Appl. Phys. B 68, 953–957 (1999) / DOI 10.1007/s003409901494

Applied Physics B
Lasers
and Optics

Reduction of the specular reflectivity from Feinberg's Cat conjugator operating in auto-oscillation mode

S. Itoh[1], **Y. Uesu**[2], **N. Oh-hori**[2], **S. Odoulov**[3]

[1] Faculty of Engineering, Tokyo Kogei University, 1583 Iiyama, Atsugi, Kanagawa, 243-02, Japan
[2] Department of Physics, Waseda University, 3-4-1 Okubo, Shinjuku-ku, Tokyo 169, Japan
[3] Institute of Physics, National Academy of Sciences, 252650, Kiev-22, Ukraine

Received: 25 November 1998/Revised version: 11 January 1999/Published online: 31 March 1999

Abstract. An incomplete correlation is discovered between the phase conjugate reflectivity in Feinberg's Cat conjugator generating a train of regular pulses with the cw incident pump wave and inhibited specular reflectivity from the sample. The frequency chirp within every spike of conjugate wave is shown to affect the efficiency of the secondary phase conjugator generating auxiliary waves in the direction of Fresnel reflection of the incident wave.

PACS: 42.65Hw; 42.65.Sf; 42.65.Yj

The specular reflection from the dielectric interface can be considerably reduced if behind this interface a phase conjugate mirror is placed (with sufficiently wide acceptance angle to collect both the transmitted wave and the phase conjugate wave backreflected from the interface [1–4]. This unusual phenomenon is related to generation of an auxiliary coherent wave propagating exactly in the direction of Fresnel reflection and possessing the phase shift of π with respect to the usual Fresnel reflection. The inhibition of the specular reflection in optical systems involving phase conjugate mirrors is similar to the coherent suppression of certain beams in different interferometers successfully used in quantum electonics for special laser cavities (Michelson, Sagnac, Fox-Smith) [5–7]. The essential difference is, however, in the fact that the systems with the phase conjugate mirrors are nonlinear and adaptive and therefore may exhibit untrivial dynamics.

We describe in this paper the results of our study of the inhibited Fresnel reflection from the Feinberg's Cat conjugator (two-interaction-region conjugator or total internal reflection conjugator) [8] operating in auto-oscillation mode [9], i.e., transforming the cw incident radiation into a phase conjugate replica with the regular pulsations. It is shown that the temporal evolution of the inhibited reflection is not correlated to the change of the phase conjugate wave intensity, that one could expect from the simple model. Moreover, the depletion of the pump wave transmitted through the sample is not correlated with the temporal envelope of the phase conjugate pulse [10]. The reason for this discrepancy is shown to be indirectly related to the strong frequency chirp within every pulse of the phase conjugate wave.

Dedicated to Prof. Dr. Eckard Krätzig, on the occasion of his 60th birthday.

1 Experimental observations

Figure 1 represents the schematic of the experimental arrangement. The traditional geometry for phase conjugation is used [8] but with the special positioning of the incident pump wave on the sample input face, aiming to excite the phase conjugate autowave (as described, for example, in [11]). The light beam from the single-frequency single-mode frequency-doubled Nd^{3+}:YAG laser with the Gaussian beam waist 0.8 mm and ultimate power 100 mW is focused onto the sample with the converging lens (focal length 150 mm). The

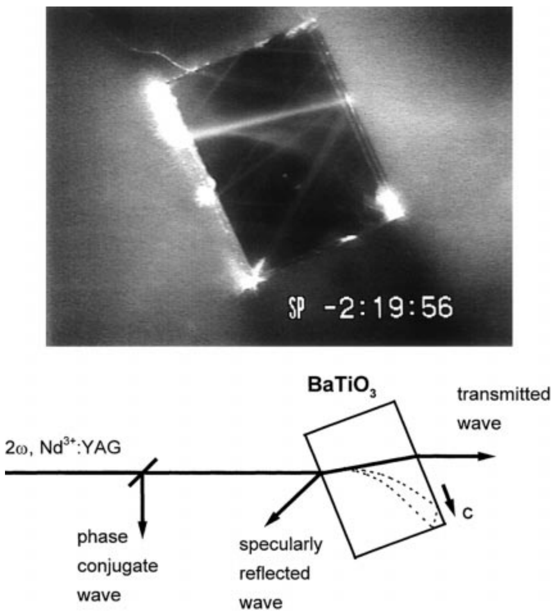

Fig. 1. Experimental setup. The picture in the inset shows the intensity distribution inside the $BaTiO_3$ sample at the maximum of the phase conjugate wave intensity

polarization of the incident wave corresponds to the extraordinary wave of the sample.

The sample of nominally undoped $BaTiO_3$ crystal measuring $3.8 \times 4.1 \times 5.4$ mm with all faces optically finished is used. A typical angular and lateral position of the beam with respect to the sample is shown in the photo inset of Fig. 1.

Two detectors are continuously monitoring the intensities of the phase conjugate wave and wave reflected from the input face of the sample (or transmitted through the sample). To check a possible frequency shift of the conjugate and reflected waves the reference wave with the frequency of the pump wave is sent to each of two detectors.

With specially selected conditions the phase conjugate wave is generated as a sequence of periodically repeating pulses shown in Fig. 2 (lower trace). The dynamics of the reflected wave are shown in the same figure as the upper trace. It is quite obvious that the change of the reflectivity occurs when the spike of the phase conjugate wave is generated. Figure 3 represents the dependence of the maximum change of the reflected wave intensity on peak pulse intensity of the phase conjugate wave. This dependence was measured using the natural statistical spread of data in a sequence of pulses of the phase conjugate wave.

The linear relationship in Fig. 3 proves that just the appearance of the phase conjugate wave causes the inhibition of the specular reflectivity. At the same time the difference in shape of pulses and the delay of the maximum intensity of the phase conjugate wave with respect to the minimum reflectivity can be easily noticed in Fig. 2.

Figure 4 shows the superposed plots of the reflected wave intensity versus intensity of the phase conjugate wave for several consecutive pulses. Note the difference in reflectivity for the increasing and decreasing phase conjugate wave intensity in the pulse. The arrow inside the graph indicates the direction of the intensity changes with increasing time. For angles of incidence smaller than the Brewster angle this direction was anticlockwise, i.e., the largest phase conjugate reflectivity is retarded with respect to the minimum of the specular reflectivity.

Similar results have been obtained for a rather wide interval of the experimental conditions (the angle of incidence from 30° to 60°, the incident beam power in the range from 2 to 50 mW). The repetition rate of the pulsation was increasing roughly linearly with the growing intensity and the pulse duration was decreasing, but the temporal envelopes for both pulses (negative for reflectivity and positive for phase conjugate intensity) were scaled in time remaining qualitatively the same.

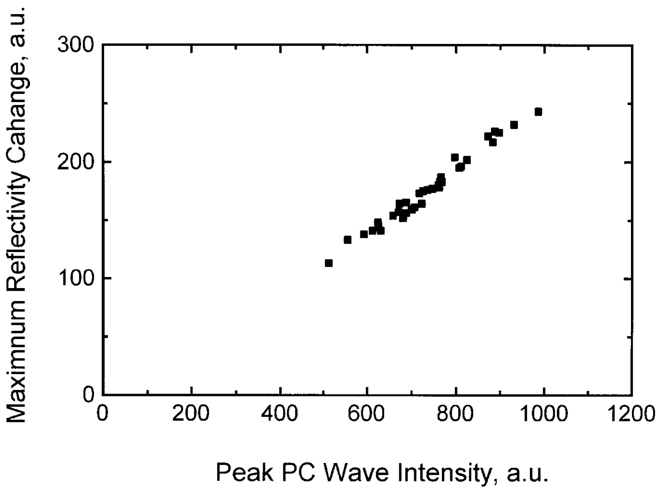

Fig. 3. Maximum deviation of the reflected wave intensity from the initial level (no photorefractive gratings, no conjugation) versus peak intensity of the phase conjugate wave

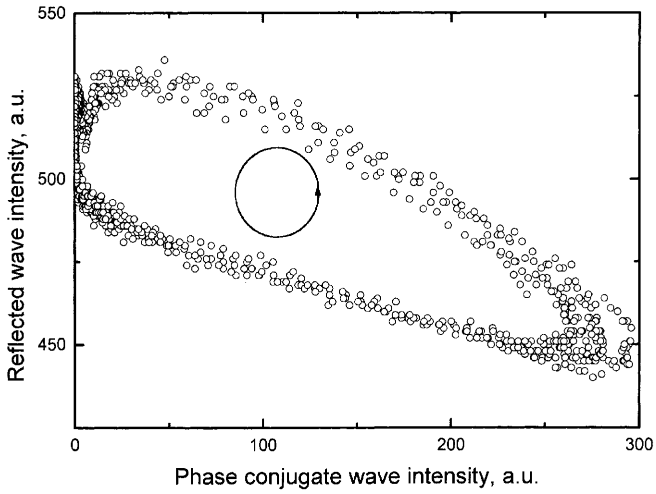

Fig. 4. Reflected wave intensity versus phase conjugate wave intensity for the sequence of pulses. The *arrow* in the *circle* inside the graph shows the direction of intensity change with increasing time

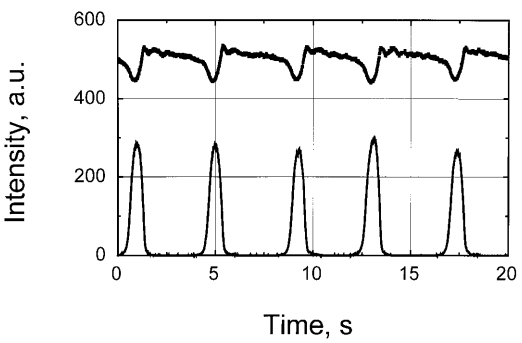

Fig. 2. Temporal variation for the phase conjugate wave intensity (*lower trace*) and reflected wave intensity (*upper trace*)

2 Discussion and verification of the model

As has been mentioned already the reduction of the specular reflectivity is caused by the destructive interference of the usual Fresnel-reflected wave and the auxiliary wave generated in the same direction inside the sample via backward-wave four-wave mixing. The origin of this auxiliary wave can be explained with the help of Fig. 5 showing the propagation direction for the incident, reflected and generated waves. Following the description of the process given in [4] we present the electric field of the j-th wave as

$$E_j = A_j \exp\left[\mathrm{i}\left(\omega t - \boldsymbol{k_j x} + \phi_j\right)\right] , \qquad (1)$$

where $A_j(x)$ is the amplitude, $\boldsymbol{k_j}$ is the wavevector, $\boldsymbol{x}$ is the propagation direction, and $\phi_j(x)$ is the phase. The wave E_1

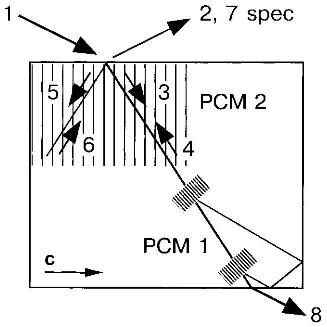

Fig. 5. Schematic representation of the propagation directions for the waves inside the sample and leaving the $BaTiO_3$ sample

incident upon the sample is partially reflected giving rise to a wave $E_2 = r'_F E_1$ and partially enters into the sample as a wave $E_3 = t_F E_1$, where r'_F and t_F are the amplitude Fresnel reflection coefficient and amplitude Fresnel transmission coefficient from the air/crystal interface, respectively.

The wave E_3 generates inside the sample the phase conjugate wave $E_4 = t_F^* r_1 A_1 \exp[\mathrm{i}(\omega - \delta)t - \mathrm{i}\boldsymbol{k_4 x} + \mathrm{i}\phi_1]$ with $\boldsymbol{k_4} = -\boldsymbol{k_3}$. Here r_1 is the amplitude reflectivity of the Cat conjugator (PCM1 in Fig. 5). In a self-pumped phase conjugator like Feinberg's Cat conjugator the longitudinal phase is preserved in conjugate wave, therefore the sign of the phase of wave 4 is the same as that of wave 1.

The frequency of the conjugate wave may differ from that of the incident wave. A comprehensive explanation of this detuning is given in [12] and attributed to simultaneous excitation of the transmission as well as reflection gratings. The phase conjugate wave E_4 is reflected from the input face back to the sample giving rise to the wave $E_5 = r_F E_4$, also with the shifted frequency $(\omega - \delta)$.

Two copropagating waves, E_3 and E_5 are recording a moving grating in the vicinity of the sample entrance face. The phase conjugate wave E_4 is, by definition, Bragg matched to this grating; the diffraction of E_4 from this grating results in the wave $E_6(x) = |t_F r_1 A_1|^2 t_F r_F^* r_2 A_1 \exp(\mathrm{i}\omega t - \mathrm{i}\boldsymbol{k_6 x} + \mathrm{i}\phi_1)$ with $\boldsymbol{k_6} = -\boldsymbol{k_5}$. As all three waves giving rise to the wave E_6 have the same phase ϕ_1 the wave E_6 also acquires the phase ϕ_1 $(\phi_6 = -\phi_5 + \phi_3 + \phi_4)$. The diffraction efficiency defines the amplitude reflectivity $r_2 = E_6/E_5^*$ of this secondary conjugator (PCM2 in Fig. 5). Note that the Doppler frequency shift, which is due to the diffraction from the moving grating, is entirely compensated for by the initial frequency shift of the readout phase conjugate wave and the diffracted wave E_6 has exactly the same frequency as the incident wave E_1 (and reflected from the input face wave E_2).

The waves E_5 and E_4 are recording also the reflection space-charge grating. The wave E_3 is Bragg-matched to this reflection grating but the phase conjugation does not occur and wave E_6 is not generated: As the grating vector $\boldsymbol{K}$ for this reflection grating is normal to the crystal C-axis the isotropic diffraction (with identical polarization of the readout and diffracted waves) from it is impossible (see, for example, [13]). At the same time the contribution of the reflection gratings recorded by the waves 5 and 6 (and also the waves 4 and 3) to the considered process can be nonzero.

In the case where the incident wave E_1 has a certain transverse structure, i.e., is not a plane wave, this transverse structure will be totally reconstructed in the wave E_6 because of two consecutive conjugations: E_4 is conjugate to E_3 and E_6 is conjugate to E_5. Thus two components propagating in the direction of the reflected wave (emerging from the crystal wave $E_7 = |t_F r_1 A_1|^2 t_F^2 r_F^* r_2 A_1 \exp(\mathrm{i}\omega t - \mathrm{i}\boldsymbol{k_7 x} + \mathrm{i}\phi_1)$ and reflected wave $E_2 = r'_F E_1$) have identical wavefronts and identical frequencies. Their interference defines the resulting reflectivity:

$$E_{\mathrm{spec}} \approx \left\{ r'_F + |t_F r_1 A_1|^2\, t_F^2 r_F^* r_2 \right\} E_1\,. \tag{2}$$

Taking into account that r'_F and r_F^* differ by π in phase the interference of two contributions will be destructive.

We put "nearly equal" sign $\approx$ in (2) because in fact one also should take into account multiple reflections from the entrance face with subsequent phase conjugation of the reflected waves as was done in [1–4]. (The wave E_6 is once more reflected into the sample in the direction of the wave E_3, the resulting wave is conjugated and reflected in direction of wave E_5, etc., etc.). Assuming relatively small phase conjugate reflectivity of the second conjugator $r_2 \ll 1$ we limit ourselves by the first nonvanishing correction term only.

Equation (2) was derived in [4] for the steady-state oscillation, i.e., for permanent cw phase conjugate wave. The reflectivities of the two phase conjugators, r_1 and r_2, are the constant values, too, in this model. To describe the case of phase conjugate autowaves it is more convenient to express the ultimate result of the reflection keeping the varying-in-time amplitude of the phase conjugate wave in a following manner:

$$E_{\mathrm{spec}} \approx E_2 + E_7 \approx \left\{ r'_F + t_F r_F^* r_2\, |E_4(t)|^2 \right\} E_1\,, \tag{3}$$

It is quite obvious that the interference term in $|E_{\mathrm{spec}}|^2$ will be also proportional to the temporal envelope of the phase conjugate wave intensity $|E_3(t)|^2$. At the same time the temporal variation of $|E_{\mathrm{spec}}|^2$ can be affected by the other factors, too.

The amplitude reflectivity of the secondary phase conjugator r_2 is a complex value. In a simple model the imaginary part of r_2 is taken to be constant in time, i.e., the phase difference between the two coherent components in the reflected waves 2 and 7 is also constant. This hypothesis should be checked experimentally because the phase of the conjugate wave may in principle be affected by different additional factors. It can change in time because of the recording of the reflection gratings by the waves E_6 and E_5. Since the transmission grating in PCM2 is recorded by the waves with different frequencies its phase shift with respect to the fringes deviates from the exact $\pi/2$ (local response appears); this in turn modifies the phase of phase conjugate wave. The question is: how large is the ultimate change of the phase difference between the waves 2 and 7? The results of the direct measurements of the temporal stability of the phase of conjugate wave will be presented further in this article.

The other reason for temporal variation of the $|E_{\mathrm{spec}}|^2$ may be related to time dependence of the real part of r_2, i.e., to the changes of the diffraction efficiency of the secondary grating. The relaxation time of the space-charge field in $BaTiO_3$ measured in this experiment at the used intensities $\tau \approx 0.2$ s is shorter than pulse duration (normally of the order of a few s). Thus we can expect that the diffraction efficiency of the grating will follow the intensity variation of the phase conjugate wave. This variation will not explain, however, the observed displacement of the minimum in reflectivity with respect to the maximum of the phase conjugate reflectivity.

Quite often the phase conjugate wave in a Cat conjugator is frequency shifted with respect to the pump wave. The moving fringes are recording a grating with smaller diffraction efficiency as compared to the frequency-degenerate case. Being a function of the frequency shift this reduction of the efficiency may lead to the additional time variation of r_2 if the frequency of the phase conjugate wave is changing during the pulse duration (frequency chirp exists). This possibility should also be checked experimentally.

To find the reason for incomplete correlation of the inhibited reflectivity and phase conjugate reflectivity we measure the temporal variation of the phase shift in these two waves using the heterodyne technique. Apart from the beam from the sample (reflected or conjugated) the reference wave from the laser is sent in the direction of the detector at a small angle with respect to the wave from the sample. The fringe pattern appear from which the narrow slit parallel to the fringe cut the intensity to be measured by the detector. If the phase difference of two waves forming the fringes is changing in time this results in variation of the intensity on the detector.

The results of the measurements of the phase variation in the phase conjugate wave and reflected wave are shown in Fig. 6a,b, respectively. The lower trace in each graph represents the temporal evolution of the pulse of phase conjugate wave.

It follows from Fig. 6 that the phase conjugate wave is shifted in frequency with respect to the pump wave and, more important, that this frequency detuning is changing with time. The processing of the data of Fig. 6a allows us to state that frequency shift is changing from a fraction of Hz at the beginning of the pulse to 5–6 Hz at the end of the pulse.

For the reflected wave (Fig. 6b), on the contrary, the frequency is not shifted at all with respect to the pump wave. Moreover, the ultimate time-variable phase shift (if it exists at all) is not larger than ($\pi/10$). This estimate is done taking into account that the variation of the intensity in the fringe pattern from the maximum to the minimum corresponds to 10 in the same units that are used in abscissa of Fig. 6b while the

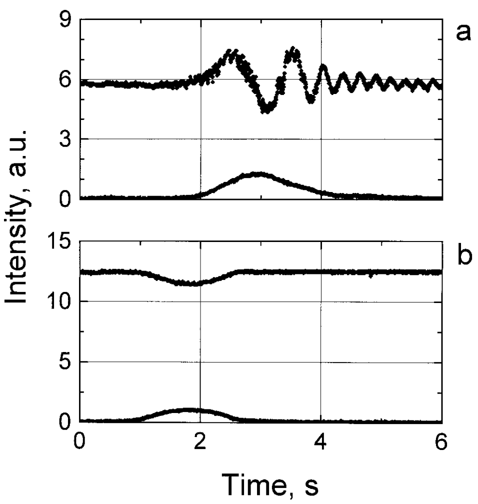

Fig. 6a,b. Beat frequency mark for the phase conjugate wave (**a**, *upper trace*) and reflected wave (**b**, *upper trace*). *Lower traces* in **a** and **b** show the corresponding temporal variation of the phase conjugate wave intensity

maximum deviation of the upper trace from constant value is no more than 1. We believe that the real phase shift is much smaller and the observed intensity variation is related to the decrease of the intensity of the reflected wave but not to the phase changes. This statement is supported by the fact that we have never seen the increase of intensity in the upper trace, when changing the position of the slit in front of the detector with respect to the fringes (adjusting it to the maximum, minimum, rising slope of decreasing slope of the fringe, etc.)

Thus from the data of Fig. 6 we conclude that (i) the phase of the auxiliary wave generated in the direction of Fresnel reflection is constant during the pulse and (ii) the frequency of the phase conjugate wave is changing during the pulse as minimum for one order of magnitude.

These results lead to the conclusion that the reason for the observed incomplete correlation of the inhibited specular reflectivity and phase conjugate intensity is related to the time variation of the diffraction efficiency of the grating responsible for secondary phase conjugation. At the beginning of the pulse the grating is recorded by the waves with relatively small frequency shift and its amplitude is rising rather quickly. However, the more intensive the phase conjugate wave becomes, the larger is the frequency shift and therefore smaller becomes the diffraction efficiency of the secondary grating.

The reduction of the space-charge amplitude E_{sc} is proportional to

$$E_{sc}(\omega) = E_{sc}(0)/\left[1+(\tau\omega)^2\right] \ , \qquad (4)$$

where τ is the characteristic relaxation time of the space-charge field. Of course, to get the detectable reduction the frequency shift should be high enough, with $\tau\omega \approx 1$. With the measured response time $\tau \approx 0.2$ s and frequency shift ω up to 20 radian per second this condition is obviously met in our case.

To check independently the validity of this explanation we study the correlation of the intensity of the transmitted light wave 8 and phase conjugate wave. At least two factors are reducing the intensity of the transmitted wave when the pulse of the phase conjugate wave is generated: first, a part of the pump wave is converted into phase conjugate wave in PCM1, and second, the pump wave is partially depleted because of the diffraction from the grating recorded in PCM2. As distinct from the inhibition of the specular reflection considered above these two processes of the transmitted wave depletion are independent and add incoherently.

According to our model the temporal envelope of the grating efficiency in PCM2 is not the same as the pulse shape of the phase conjugate wave: the diffraction efficiency of PCM2 grating is considerably decreased when the phase conjugate intensity reaches its maximum. The frequency shift ω of the phase conjugate wave in the maximum of the pulse is already too high (about 6 radian per second, see Fig. 6a) and the diffraction efficiency which is proportional to E_{sc}^2 is approximately 6 times smaller than its maximum value; it is further decreasing rapidly in time because of the increasing frequency shift ω. So one can expect that the contribution of the diffraction from the grating PCM2 to the reduction of the transmitted intensity should be much smaller for the decreasing part of the pulse of phase conjugate wave as compared to its increasing part. If the reduction of the transmitted wave intensity $|E_8|^2$ is caused by the conversion of the incident wave intensity $|E_1|^2$ into the phase conjugate wave intensity $|E_4|^2$ the linear relationship should be expected between these quantities

$$|E_8|^2 = |E_1|^2 - |E_4|^2 \ . \tag{5}$$

Figure 7 shows the dependence of the transmitted wave intensity on the intensity of the phase conjugate wave for several pulses, constructed in a similar way to the dependence of Fig. 4. The arrows in the circle show the direction of the intensity changes for increasing time. The loop shape of this dependence points to incomplete correlation, as in the case of inhibited specular reflection. At the same time this loop is quite different from that of Fig. 4: the part corresponding to the decreasing slope of the phase conjugate pulse is very close to the linear dependence. This is just what we expected from our model.

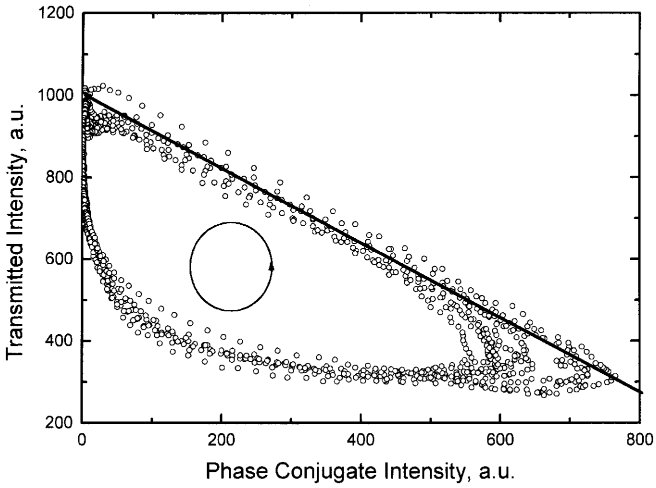

Fig. 7. Transmitted wave intensity versus phase conjugate wave intensity for the sequence of pulses. The *arrow* in the *circle* inside the graph shows the direction of intensity change with increasing time

Acknowledgements. S. Odoulov acknowledges the hospitality of the Laboratory of Nonlinear Optics, Waseda University during his stay as an invited professor in Tokyo and partial support of Civilian Research and Development Foundation via grant UP2-322 and the Ministry of Science, Technology and Industrial Politic, Ukraine via grant 4Φ/356. We are grateful to our reviewers for valuable suggestions.

References

1. A.T. Friberg, P.D. Drummond: J. Opt. Soc. Am. **73**, 1216 (1983)
2. P.D. Drummond, A.T. Friberg: J. Appl. Phys. **54**, 5618 (1983)
3. I. Lindsay, J.C. Dainty: Opt. Commun. **59**, 405 (1986)
4. D. Pepper: Phys. Rev. Lett. **62**, 2945 (1989)
5. S.G. Rozuvan, E.A. Tikhonov: Russ. J. Quantum Electron. **25**, 337 (1995)
6. P.W. Smith: IEEE J. Quantum Electron. **QE-1**, 343 (1965); (Patent application by A.G. Fox)
7. W.W. Rigrod: IEEE J. Quantum Electron. **QE-6**, 9 (1970)
8. J. Feinberg: Opt. Lett. **7**, 486 (1982)
9. V.I. Sugakov: World Sc. Ser. on Nonlinear Science, Ser. A, **33**, 1 (1998)
10. S. Itoh, S. Imamura, N. Kozumi, H. Saitoh, N. Oh-hori, A. Ueno, Y. Uesu: Paper WPE01, Proceedings of Topical Meeting on Photorefractive Materials, Effects and Devices (PR'97), Chiba, Japan, ed. by Waseda University, Tokyo, pp. 246–249
11. R. Eason, M. Gower: Opt. Commun. **59**, 77 (1986)
12. D. Gauthier, P. Narum, R. Boyd: Phys. Rev. Lett. **58**, 1640 (1987)
13. M.P. Petrov, S.I. Stepanov, A.V. Khomenko: Photorefractive Crystals in Coherent Optical Systems (Springer, Berlin, Heidelberg 1991)