Y. X. Wang · M. Gaster

Effect of surface steps on boundary layer transition

Abstract An experimental study has been carried out to examine the effect of a sharp-edged step on boundary layer transition. The transition position and disturbance spectra in the boundary layer for different step heights and free-stream velocities were measured by hot-wire anemometry. A correlation between the transition Reynolds number and the relative step height has been established for both backward-facing and forward-facing steps. The transition position is associated with the "N-factor" that defines the integrated growth of instability waves at transition. The boundary layer over a step has an earlier transition position than that on a smooth plate, since the instability waves amplify more rapidly than those on a smooth surface. The transition N-factor for the flow containing a step, calculated using the amplification rates on a smooth plate, will, therefore, be smaller than that on surfaces without a step. The observed reduction of the N-factor occurring with a step has been shown to correlate with the height of the step, thus, providing an empirical tool that can be used to estimate the transition position when steps occur. An appropriate value of N can be determined from knowledge of the step height.

1 Introduction

Boundary layer transition is strongly influenced by any imperfections that exist on aerodynamic surfaces. These imperfections, or roughness elements, can occur in various forms and are of different sizes. They may include

Y. X. Wang (✉)
Glasgow University, Glasgow,
G12 8QQ, Scotland, UK
E-mail: y.wang@aero.gla.ac.uk

M. Gaster
Queen Mary, University of London,
London, E1 4NS, UK

regions of waviness, bulges, steps, gaps at junctions, surface contamination from insect debris, ice and dirt particles of various sorts. Although modern manufacturing and maintenance procedures makes it possible to provide reasonably good operational surfaces, some imperfections are unavoidable, particularly those arising from such things as the installation of inspection panels. They often arise in the form of sharp-edged steps. The influence of this type of surface imperfection on transition has been studied and a simple engineering tool that can be used in design processes to estimate the position of transition when steps arise has been devised.

The importance of roughness on transition has been recognised for many years and, mostly, the influence is estimated empirically using the results of experiments made many years ago. Fage ([1943]) collected experimental wind tunnel data of the measured transition positions for 2D smooth bulges, hollows and surface ridge corrugations and established criteria for the critical heights of these imperfections. Tani ([1961]) reviewed his own experimental results on the effect of spanwise trip wires and presented the correlation of transition Reynolds number with the relative roughness height, where the height was normalised by the local displacement thickness. His results showed that the transition Reynolds number was a simple function of roughness height. The relationship had universal validity, being independent of both pressure gradient and free-stream turbulence, provided that the pressure gradient was weak. A review of the early experimental results on transition caused by roughness has also been given by Schlichting ([1979]). Analytical treatment of this problem has also been carried out by using a combination of an interactive boundary layer procedure coupled with the e^N method by Nayfeh et al. ([1988]), Cebeci and Egan ([1989]) and Masad and Lyer ([1994]).

Currently, the most successful and widely used transition prediction tool in engineering is the e^N method, proposed independently by Smith and Gamberoni ([1956]) and Van Ingen ([1956]). This method is based on the idea that the position of the transition can be

correlated with the position on the aerofoil, where the overall amplification of Tollmien–Schlichting waves has reached a particular level, namely, e^N, where N has been found from experiments to be consistently close to 9. The rationale for this approach is that the amplification rate is very large close to breakdown, so that the estimated transition position is relatively insensitive to the chosen value of N.

In this paper, the effect of steps on the transition location of the boundary layer over a flat plate has been studied. Both backward-facing and forward-facing steps were used. The transition Reynolds number was measured for a variety of step heights and free-stream velocities in the expectation that a behaviour pattern would emerge which would be a useful guide to the effect of different sized steps on transition.

2 Experimental setups

The experiments were conducted in the low-turbulence wind tunnel of the Engineering Department, Queen Mary, University of London. The dimensions of the test section was 3×3×6 ft (0.915×0.915×1.83 m) and the velocity in the working section could be set within the range 0.5–40.0 m/s. The turbulence intensity measured in the empty tunnel was less than 0.01% of the free-stream velocity within the frequency range 4 Hz–4 kHz at a free-stream velocity of 18 m/s.

The measurements were made on a flat plate that was constructed from two sections in such a way that, when bolted together, a step was formed at 300 mm from the leading edge. The plates were bolted together with ten clamping bolts and ten jacking screws so that either forward-facing or backward-facing steps could be set up with a specified step height. The model had an elliptic nose region covering about 40 mm and extended 1,000 mm downstream from the step to the trailing edge

flap and tab that were used to regulate the stagnation point at the nose.

The model was mounted vertically in the wind tunnel and spanned the whole test section. Initially, the two sections were set up to provide a single smooth surface. A near zero pressure gradient was set over the plate by adjusting the incidence of the plate together with the trailing edge flap and tab. A schematic diagram of the plate and associated data acquisition apparatus is shown in Fig. 1.

Velocities within the boundary layer in the streamwise direction were measured using a DISA 55M01 constant temperature hot-wire anemometer. A boundary layer type probe with a gold-plated tungsten wire element of diameter 2.5 μm and an effective length of 0.55 mm was used. The hot-wire probe support was part of a three-dimensional traverse system, which was controlled by a computer to move in the streamwise (x), wall-normal (y) and spanwise (z) directions.

The step heights were set over the range of 0–1.0 mm for the backward-facing step and 0–2.0 mm for the forward-facing step. The free-stream velocity of the wind tunnel was varied over the range 16–34 m/s.

3 Transition detection methods

A state measure of the flow in the transition zone is intermittency, where the flow consists of a series of turbulent regions embedded within a basically laminar flow. The signal from a hot-wire anemometer placed in the boundary layer within this zone displays this intermittent behaviour very clearly on an oscilloscope screen. The degree of intermittency, or the proportion of the time series containing turbulence, provides an indication of the completeness of the transition process at that position. A rough indication of the transition state can, therefore, be formed by a simple visual inspection of the

**Fig. 1** Schematic of step model and the data acquisition system
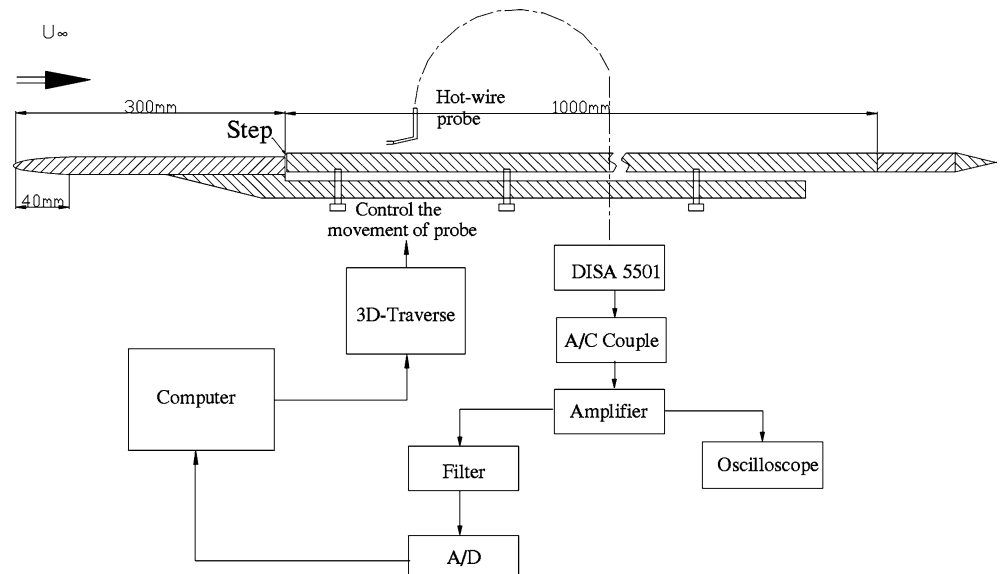
hot-wire signal displayed on an oscilloscope. But of course, this process tends to be subjective, so that different experimenters provide slightly different interpretations of the signal and, hence, different estimates of the transition position. In many boundary layer transition experiments, the transition position has been determined by observations of the beginning of turbulent bursts in the signals from a hot-wire placed near the wall, or by a surface probe detecting the pressure fluctuations. The uncertainty of the transition position determination by this method was considered to be no better than ± 25 mm (Klebanoff and Tidstrom 1972) in typical low-speed wind tunnel experiments.

More sophisticated intermittency analysis of transitional flows to discriminate between turbulent and laminar flows can be found in the literature. For example, Hedley and Keffer (1974) used a conditional sampling and averaging method for deciding when a fluid motion can be considered to be turbulent. The detailed intermittent signal processing methods, including the selection of the detector function, the generation of criterion function and the establishment of the threshold level, were presented. A conditional sampling technique was applied by Kuan and Wang (1990) to study the boundary layer transition. They introduced a dual-slop method to find the threshold value and obtained the intermittency distribution across the boundary layer at various streamwise locations. The transition positions that were detected by using these sophisticated methods are more accurate and objective. However, the applications of these methods also cause complexities both in experimental arrangement and data processing.

Many experimental results have shown that, in the presence of roughness, the route to transition is via a K-type breakdown (Klebanoff and Tidstrom 1972; Breuer et al. 1996). This transition process is characterised by the development of a series of spikes in the time series of the hot-wire signal. The spikes correspond to the passage of the rolled-up shear layer with a steep gradient passing over the hot-wire. The number of the spikes observed increases as the disturbances propagate downstream. Recent experiments and DNS calculations (Bake et al. 2000) have indicated that, even when breakdown was of the subharmonic type, spikes were still observed in the later stages of transition. This suggested that the first appearance of spikes could be used as a transition discriminator. In the work presented here, the position of transition was, therefore, taken as being the position where a spike was first observed. The hot-wire probe was positioned at the edge of the boundary layer and then moved gradually downstream. The output signal was displayed on an oscilloscope and the station of the occurrence of the first spike was noted. Normally, a period of 5 s of sample time was used at each streamwise position. It was found that consistent estimates of the transition position could be made to an accuracy of ± 5 mm. But the use of any other criteria would produce different estimates of the transition location. Typical time series signals are shown in Fig. 2. These signals were collected at the free-stream velocity of 28 m/s for a backward-facing step with a height of 0.7 mm. At a streamwise distance of 735 mm from the leading edge, the first appearance of a spike was detected by a hot-

Fig. 2a–c Time series signal plots for a backward-facing step, $U_\infty = 28$ m/s, $h = -0.7$ mm. **a** $X = 730$ mm. **b** $X = 735$ mm, inception position of transition. **c** $X = 740$ mm
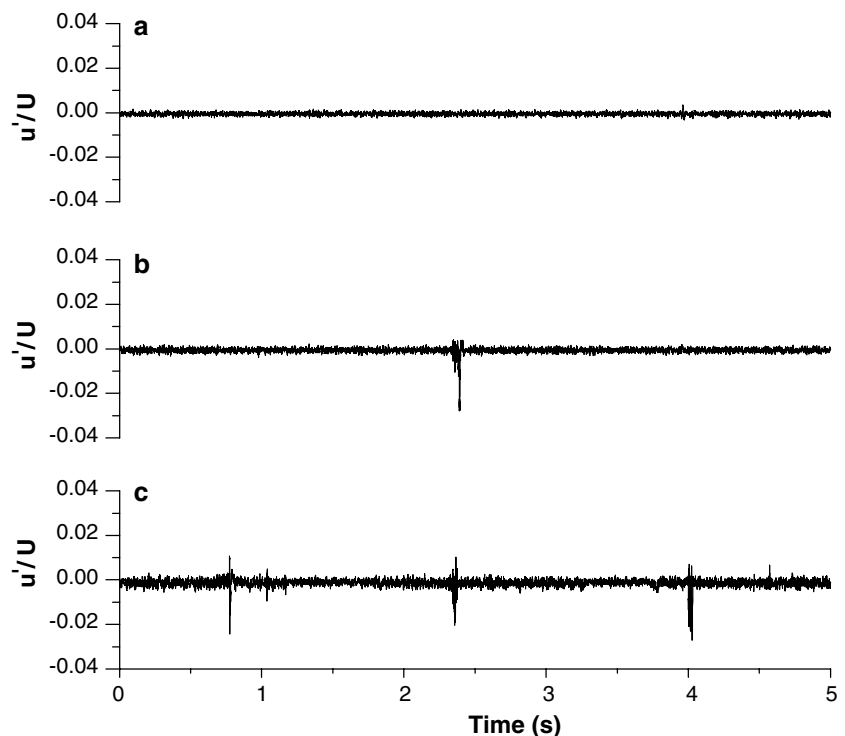
wire that was placed at the edge of boundary layer (about 3.2 mm from the model surface at this streamwise position), as shown in Fig. 2b. At a distance of 5 mm upstream of this position, the flow was laminar (Fig. 2a). However, at the streamwise position of 5 mm downstream, three spikes were detected in a period of 5 s of sample time (Fig. 2c).

4 Results and discussion

4.1 Measurements of transition positions

The transition position on the smooth surface of the flat plate without any steps was first detected by the technique described above. Transition positions were found to be at 1,060 mm from the leading edge at a free-stream velocity of 34 m/s and at 1,095 mm for a free-stream velocity of 33 m/s. The corresponding transition Reynolds number was 2.4×10^6 for both flow speeds. This value is slightly lower than the values obtained in other transition experiments carried out in different low-turbulence wind tunnels using other detection techniques. Tani (1961) obtained a value of 2.6×10^6 using a surface pitot tube. The pitot pressure is found to increase suddenly as the probe passes through the transition zone, and this sudden rise was used to define the transition position. Klebanoff and Tidstrom (1972) detected the transition Reynolds number to be 2.8×10^6 by using a hot wire to detect the position of the start of turbulent bursts. They also claimed that this position was generally a few inches farther downstream from the region where the spikes first appeared. These variations in the measured transition Reynolds number are not very surprising in view of the different methods used to determine the inception point of transition, coupled with the variations that also must arise from the levels of turbulence intensity that existed in the different wind tunnels.

Generally, the presence of a smooth 2D roughness causes the transition position to move progressively closer to the location of the roughness element as the height is increased. With a fixed height, the transition position also moves gradually towards the location of roughness with increasing free-stream velocity. This pattern of behaviour was also observed in the current experiment for the steps of both forms. The transition detection scheme previously discussed was used to determine the variation in the transition position with flow speed and roughness height, for both forward-facing and backward-facing steps. The results from these experiments are shown in Fig. 3.

The comparison of the measurements for the two types of steps tested indicated that transition was much more sensitive to the backward-facing step than the forward-facing geometry. At the same absolute step height of 0.5 mm and a free-stream velocity of 34 m/s, transition occurred at 705 mm for the backward-facing step, where the corresponding transition Reynolds number was 1.6×10^6, whereas for the forward-facing step, the transition position was at 1,030 mm, corresponding to a transition Reynolds number of 2.33×10^6. Dovgal et al. (1990, 1994) observed in their receptivity experiments that the magnitude of the disturbance excited on a backward-facing step was almost twice as large as that on a forward-facing step of the same height. Our current results are in accordance with their findings. The evolution of a small amplitude instability wave passing over a shallow bump with both a sharp leading edge and a sharp trailing edge was experimentally investigated by Wang (2004). It was found that, in the presence of the bump, the mean flow was distorted much more severely in the downstream region than that in the upstream region. The downstream mean flow distortion range was also much larger than the upstream distortion range. In the downstream region, the instability wave was amplified more significantly and its streamwise growth distance was much

Fig. 3 Transition positions at different step heights and free-stream velocities
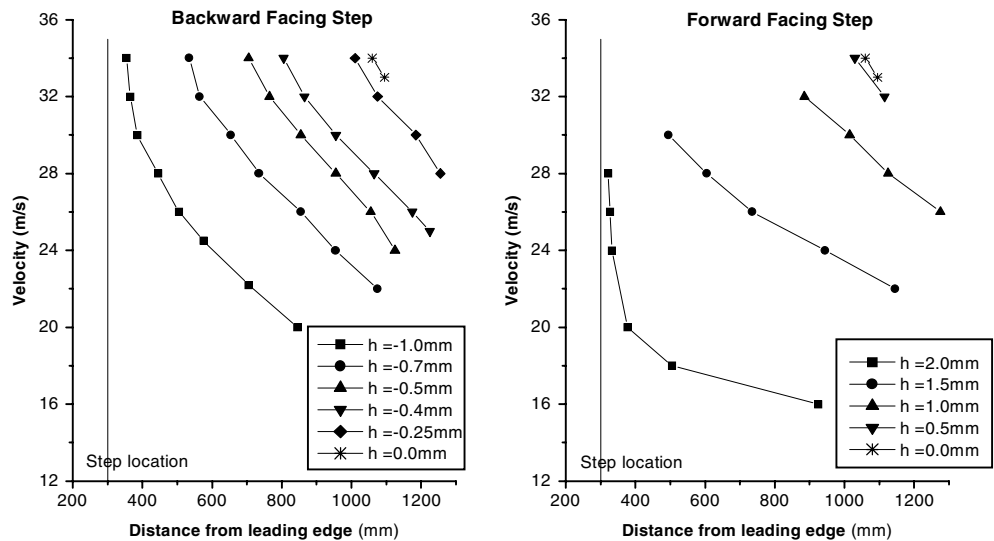
Fig. 4 Variations of transition Reynolds number with relative step height
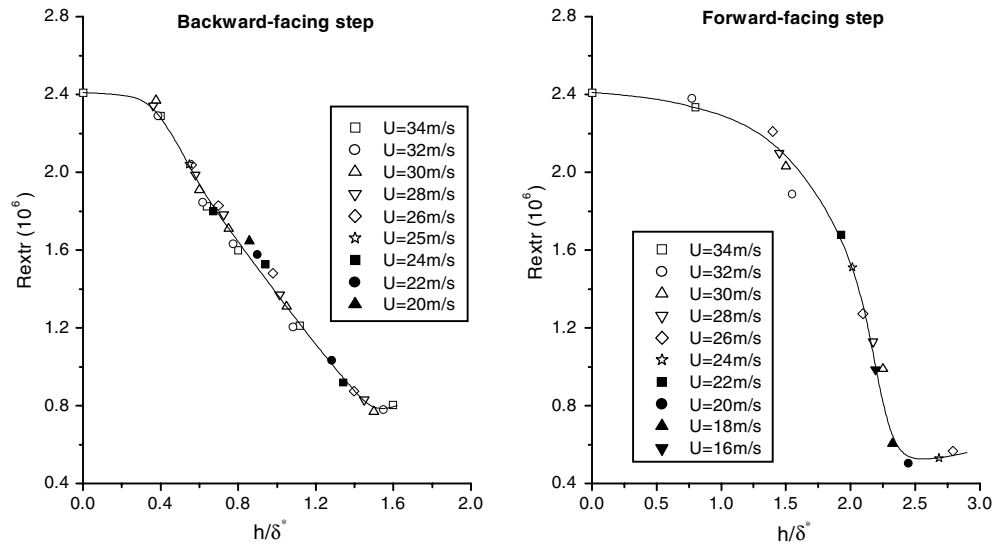
longer than that in the upstream region. So, the enhancement of transition was much more significant in the downstream region. The flows over the forward-facing step and backward-facing step are, somehow, similar to the flows in the upstream region and downstream region of the bump.

4.2 Transition Reynolds number versus step height

The transition velocity/position data were converted to transition Reynolds numbers (Re_{xtr}) and the step heights were made non-dimensional (h/δ^*). The displacement thickness, δ^*, was calculated on the basis of the Blasius mean flow boundary layer over the smooth plate at the location of the step. The experimental points collapsed onto a single curve, as shown in Fig. 4. This figure provides the unique relationship between the non-dimensional step size and the transition Reynolds number for each type of step. The shape of this curve defining was, apparently, somewhat different for the backward-facing and forward-facing steps.

This curve is useful in practice for design purposes when the prediction of transition on surfaces with steps is required. A similar correlation was first obtained by Dryden (see Tani 1961) by analysing Tani's experimental data, where the 2D roughness element was a cylindrical wire of circular cross section. He found that the experimental points lay on a single curve, as long as the transition position was not too close to the roughness element, $x_t/x_k > 1.1$, where x_t and x_k represent the distance of the transition position and the roughness position from the leading edge, respectively. As x_t approached x_k, the experimental points began to depart from this single curve (Tani 1961; Schlichting 1979).

It appears that the actual shape of the various 2D forms of roughness, circular cylinder, forward-facing or backward-facing step, etc., has an impact on the relationship between the roughness size and the transition Reynolds number. A comparison of the various curves found experimentally is shown in Fig. 5. The data for the cylindrical wire were extracted from Tani's paper. The cylindrical wire is obviously more efficient than the steps in triggering transition.
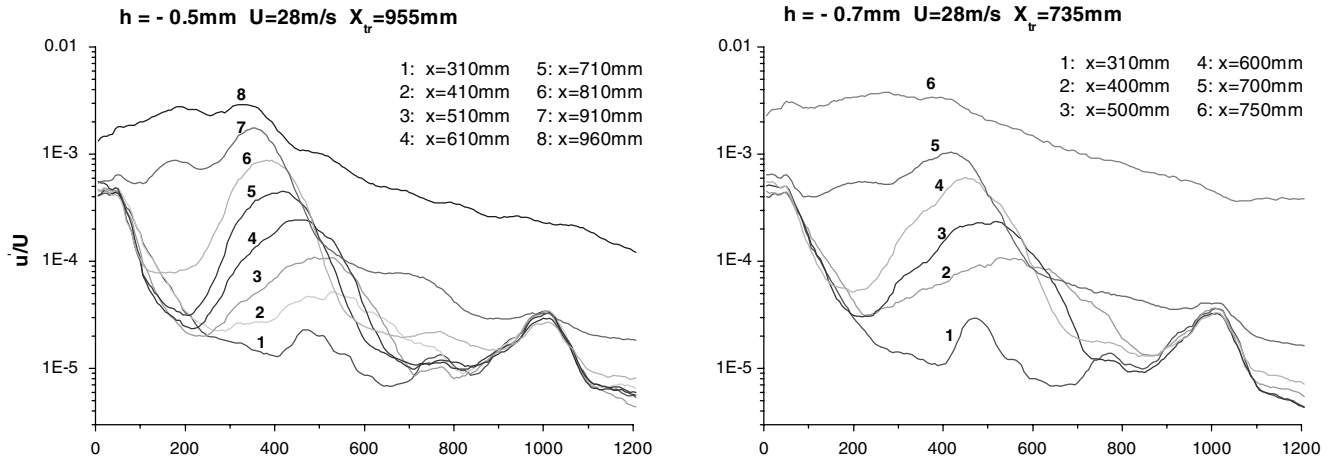
4.3 Spectra distribution and comparison with stability theory

In this experiment, the observed boundary layer perturbations that were detected by the hot-wire were excited by the wind tunnel background environment containing both acoustic disturbances and turbulence. These were generally broadband in nature and contained no significant periodic components. In order to understand the behaviour of transition induced by steps in a quantitative manner, the spectra of the excited distur-
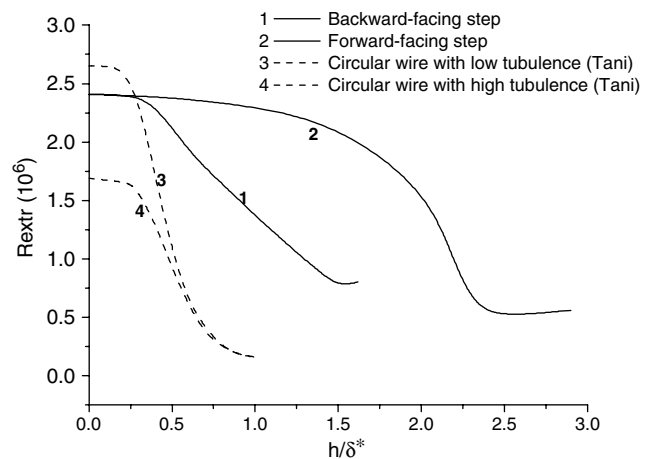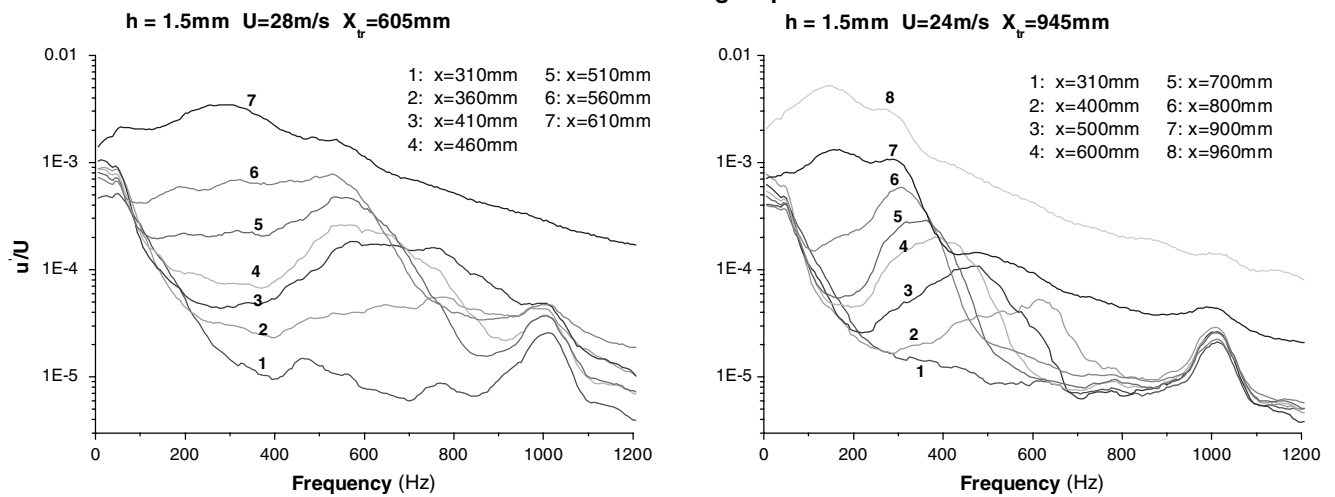

Fig. 5 Comparison of correlation curves for different types of roughness element
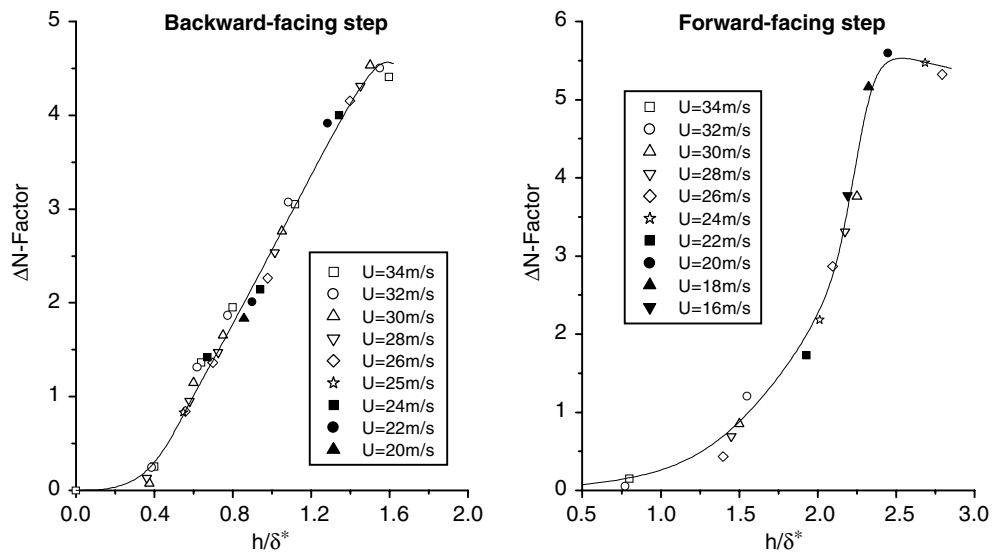
Backward facing step

h = - 0.5mm U=28m/s X$_{tr}$=955mm

1: x=310mm   5: x=710mm
2: x=410mm   6: x=810mm
3: x=510mm   7: x=910mm
4: x=610mm   8: x=960mm

h = - 0.7mm U=28m/s X$_{tr}$=735mm

1: x=310mm   4: x=600mm
2: x=400mm   5: x=700mm
3: x=500mm   6: x=750mm

Forward facing step

h = 1.5mm U=28m/s X$_{tr}$=605mm

1: x=310mm   5: x=510mm
2: x=360mm   6: x=560mm
3: x=410mm   7: x=610mm
4: x=460mm

h = 1.5mm U=24m/s X$_{tr}$=945mm

1: x=310mm   5: x=700mm
2: x=400mm   6: x=800mm
3: x=500mm   7: x=900mm
4: x=600mm   8: x=960mm

Fig. 6 Spectra of disturbances at different step heights and free-stream velocities

Fig. 7 Correlation curves of the reduction of *N*-factor with relative step height

Backward-facing step

U=34m/s
U=32m/s
U=30m/s
U=28m/s
U=26m/s
U=25m/s
U=24m/s
U=22m/s
U=20m/s

Forward-facing step

U=34m/s
U=32m/s
U=30m/s
U=28m/s
U=26m/s
U=24m/s
U=22m/s
U=20m/s
U=18m/s
U=16m/s

bances in the boundary layer were measured. Figure 6 shows the frequency spectra at different distances downstream from the step. The spectra are presented in the terms of intensity in semi-log form to cover a wide dynamic range. The measurements were made with a hot-wire probe placed 1.0 mm off the surface. The transition position (X_{tr}) corresponding to each step height and free-stream velocity are also indicated on this figure. At a small distance downstream from the transition position, the spectrum began to show spectral broadening that can be associated with the occurrence of turbulence.

It is convenient to discuss the behaviour of the spectra in three frequency bands. Low frequency disturbances, $f < 50$ Hz; a mid-range band covering the frequencies of 200–700 Hz, dependent on the streamwise distance and free-stream velocity; and a high frequency portion around 1,000 Hz. The low frequency band of disturbance has also been commonly seen in other wind tunnel experiments (Klebanoff and Tidstrom 1972; Dovgal et al. 1990, 1994; Breuer et al. 1996; Haggmark et al. 2000). It was also observed in other experiments carried out in this particular low-turbulence wind tunnel (Shaikh 1997). There were no significant changes in magnitude with the streamwise distance and free-stream velocity. It was attributed to the wind tunnel's background noise. The magnitude of the high frequency band was much smaller and almost constant at different streamwise positions. It was most likely caused by the vibration of the hot-wire prong (Shaikh 1993). The mid-range frequency band was found to be consistent with the unstable region of Tollmien–Schlichting waves predicted by instability theory. For example, at a streamwise distance of 510 mm and a velocity of 28 m/s, according to instability theory, the unstable disturbance frequency band is 200–550 Hz, agreeing with the experimental measurements. In the lower part of this frequency range, about 200–300 Hz, the associated disturbances just become unstable and some propagation distance was needed before the disturbances of appreciable magnitude could be detected. So, the measured disturbances were still quite small in this low frequency range.

4.4 N-factor decrease versus step height

Currently, the most commonly used transition prediction technique is the e^N method. In this approach, it is assumed that transition occurs when the N-factor reaches about 9 (with a scatter from 7 to 11, depending on circumstances). Theoretical N-factor calculations were carried out by using the fast N-factor calculation code (Gaster 1997) at different free-stream velocities on the flat plate. The final value of the N-factor was obtained from the envelope of the overall maximum amplifications of different frequencies in the frequency range 20–900 Hz. The N-factor at the transition position measured on a smooth flat plate was found to be 7.4. In the presence of the step, the transition position moved forward and the corresponding N-factor was, therefore, reduced. The decrease of the calculated N-factor at transition, denoted as ΔN, was calculated to be the following:

$$\Delta N = N_{smooth} - N_{step}$$

where N_{smooth} is the N-factor calculated at the transition position for the smooth surface and N_{step} is the N-factor calculated at the transition position in the presence of the step.

The variations of the reduction in the N-factor, ΔN, with non-dimensional step height are shown in Fig. 7 for a range of flow speeds. Because of the relationships between the transition Reynolds number and the step height shown in Fig. 4, it is not surprising that the reduction of the N-factor against the non-dimensional step height collapses onto single curves for backward-facing and forward-facing steps. This curve provides a useful guide to the expected movement of transition caused by steps and extends the e^N transition prediction method to boundary layers with steps.

5 Conclusions

The effect of steps of various sizes on the boundary layer transition was investigated experimentally. The presence of a sharp-edged step on an otherwise smooth surface of a flat plate hastened the occurrence of laminar–turbulent boundary layer transition. The transition position moved progressively forward to the position of the step with an increase of step height or free-stream velocity. The transition movement was found to be greater for a backward-facing step than for a forward-facing step under the same absolute height and free-stream conditions.

The variation of the transition Reynolds number caused by the presence of steps of different relative height fell onto a single monotonic curve. The shape of this curve was different for the backward-facing and forward-facing steps.

The unstable frequency range of the measured disturbance was found to be consistent with that predicted by linear stability theory.

The presence of a step caused a reduction of the transition N-factor. This decrease of the N-factor was a unique function of the relative step height for each type of step. This correlation curve suggests that it is possible to extend the application range of the widely used e^N prediction method from smooth surfaces to surfaces with step-like sharp discontinuities. Before this can be done, however, the influence of pressure gradients needs to be investigated.

References

Bake S, Ivanov A, Kachanov Y (2000) Resemblance of K- and N-regimes of boundary-layer transition at late stages. Eur J Mech B–Fluids 19:1–22

Breuer KK, Dzenitis EG, Gunnarsson J, Ullmar M (1996) Linear and nonlinear evolution of boundary layer instabilities generated by acoustic receptivity mechanisms. Phys Fluid 8:1415–1423

Cebeci T, Egan DA (1989) Prediction of transition due to isolated roughness. AIAA J 27(7):870–875

Dovgal AV, Kozlov VV (1990) Hydrodynamic instability and receptivity of small-scale separation regions. In: Arnal D, Michel R (eds) Laminar–turbulent transition, vol 3. Springer, Berlin Heidelberg New York, pp 523–531

Dovgal AV, Kozlov VV, Michalke A (1994) Laminar boundary layer separation: instability and associated phenomena. Prog Aerospace Sci 30:61–94

Fage A (1943) The smallest size of a spanwise surface corrugation which affects boundary layer transition on an aerofoil. British Aeronautical Research Council, report and memoranda no 2120

Gaster M (1997) Rapid estimator of eigenvalues for N-factor calculation. Report to DERA, 1997

Haggmark CP, Bakchinov AA, Alfredsson PH (2000) Experiments on a two-dimensional laminar separation bubble. Phil Trans R Soc Lond A 358:3193–3205

Hedley TB, Keffer JF (1974) Turbulent/non-turbulent decisions in an intermittent flow. J Fluid Mech 64:625–644

Klebanoff PS, Tidstrom KD (1972) Mechanism by which a two-dimensional roughness element induces boundary layer transition. Phys Fluid 15(7):1173–1188

Kuan CL, Wang T (1990) Investigation of the intermittent behavior of transitional boundary layer using a conditional averaging technique. Exp Thermal Fluid Sci 3:157–173

Masad JA, Lyer V (1994) Transition prediction and control in subsonic flow over a hump. Phys Fluid 6(1):313–327

Nayfeh AH, Ragab SA, Al-Maaitah A (1988) Effect of bulges on the stability of boundary layers. Phys Fluid 31(4):796–806

Schlichting H (1979) Boundary layer theory, 7th edn. McGraw-Hill, New York

Shaikh FH (1993) Turbulent spots in a transitional boundary layer. PhD thesis, Churchill College, Cambridge University

Shaikh FH (1997) Investigation of transition to turbulence using white-noise excitation and local analysis techniques. J Fluid Mech 348:29–83

Smith AMO, Gramberoni N (1956) Transition, pressure gradient and stability theory. Douglas Aircraft Co., technical report no ES-26388

Tani I (1961) Effect of two-dimensional and isolated roughness on laminar flow. In: Lachmann GV (ed) Boundary layer and flow control, vol 2. Pergamon Press, Oxford, pp 637–656

Van Ingen JL (1956) A suggested semi-empirical method for the calculation of the boundary layer transition region. Department of Aeronautics Engineering, Deft Institute of Technology, Delft, The Netherlands, report VTH-74

Wang YX (2004) Instability and transition of boundary layer flows disturbed by steps and bumps. PhD thesis, Queen Mary, University of London