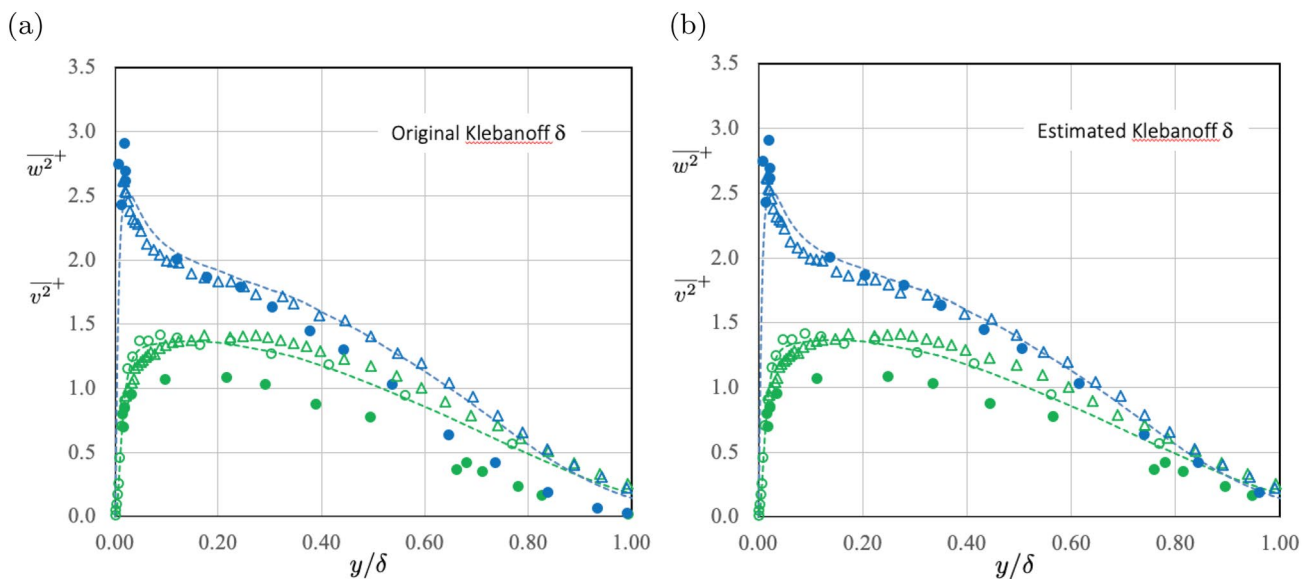
As to the spanwise turbulence levels, Fig. 6 demonstrates that the K55 levels agree well with the other data near the wall, and the agreement in the outer layer improves considerably when using the 99% thickness (comparing Fig. 6a and b).

### 3.4 Shear stress distribution

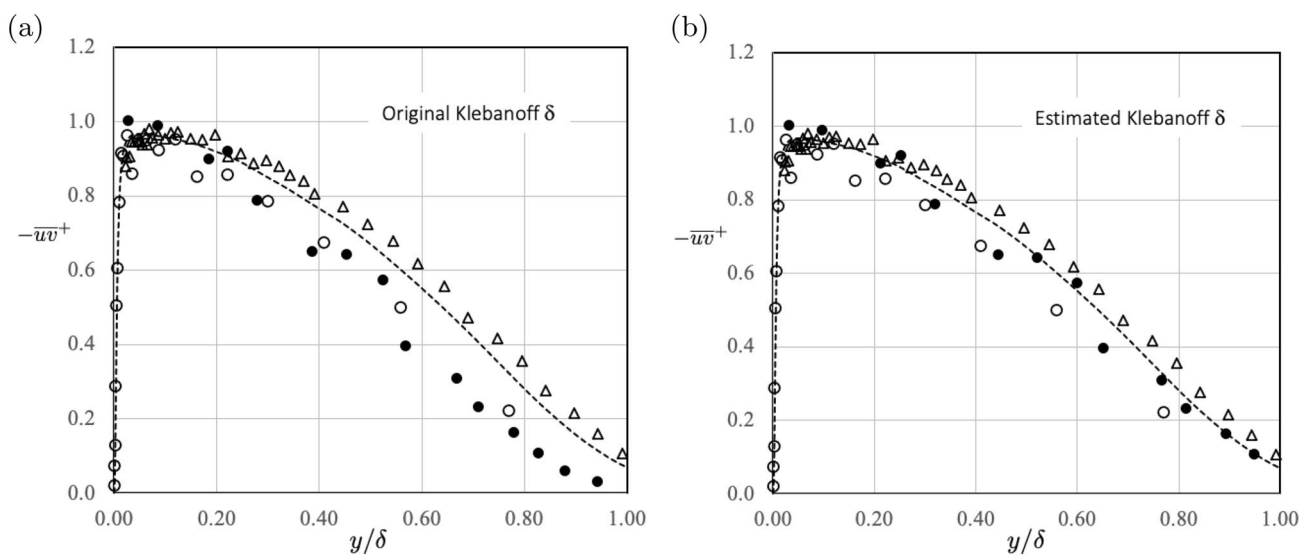
The shear stress follows the same trend as the spanwise stress, in that the K55 levels agree well with the other data near the wall. They then diverge from the consensus levels for  $y/\delta > 0.2$ , although the differences in the outer layer decrease when using the 99% thickness (comparing Fig. 7a and b). Notably, the DeGraaff and Eaton (2000) data fall below the consensus levels by about 10–15% in the outer layer.



**Fig. 5** Comparison in inner scaling for  $\overline{u^2}^+$ . •, Klebanoff (1955)  $Re_\tau = 2406$ ; ○, DeGraaff and Eaton (2000)  $Re_\tau = 1692$ ; □, DeGraaff and Eaton (2000)  $Re_\tau = 4336$ ; △, Osaka, Kameda and Mochizuki (1998)  $Re_\tau = 1750$ ; - - - -, Sillero et al. (2013)  $Re_\tau = 1848$



**Fig. 6** Comparison in outer scaling for  $v^{2+}$  (green) and  $w^{2+}$  (blue). •, Klebanoff (1955)  $Re_\tau = 2406$ ; o, DeGraaff and Eaton (2000)  $Re_\tau = 1692$ ;  $\Delta$ , Osaka, Kameda and Mochizuki (1998)  $Re_\tau = 1750$ ; - - - -, Sillero et al. (2013)  $Re_\tau = 1848$



**Fig. 7** Comparison in outer scaling for  $-\overline{uv}^+$ . •, Klebanoff (1955)  $Re_\tau = 2406$ ; o, DeGraaff and Eaton (2000)  $Re_\tau = 1692$ ;  $\Delta$ , Osaka, Kameda and Mochizuki (1998)  $Re_\tau = 1750$ ; - - - -, Sillero et al. (2013)  $Re_\tau = 1848$

### 4 Conclusions

The Klebanoff K55 data (Klebanoff 1955) displays some serious shortcomings, and should not be used as a reference standard to compare with other experiments and computations. The distributions of  $\overline{u^2}^+$ ,  $v^{2+}$  and  $-\overline{uv}^+$ , all fall well below the current consensus levels in the outer layer, even when the “correct” boundary layer thickness is used. In addition, the inner peak in  $\overline{u^2}^+$  is further from

the wall than is now commonly accepted. Only  $w^{2+}$  is in line with expectations. Apart from possible measurement errors, the discrepancies appear to be due to the slow decay of the effects of the upstream roughness used to artificially thicken the boundary layer.

It is therefore strongly recommended that in the future authors should use, instead of K55, materials drawn from validated databases like those referenced in the Introduction.

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**Author Contributions** AJS is solely responsible for the content of this manuscript.

**Data availability** All data and materials are available in the open literature.

## Declarations

**Conflict of interest** The authors declare that they have no conflict of interest.

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