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Controlled generation of single photons from a strongly coupled atom-cavity system

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Abstract. We propose a new method for the generation of single photons. Our scheme will lead to the emission of one photon into a single mode of the radiation field in response to a trigger event. This photon is emitted from an atom strongly coupled to a high-finesse optical cavity, and the trigger is a classical light pulse. The device combines cavity-QED with an adiabatic transfer technique. We simulate this process numerically and show that it is possible to control the temporal behaviour of the photon emission probability by the shape and the detuning of the trigger pulse. An extension of the scheme with a reloading mechanism will allow one to emit a bit-stream of photons at a given rate.

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Atoms, ions or photons in a superposition of different states are the basic building blocks of quantum information processing, quantum communication, or quantum cryptography, where they act as elementary quantum bits (qubits) [1]. Simple quantum-logic operations [2, 3] in future quantum-logic networks [4] and the transmission of quantum information from one place to another [5], such as the teleportation of a quantum state [6–8], are based on the entanglement of different qubits or even different types of qubits. In a quantum network, for example, single atoms or ions are entangled with individual photons. To generate such an entanglement in a controlled way, a triggered source for single photons will be needed.

So far, most schemes used for the generation of single photons rely on spontaneous emission events or on parametric down-conversion. However, these processes produce photons at more or less random times. Only recently, evidence for a single-photon turnstile device has been demonstrated by Kim et al. [9]. They employed the Coulomb blockade mechanism in a quantum dot to trigger the emission of a single photon, radiated in an essentially random direction. Here, we adopt a different approach. Our studies are based on proposals by Law et al. [10, 11], where a single atom strongly coupled to an optical cavity is used as the active medium generating the photon. The cavity defines the active mode and ensures photon emission into a well-defined direction. The main aspects of the proposed mechanism can be explained analytically, but numerical simulations are necessary to analyze the flexibility and the limits of the excitation process.

Figure 1 shows the excitation scheme for a single-photon emission on the energy scale of the atomic bare states. We consider a Λ-type three-level atom with two long-lived states, $|u\rangle$ and $|g\rangle$, typically two Zeeman or hyperfine states of the atomic ground state, and an electronically excited state, $|e\rangle$. The atom is inside a single-mode optical cavity, with states $|0\rangle$ and $|1\rangle$ denoting a cavity field with zero and one photon, respectively. The cavity frequency is close to the atomic transition frequency between states $|e\rangle$ and $|g\rangle$, but far off resonance from the $|e\rangle$ to $|u\rangle$ transition. Hence, only the product states $|e, 0\rangle$ and $|g, 1\rangle$ are coupled by the cavity mode. The coupling constant g is time independent for an atom at rest. Note that the cavity does not couple states with equal photon number, i.e. $|e, 0\rangle$ with $|u, 0\rangle$, and $|e, 0\rangle$ with $|g, 0\rangle$.

Initially, the system is prepared in state $|u, 0\rangle$. To trigger a photon emission, the atom is exposed to a light pulse

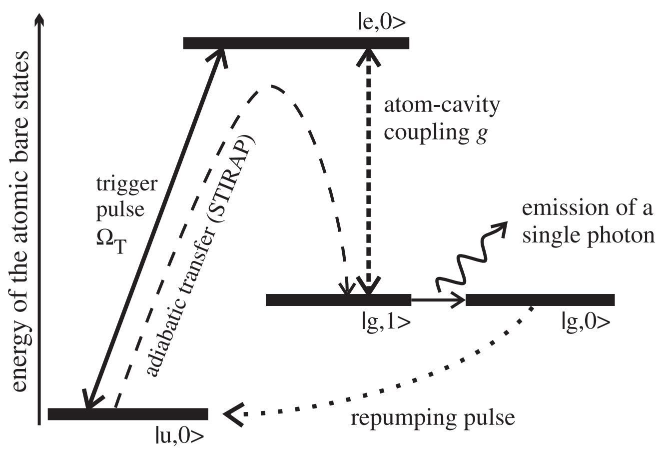

Fig. 1. Scheme of the atomic levels coupled by the trigger pulse, the cavity, and a possible repumping pulse

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crossing the cavity transverse to its axis. This pulse has Rabi frequency Ω_T and is near resonant with the transition between states $|u, 0\rangle$ and $|e, 0\rangle$, thereby coupling these states. Provided the trigger pulse rises sufficiently slowly, an adiabatic evolution of the atom-cavity system is assured. If the two-photon resonance condition is fulfilled, a STIRAP-type adiabatic passage [12, 13] takes place and a transition from $|u, 0\rangle$ to $|g, 1\rangle$ is realized. The process generates a single photon in the cavity mode, and the subsequent decay of the cavity field leads to the emission of a single-photon pulse. In the end, the state vector of the atom-cavity system is $|g, 0\rangle$, which is decoupled from any further interaction.

To analyze the excitation scheme in more detail, we assume a Raman-resonant excitation, where Δ is the common detuning of the trigger pulse and the cavity mode from the intermediate level $|e, 0\rangle$. The combined atom-cavity system is examined in the interaction picture. In the basis of the uncoupled states, $\{|u, 0\rangle, |e, 0\rangle, |g, 1\rangle\}$, the interaction Hamiltonian reads

$$H = -\frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_T & 0 \\ \Omega_T^* & 2\Delta & 2g \\ 0 & 2g^* & 0 \end{pmatrix}, \tag{1}$$

where $2g$ is the cavity-induced resonant single-photon Rabi frequency on the $|e, 0\rangle \longleftrightarrow |g, 1\rangle$ transition. If the time dependence of the Hamiltonian is neglected, the state vector of the system, $|\Psi\rangle$, can be expressed as a time-independent superposition of the three eigenstates (or dressed states, see [14]) of Hamiltonian (1):

$$\begin{aligned} |a^0\rangle &= \cos\Theta|u, 0\rangle - \sin\Theta|g, 1\rangle, \\ |a^+\rangle &= \cos\Phi\sin\Theta|u, 0\rangle - \sin\Phi|e, 0\rangle + \cos\Phi\cos\Theta|g, 1\rangle, \\ |a^-\rangle &= \sin\Phi\sin\Theta|u, 0\rangle + \cos\Phi|e, 0\rangle + \sin\Phi\cos\Theta|g, 1\rangle, \end{aligned} \tag{2}$$

where the mixing angles Θ and Φ are given by

$$\tan\Theta = \frac{\Omega_T}{2g} \quad \text{and} \quad \tan\Phi = \frac{\sqrt{4g^2 + \Omega_T^2}}{\sqrt{4g^2 + \Omega_T^2 + \Delta^2} - \Delta}, \tag{3}$$

with Ω_T and g assumed to be real. The corresponding eigenfrequencies are

$$\omega^0 = 0 \quad \text{and} \quad \omega^{\pm} = \frac{1}{2}\left(\Delta \pm \sqrt{4g^2 + \Omega_T^2 + \Delta^2}\right). \tag{4}$$

When a trigger pulse is applied, the Hamiltonian and the dressed-state basis, $\{|a^0\rangle, |a^+\rangle, |a^-\rangle\}$, are changing as a function of time. Provided that all parameters change slowly, the state vector, $|\Psi\rangle$, adiabatically follows the dressed-state basis and can still be expressed as an invariant superposition of $|a\rangle$-vectors throughout the interaction.

Before the trigger pulse is applied ($\Omega_T = 0$), the atom is only coupled to the cavity mode ($2g > 0$), i.e. $\tan\Theta = 0$. Therefore, preparing the atom initially in $|u, 0\rangle$ is equivalent to a preparation of the atom-cavity system in state $|a^0\rangle$. It is obvious from (2) and (3) that any interaction with the trigger pulse leading to $\Omega_T \gg 2g$, i.e. $\tan\Theta \gg 1$, implies the evolution of $|a^0\rangle$ and, hence, the state vector, $|\Psi\rangle$, into state $|g, 1\rangle$. Therefore, the atom is transferred to the other long-lived state, $|g\rangle$, and a photon is generated in the cavity mode at the same time. As adiabatic following must be assured, the slope of the trigger pulse must be sufficiently small to fulfill the adiabaticity constraint [15],

$$\left|\langle a^{\pm}|\tfrac{\mathrm{d}}{\mathrm{d}t}|a^0\rangle\right| \ll |\omega^0 - \omega^{\pm}| \quad \text{or} \quad |\dot{\Theta}| \ll |\omega^0 - \omega^{\pm}|. \tag{5}$$

For a Gaussian trigger pulse, $\Omega_T(t) = \Omega_0\exp(-(t/\Delta\tau)^2)$, with a duration (FWHM) of $\Delta\tau\sqrt{4\ln 2}$, and an amplitude comparable to the atom-cavity coupling, $\Omega_T(\Delta\tau) \approx 2g$, the excitation is adiabatic if

$$\Delta\tau g \gg 1/\sqrt{2}. \tag{6}$$

This constraint yields a first lower limit for the time interval required to generate one photon, because the population transfer will not work reliably with trigger pulses shorter than the inverse coupling constant, g^{-1}. A second lower limit is the cavity-decay time, $(2\kappa)^{-1}$, which is needed for the emission of the photon out of the cavity.

Up to this point, the finite lifetimes of the cavity field, κ^{-1}, and of the atom's excited state, γ^{-1}, have been omitted in the analytical treatment. To include these two decay channels, we chose $|U\rangle = |u, 0\rangle$, $|E\rangle = |e, 0\rangle$, and $|G\rangle = |g, 1\rangle$ as basis states and simulate the emission process numerically in the density matrix representation. Note that the non-interacting state $|g, 0\rangle$ is not included in this representation. The time evolution of the atom-cavity system is given by the master equation [14],

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho = -\frac{i}{\hbar}\left[H, \varrho\right] - \Gamma\varrho, \tag{7}$$

where the linear operator Γ describes the effect of all relaxation processes. The elements of $\Gamma\varrho$ are expressed in terms of the relaxation constants, $\gamma_E = \gamma$, $\gamma_G = 2\kappa$, $\gamma_U = 0$ (there is no loss from state $|u, 0\rangle$), and the Einstein coefficients, A_{ki}, for spontaneous transitions between the basis states:

$$\left[\Gamma\varrho\right]_{ij} = \frac{1}{2}(\gamma_i + \gamma_j)\varrho_{ij} - \delta_{ij}\sum_k \varrho_{kk}A_{ki}. \tag{8}$$

The first term on the right-hand side of (8) encompasses only losses and dampings of the ϱ_{ij}'s, whereas the second term takes into account the incoherent population flux into basis states. In the present situation, only state $|u, 0\rangle$ is populated by spontaneous emission from $|e, 0\rangle$. For simplicity we assume that this transition contributes 50% to the total decay rate of state $|e, 0\rangle$, i.e. the only non-zero Einstein coefficient is $A_{EU} = \frac{1}{2}\gamma$. The other 50% is due to the direct transition to the non-basis state $|g, 0\rangle$. This decay channel is already included in γ and, hence, taken into account by the first term. Note that the spontaneous decay process $|e, 0\rangle \longrightarrow |g, 1\rangle$ is neglected because of the typically small solid angle of the cavity mode ($\Omega/4\pi \approx 10^{-4}$). Finally, the transient photon emission rate, $2\kappa\varrho_{GG}$, is obtained from the numerical solution of (7).

Results of the simulation are shown in Fig. 2, where a trigger pulse with a Gaussian shape and realistic values for γ and κ were chosen. It is obvious that the generated photon is already emitted from the cavity before the Rabi frequency of the trigger, Ω_T, gets larger than $2g$. This early photon emission occurs if the cavity lifetime, $(2\kappa)^{-1}$, is short with respect

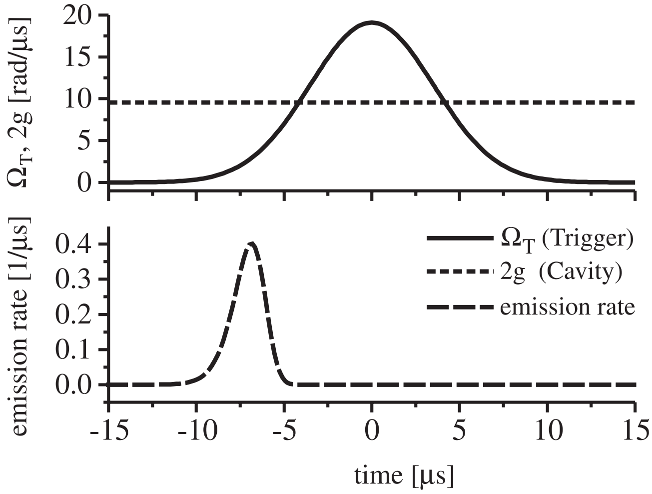

Fig. 2. Numerical simulation of a single-photon pulse. The time evolution of the trigger pulse, $\Omega_T(t)$, and the constant coupling to the cavity mode, $2g$, are shown in the upper part. Below, the corresponding photon emission rate out of the cavity is shown, assuming a cavity-decay constant of $2\kappa = 2\pi \times 1.5$ MHz and a spontaneous emission rate constant of $\gamma = 2\pi \times 6$ MHz. The integral of the emission rate yields a single-photon emission probability of 92%

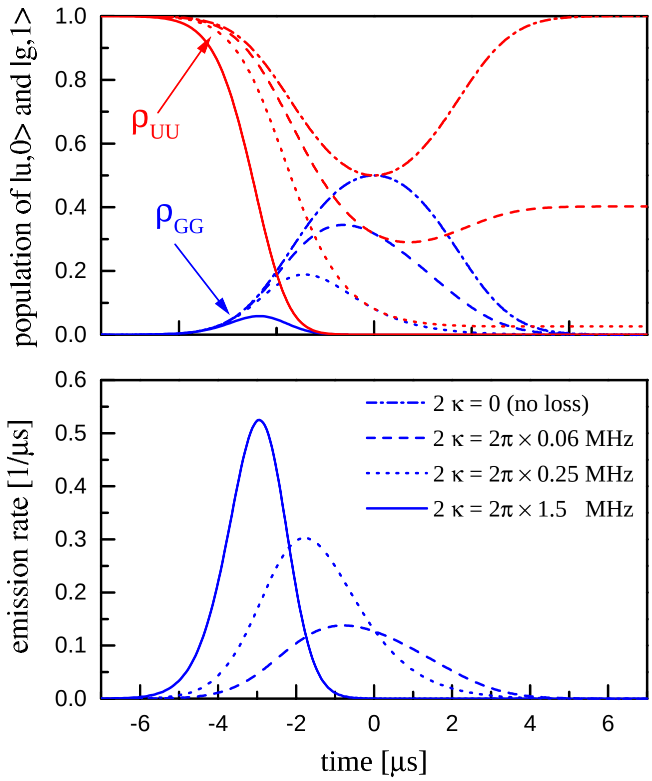

Fig. 3. Population of states $|u,0\rangle$ and $|g,1\rangle$ (*upper part*) and photon-emission probability (*lower part*) as a function of time for $\gamma = 2\pi \times 6$ MHz and different values of κ in response to a Gaussian trigger with $\Delta\tau = 5\,\mu$s and a peak Rabi-frequency $\Omega_0 = 2g = 2\pi \times 9$ MHz

to the duration of the trigger pulse. Once the photon is emitted, the remaining system is in state $|g,0\rangle$, which is decoupled from any further interaction. Therefore it is not really necessary to reach the final condition $\Omega_T \gg 2g$, and the trigger pulse could be switched off even non-adiabatically after the photon emission has taken place.

In Fig. 3, the populations of the states $|u,0\rangle$ and $|g,1\rangle$ are plotted together with the photon-emission rate as a function of time for different values of κ. In the loss-free case, neither a photon is emitted nor a photon remains inside the cavity mode, since the initial condition (coupling to the cavity and weak trigger pulse) is restored towards the end of the interaction. This coherent population return [16] to the initial state, $|u,0\rangle$, occurs with the falling edge of the trigger pulse. It takes place if the cavity-decay time, $(2\kappa)^{-1}$, is longer than the trigger pulse duration. With increasing cavity-decay constant, 2κ, the emission probability not only rises, but also narrows and shifts towards earlier times. From this, one might be tempted to use a bad cavity with large loss for optimum performance. However, since the decay of the cavity field directly affects state $|g,1\rangle$, the off-diagonal density-matrix element ϱ_{UG} is also damped and, hence, the coherence between states $|u,0\rangle$ and $|g,1\rangle$ gets lost. Since an incoherent superposition of these states does not project exclusively onto the dressed state $|a^0\rangle$ (see (2)), some population is transferred to the other dressed states, $|a^{\pm}\rangle$, which contain a contribution from $|e,0\rangle$. Hence, the excitation can be lost by spontaneous decay. This loss is expected to be small in the case of strong coupling, $g > (\kappa, \gamma/2)$. For example for the parameters of Fig. 3, the fraction of transient population in the excited state, $|e,0\rangle$, is less than 10^{-3} throughout the whole process. Therefore spontaneous emission loss in transverse modes other than the cavity mode is negligible and will not degrade the process significantly.

We now consider the more general case where the trigger pulse detuning, Δ_T, and the cavity detuning, Δ_C, from the appropriate atomic resonances are not necessarily equal. Results are displayed in Fig. 4, which shows the overall photon emission probability, P_{Emission}, and the overall loss probability, P_{Loss}, as functions of Δ_T and Δ_C. Both probabilities are calculated from the time integral of the corresponding emission and loss rate, i.e. $P_{\mathrm{Emission}} = \int 2\kappa\varrho_{GG}\mathrm{d}t$ and $P_{\mathrm{Loss}} = \int \frac{1}{2}\gamma\varrho_{EE}\mathrm{d}t$, respectively. It is evident from Fig. 4a, that the emission probability as well as the losses have two maxima for $\Omega_0 \ll g$. The reason for this is the cavity-induced coupling of states $|g,1\rangle$ and $|e,0\rangle$, producing a new doublet of non-degenerate states. This is the well-known vacuum-Rabi splitting [17], now probed by a weak light pulse coupled to the atom [18]. The splitting survives only for weak driving, but changes drastically for larger trigger amplitudes, $\Omega_0 \geq g$, as shown in Fig. 4c. In this case, the photon emission probability is close to unity if the excitation is Raman resonant ($\Delta_T = \Delta_C$) and the losses are vanishingly small. We emphasize that this holds true even in the case of a resonant excitation ($\Delta_T = \Delta_C = 0$), i.e. an influence of the vacuum-Rabi splitting is neither visible in the emission probability nor in the spontaneous emission losses, as shown in the second row of Fig. 4. For $\Delta_T \neq \Delta_C$, these losses reach their maximum if the trigger pulse is resonant with the atomic transition ($\Delta_T = 0$), and the bandwidth of the losses power-broadens with increasing amplitude of the trigger pulse (from left to right). As mentioned above, the losses are vanishingly small only if the Raman resonance condition, $\Delta_T = \Delta_C$, is fulfilled.

The influence of the trigger amplitude and the common detuning, $\Delta = \Delta_T = \Delta_C$, on the shape of the photon emission probability is shown in the last row of Fig. 4. The duration

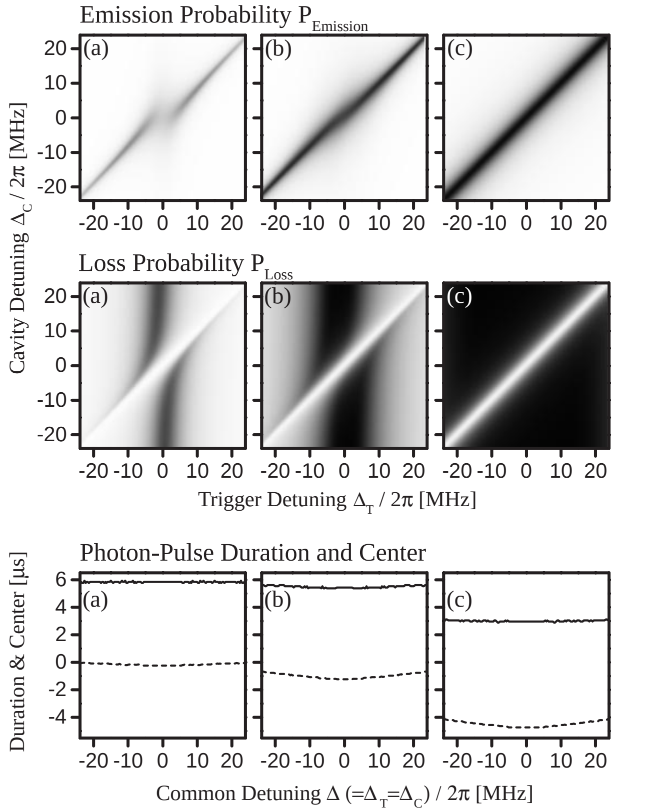

Fig. 4a–c. Photon emission probability, loss of excitation in transverse radiation modes, photon-pulse duration (FWHM) and center as functions of both cavity detuning, Δ_C, and trigger-pulse detuning, Δ_T, from the atomic resonance. The trigger pulse is $\Delta\tau = 5\,\mu\text{s}$ long. The three columns correspond to three different peak Rabi-frequencies of the trigger pulse (**a** $\Omega_0 = 2\pi \times 0.64\,\text{MHz}$, **b** $\Omega_0 = 2\pi \times 1.6\,\text{MHz}$, **c** $\Omega_0 = 2\pi \times 6.4\,\text{MHz}$). The cavity-decay constant is $\kappa = 2\pi \times 0.75\,\text{MHz}$, the spontaneous emission constant is $\gamma = 2\pi \times 6\,\text{MHz}$ and the atom-cavity coupling constant is $g = 2\pi \times 4.5\,\text{MHz}$. The probabilities are shown in shades of gray, ranging from 0.0 (*white*) to 1.0 (*black*). The duration (*solid lines, last row*) and the center position (*dashed lines, last row*) of the emission are only shown for a Raman resonant excitation, i.e. for $\Delta_T = \Delta_C$. The center of the emission probability precedes the center of the trigger pulse in **b** and **c**

(FWHM) and center of the time-dependent emission probability vary only slightly as a function of Δ, but show a strong dependence on the trigger-pulse amplitude.

Figure 5 shows the photon-emission probability, the duration and the center of the emission as functions of the trigger pulse amplitude, Ω_0, for a resonant excitation, $\Delta_T = \Delta_C = 0$. The photon-emission probability tends towards an asymptotic limit of $P_{\text{Emission}} = 95\%$ and does not reach unity with increasing trigger pulse amplitude. This limit is caused by the damping of ϱ_{UG}, which was already discussed above. Note that the emission probability is already larger than 90% if the peak Rabi-frequency of the trigger, Ω_0, exceeds $g/2$.

The single-photon pulse duration varies between two limits. Figure 5 shows that it does not exceed the FWHM, $\Delta\tau\sqrt{2\ln 2}$, of the trigger-pulse *intensity*. A longer photon pulse is not possible due to the coherent population return mentioned above. Of course, the lower limit of the pulse duration is given by the cavity decay time. It can also be seen from the figure that the center of the emission shifts towards earlier times with increasing amplitude of the trigger pulse. In the case of a Gaussian trigger-pulse, a lower bound for the center of the emission does not exist, since the emission occurs as soon as $\Omega_T(t)$ and g are of comparable magnitude.

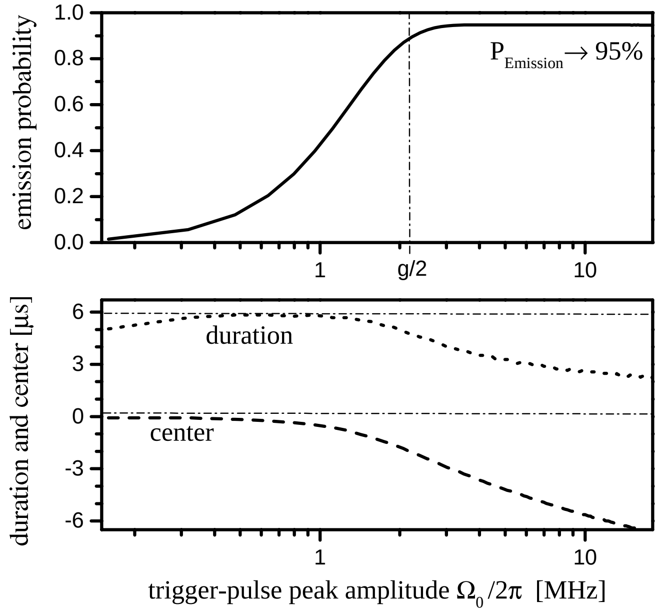

Fig. 5. The photon-emission probability (*solid line*) and its duration (FWHM, *dotted line*) and center position (*dashed line*), shown as a function of the peak Rabi-frequency Ω_0, for an atom-cavity coupling constant $g = 2\pi \times 4.5\,\text{MHz}$, resonant excitation, $\Delta_T = \Delta_C = 0$, a cavity-decay constant $\kappa = 2\pi \times 0.75\,\text{MHz}$ and a trigger-pulse width of $\Delta\tau = 5\,\mu\text{s}$. This corresponds to a duration (FWHM) of the trigger-pulse *intensity* of $\Delta\tau\sqrt{2\ln 2} = 5.9\,\mu\text{s}$, which is also the upper limit of the photon-pulse duration. The lower limit is given by the cavity decay-time, $(2\kappa)^{-1}$. These two limits are indicated by the *horizontal dash-dotted lines*. For the parameters chosen here, the emission probability exceeds 90% if $\Omega_0 > g/2$. This regime is indicated by the *dash-dotted vertical line*

We finally discuss two possible applications. First, a repetitive emission of single photons might be desirable. Hence, the emission cycle has to be completed by an independent re-pumping scheme, which brings the system back to the initial state, $|u, 0\rangle$. In Fig. 1, a possible re-pumping mechanism with a microwave π-pulse is indicated. Alternatively, one could think of a cavity-independent Raman re-pumping scheme. With the emission cycle closed, the device could produce a bit-stream of single photons as proposed in [11].

Second, an interesting aspect of the scheme is its possible use for quantum teleportation: a photon is only generated if the atom is initially prepared in state $|u\rangle$. An atom that resides already in state $|g\rangle$ cannot emit a photon. But in both cases the system reaches the same final state $|g, 0\rangle$. It follows that any initial superposition state of the form

$$|\Psi_i\rangle = \alpha|u\rangle + \beta|g\rangle$$

is transformed into

$$|\Psi_f\rangle = |g, 0\rangle \otimes (\alpha|\text{one photon}\rangle + \beta|\text{no photon}\rangle) ,$$

and an entanglement between the photon and the atom is not established. Instead, the information about the initial atomic state is encoded in the emitted light field. In a setup where the emitted photon is captured by a second remote atom, the initial atomic state might be reinstalled and the quantum state is teleported to the other atom.

We conclude by noting that the proposed scheme requires a single atom localized in the center of a high-finesse optical cavity. This can easily be achieved with a technique recently reported in [19, 20]. With an individual atom in the cavity, the photon emission probability – and therefore the shape of the photon wavepacket – can be controlled by varying the shape and the amplitude of the trigger pulse. From our numerical simulations, we expect a photon-emission probability close to unity. It should also be possible to repeat the photon-emission process at a predefined rate using an independent re-pumping scheme to recycle the atom. Moreover, the emission process depends also on the initial state vector of the atom, so that a quantum teleportation of internal atomic states could be realized.

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