A pneumatic droplet-on-demand generator

S. Cheng, S. Chandra

Abstract The design of a pneumatic droplet generator to produce small (~0.2 mm diameter) water droplets on demand is described. It consists of a cylindrical, liquid-filled chamber with a small nozzle set into its bottom surface, connected to a gas cylinder through a solenoid valve. Rapidly opening and closing the valve sends a pressure pulse to the liquid, ejecting a single droplet through the nozzle. Gas in the chamber escapes through a vent hole so that the pressure drops rapidly and more droplets do not emerge. We photographed droplets as they emerged from the nozzle, and recorded pressure fluctuations in the chamber. We determined the duration of the pressure pulse required to generate a single drop; longer pulses produced satellite drops. The length of the water jet when its tip detached and the diameter of the droplet that formed could be predicted using results from linear stability analysis. The peak pressure in the cavity could be increased by raising the supply pressure, increasing the width of the pressure pulse, or by reducing the size of the pressure relief vent.

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Introduction

Many engineering applications require the production of small, uniform sized droplets. Droplet generators were first developed as research tools to investigate droplet dynamics, spray cooling and droplet combustion, or to calibrate particle size measuring instruments. They have since found use in many industrial applications such as ink-jet printing, dispensing controlled volumes of pharmaceuticals, and deposition of adhesive or solder droplets

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S. Cheng, S. Chandra (⊠) Centre for Advanced Coating Technologies, Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, M5S 3G8, Canada E-mail: chandra@mie.utoronto.ca Tel.: +1-416-9785742 Fax: +1-416-9787753

Materials and Manufacturing Ontario (MMO) and Teck Cominco Product Technology Centre of Mississauga, Ontario, Canada offered financial support for this research. Their assistance is gratefully acknowledged on circuit boards. In recent years there has been great interest in droplet based manufacturing techniques for rapid prototype production. One method is to place droplets of a binder material on a layer of metal or ceramic powder to form a solid part (Sachs et al. 1992). In another technique molten droplets—which may be of metal, wax or polymer—are deposited on a solid substrate, and by moving both the droplet stream and substrate, components with complex shapes are built up (Weiss et al. 1997; Orme and Smith 2000). Development of all these technologies requires a reliable method of generating droplets on demand.

The easiest method of forming droplets is to drive a liquid jet through a vibrating orifice; fluid instabilities break up the jet at regular intervals producing a stream of droplets whose size depends on the oscillation frequency. Traditionally the nozzle was a stainless steel needle vibrated by connecting it to a loudspeaker cone (Pederson 1970). A contemporary design (Switzer 1991), based on the same principle, places a piezoelectric tube around a section of flexible tubing through which liquid is forced. Applying electrical pulses to the piezoelectric crystal makes it contract periodically, imposing a disturbance on the liquid jet. This method, though easy to implement, does not permit production of single droplets on demand. To select only a few droplets from the continuous stream the liquid is charged and superfluous droplets deflected to a collector by applying an electric field (Liu and Orme 2002).

A droplet-on-demand generator requires a different design. A typical configuration (Yang et al. 1997) consists of a cylindrical chamber filled with liquid, with a small nozzle set into one wall. One of the flat walls of the chamber is made from a disk of piezoelectric ceramic. Application of a voltage pulse to the piezoelectric disk makes it flex, forcing a droplet out through the nozzle. Though piezoelectric droplet generators are conceptually quite simple, experience has shown that they are quite complicated to operate. High voltage pulse generators, required to drive the piezoelectric element, are expensive. The droplet generator has to be completely filled and primed very carefully, since even small air bubbles in the liquid can stop it from working. The pressure in the chamber and the force with which the piezoelectric element is clamped affect operation and must be controlled precisely. Voltage and pressure settings have to be determined for each liquid since they vary with properties such as viscosity and surface tension. The piezoelectric crystal is quite fragile and cannot withstand high temperatures. To



Fig. 1. Schematic diagram of droplet generator

produce droplets of molten materials the piezoelectric components have to be shielded from high temperatures, further complicating the design (Orme and Smith 2002; Liu and Orme 2002).

Ideally a droplet-on-demand generator should be robust, easy to operate, and cheap to fabricate. It should be insensitive to liquid properties and able to operate even if the chamber contains only a small amount of liquid. Finally, it should be able to withstand high temperatures.

This paper describes a pneumatic droplet-on-demand generator that we have developed (Chandra and Jivraj 2002), which works by applying pulses of pressurised gas to liquid contained in a chamber, forcing out droplets through a nozzle in the bottom plate of the generator. A solenoid valve is rapidly opened and closed to create pressure pulses. There are no moving parts in contact with the liquid, making the generator simple to build, sturdy, and easy to adapt to high temperature applications. We conducted an experimental study to determine the effect of varying the amplitude and duration of the pressure pulse on the size and number of droplets produced. We photographed droplets as they emerged from the nozzle and measured the pressure variation in the droplet generator chamber. The intent was to determine conditions that ensure production of single droplets on demand.

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Experimental method

The droplet generator body consisted of a stainless steel cylinder 50.8 mm (2 in) o.d., 20.3 mm (0.8 in) i.d. and 63.5 mm (2.5 in) long (see Fig. 1). Both ends of the cylinder were capped with 6.35 mm (0.25 in) thick stainless steel plates bolted on using polytetrafluoroethylene O-rings for seals. A cylindrical synthetic sapphire nozzle with an o.d. of 2.0 mm and orifice of 0.102 mm (0.004 in)



Fig. 2. Schematic diagram of experimental apparatus

(model 2-80-004, Swiss Jewel Company, Philadelphia, Penn.) was inserted into a hole in the bottom plate of the droplet generator and sealed in place using ceramic cement (Omegabond CC, Omega Inc, Stamford, Conn.). The diameter of the nozzle was too small for liquid to flow through it under the effect of gravity alone, but droplets could be forced out by applying a pressure pulse from a nitrogen tank. Figure 2 shows a schematic diagram of the experimental apparatus. The gauge pressure of the nitrogen supply line was maintained in the range 70~400 kPa (10~60 psig) and a pressure pulse was delivered by rapidly opening and closing a solenoid valve (model 8262G208, Asco Valve Canada, Brantford, Ont.) mounted in the gas line. The valve was activated by applying a 12-V DC pulse for approximately 4–5 ms. The response time for the valve to open fully was specified by the manufacturer as 90 ms, which means that it only opened partially when triggered in our experiments. We operated the solenoid at rates of up to 3 Hz. A 1-l buffer tank was connected in the gas supply line to dampen pressure fluctuations when the solenoid valve was actuated.

It was difficult to open and close the solenoid valve sufficiently rapidly to eject a single droplet. Experiments showed that a long liquid jet emerged from the nozzle that then broke up into several drops. To relieve the pressure inside the chamber very rapidly the gas line was connected to the droplet generator chamber through a stainless steel T-junction, one of whose outlets was connected to the upper plate of the droplet generator while the other was open to the atmosphere through a length of stainless steel tubing whose end was either left open or capped with an adjustable valve (see Fig. 1). When a gas pulse was applied sufficient pressure built up inside the chamber to force out a single droplet. The gas then escaped to the atmosphere though the vent, relieving pressure inside the chamber and preventing further droplets from escaping. By adjusting the electronic pulse width the opening time of the solenoid valve, and thereby the duration of pressure rise in the chamber, could be varied.

To measure pressure variations in the cavity of the droplet generator a high-speed dynamic pressure transducer (type 601B1, Kistler Instrument Corp., Amherst, N.Y., USA) was mounted in a hole drilled into the top cover plate with its face flush with the plate surface. The effective measuring range of the pressure transducer was 0–69 kPa. It had a response time of 2 μ s (rise time from 10 to 90%) and was certified by the manufacturer to be calibrated within an accuracy of $\pm 0.04\%$. The signal from the pressure transducer was amplified (type 5010B dual mode amplifier, Kistler Instrument Corp.) and recorded using a computer based LabVIEW control system (National Instruments, Austin, Tex., USA) with a data acquisition board (PCI-MIO-16E-4, National Instruments). The same board was also used to drive the solenoid valve and to control the camera and flash used to photograph droplets.

To observe droplets as they emerged from the nozzle a high magnification photography system was required since the area photographed was less than 1.5 mm across. A digital still camera (Nikon E3, Nikon Canada, Mississauga, Ont., Canada) was equipped with a long-range microscope (Questar QM100, Company Seven, Astro-Optics Division, Montpelier, Md., USA), which had a resolution of 1.1 μ m at a distance of 0.15 m. A high-energy strobe (Model X1200, Perkin-Elmer Optoelectronics, Fremont, Calif., USA) with a flash duration of 10 μ s was used to provide illumination. Light from the strobe was carried through optical fibre cables to a square (50 mm×50 mm) diffuser panel placed 50 mm from the nozzle, pointing up towards it at an angle of approximately 45° (see Fig. 2).

To take a photograph the LabVIEW control system was used to send a signal to the camera. After a delay of approximately 100 ms its shutter lifted and stayed open for 250 ms. Next the computer control system activated the solenoid valve for 4–5 ms, creating a gas pressure pulse. A droplet emerged from the nozzle approximately 7 ms after the solenoid valve first opened. The flash was then triggered, exposing the CCD chip of the camera and capturing a single image of the droplet. The picture was transferred to a computer for analysis using image analysis software (Image, National Institutes of Health, Bethesda, Md., USA). By varying the flash delay different stages of droplet formation were captured, and the entire process reconstructed by placing successive frames in sequence.

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Results and discussion

Figures 3, 4, 5, 6 and 7 show photographs of water droplets emerging from the nozzle of the droplet generator when a pressure pulse was applied, for varying opening times of the solenoid valve. In all cases the nitrogen supply pressure was 138 kPa (20 psig), the nozzle diameter was 0.102 mm (0.004 in) and the needle valve controlling the size of the vent from the droplet generator chamber was kept fully open. Each figure shows successive stages of droplets emerging from the nozzle. The time of each frame (*t*) is shown below it, measured from the instant liquid first



Fig. 3. Emerging of a water jet from the droplet generator nozzle using a 4.56 ms pulse to trigger the solenoid valve. No droplet detached from the tip of the jet

came out of the nozzle. In all these images the view of the droplet is foreshortened because of the camera being tilted upwards. To compensate for this distortion when making length measurements we used the circular hole in which the nozzle was set (visible as an ellipse in the pictures) as a calibration scale. Since the hole diameter was known, by measuring the major and minor axes of the ellipses in photographs we calculated the degree of foreshortening. All vertical length measurements were then corrected to compensate for this. Typically measured lengths were increased by 10–15%; Cheng (2002) has given further details.

Figure 3 shows the case when a pulse width of 4.56 ms was applied to the solenoid valve. A liquid jet came out of the nozzle and reached its maximum length of 0.80 mm in approximately 2.0 ms, which was shorter than the critical length for break-up. Surface tension then pulled the jet back, and it withdrew without a drop detaching from it.

Figure 4 shows photographs of droplet formation when a pulse of 4.61 ms was applied to open the solenoid valve. In this case the jet became long enough



Fig. 4. Formation of a single water droplet using a 4.61 ms pulse to trigger the solenoid valve

Fig. 5. Formation of two water droplets using a 4.65 ms pulse to trigger the solenoid valve

(1.06 mm) that the tip began to neck off (t=2.7 ms) and then detached. The remaining portion of the water jet then withdrew back into the nozzle, while the droplet of 0.23 mm diameter that detached continued to fall.

Increasing the pulse width further made the liquid jet longer and increased the number of droplets that detached. A pulse of 4.65 ms produced two droplets—a large one with 0.23 mm diameter followed by a smaller satellite droplet with 0.14 mm diameter (Fig. 5). The first droplet detached when the jet length was 1.26 mm (t=2.7 ms), and the second satellite droplet detached when the jet was 0.82 mm long (t=2.9 ms).

Increasing the pulse to 4.70 ms produced three droplets, a large one of 0.23 mm diameter and two satellite drops of 0.13 mm diameter (Fig. 6). The water jet had a length of 1.47 mm when the first, biggest, droplet detached (t=2.7 ms), 1.01 mm when the second droplet detached (t=2.9 ms) and 0.88 mm when the third satellite droplet detached (t=3.2 ms).

Finally, an even longer pulse of 4.94 ms resulted in a long jet issuing from the nozzle that broke up into a stream of droplets (Fig. 7).

The length of the jet (L_j) from which a droplet detaches can be predicted using linear stability analysis (Yim 1996), and is given by

to trigger the solenoid valve

$$\frac{L_{j}}{d_{j}} = C \left[\sqrt{We} + \frac{3We}{Re} \right]$$
(1)

where d_j is the jet diameter, assumed to equal that of the nozzle and the Weber number (*We*) and Reynolds number (*Re*) are defined as:

$$We = \frac{\rho v_j^2 d_j}{\sigma} \tag{2}$$

$$Re = \frac{\rho v_j d_j}{\mu} \tag{3}$$

 ρ , μ and σ are the liquid density, viscosity and surface tension respectively, and v_j the jet velocity. The parameter *C* depends on nozzle diameter and length, inlet and outlet conditions of the nozzle, and the liquid used. Grant and Middleman (1966) recommended an average value of 13.4. The diameter of the droplet (d_d) that detaches is given by (Yim 1996):

$$\frac{d_{\rm d}}{d_{\rm j}} = \left(1.5\pi\sqrt{2+3Oh}\right)^{1/3} \tag{4}$$



Fig. 6. Formation of three water droplets using a 4.70 ms pulse to trigger the solenoid valve

where *Oh* is the Ohnesorge number:

$$Oh = \frac{\mu}{\sqrt{\rho\sigma d_{j}}} \tag{5}$$

From photographs (see Fig. 4) of a single water droplet detaching from the tip of a water jet emerging from a 0.102 mm nozzle, we estimated the jet velocity to be 0.58 m/s. Using these values for d_j and v_j respectively, we obtain Re=58.8, We=0.47 and Oh=0.0102. Equations (1) and (4) then give $L_j=0.96$ mm (measured $L_j=1.06$ mm) and $d_d=0.19$ mm (measured $d_d=0.23$ mm). Satellite droplets are created by non-linear effects that cannot be modelled using linear stability analysis.

Figure 8 shows the pressure variation in the cavity of the droplet generator during the formation of a water droplet, measured with a pressure transducer. It corresponds to the case where just one water droplet was formed, as shown in Fig. 4. The supply pressure was 138 kPa (20 psig), the width of the pulse used to open the solenoid valve was 4.61 ms and the release needle valve was kept fully open. The timing of the electrical pulse sent to open the solenoid valve is also shown in the figure. Time t=0 corresponds to the start of the pulse. The pressure in the cavity oscillated at a constant frequency while its amplitude gradually decreased. The

gauge pressure was initially positive, but then became negative for a short time as it oscillated, before going back to zero. Water first emerged from the nozzle at t=7.5 ms and 2.7 ms later a single droplet detached while the remaining liquid started to withdraw (see Fig. 4) at a time corresponding to the gauge pressure in the cavity becoming negative (see Fig. 8).

When the solenoid valve was opened for a longer time the amplitude of the pressure pulse increased, because the time for which nitrogen flowed into the chamber of the droplet generator was greater. Figure 9 shows measured pressure variations for solenoid valve opening times of 4.56, 4.61, 4.65 and 4.70 ms, corresponding to Figs. 3, 4, 5 and 6 respectively. As pulse width widened the peak pressure increased and the time at which the peak pressure occurred was delayed. Greater pressure in the chamber resulted in a longer jet of water emerging from the nozzle and therefore a larger number of droplets. In all cases liquid was first visible at the nozzle outlet at t=7.5 ms, corresponding to a chamber pressure of 3.7 kPa.

The hydrostatic pressure in the chamber was relatively small: typically we operated with less than 10 mm of water in the chamber, giving a head of 0.1 kPa at the nozzle. A quick estimate of the gauge pressure (Δp) required to overcome liquid surface tension and force a droplet out of the nozzle is given by:

$$\Delta p = \frac{2\sigma}{r_{\rm i}} \tag{6}$$

where r_j is the radius of curvature of the liquid surface, assumed to be equal to that of the nozzle orifice $(r_j=0.051 \text{ mm})$, giving $\Delta p=2.9$ kPa. Since this is the minimum value required to overcome surface tension, the measured value of 3.7 kPa when the liquid first emerged from the nozzle (see Fig. 9) appears reasonable. In all cases the liquid jet began to withdraw into the nozzle 2.7 ms after it first emerged, corresponding to the moment when the chamber gauge pressure became negative. The peak pressure during the second oscillation was less than 2.9 kPa and therefore no further droplets were ejected.

To get only a single drop it is essential to have the liquid jet withdraw immediately after the first drop detaches. Obtaining a negative gauge pressure is therefore very important to the functioning of the droplet generator. To understand how the pressure in the droplet generator oscillates when a pressure pulse is applied we modelled it as a Helmholtz resonator, which is an air filled enclosure connected to a short, open neck as shown in Fig. 10. When the solenoid valve is suddenly opened and closed air in the neck of resonator oscillates somewhat like a solid piston, alternately compressing and expanding the air in the enclosed volume. The resonator oscillation frequency (F) is given by (Irwin and Graf 1979; Harris 1991):

$$F = \frac{c}{2\pi} \sqrt{\frac{A}{lV}} \tag{7}$$

where A is the cross-sectional area of the neck and l its effective length (defined as $l = L + 0.8\sqrt{A}$, with L the neck length), V the volume of the cavity and c the speed of sound in air, assumed to be 343 m/s.



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Fig. 8. Measured pressure variation inside the droplet generator cavity when a 4.61 ms pulse was used to trigger the solenoid valve

We varied the volume of the resonator by changing the amount of water in the droplet generator. A set of experiments were conducted with a supply pressure of 138 kPa (20 psig), electronic pulse width to the solenoid valve set

Fig. 7. Formation of multiple droplets using a 4.94 ms pulse to trigger the solenoid valve

at 4.70 ms, with a fully open venting tube (90 mm long, 5.1 mm i.d.) without any valve mounted at the end. The volume of the empty cavity was 20.6 ml. Figure 11a-e shows pressure curves with the droplet generator containing various amounts of water. When the cavity was almost full of water the negative portion of the pressure curve increased. As the volume of water was increased, and thereby that of the resonator cavity (V) reduced, the resonator frequency increased as predicted by Eq. (7). Figure 12 shows comparisons between the measured frequencies and those calculated from Eq. (7). Error bars on the theoretical values represent uncertainty in measurements of the liquid volume (± 0.5 ml). Agreement is fairly good; a possible reason for discrepancies may be that the bottom surface of the droplet generator air cavity was liquid whereas the model assumed all resonator walls to be rigid.

Figure 13 shows the effect of varying the gas supply pressure and pulse width. Three different supply pressures are shown: 69 kPa (10 psig), 138 kPa (20 psig) and 207 kPa (30 psig), and two different pulse widths: 5.64 ms and 6.11 ms. As the pulse width was raised, the peak pressure reached during the pulse increased linearly with pulse time. Increasing the supply pressure changed the amplitude of the pressure curve significantly, but not its



Fig. 9. Measured pressure variation inside the droplet generator cavity when pulses varying from 4.56 to 4.70 ms in width were used to trigger the solenoid valve



Fig. 10. Modelling the droplet generator cavity as a Helmholtz resonator

frequency and phase. The peak pressure in the droplet generator was not a linear function of the supply pressure. As seen in Fig. 13, doubling supply pressure increased the maximum pressure by a factor of 3.6, whereas tripling supply pressure raised the peak pressure by a factor of 10.2. Consequently, when supply pressure is high small variations in solenoid pulse width will produce large changes in peak pressure, making droplet generation less repeatable. Also, as supply pressure is increased we no longer obtain a negative gauge pressure (see Fig. 13). Operating with a low supply pressure is therefore better for obtaining individual drops.

We examined the effect of varying vent hole size. For most experiments a 10-turn needle valve was mounted on the outlet of the vent; the results shown in Figs. 3, 4, 5, 6, 7, 8 and 9 were obtained with the needle valve mounted and kept fully open. Subsequent tests showed that closing the ball valve by one to nine turns had little effect on pressure variation in the droplet generator and it was only when the valve was fully shut that the peak pressure increased significantly. It was concluded that the resistance to flow in



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Fig. 11. Pressure variation inside the droplet generator cavity when it was filled with: **a** 0 ml, **b** 4 ml, **c** 8 ml, **d** 12 ml and **e** 16 ml of water The supply pressure was 138 kPa (20 psig), pulse width to the solenoid valve was 4.70 ms



Fig. 12. Variation of pressure oscillation frequency with air volume in droplet generator cavity. The supply pressure was 138 kPa (20 psig), pulse width to the solenoid valve was 4.70 ms

the small passages in the needle valve was already so large that small adjustments in opening size had little effect. We therefore replaced the needle valve with a quarter-turn ball valve that had a large flow passage and less resistance to flow.

Figure 14 shows measured pressure curves in the chamber when a ball valve was attached to the outlet vent. The valve handle could be set to any position between



Fig. 13. Pressure variation for different combinations of pulse width and supply pressure



Fig. 14. Pressure variation in the droplet generator cavity with a ball valve mounted on the exhaust vent. The valve handle position was set at four different positions from fully closed to fully opened. The supply pressure was set to 138 kPa (20 psig) and the solenoid valve pulse width to 4.70 ms

 0° (fully closed) to 90° (fully opened). The supply pressure was set to 138 kPa (20 psig) and the pulse width to 4.70 ms. When the valve was fully open the peak pressure was relatively low (3.1 kPa) and then decreased sharply to -3.5 kPa. As the ball valve was closed the pressure curve shifted upwards. By adjusting the valve opening we could select the maximum and minimum pressures.

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Summary and conclusions

We have designed a pneumatic droplet generator to produce water droplets on demand. It consists of a

cylindrical, liquid-filled chamber with a small nozzle set into its bottom surface, connected to a gas cylinder through a solenoid valve. Opening the valve for 4–5 ms sends a pressure pulse to the liquid, ejecting a single droplet through the nozzle. Pressure in the chamber then drops rapidly as gas escapes through a vent hole and prevents more droplets from escaping. Application of a gas pulse creates pressure oscillations in the chamber, so that gauge pressure first increases and pushes out liquid, and then becomes negative and retracts the liquid jet. Obtaining negative pressure is important for producing a single droplet on demand.

We photographed droplets as they emerged from the nozzle, and recorded pressure fluctuation in the chamber. We determined the duration of the pressure pulse required to generate a single drop; longer pulses produced satellite drops. The length of the water jet when its tip detached and the diameter of the droplet formed could be predicted using linear stability analysis. The frequency of pressure fluctuations in the droplet generator cavity was calculated by modelling it as a Helmholtz resonator. As the volume of water in the cavity was increased, so that the gas volume decreased, the oscillation frequency diminished. The peak pressure in the cavity could be increased by raising the supply pressure, increasing the duration of the pressure pulse, or by reducing the size of the pressure relief vent.

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