

# Effect of partial coherence on four-wave mixing in photorefractive materials via reflection grating approximation

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**Abstract.** We investigate the effect of beam coherence on four-wave mixing via reflection gratings in photorefractive media. For the case of phase conjugation, the results of our theoretical analysis indicate that partial coherence always leads to a drop of signal gain and phase conjugate reflectivity in non-depleted cases. In general, the mutual coherence of the signal beam and the pump beam can be enhanced due to the process of wave mixing. The mutual coherence of the phase conjugate beam and one of the pump beams depends on the beam intensity ratio as well as the optical path difference. This is distinctly different from the four-wave mixing case with a transmission grating.

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Wave mixing in photorefractive (PR) crystals is a fundamental nonlinear optical process which is responsible for many applications, such as signal processing, optical communications, optical networks, optical computing, etc. [1]. For reasons of mathematical simplicity, theoretical study in this area has been focused on wave mixing with monochromatic waves, or waves with full coherence. However, for some applications, such as double phase conjugation [2–4], achromatic volume holography [5], or optical phase conjugation through turbulent media (i.e. sea water or the atmosphere) [6], the effect of beam coherence becomes important in the coupling process if the coherence of the beams is limited either by the intrinsic properties of the light source (e.g. beams from two different lasers) or due to the propagation delay (e.g. the path difference between the beams is difficult or impossible to equalize). Thus, knowledge of the state of coherence during and after coupling is essential in these applications.

Two-wave mixing (TWM) in PR crystals with partially coherent waves has been studied by previous researchers [7–11]. Cronin-Golomb et al. [7, 8] studied the effect of partial spatiotemporal (3-dimensional) coherence in photorefractive two-wave mixing theoretically and experimentally. They found that the spatial coherence could be improved for amplified and deteriorated for deamplified waves. Bogodaev et al. studied two-wave mixing with partially spatiotemporal (1-dimensional) coherent waves in transmission grating cases [9]. Yi et al. studied two-wave mixing with partially spatiotemporal (1-dimensional) coherent waves with contradiirectional beams [10, 11]. They also studied TWM with partial coherent waves in high-speed media [12]. According to these studies, the coherence of the beams can be improved due to the wave mixing.

There are four gratings recorded in the photorefractive material in the four-wave mixing (FWM) scheme, viz. transmission grating, reflection grating and two  $2k$  gratings. Usually, only one-grating is taken into account when it is dominant. This is the so-called one-grating approximation. FWM is responsible for many modes of phase conjugation (PC), including stimulated photorefractive scattering (SPS), self-pumped phase conjugation (SPPC), and mutually pumped phase conjugation (MPPC) [2–4]. The non-requirement for coherence of the two pump beams in FWM means it has great potential in many applications, e.g. optical interconnecting, laser phase locking, and laser beam cleaning [13–17]. However, in some configurations, it was found experimentally that the performance of FWM is very sensitive to the degree of mutual coherence of the two pump beams [18]. The effect of beam coherence in MPPC was studied theoretically and experimentally without taking into account the coupling and propagation of mutual coherence and it was found that the performance of the phase conjugator can be decreased or enhanced depending on the contribution of the reflection gratings [19, 20]. FWM with partially spatiotemporal (1-dimensional) coherent waves via transmission grating approximation (TGA) was recently studied by Krolkowski [21] and our group [22] by taking into account the coupling and propagation of the mutual coherence. It was found that the mutual coherence of the signal and pump beam could be en-

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hanced or decreased depending on the coupling constant and the signal–pump beam ratio [21,22]. For the case of phase conjugation, the PC beam and the pump beam remain in full coherence during the propagation [22].

In fact, when the two pump beams are partially coherent, reflection (and/or  $2k$ ) gratings must be taken into account. In the case of transmission grating interaction, the optical path difference between the interfering waves of the four waves remains approximately the same as the four waves propagate through the photorefractive medium, especially when the incident angles of the four waves are small. Only one free variable is adequate to describe the second-order statistical properties of the four beams, which include the intensities and the normalized mutual coherence. But this is not the case when the reflection grating is present. In the case of reflection grating interaction, the optical path differences between the four interfering waves change significantly as the four waves propagate through the PR medium. We need at least two variables to describe the second-order statistical properties of the four beams [23]. Another important issue in reflection grating approximation (RGA) is that the boundary conditions on the second-order statistical properties, i.e. mutual coherence, are not easily obtained. In this paper, we propose a theoretical model to analyze the effect of beam coherence on nonlinear optical FWM and the formation of index gratings in PR media by taking into account the propagation and coupling of the mutual coherence via RGA in cases where there is no pump depletion. We limit our consideration to reflection grating approximation only. Since here we investigate FWM with one-dimensional spatiotemporal coherent beams, the contribution of the  $2k$  gratings can be regarded as part of the reflection grating contribution.

## 1 Theory

Referring to Fig. 1, we consider the process of optical four-wave mixing with RGA in a nonlinear medium. Assuming that all the partially coherent waves have the same central frequency  $\omega_0$ , all the waves are polarized perpendicular to the plane and the waves form two pairs of counter-propagating beams with  $k_3 = -k_2$ ,  $k_4 = -k_1$ ; the coupled wave equations for the slowly varying amplitudes  $A_j(z, t)$  in a purely diffusive photorefractive medium can be written as

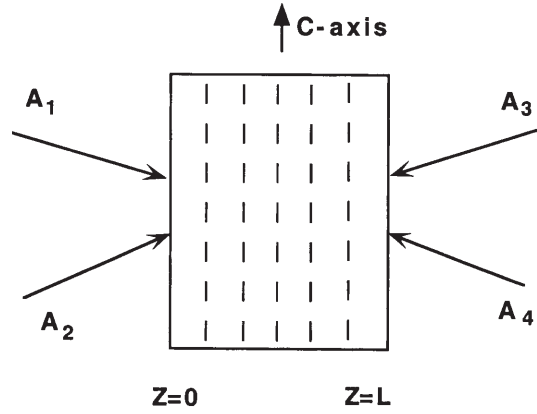
$$\frac{\partial A_1(z, t)}{\partial z} + \frac{1}{v} \frac{\partial A_1(z, t)}{\partial t} = -\frac{\gamma}{2} \frac{Q(z, t) A_3(z, t)}{I_0} \quad (1)$$

$$\frac{\partial A_2(z, t)}{\partial z} + \frac{1}{v} \frac{\partial A_2(z, t)}{\partial t} = -\frac{\gamma}{2} \frac{Q(z, t) A_4(z, t)}{I_0} \quad (2)$$

$$\frac{\partial A_3(z, t)}{\partial z} - \frac{1}{v} \frac{\partial A_3(z, t)}{\partial t} = -\frac{\gamma}{2} \frac{Q^*(z, t) A_1(z, t)}{I_0} \quad (3)$$

$$\frac{\partial A_4(z, t)}{\partial z} - \frac{1}{v} \frac{\partial A_4(z, t)}{\partial t} = -\frac{\gamma}{2} \frac{Q^*(z, t) A_2(z, t)}{I_0} \quad (4)$$

where  $\gamma$  is the intensity coupling constant,  $v$  is the group velocity, and  $I_0(z) = I_1(z) + I_2(z) + I_3(z) + I_4(z)$  is the total



**Fig. 1.** Four-wave mixing in a photorefractive medium via reflection grating approximation. The gratings are formed by beam pair ( $A_1, A_3$ ) and/or ( $A_2, A_4$ ). We designate  $A_1$  as the signal beam,  $A_2$  and  $A_3$  are the pump beams, and  $A_4$  is the phase conjugate beam

intensity at position  $z$ .  $Q(z, t)$  is a measure of the index grating. In purely diffusive photorefractive media (e.g. BaTiO<sub>3</sub>, SBN or KNSBN without an applied field), the dynamic index grating is described by the following relaxation equation

$$\tau_p \frac{\partial Q(z, t)}{\partial t} + Q(z, t) = A_1(z, t) A_3^*(z, t) + A_2(z, t) A_4^*(z, t)$$

where  $\tau_p$  is the total relaxation time constant of the reflection grating. If we assume that the temporal behavior of each wave's complex amplitude is a stationary random process with a coherence time  $\delta\omega^{-1}$  which is substantially less than the relaxation time of the material (i.e.  $\delta\omega\tau_p \gg 1$ ) [24], then we can replace the dynamic grating amplitude  $Q(z, t)$  with its ensemble average  $\langle Q(z, t) \rangle = \langle A_1(z, t) A_3^*(z, t) + A_2(z, t) A_4^*(z, t) \rangle$  [11].

For convenience in our later discussion, we now briefly give some notation and definitions for the statistical properties of the four optical waves.  $\Gamma_{mn}(z, \tau) = \langle A_m(z, t_1) A_n^*(z, t_2) \rangle$  represents the self-coherence functions ( $m = n$ ) and mutual coherence functions ( $m \neq n$ ) of the four waves.  $\tau$  is a time delay,  $\tau = t_1 - t_2$ . With these definitions, one can easily obtain:  $Q(z, t) = \Gamma_{13}(z, 0) + \Gamma_{24}(z, 0)$ . Being independent of time,  $Q(z, t)$  in the above equations can be written as  $Q(z)$ . Note that  $Q(z)$  is the sum of the two mutual coherence functions of the four beams at position  $z$ . Using (1)–(4) and the above definitions, we obtain a set of differential equations describing the coupling and propagation of the self-coherence and the mutual coherence functions during the four-wave mixing process,

$$\frac{\partial \Gamma_{11}(z, \tau)}{\partial z} = -\frac{\gamma}{2I_0} [Q^*(z) \Gamma_{13}(z, \tau) + Q(z) \Gamma_{13}^*(z, -\tau)] \quad (5)$$

$$\frac{\partial \Gamma_{22}(z, \tau)}{\partial z} = -\frac{\gamma}{2I_0} [Q^*(z) \Gamma_{24}(z, \tau) + Q(z) \Gamma_{24}^*(z, \tau)] \quad (6)$$

$$\frac{\partial \Gamma_{33}(z, \tau)}{\partial z} = -\frac{\gamma}{2I_0} [Q^*(z) \Gamma_{13}(z, \tau) + Q(z) \Gamma_{13}^*(z, -\tau)] \quad (7)$$

$$\frac{\partial \Gamma_{44}(z, \tau)}{\partial z} = -\frac{\gamma}{2I_0} [Q^*(z) \Gamma_{24}(z, \tau) + Q(z) \Gamma_{24}^*(z, \tau)] \quad (8)$$

$$\frac{\partial \Gamma_{13}(z, \tau)}{\partial z} = -\frac{2}{\nu} \frac{\partial \Gamma_{13}(z, \tau)}{\partial \tau} - \frac{\gamma}{2I_0} Q(z) [\Gamma_{11}(z, \tau) + \Gamma_{33}(z, \tau)] \quad (9)$$

$$\frac{\partial \Gamma_{24}(z, \tau)}{\partial z} = -\frac{2}{\nu} \frac{\partial \Gamma_{24}(z, \tau)}{\partial \tau} - \frac{\gamma}{2I_0} Q(z) [\Gamma_{22}(z, \tau) + \Gamma_{44}(z, \tau)] \quad (10)$$

Here we only keep the six equations that have contributions to the FWM with reflection gratings approximation. From the above set of differential (5)–(10), we find that  $\Gamma_{11}(z, \tau) - \Gamma_{33}(z, \tau) = \text{const.}$  and  $\Gamma_{22}(z, \tau) - \Gamma_{44}(z, \tau) = \text{const.}$  That means that the set of coupled equations is consistent with the conservation of energy. Note that solving (5)–(10) is a two-point boundary-value problem. If the complete boundary conditions are available, the self-coherence and mutual coherence in the set of (5)–(10) can be solved. Unfortunately, we can only obtain the self-coherence of the four waves and the mutual coherent functions before the four waves enter the medium. A complete knowledge of the mutual coherence of the four waves at the two boundaries is often not available. In non-depleted pump approximation with  $A_4(L, t) = 0$ , if we assume the statistical properties of the pump beams are not affected by the wave mixing, then complete information about the boundary conditions can be obtained. Assuming that all four beams are derived from the same source (i.e. the same laser) and the spectral distribution of the source wave is Gaussian, the normalized modulus of the self-coherence function can be written,

$$\Gamma_s(\tau) = \exp \left[ -\left( \frac{\pi \Delta \nu \tau}{2\sqrt{\ln 2}} \right)^2 \right] \quad (11)$$

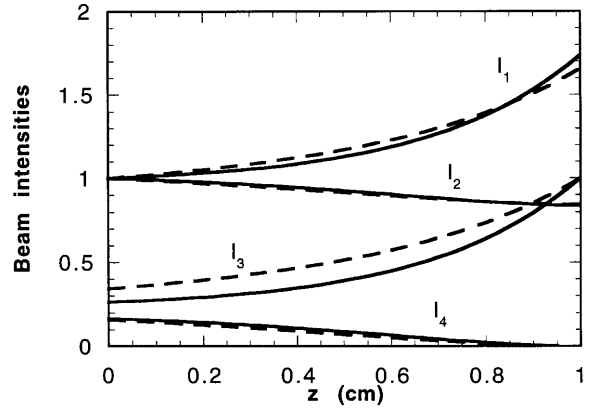
where  $\Delta \nu$  is the bandwidth, and  $\tau$  is the time delay. Assuming the two pump beams have the same intensities and the intensity ratio of the pump beam to the signal beam is  $\beta$ , the boundary conditions at  $z = 0$  and at  $z = L$  for the four waves are

$$\begin{aligned} \Gamma_{11}(0, \tau) &= \Gamma_s(\tau), \\ \Gamma_{22}(0, \tau) &= \Gamma_{33}(L, \tau) = \beta \Gamma_s(\tau), \\ \Gamma_{13}(0, \tau) &= \sqrt{\beta} \Gamma_s(\tau + \Delta t), \\ \Gamma_{44}(L, \tau) &= 0, \\ \Gamma_{24}(L, \tau) &= 0, \end{aligned}$$

where  $\Delta t$  is the time delay between the signal wave  $A_1$  and the pump wave  $A_3$  at  $z = 0$ . At the input  $z = 0$ ,  $\Gamma_{13}(0, \tau)$  determines the initial mutual coherence function of the signal beam and pump beam. In the above, we assume  $I_4(L, t) = 0$ .

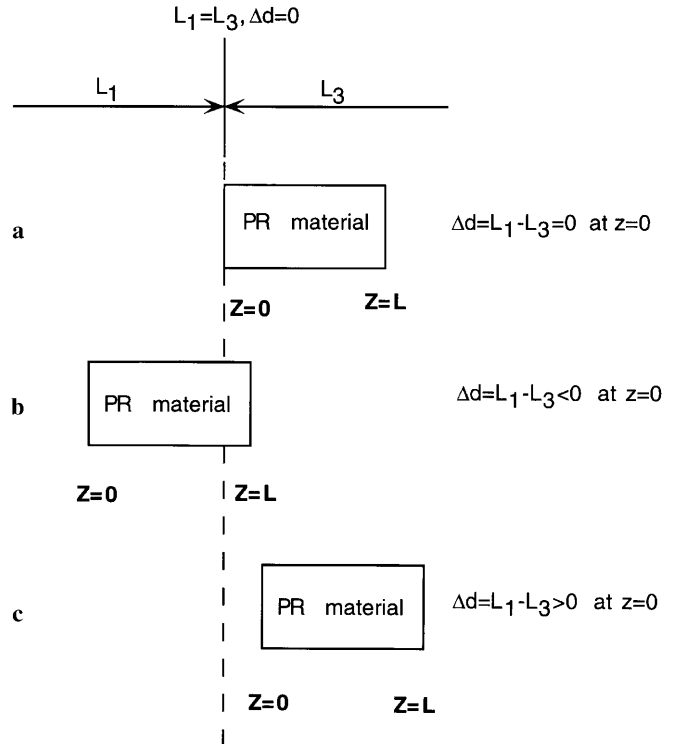
## 2 Numerical results and discussions

After we obtain the complete boundary conditions, we can use the relaxation method to solve the set of differential equations (5)–(10) numerically [25]. We first assume an arbitrary

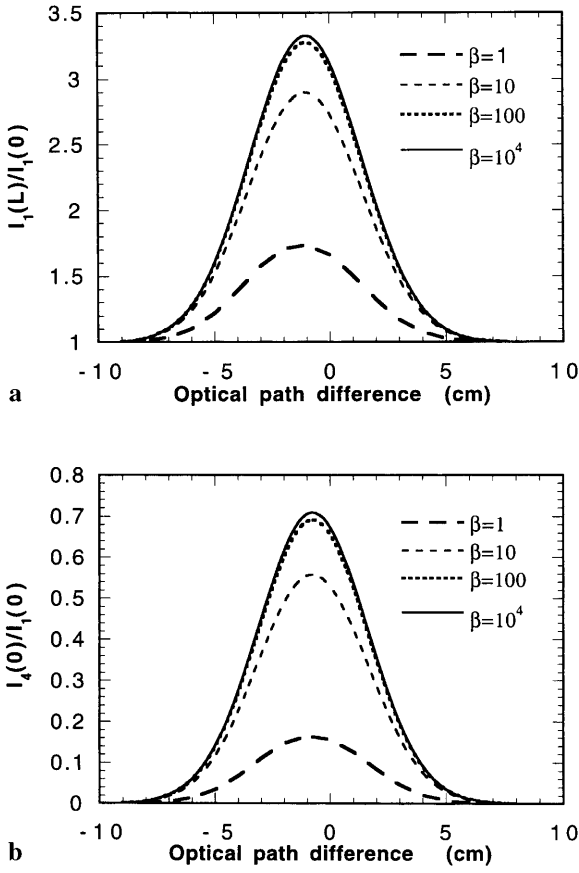


**Fig. 2.** Beam intensities in the photorefractive medium. *Solid lines*: fully coherent waves; *dashed lines*: partially coherent waves.  $I_1 = I_2 = I_3 = 1$ ,  $I_4 = 0$ ,  $n = 2.3$ ,  $L = 1$  cm,  $\gamma = -5$  cm<sup>-1</sup>,  $\Delta \nu = 1.8 \times 10^9$  Hz

reflection index grating and an arbitrary mutual coherence. Then we calculate the distribution of the gratings, the intensities of the four beams and the mutual coherence of the beams using the set of coupled equations (5)–(10). We then compare the calculated results with the initial assumptions and the boundary conditions. If the error is unacceptable, we can continue the calculations until we get satisfactory results. In our calculations, we set an error of  $10^{-5}$ . We find the results are always convergent in the non-depleted pump cases. The calculated results are shown in Figs. 2–7. Figure 2 shows the variation of the beam intensity in the PR medium. In the cal-



**Fig. 3.** Illustration of the optical path difference between the signal beam and the pump beam.  $L_1$  and  $L_3$  are the optical paths of the signal and pump beams.  $\Delta d = 0$  at  $z = 0$  means the position of  $L_1 = L_3$  is at  $z = 0$ ;  $\Delta d < 0$  ( $\Delta d > 0$ ) at  $z = 0$  means the position of  $L_1 = L_3$  is at  $z > 0$  ( $z < 0$ )



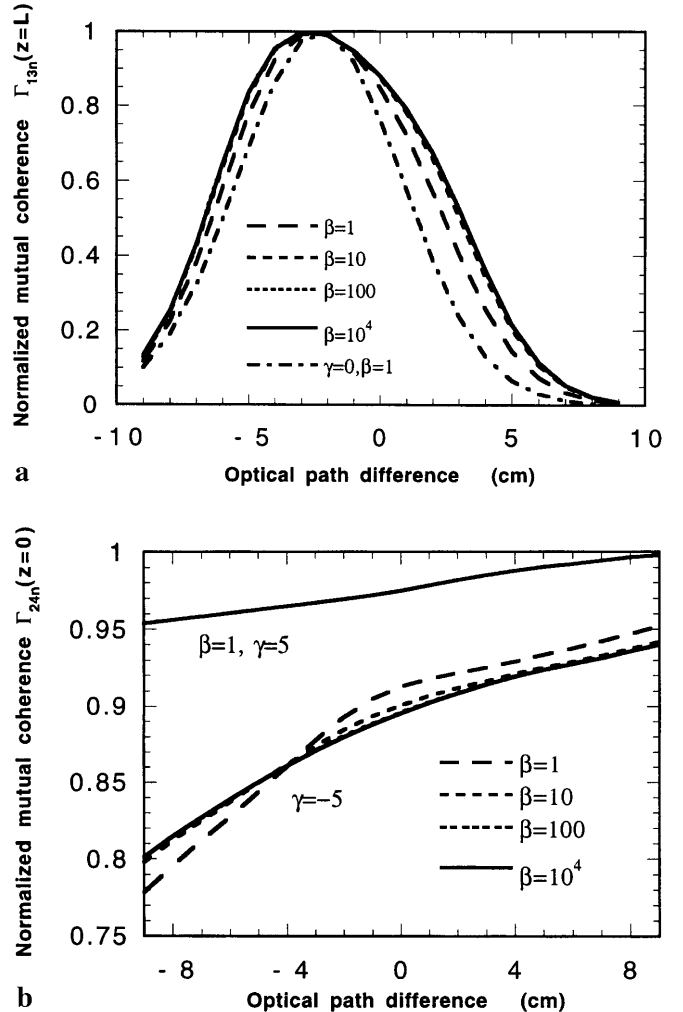
**Fig. 4.** Dependence of the signal gain and the phase conjugate reflectivity on the optical path difference. **a** Signal gain. **b** Phase conjugate reflectivity;  $\beta$  is the pump–signal ratio. The other parameters are the same as in Fig. 2

ulation, we use  $L = 1$  cm,  $n = 2.3$ ,  $\Delta\nu = 1.8 \times 10^9$  Hz and  $\gamma = -5$  cm $^{-1}$ . The intensities of the four beams are  $I_1 = I_2 = I_3 = 1$ ,  $I_4 = 0$ . The optical path difference between the pump beam ( $A_3$ ) and the signal beam ( $A_1$ ) at  $z = 0$  is chosen to be zero (see Fig. 3a). The solid lines are for monochromatic waves, dashed lines are for partially coherent waves. We note that with a non-depleted pump approximation, partial coherence can lead to a drop of signal gain and phase conjugate reflectivity. For convenience, we define the optical path difference of the signal beam and the pump beam as  $\Delta d = L_1 - L_3$ , where  $L_3$  is the optical path of the pump beam  $I_3$ , and  $L_1$  is the optical path of the signal beam  $I_1$  shown in Fig. 3. Both  $L_1$  and  $L_3$  are measured from the output end of the laser. Figure 3a shows  $\Delta d = 0$  at  $z = 0$ , meaning  $L_1 = L_3$  at  $z = 0$ ; b shows  $\Delta d < 0$  at  $z = 0$ , meaning  $L_1 = L_3$  at  $z > 0$ ; c shows  $\Delta d > 0$  at  $z = 0$ , meaning  $L_1 = L_3$  at  $z < 0$ . In the following, when we mention the optical path difference between the signal beam and the pump beam, we mean the optical path difference at  $z = 0$ . For convenience in our later discussion, when we mention the mutual coherence of the signal beam and the pump beam, we mean the mutual coherence of beams  $A_1$  and  $A_3$ ; when we mention the mutual coherence of the PC beam and the pump beam, we mean  $A_4$  and  $A_2$ , because in RGA only these two make contributions to the wave mixing. Figure 4a,b shows the signal gain and phase conjugate reflectivity as functions of the optical path difference between the signal beam ( $A_1$ ) and the pump beam ( $A_3$ )

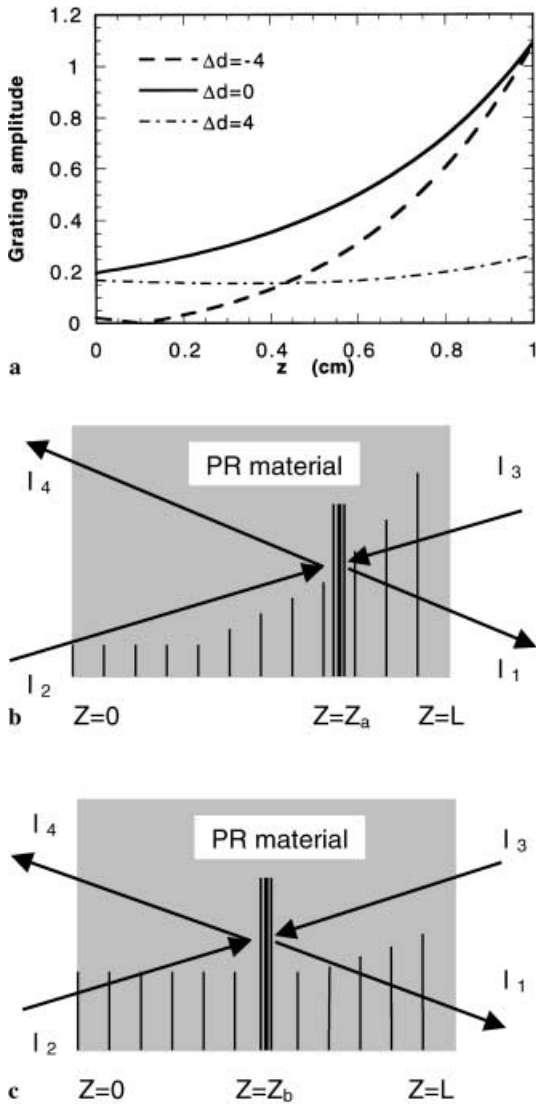
at the signal wave entrance boundary ( $z = 0$ ) at various beam ratios  $\beta$ . In this plot, the parameters are  $L = 1$  cm,  $n = 2.3$ ,  $\Delta\nu = 1.8 \times 10^9$  Hz,  $\gamma = -5$  cm $^{-1}$ . Note that increasing the beam ratio ( $\beta$ ) can lead to an increase of the signal gain and the phase conjugate reflectivity. We also note that the maximum signal gain and phase conjugate reflectivity occur when the optical path difference is negative (i.e. the zero optical path difference occurs in the PR medium, case Fig. 3b).

The normalized mutual coherence of the signal beam and pump beam at  $z = L$  is shown in Fig. 5a as a function of the optical path difference (as defined in Fig. 3). By comparing with the case of  $\gamma = 0$ , we find that wave coupling can enhance the normalized mutual coherence of the signal beam and the pump beam. We also note that the normalized mutual coherence increases with the beam intensity ratio until it reaches saturation. Note that the curves for  $\beta = 100$  and  $\beta = 10^4$  overlap. This is similar to the results obtained previously in TWM with partial coherent beams [10, 11].

The normalized mutual coherence of the PC beam and the pump beam at  $z = 0$  as a function of the optical path difference under various conditions is shown in Fig. 5b. We



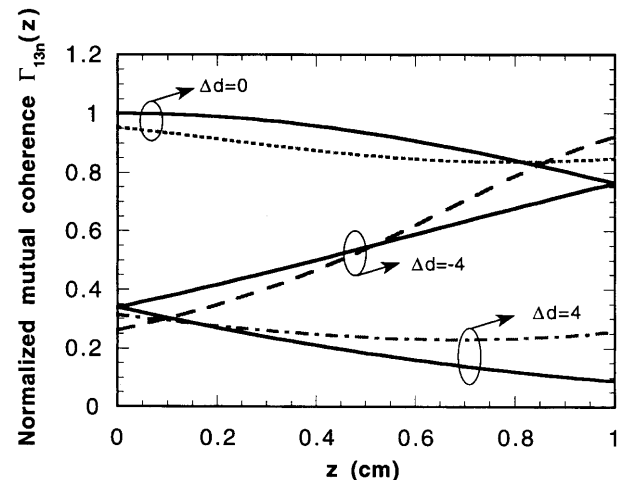
**Fig. 5.** **a** Dependence of the normalized mutual coherence of the signal beam and the pump beam on the optical path difference with respect to the signal entrance plane ( $z = 0$ ). **b** Dependence of the normalized mutual coherence of the phase conjugate beam and the pump beam on the optical path difference with respect to  $z = 0$



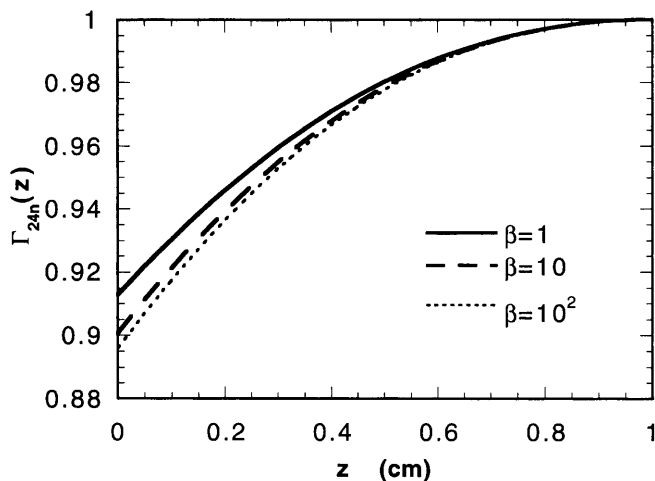
**Fig. 6.** **a** Index gratings recorded in the photorefractive medium under various conditions. Note that when  $\Delta d < 0$  (i.e.  $\Delta d < 0 = -4$ ) the amplitude of the grating in the rear part is much higher than that in the front part. When  $\Delta d > 0$  (i.e.  $\Delta d = 4$ ) the amplitude of the grating in the rear part is comparable to that in the front part. **b**  $\Delta d = -4$ , the equivalent grating is located at  $z = z_a > L/2$ ; note that the optical path difference between  $I_2$  and  $I_4$  at  $z = 0$  is larger than  $nL$ . **c**  $\Delta d = 4$ , the amplitude of the front grating is comparable to that in the rear part, the equivalent grating is located at  $z = z_b \sim L/2$ ; Note that the optical path difference between  $I_2$  and  $I_4$  at  $z = 0$  is about (or less than)  $nL$ .

find that when  $\Delta d > -4$  cm, increasing the beam ratio [ $\beta = I_3(L)/I_1(0)$ ] will lead to a decrease in the normalized mutual coherence. We note that the normalized mutual coherence  $\Gamma_{24}(0)$  is an increasing function of the optical path difference of the signal beam and the pump beam. In other words, a higher normalized mutual coherence can be obtained when  $\Delta d > 0$ , which corresponds to the case when a zero optical path difference occurs in front of the signal entrance plane ( $z = 0$ ) (see Fig. 3c). This can be explained from the distribution of the grating strength in the medium under different conditions (see Fig. 6a). If the optical path difference between the signal beam and the pump beam is negative (e.g.  $\Delta d = -4$  cm, the case in Fig. 3b), the grating amplitude is stronger at the rear ( $z = L$ ) of the medium. The intensity of

the phase conjugate beam is the sum of the diffracted beams from various parts of the grating in the medium. Thus, in this case, the PC beams mainly gain from the rear gratings, viz. the amount of the diffracted intensity from the rear gratings is predominant. We can consider that the PC beam gains its energy from the pump beam ( $A_2$ ) via a grating recorded at position  $L/2 < z = z_a$ , as illustrated in Fig. 6b. The equivalent optical path difference between the PC beam ( $A_4$ ) and the pump beam ( $A_2$ ) at  $z = 0$  is larger than  $nL$ , where  $L$  is the thickness of the PR material, and  $n$  is the refractive index of the material. Now, on the other hand, if the optical path difference between the signal beam and the pump beam is positive (e.g.  $\Delta d = 4$  cm, the case Fig. 3c), the amplitude of the grating recorded in the front is comparable to (or larger than) that recorded in the rear. Thus, the amount of the intensity of the PC beam diffracted from the front portion of the grating is comparable to (or larger than) that diffracted from the rear portion of the grating. Thus, we can consider that the PC beam gains its energy from the pump beam via a grating recorded at  $z = z_b$ , as illustrated in Fig. 6c. Note that in this case the equivalent optical path difference at  $z = 0$  is approximately  $2nz_b$  which is reduced compared with the former case. Therefore, a positive optical path difference between the signal beam and the pump beam leads to an increase in the normalized mutual coherence of the PC beam and the pump beam. When the grating amplitude is stronger near  $z = 0$ , the normalized mutual coherence  $\Gamma_{24n}(z = 0)$  can be substantially enhanced [see Fig. 5b, where  $\gamma = 5$ , the front part of the grating is stronger than the rear part, but note that the energy is coupled from the signal beam to the pump beam ( $A_3$ )]. However from Fig. 4b we note that a larger optical path difference between the signal beam and the pump beam also leads to a decrease in the phase conjugate reflectivity. It is very clear that the normalized mutual coherence of the PC beam and the pump beam is no longer unity in Fig. 5b. It is very different to the case of TGA where the PC beam and the pump beam remain fully coherent during the propagation [22]. This difference is also due to the contradirectional propagation of the pump beam  $A_2$  and the phase conjugate beam  $A_4$ .



**Fig. 7.** Distribution and propagation of the normalized mutual coherence of the signal beam and the pump beam under various conditions. The pump-signal ratio is 1.  $\Delta d$  is the optical path difference between the signal and the pump beam at  $z = 0$ . Note that the normalized mutual coherence at  $z = L$  is enhanced



**Fig. 8.** Distribution and propagation of the normalized mutual coherence of the PC beam and the pump beam under various conditions. The optical path difference ( $\Delta d$ ) at  $z = 0$  is zero

Figure 7 shows the normalized mutual coherence function  $\Gamma_{13n}(z)$  of the signal beam and the pump beam as a function of  $z$  for various path differences between the signal and the pump beams with and without coupling. The parameters are the same as in Fig. 2;  $I_1 = I_2 = I_3 = 1$ ,  $I_4 = 0.0$ ,  $\gamma = -5 \text{ cm}^{-1}$ , the solid (broken) lines indicate mutual coherence without (with) coupling. Note that when the optical path difference is zero at  $z = 0$ , wave mixing leads to a decrease in the mutual coherence of the signal beam and the pump beam at the front boundary ( $z = 0$ ) and an increase in the mutual coherence at the rear boundary ( $z = L$ ). When the optical path difference at  $z = 0$  is negative ( $\Delta d = -4 \text{ cm}$ ), the mutual coherence at the rear boundary is higher than that at the front boundary. When the optical path difference at  $z = 0$  is positive ( $\Delta d = 4 \text{ cm}$ ), the mutual coherence  $\Gamma_{13}$  can be kept nearly constant during the propagation. From this plot we can clearly see that wave mixing can enhance the normalized mutual coherence of the signal beam and the pump beam at the rear boundary.

The normalized mutual coherence of the phase conjugate beam and the pump beam  $\Gamma_{24n}$  as a function of  $z$  is shown in Fig. 8. In this plot, we assume the optical path difference between the signal beam and the pump beam at  $z = 0$  is zero. Other parameters are the same as in Fig. 2. We note that increasing the pump–signal beam ratio can decrease the normalized mutual coherence of the phase conjugate beam and the pump beam at the front boundary ( $z = 0$ ). However from Fig. 4b, we note that increasing pump–signal beam ratio can increase the phase conjugate reflectivity.

### 3 Conclusion

In summary, we studied the effect of partial coherence on FWM via a reflection grating approximation (RGA). We find that partial coherence always leads to a drop in the signal gain

and phase conjugate reflectivity using non-depleted pump approximation. Wave mixing can enhance the coherence of the signal beam and the pump beam. Higher coherence of the PC beam ( $A_4$ ) and the pump beam ( $A_2$ ) can be obtained when the optical path difference between the signal beam ( $A_1$ ) and the pump beam ( $A_3$ ) at the signal entrance boundary is positive. This always leads to a drop in the phase conjugate reflectivity. The normalized mutual coherence of the PC beam and the pump beam is no longer unity, which is different to the TGA case. Increasing the pump–signal ratio can enhance the PC reflectivity, but decrease the normalized mutual coherence of the PC beam and the pump beam. Four-wave mixing with partially coherent waves is a very complicated phenomenon. Undoubtedly, FWM with partially coherent waves through TGA and RGA can shed some light on how mutual coherence evolves and propagates in the four-wave mixing process and how it affects the wave mixing. The effect of partial coherence on four-wave mixing taking into account the propagation and coupling of the mutual coherence when all the gratings are present is the subject of a future publication.

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