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Polarization quantum states of light in nonlinear distributed feedback systems; quantum nondemolition measurements of the Stokes parameters of light and atomic angular momentum

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Abstract. A possibility of the formation of polarization-squeezed light in the case of interaction of two orthogonally polarized modes in a spatio-periodically nonlinear Kerr-like medium is considered. It is shown that the fluctuations of the Stokes parameters of light could be less than their fluctuations in a coherent state at the output of the medium. An analysis shows that the light polarization degree is not fixed in such a nonclassical state of light. We also introduce a new nonlinear parameter of light polarization associated with the fluctuation variances of three Stokes parameters. The value of the parameter is examined for different quantum states of light. A procedure for the quantum nondemolition (QND) measurement of the Stokes parameters of light is described for the first time. We show that the precision measurement of the Stokes parameter depending on phase could be used for the QND measurement of the phase difference of two orthogonally polarized modes. The general description of the measurement procedure under study allows us to propose also a scheme for the QND measurement of angular momentum of atomic systems.

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At present, the generation of squeezed states, nonclassical statistics of photons and the realization of quantum nondemolition (QND) measurements is prompting the study of many nonlinear systems, which typically result from the energy exchange and quantum interference between two (or more) conjugated modes [1]. That leads to suppression of fluctuations of one of the observable variables of the field while, for another variable, the fluctuations are enhanced. In fact, the competition and energy exchange between travelling and scattered waves become possible in the case of Bragg diffraction in a spatially periodic structure, that is, in a system of distributed feedback (DFB) [2].

The high-efficiency systems of nonclassical light generation, based on a different type of DFB system, have been described on a number of occasions to data [3]. The best method of obtaining such DFB in optics can be established by use of nonlinear optical fibers of two special types, namely fibers with their dielectric constant periodically varying along the direction of propagation of light, and also fibers with two tunnel-coupled cores [4]. On the other hand, the DFB system is obtained in atomic optics for a collection of ultracold two-level atoms via nonlinear Bragg diffraction in a periodic structure being a standing wave of a laser field [5, 6]. In this case, the formation of nonclassical states for boson-like atoms occurs, and the analogy with the light travelling in the above mentioned optical fibers takes place.

Another aspect of the quantum states under study is the polarization quantum state of light. Recently, a quantum analysis of such a state has been intensively developed [7–10]. The discussion has generally focused on calculations of the Stokes parameters, which are associated with Hermitian Stokes operators and have an explicit physical interpretation.

From the standpoint of wanting to apply the use of quantum-squeezed light in high-precision polarimetry and ellipsometry, the polarization-squeezed light has a special interest. In fact, for such a state, the level of quantum fluctuations of one of the Stokes parameters is smaller than in the coherent state. It has been shown in [10] that polarization-squeezed light can be generated in cubic nonlinear anisotropic media when there is an anisotropy of the nonlinear correction to its refractive index.

In the present paper, we consider another possible method of generating polarization-squeezed light in a spatially periodic nonlinear medium [11]. It is shown that both the linear and nonlinear energy exchange between orthogonally (along the x- and y-axis, and/or two circularly) polarized modes results in the formation of a nonclassical polarization of light in such a DFB system.

At the same time, to observe this new nonclassical state of light, it is necessary to select an appropriate quantum measurement procedure. The QND measurement method [12] seems to be a suitable procedure for that (see [13–15]). But earlier analysis has shown that the QND measurement method cannot be directly applied to our problem because certain conditions have to be satisfied for it to be valid (see [4, 13, 14]).

We describe herein the procedure of QND measurement especially for the Stokes parameters of light developed by us (see also [16]). In general, this presentation could be applied to any other observables of SU(2) algebra (for example, to the angular momentum of an atomic system); however, the phase sensitivity of the Stokes parameters of light under study allowed us to consider the measurements of the phase difference between two polarization modes by the QND procedure (cf. [17]).

The same technique is proposed by us for detection of angular momentum in atomic systems as well. Auxiliary questions concerning the polarization light formation in an anisotropic nonlinear medium are shown in an Appendix.

1 Classical and quantum description of the polarization states of light

In classical optics, one way to describe light polarization is linked with the theory behind the Stokes parameters (see, for example, [18]). In principle, the Stokes parameters could be simultaneously and exactly measured by a set of optical elements, for instance two photodetectors, polarization and a phase plate [19]. A clear geometrical interpretation of the Stokes parameters makes this method of light polarization even more attractive. From a mathematical point of view, the polarization state of a light can be given using the Poincaré sphere in the Stokes space of $S_1,\ S_2,\ S_3$. Each point on the sphere corresponds to a definite polarization state, whose variation is characterized by movement of the image point.

In classical optics furthermore, in order to describe quantitatively the polarization state of light, we introduce the degree of polarization $\mathcal{P}$, which is equal to the ratio of the intensity of the polarized part of the radiation I_{pol} to the total intensity I_{tot} thus:

$$\mathcal{P} = I_{\mathrm{pol}}/I_{\mathrm{tot}} = \frac{\sqrt{\langle S_1\rangle^2 + \langle S_2\rangle^2 + \langle S_3\rangle^2}}{\langle S_0\rangle}\,, \tag{1}$$

where S_0 is the Stokes parameter that determines the total intensity of the modes; sign $\langle \ldots \rangle$ denotes here and below the average value. For completely polarized light, $\mathcal{P} = 1$, while for partially polarized light $0 < \mathcal{P} < 1$, and a transition to unpolarized light corresponds to compression of the Poincaré sphere to zero radius (see also [9]).

From a quantum physics standpoint, the difference between polarized (elliptically in the general case) monochromatic light and unpolarized light is that the polarized case results in a "pure" state, in other words a coherent mixture of two components which are polarized in two perpendicular directions (where the amplitudes of these components are summed), whereas unpolarized light represents a "mixed" state, in other words an incoherent mixture (where the intensities of the components are summed) [18, 20].

In the language of wave functions (with photons existing only in the momentum representation [21]), for "pure" states we have linear combinations, which can be used to calculate the total probability of the photon polarization state, and for mixed states we can sum the squares of the absolute values of the wave functions, i.e. the individual probabilities. Moreover, although no type of polarization is dominant, right- and left-circularly polarized photons are the most easily defined in the terms of spin operators. It is well known [22] that to characterize in general the photon polarization state, we can define an arbitrary polarization state (in terms of wave functions) as a linear combination of the states for two orthogonally polarizated modes. In this case, any transformation of the light polarization state could be geometrically represented by the Poincaré sphere as a rotation in a 3D-configuration space of the $S_1,\ S_2,\ S_3$ parameters.

Another way to describe the polarization state of light in quantum optics is to introduce the measurable (observable) quantities associated with the operators of the Stokes parameters S_j, where $j = 0, 1, 2, 3$ (see for example [20]). The presence of fluctuations, which are unavoidable in quantum theory, results in uncertainty for these characteristics of the light polarization. Moreover, the uncertainty product for variances of the Stokes parameters fluctuations occurs (see below) in contrast to the classical description. In fact, in quantum optics, an exact and simultaneous measurement of the Stokes parameters is absolutely impossible: the situation is the same as for momentum (orbital, angular, and/or spin) in the traditional quantum mechanics of particles. So, we are dealing with, principally, the quantum fluctuations of the light vector. Thermal fluctuations, which exist in the classical description as well, are outside our consideration because they can usually be associated with technical fluctuations, in particular due to the quality of the polarizer device used to produce the polarization state of optical field.

The description of the partial polarization of light using the real Stokes parameters corresponds to the close relationship between the classical and quantum approaches for given polarization properties. However, a quantum theory results in the existence of specific nonclassical polarization states, for which a general analysis is presented in the next section.

2 Polarization-squeezed states of light

2.1 Nonclassical polarization states

Let us characterize the polarization state of the two-mode field under consideration by the Stokes operators:

$$S_0 = a_x^+ a_x + a_y^+ a_y\,, \tag{2a}$$

$$S_1 = a_x^+ a_x - a_y^+ a_y\,, \tag{2b}$$

$$S_2 = a_x^+ a_y e^{\mathrm{i}\theta} + a_y^+ a_x e^{-\mathrm{i}\theta}\,,\ \text{and} \tag{2c}$$

$$S_3 = \mathrm{i}(a_y^+ a_x e^{-\mathrm{i}\theta} - a_x^+ a_y e^{\mathrm{i}\theta})\,, \tag{2d}$$

where the parameter θ is the classical phase (see below) and a_j (a_j^+) is the photon annihilation (creation) operator, respectively, for the j-th polarized component of light. The operators $a_j(a_j^+)$ obey the well known commutation relations:

$$\left[a_j;\, a_k^+\right] = \delta_{jk},\ \text{with}\ \ j, k = x, y \tag{3}$$

The Stokes operators obey the commutation relations of the SU(2) algebra:

$$[S_2;\, S_3] = 2\mathrm{i}S_1,\ [S_1;\, S_2] = 2\mathrm{i}S_3,\ [S_3;\, S_1] = 2\mathrm{i}S_2 \tag{4a}$$

$$[S_0;\, S_j] = 0,\ \ (j = 1, 2, 3). \tag{4b}$$

The relationships given in (4a) reduce to the so-called Schrödinger–Robertson uncertainty relation [23]:

$$\langle \Delta S_j^2(z)\rangle\langle \Delta S_k^2(z)\rangle \geq |\langle S_m(z)\rangle|^2 /1 - r_{jk}^2\,, \tag{5}$$

with $j,k,m = 1,2,3$ but $j \neq k \neq m$, where $\langle \Delta S_j^2(z) \rangle \equiv \langle \Delta S_j^2(z) \rangle - \langle S_j(z) \rangle^2$ is the variance of the fluctuations of the j-th Stokes parameter, and r_{jk} is the correlation coefficient between the Stokes parameters S_j and S_k, which is defined by

$$r_{jk} = \frac{\langle S_j S_k \rangle + \langle S_k S_j \rangle - 2\langle S_j \rangle \langle S_k \rangle}{2(\langle \Delta S_j^2 \rangle \langle \Delta S_k^2 \rangle)^{1/2}} . \quad (6)$$

According to the form of (5), the Stokes parameters cannot be simultaneously and exactly measured in a quantum optics.

We assume that the interacting waves are initially in a coherent state at the input of the nonlinear medium, i.e.

$$a_j|\alpha_j\rangle = \alpha_j|\alpha_j\rangle \ (j = x, y) , \text{ and} \quad (7a)$$

$$|\xi\rangle = |\alpha_x\rangle|\alpha_y\rangle \quad (7b)$$

where α_j and $|\alpha_j\rangle$ is the eigenvalue and the eigenstate of the a_j operator, and $|\xi\rangle$ is the state of the total field containing two modes. If we take the definitions in (2) it is easy to obtain that

$$\langle \Delta S_j^2 \rangle = |\alpha_x|^2 + |\alpha_y|^2, \quad r_{jk} = 0, \quad (j = 1,2,3) . \quad (8)$$

It follows from the expressions in (8) that the level of the fluctuations of the Stokes parameters is defined by the sum of the average photon numbers $\langle n_x \rangle = |\alpha_x|^2$ and $\langle n_y \rangle = |\alpha_y|^2$ in the coherent modes.

Taking into account (5), we can write conditions for the squeezed states of light in terms of the Stokes parameter variances:

$$\langle \Delta S_j^2(z) \rangle \lesseqgtr |\langle S_m(z) \rangle| / (1 - r_{jk}^2)^{1/2} , \text{ and}$$
$$\langle \Delta S_k^2(z) \rangle \gtreqless |\langle S_m(z) \rangle| / (1 - r_{jk}^2)^{1/2}$$
$$\text{with } j,k,m = 1,2,3 \text{ but } j \neq k \neq m . \quad (9)$$

Thus, in quantum optics, the state of polarization may be presented on the Poincaré sphere with coordinates $\langle S_1 \rangle$, $\langle S_2 \rangle$, $\langle S_3 \rangle$ (see Fig. 1). Here a fluctuational uncertainty can be associated with a particular region of uncertainty in the Stokes parameters $S_{1,2,3}$ around their mean values [10, 11]. From the physical point of view, the inequalities in (9) mean that the ball-shaped region of uncertainty (for coherent states, (7) and (8)) of the Stokes parameters is transformed to the ellipsoid of the uncertainty for squeezed states (9). This type of nonclassical polarization state has been called the polarization-squeezed (PS) state [11].

Let us now discuss which nonclassical states under study have the properties that are similar to those of light with squeezed fluctuations of the Stokes parameters. First of all, we consider the parameters $S_0(z)$ and $S_1(z)$, which are defined by the relations (2a) and (2b). It is well known that fluctuations in these parameters can be suppressed to values below the level corresponding to the coherent state, owing to a correlation or anticorrelation among the photons in the two modes. Such a situation is obtained, for example, in parametric processes and four-wave mixing [24], as well as for other schemes of multiwave scattering [25], where correlated photons are created in pairs.

The physical properties of squeezed light with suppression of the fluctuations of $\langle \Delta S_2^2(z) \rangle$ or $\langle \Delta S_3^2(z) \rangle$ can be elucidated in the simplest case by assuming that the field of one of the modes is classical. Then, for example, replacing a_x by the classical quantity $|A_x| \exp(i\varphi_x)$, where φ_x is the phase, we have (see also (2c), (2d)):

$$S_2(z) = Q_y(z)|A_x| , \quad (10a)$$

$$S_3(z) = P_y(z)|A_x| \quad (10b)$$

where $Q_y(z) = \mathrm{e}^{-\mathrm{i}\Theta} a_y(z) + \mathrm{e}^{-\mathrm{i}\Theta} a_y^+(z)$, and $P_y = \mathrm{i}[\mathrm{e}^{\mathrm{i}\Theta} a_y^+(z) - \mathrm{e}^{-\mathrm{i}\Theta} a_y(z)]$, with $\Theta = \varphi_x - \theta$, are the Hermitian quadrature components for the mode described by a_y. It is seen from (10a) and (10b) that polarization-squeezed light is closest to quadrature-squeezed states of a field in this case. Experiments devised to obtain light with such characteristics are well known (see, for example, [1]).

When polarization-squeezed light is compared with quadrature-squeezed light, the specific features associated with the vector characteristics of the field must be borne in mind. For example, in a cubic nonlinear medium, quadrature-squeezed states can be obtained in practice only in processes involving the self-interaction and cross-interaction of waves [26], while polarization-squeezed light can form in such media only when there is anisotropy in the nonlinear correction to the refractive index [10]. In the latter case, there is no power conversion between polarization modes propagating in the nonlinear medium. Therefore, both the total number of photons and the difference between the numbers of photons in the modes are maintained (see Appendix). In contrast, the main purpose of the present paper is to demonstrate the possibility of obtaining a new class of polarization-squeezed light, in which there is an energy exchange between the polarization modes.

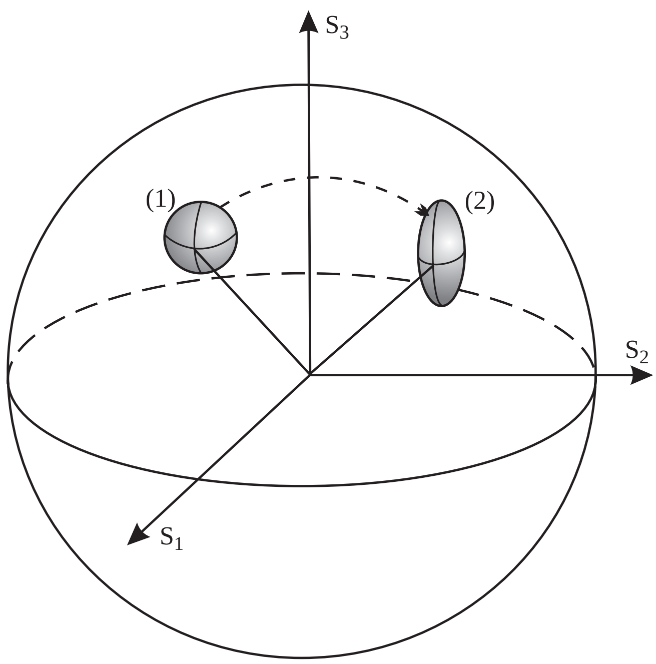

Fig. 1. Transformation of a coherent state (1) into a polarization-squeezed state (2) on the Poincaré sphere

2.2 Basic equations and quantum fluctuations

We describe here the propagation of polarized modes in the periodically-inhomogeneous nonlinear optical fiber (see [27, 28]) by Heisenberg equations for the evolution of the annihilation operators a_x and a_y related to the two orthogonal components of the polarization of light [11]. We have

$$\mathrm{i}\hbar\, \mathrm{d}a_j/\mathrm{d}t = \left[a_j; \mathrm{H}_{\mathrm{int}}\right] \;\; (j = x, y)\,, \tag{11}$$

with the interaction Hamiltonian

$$\mathrm{H}_{\mathrm{int}} = (\hbar c/n_1)\big(\beta a_x^+ a_y + \beta a_y^+ a_x + 0.5R(a_x^{+^2} a_x^2 + a_y^{+^2} a_y^2) + (R/3) a_x^+ a_x a_y^+ a_y\big)\,, \tag{12}$$

where $\hbar$ is the Planck constant, c is the light velocity in a vacuum and β is the linear coefficient of the mode coupling. Further, $R \equiv 4\pi\hbar\omega k_0 n_2/n_1 s\varepsilon_0 V$ is the nonlinear coefficient (with $k_0 = \omega/c$), n_1 and n_2 are the linear and nonlinear refractive index respectively, the parameter s is determined by the field distribution over the fiber area, and V is the quantization volume.

The solutions of the equations (11) can be found in the form [3]:

$$a_{x,y}(z) = (c_1(z)\exp(\mathrm{i}\beta z) \pm c_2(z)\exp(-\mathrm{i}\beta z))/\sqrt{2}\,, \tag{13}$$

where time t is replaced by $-zn_1/c$, and $c_{1,2}(z)$ are the slowly varying operators that are determined by the following expressions:

$$c_{1,2}(z) = \exp\left\{\mathrm{i}\sigma(5c_{1,2}^+ c_{1,2} + 6c_{2,1}^+ c_{2,1})/6\right\} c_{1,2}\,. \tag{14}$$

Here and below, $c_{1,2} \equiv c_{1,2}(z=0)$ is the operator value at the input of the nonlinear medium, and $\sigma = Rz$. It is easy to show that the commutation relations (3) are valid for operators $a_{x,y}$.

Let us now consider the fluctuations of the Stokes parameters S_2 and S_3 (2c) and (2d), at the output of the Kerr-like nonlinear medium. Taking into account the relations (13) and (14), and setting $\langle n_x\rangle \equiv \langle n\rangle \neq 0$ and $\langle n_y\rangle = 0$ at the input of the optical fiber, we obtain the following expressions for variances $\langle\Delta S_{2,3}^2\rangle$:

$$\langle\Delta S_2^2\rangle = \langle n\rangle(1 + \kappa^2\sin^2\theta + \kappa\sin 2\theta) \text{ and} \tag{15a}$$

$$\langle\Delta S_3^2\rangle = \langle n\rangle(1 + \kappa^2\cos^2\theta - \kappa\sin 2\theta)\,, \tag{15b}$$

where we have introduced an effective nonlinear parameter $\kappa \equiv \Psi_{\mathrm{eff}} = \Psi\cos(\beta 2z)$. Within these expressions, $\Psi = \sigma\langle n\rangle/6 \equiv Rz\langle n\rangle/6$, and $\theta = \phi - k_0 z$, and together they represent the initial phase difference between two orthogonally polarized waves that is determined by ϕ and the phase shift $k_0 z$ due to wave number differences in two modes, on the assumption that $\sigma^2\langle n\rangle \ll 1$ (see Appendix). In the general case, the variances $\langle\Delta S_{2,3}^2\rangle$ in (15a) and (15b) depend on the κ parameter, in other words Ψ and $2\beta z$, and on phase θ.

The 3D dependences of the variances given in (15a) and (15b) are shown in Fig. 2. In general, it is obvious from Fig. 2a and Fig. 2b that the normalized variances of the Stokes parameters $\sigma_{2,3}^2 \equiv \langle\Delta S_{2,3}^2\rangle/\langle n\rangle$ demonstrate the oscillation behavior with parameter β and phase

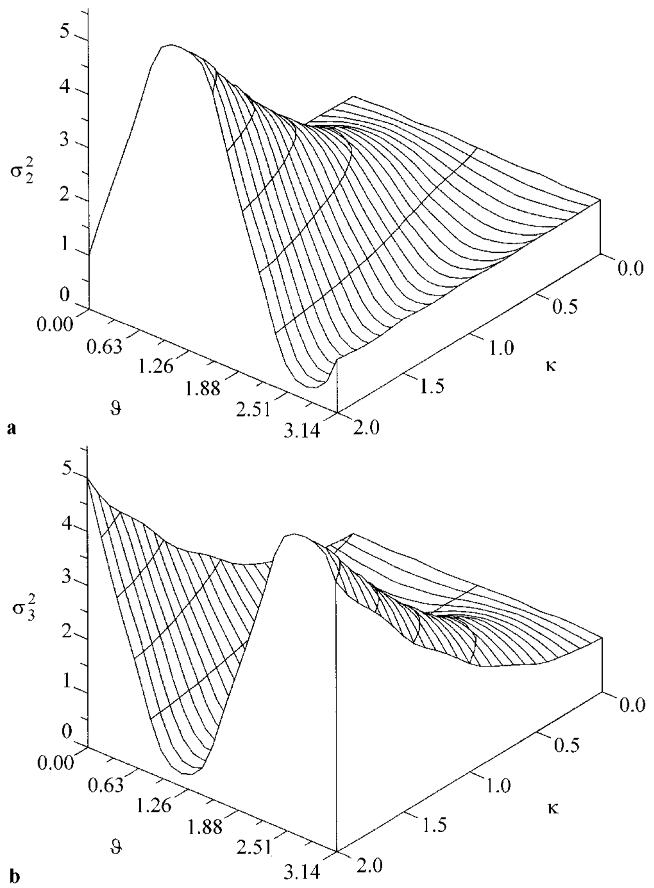

Fig. 2a,b. 3D dependences for normalized variance $\sigma_j^2 \equiv \langle\Delta S_j^2(z)\rangle/\langle n\rangle$ $(j = 2, 3)$ of the Stokes parameters: **a** $S_2(z)$ plotted against the effective nonlinear parameter κ and phase θ; **b** $S_3(z)$ plotted against κ and θ. The value $\sigma^2 = 1$ corresponds to the coherent level of the Stokes parameter variances

θ. Specifically, when $\kappa = 2$ and $\theta = 1.2$ rad, the minimal value of σ_2^2 corresponds to maximum value of σ_3^2. Thus, if the value of σ_2^2 or σ_3^2 is smaller than the coherent noise level $\sigma_{2,3}^2 = 1$, then polarization-squeezed light is formed. The Stokes parameter variances given in (15a) and (15b) are the same as for coherent states at the output of the medium when the effective nonlinear parameter $|\kappa| = 0$.

Expressions (15a) and (15b) reach their limiting values when $\tan(2\theta) = -2/\kappa$. The minimum and maximum values are:

$$\langle\Delta S_2^2\rangle_{\min} = \langle n\rangle\left(\sqrt{1+\kappa^2/4} - |\kappa|/2\right)^2 \text{ and} \tag{16a}$$

$$\langle\Delta S_3^2\rangle_{\max} = \langle n\rangle\left(\sqrt{1+\kappa^2/4} + |\kappa|/2\right)^2\,, \tag{16b}$$

and in this case the product

$$\langle\Delta S_2^2\rangle_{\min}\langle\Delta S_3^2\rangle_{\max} = \langle n\rangle^2 \tag{17}$$

is minimal and we have an ideal squeezing.

Let us now consider the fluctuations of the Stokes parameter S_1. The expression for $\langle\Delta S_1^2\rangle$ obtained under the same approximation as (15a) and (15b) has a form:

$$\langle \Delta S_1^2 \rangle = \langle n \rangle \left(1 + \Psi^2 \sin^2(2\beta z)\right) \quad (18)$$

It is easy to see that $\langle \Delta S_1^2 \rangle$ is also greater than the variance for the coherent state excluding the value of distance z which satisfies the condition $2\beta z = \pi(1+m)$ ($m = 0, 1, 2, \ldots$).

3 A nonlinear parameter to describe the light polarization state in quantum optics

The parameter $\mathcal{P}$, introduced above at (1), characterizes the degree of polarization and needs to be discussed separately in quantum optics. Indeed, a recent study in [7, 10, 11] of the problem showed the nonlinear behavior of the quantity for nonclassical states of light. The result can be recognized taking into account, first, the definitions in (2); secondly, the relations (13) and (14); and also, thirdly, the fact that the initial modes are in a coherent state. Then we obtain for the degree of polarization (1) the following expression:

$$\mathcal{P} = \mathrm{e}^{-\langle n \rangle \sigma^2/72} \approx 1 - \langle n \rangle \sigma^2/72 \,. \quad (19)$$

The fact that the value of the polarization degree (19) could be different from unity in principle is a purely quantum effect (see also [7, 8, 10, 11]). The dependence $\mathcal{P}$ on the average photon number $\langle n_x \rangle \equiv \langle n \rangle$ of a polarization mode at the input of an optical fiber (when $\langle n_y \rangle = 0$) is a nonlinear quantum effect in contrast to the traditional interpretation of the light polarization degree as a fixed parameter (cf [7, 10] and [18, 20, 21]). But in a real experiment, we have found $\langle n \rangle \sigma^2 \ll 1$, and this means that, to the same degree of accuracy as obtained for (15) and (16), we have $\mathcal{P} = 1$. We obtained a similar result for $\mathcal{P}$ for the nonlinear optical process considered in the Appendix (see also [10]).

It is important to note that although the determination of the polarization degree $\mathcal{P}$ via (1) is considered for many cases [8–11], the formula in (1) cannot be directly applied for some quantum states of light. An alternative determination of $\mathcal{P}$ in terms of the coherence matrix elements [7] corresponds to formula (1) and in the terms of the Stokes parameter operators.

The main problem to introduce the polarization degree into quantum theory is determined by the vacuum states of light. In fact, for the two-mode vacuum state $|\xi\rangle = |0\rangle_x |0\rangle_y$, the average values of the Stokes parameters for polarized modes (given by (7b)) are equal to zero ($\langle S_j \rangle = 0$), and so the expression (1) is not applied to describe the state of light. On the other hand, the determination of the polarization degree for a vacuum as a limit value for coherent (and/or Fock) states, with the average number of photons being zero, results in a completely polarized light, i.e. $\mathcal{P}_{\text{vac}} = 1$. This fact cannot, however, be physically interpreted.

Let us now define the nonlinear parameter for light polarization in the quantum case by the following modification of expression (1):

$$P = \left(\sum_{j=1}^{3} \langle S_j \rangle^2 \Big/ \sum_{j=1}^{3} \langle S_j^2 \rangle \right)^{1/2} , \quad (20)$$

where this formula (20) differs from expression (1) by the renormalization factor only. However, the difference is major and results in a quantum correction of the introduced nonlinear parameter (20). Indeed, in contrast to the classical optics approach (see for example [18]), where

$$\sum_{j=1}^{3} S_j^2 \le S_0^2 \,, \quad (21a)$$

we now have an opposite inequality

$$\sum_{j=1}^{3} S_j^2 = S_0 (S_0 + 2) > S_0^2 \,. \quad (21b)$$

In a quasiclassical approximation for a large photon number, that is, when $\langle S_0 \rangle = n_0 \equiv \langle n_x \rangle + \langle n_y \rangle \gg 1$, the nonlinear parameter (20) can be associated with the light polarization degree thus:

$$\sum_{j=1}^{3} \langle S_j^2 \rangle \approx \sum_{j=1}^{3} \langle S_j \rangle^2 \,, \quad (22)$$

$$P \approx \mathcal{P} \quad (23)$$

It is useful to introduce the depolarization degree parameter D as follows:

$$D = \sqrt{1 - P^2} = \left(\sum_{j=1}^{3} \langle \Delta S_j^2 \rangle \Big/ \sum_{j=1}^{3} \langle S_j^2 \rangle \right)^{1/2} , \quad (24)$$

and by this means the nonlinear parameter of depolarization (24) is coupled with the normalized variance of fluctuations for the three Stokes parameters.

Let us examine the definitions (20) and (24) for different quantum states of two orthogonally polarized modes of light. Firstly, for the coherent state we obtain:

$$P_{\text{coh}}^2 = n_0/(n_0 + 3), \text{ and} \quad (25a)$$

$$D_{\text{coh}}^2 = 3/(n_0 + 3) \,, \quad (25b)$$

where $n_0 \equiv \langle n_x \rangle + \langle n_y \rangle$ is the total number of photons in two polarized modes together. In the quasiclassical approximation ($n_0 \gg 1$), we have

$$P_{\text{coh}}^2 \approx 1 \,, \quad (26)$$

and so, in the coherent state, the nonlinear polarization parameter of light differs from unity for quantum optics but is almost identical to the quasiclassical approximation. For another limit, $n_0 = 0$ (the case of vacuum polarized modes), we have:

$$P_{\text{vac}}^2 = 0 \,, \text{ and} \quad (27a)$$

$$D_{\text{vac}}^2 = 1 \,, \quad (27b)$$

and this means that a vacuum state of light is completely unpolarized.

For the Fock state of two polarized modes (see (7b)), i.e. when $|\xi\rangle = |n_x\rangle|n_y\rangle$, we have:

$$P_F^2 = (\langle n_x\rangle - \langle n_y\rangle)^2/(n_0^2 + 2n_0)\,, \text{ and} \tag{28a}$$
$$D_F^2 = 2(2\langle n_x\rangle\langle n_y\rangle + n_0)/(n_0^2 + 2n_0)\,. \tag{28b}$$

In the case where $\langle n_x\rangle = \langle n_y\rangle$, the polarization parameter given by (28a) corresponds with the vacuum state (27a). For a linearly polarized light, where $\langle n_y\rangle = 0$, we obtain:

$$P_F^2 = \langle n_x\rangle/(2 + \langle n_x\rangle)\,, \text{ and} \tag{29a}$$
$$D_F^2 = 2/(2 + \langle n_x\rangle)\,. \tag{29b}$$

In the limit of a vacuum field ($\langle n_x\rangle = 0$) the expressions (29a) and (29b) are reduced to the same result as above in (27a) and (27b). On the other hand, in a quasiclassical approximation for a linearly polarized optical field ($\langle n_x\rangle \gg 1$), we have a result similar to the coherent state given in (26). In summary then, the polarization parameters of light (20) and (24) density from our work lead to physically reasonable results.

Let us now calculate the parameters (20) and (24) for polarization-squeezed light. In the case of an anisotropic medium of cubic nonlinearity, where such a non-classical state of light can be generated (see Appendix), we have:

$$P_{sq}^2 = P_{coh}^2 - 8\langle n_x\rangle\langle n_y\rangle W/(n_0^2 + 3n_0)\,, \text{ and} \tag{30a}$$
$$D_{sq}^2 = D_{coh}^2 + 8\langle n_x\rangle\langle n_y\rangle W/(n_0^2 + 3n_0)\,, \tag{30b}$$

where the first terms on the right-hand side of the equations are described by expression (25a) and (25b), and the parameter W is determined within the Appendix at (A.4). The second terms in (30a) and (30b) determine the quantum additions due to the redistribution of quantum fluctuations for a polarization squeezing (see also (A.4)). Thus for squeezed light, the value of the polarization nonlinear parameter P_{sq} is less than for the coherent state. This result corresponds to the uncertainty product given at (17) because the suppression of fluctuations for one of the Stokes parameters is accompanied by an enhancement of fluctuations for another parameter. The nonlinear parameter of depolarization D increases for the same reason as a result (see Fig. 1). In practice, the magnitude of W is much less than unity (see (A.4)), and so the difference of the P and D parameters for the two cases, i.e. for coherent and squeezed light, is very small (see also [11]).

The principal result obtained from our analysis reduces to the fact that a completely polarized light cannot be generated in quantum optics, as shown by (19). At the same time, nonclassical states of optical fields can be described by nonlinear parameters of polarization (at (20)) and depolarization (at (24)) associated with the total variances of fluctuations for the Stokes parameters of light.

4 Quantum nondemolition measurements for the Stokes parameters

4.1 General description of the SU(2) algebra observables measurement

Here we formulate the general principles for the SU(2) algebra observables using the Stokes parameters from (2). The conditions to obtain QND measurements for the case under consideration can be formulated as follows. We define the parameter S_m as a nondemolished measured (signal) Stokes parameter, but the parameter S_p as a measuring (probe) parameter, and finally the parameter S_i as an auxiliary parameter, where $m,\ p,\ i = 1, 2, 3,$ but $m \neq p \neq i$.

In general for QND measurement, it is supposed that a measured Stokes parameter S_m interacts with the probe parameter S_p in some physical system (we will call it a QND apparatus) [12]. For this type of quantum measurement, it is necessary to fulfill the following basic conditions [16]:

1. the measured Stokes parameter S_m^{in} must be conserved, in other words a process of measurement has to add the minimum noise to the output Stokes parameter S_m^{out};
2. the detected value of the probe Stokes parameter S_p^{out} must contain "full information" about the measured value of S_m^{in} at the QND apparatus input; and
3. any observable (in our case the S_0 and S_i operators) must commute with the operators of measured and probe quantities, and from the physical point of view, this condition means that the measurement process must be separated from any destructive feedback from these observables.

We introduce the following correlation coefficients between the Stokes parameters to describe the abovementioned QND conditions (see also [3, 13, 16]):

$$K_1 = \frac{|\langle S_m^{in} S_m^{out}\rangle + \langle S_m^{out} S_m^{in}\rangle - 2\langle S_m^{in}\rangle\langle S_m^{out}\rangle|^2}{4\langle(\Delta S_m^{in})^2\rangle\langle(\Delta S_m^{out})^2\rangle}\,, \tag{31a}$$
$$K_2 = \frac{|\langle S_m^{in} S_p^{out}\rangle + \langle S_p^{out} S_m^{in}\rangle - 2\langle S_m^{in}\rangle\langle S_p^{out}\rangle|^2}{4\langle(\Delta S_m^{in})^2\rangle\langle(\Delta S_p^{out})^2\rangle}\,, \tag{31b}$$

$$R_1(S_i^{out}, S_m^{out}) \equiv (r_{im}^{out})^2\,, \tag{32a}$$
$$R_2(S_i^{out}, S_p^{out}) \equiv (r_{ip}^{out})^2\,, \text{ and} \tag{32b}$$
$$R_3(S_m^{out}, S_p^{out}) \equiv (r_{mp}^{out})^2\,, \tag{32c}$$

In an ideal case of QND measurement, it is clear that $S_m^{in} = S_m^{out}$, $S_p^{out} = \lambda_2 S_m^{in}$ (where λ_2 is the QND gain) and thus, $K_{1,2} = 1$, $R_{1,2} = 0$, and $R_3 = 1$. However, experimentally it is difficult to satisfy the criteria (31a), and (31b) simultaneously. Therefore such a type of QND measurement turns out to be always nonideal (see below).

4.2 QND measurement for the parameter S_1

Now we assume that the Stokes parameter $S_m^{in} \equiv S_1^{in}$ can be measured by a probe with parameter $S_p^{out} \equiv S_3^{out}$ ($S_i \equiv S_2$) without any demolition. To obtain the necessary linear coupling between the measured Stokes parameters S_1^{in} and the probe parameter, for example S_3^{out}, we consider, for example, using two elements, namely a phase plate ϕ and a linear spatio-periodical optical fiber. The propagation of the two orthogonally polarized modes in such a system is described by the equations in (11) and the Hamiltonian (12), where nonlinear terms are omitted. For the case of a special selection of phase combination when $\theta = 0$, a coupling of the Stokes operators in (2) is given by the following expressions:

$$S_{0,2}^{out} = S_{0,2}^{in}\,, \text{ and} \tag{33a,b}$$
$$S_{3,1}^{out} = \lambda_1 S_{3,1}^{in} \pm \lambda_2 S_{1,3}^{in} \tag{33c,d}$$

where $\lambda_1 \equiv \cos(g)$, and for this case $\lambda_2 \equiv \sin(g)$ and the value of g therein is $2\beta z$ and defines an efficiency of the linear transformation of the Stokes parameters. S_j^{in} and S_j^{out} $(j = 1, 2, 3)$ are the Stokes parameters at the input and the output of the QND apparatus, respectively.

Let us consider the coefficients given in (31) and (32) for the case $m = 1$, $p = 3$, which characterizes the perfomance of QND apparatus in a linear system. For the case $\lambda_2 \to 0$ with $(|\lambda_2| \equiv \sin(g) \approx g \ll 1)$, from the expressions (33b,c) the relevant correlation coefficients $K_{1,2}$ and $R_{1,2}$ are obtained in the form:

$$K_1 \approx 1\,, \tag{34a}$$

$$K_2 \approx r_{13}^2 + \left(1 + \frac{\lambda_1^2 V_3}{\lambda_2^2 V_1} + 2\frac{\lambda_1}{\lambda_2} r_{13}\sqrt{V_3/V_1}\right)^{-1}, \text{ and} \tag{34b}$$

$$R_1 \approx R_2 \approx r_{12}^2\,. \tag{34c}$$

Here, $V_{1,3} \equiv \langle(\Delta S_{1,3}^{\text{in}})^2\rangle$; and r_{13}, r_{12} are the correlation coefficients of (6) between S_1^{in} and S_3^{in}, and between S_1^{in} and S_2^{in}, respectively, at the input of the QND apparatus. It follows from relation (34a) that we have an approximately nondemolition value of the S_1^{in} parameter.

However, at the same time we must require the following inequalities to be fulfilled for its measurement:

$$V_3 \ll (\lambda_2/\lambda_1)^2 V_1 \approx g^2 V_1\,, \tag{35a}$$

$$r_{13}^2 \ll 1\,, \text{ and} \tag{35b}$$

$$r_{12}^2 \ll 1 \tag{35c}$$

Since $g^2 \ll 1$, the condition (35a) means that the probe Stokes parameter variance V_3 must be essentially less at the input to the linear system than the variance V_1 of the measured quantity. We also should require the absence of correlation between S_1^{in} and S_3^{in} and also between S_1^{in} and S_2^{in} for obtaining a "good" QND measurement. It is easy to show that, to fulfill the conditions of (35), we have to satisfy the demand of an ideal measurement, $R_3 \cong 1$, at the same time. Thus, the coefficient R_3 does not have any relevance to the introduction of any new limitation in the measurement procedure, and so the coefficient R_3 can in practice be ignored.

The setup for QND measurement of the S_1^{in} parameter is shown in Fig. 3. The procedure consists of several steps. First, the radiation has to propagate through the medium with an anisotropic cubic nonlinearity (NL) to generate squeezed light. Secondly, the $S_{1\text{b}}$ and $S_{3\text{b}}$ are the values of the Stokes parameters at the input of the QND apparatus, being formed before the process of measurement. Finally, a linear system L has to be included in the process of measurement as well, in order to control the energy exchange between the two modes in the system.

The propagation of the two orthogonally polarized modes b_x and b_y in the nonlinear optical medium under consideration is analyzed in the Appendix. It is shown that the polarization-squeezed (PS) light is exactly formed in the anisotropic nonlinear medium NL (Fig. 3), and the light satisfies the QND measurement conditions (31) and (32) for the Stokes parameter S_1^{in}. At the same time, the S_1 value, i.e. the difference in the photon numbers for the two orthogonal modes, is a conserved value during the process of propagation of the field through an anisotropic nonlinear medium–in other words, $S_{1\text{b}} = S_1^{\text{in}}$.

Using the limiting value of the variance $\langle(\Delta S_3^{\text{in}})^2\rangle$ of the probe Stokes parameter (see (A.5)), we can rewrite the inequality in (35a) as a condition for nonlinear phase shift, thus:

$$\Psi_{\text{n}1} \gg 2ctg(2g) \approx 1/g\,. \tag{36}$$

The expression (36) has a simple physical meaning. The efficiency of the nonlinear interaction of the orthogonally polarized modes, which is defined by the nonlinear phase shift $\Psi_{\text{n}1}$ in the NL region, must be greater than the corresponding efficiency of the energy exchange between the two modes described by the parameter g of the linear system L.

For an ideal squeezing, the decrease of variance V_3 is accompanied by an increase of variance V_2 for conjugate Stokes parameter S_2 (cf. (17)). Thus, precise measurement of the Stokes parameter $S_{1\text{b}} = S_1^{\text{in}}$ is obtained through a preliminary redistribution of quantum fluctuations from the probe parameter S_3^{in} to the conjugate parameter S_2^{in} in an anisotropic medium of cubic nonlinearity NL. The latter is isolated from the measuring process (see expression (33b)) occuring in the linear system L (Fig. 3).

The 3D-dependencies (see (31a) and (31b)) of the correlation coefficients K_1 and K_2 against both the nonlinear phase shift $\Psi_{\text{n}1}$ (in anisotropic medium NL) and the linear coupling coefficient g (in linear system L) are shown in Fig. 4a and Fig. 4b. We also assume that the conditions of (35b) and (35c) are valid, i.e. $r_{13} = r_{12} = 0$, and so the variance of the Stokes parameter S_3^{in} is determined by the expression (A.5) at the entrance to linear system. Comparing Fig. 4a with Fig. 4b, we can see that the correlation coefficients K_1 and K_2 do not take a value of unity at the fixed magnitudes of g and $\Psi_{\text{n}1}$, and this, as we mentioned above, is as a result of imperfections in the measurement procedure. However, with an increase in the nonlinear phase shift $\Psi_{\text{n}1}$, we have that the K_1 and K_2 curves become steeper as a function of g. For example, for $\Psi_{\text{n}1} \approx 9.5$ and $g = \pi/6$ (when condition (36) still holds true), the numerical magnitudes of the correlation coefficients are $K_1 \approx 0.99$ and $K_2 \approx 0.97$ (see Figs. 4a,b). Thus, the accuracy of the measurement of the S_1^{in} Stokes parameter appears acceptably high.

It should be pointed out that a QND measurement cannot be obtained for the Stokes parameter S_1^{in} by a QND apparatus including a single linear system only. In fact, when $\Psi_{\text{n}1} = 0$, the correlation coefficients $K_1 \approx 1$, $K_2 \to 0$ when the value $\lambda_2 \to 0$, and $K_1 \to 0$ and $K_2 \approx 1$ when $\lambda_1 \to 0$, for the limit

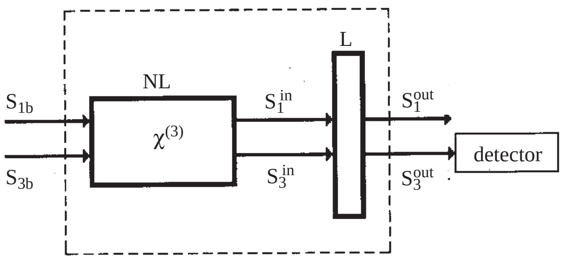

Fig. 3. Setup of QND measurement of the S_1^{in} Stokes parameter. The parameter $S_{j\text{b}}$ $(j = 1, 3)$ is the input Stokes parameter at the input of the QND apparatus and consists of nonlinear medium NL and linear system L; S_j^{out} is the Stokes parameter at the output of the linear system L

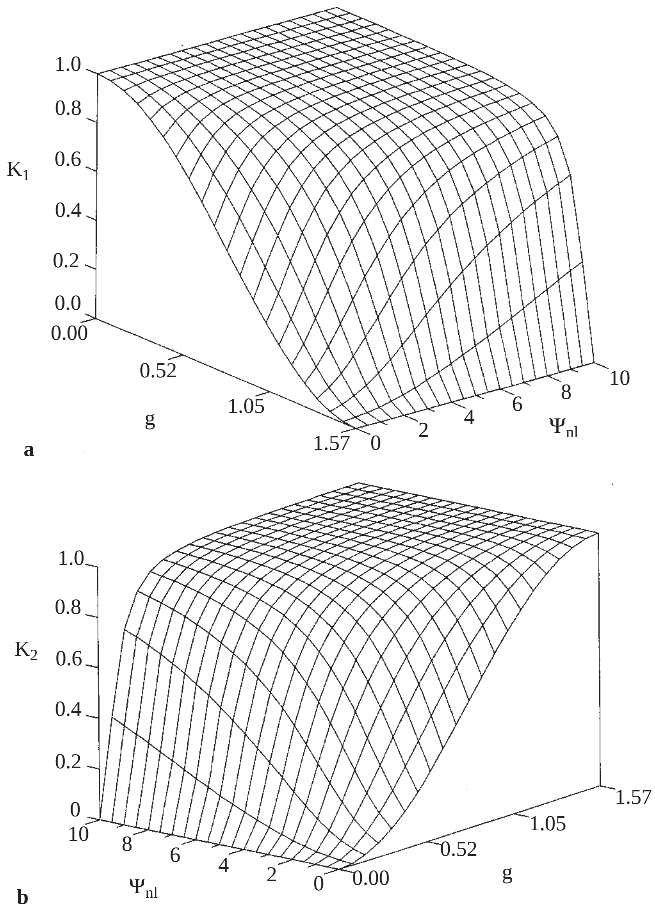

Fig. 4a,b. Correlation coefficients K_1 **a** and K_2 **b** plotted, respectively, against the nonlinear linear phase shift Ψ_{nl} and linear coupling parameter g. The magnitudes of the parameters used for calculations are the limiting phase value $\phi = -0.5\arctan(2/\Psi_{nl})$, and the coefficient of nonlinear medium anisotropy $\gamma = (\gamma_1 + \gamma_2)/2$ (see Appendix for the definition γ_1 and γ_2

cases considered above (see Fig. 4a and Fig. 4b). Moreover, the conditions of (35) are not satisfied in the case when the nonlinear medium has no anisotropy of a cubic nonlinearity type and so the PS light does not even arrive at the input to a linear system.

4.3 QND measurements for the S_3 and S_2 parameters

Now let us briefly consider the QND measurement for the Stokes parameter S_3 (and/or S_2). For this purpose the linear coupling between the measured S_3 and probe S_1 parameters is obtained by the linear spatio-periodically optical fiber (see (33c,d)). Therefore, we can offer a scheme for QND measurement of the S_3 Stokes parameter by modifying the scheme of the S_1 measurement.

The corresponding setup is shown in Fig. 5. The QND apparatus consists of two linear systems `L1` and `L2` and also the anisotropic medium of cubic nonlinearity `NL` placed between them. The schematic in Fig. 5 differs from the earlier one in Fig. 3 due to existence of an additional linear device `L1`. Such a preliminary linear system "rotates" the measured Stokes parameter S_{3c} and transforms it into the photon number difference (i.e. into parameter S_{1b}) at the input of the NL medium. As a result, we have

$$S_{2b} = S_{2c}\,, \tag{37a}$$
$$S_{1b} = S_{3c}\,, \text{ and} \tag{37b}$$
$$S_{3b} = -S_{1c}\,. \tag{37c}$$

Then the value S_{1b} can be measured without destruction according to the procedure considered above, and we have

$$S_1^{in} = S_{1b} = S_{3c}\,. \tag{38}$$

In an ideal case of QND measurement, the transformation of the Stokes parameters produced by the linear system `L2` results in the expressions

$$S_1^{out} = S_1^{in} = S_{3c}\,, \text{ and} \tag{39a}$$
$$S_3^{out} = S_1^{in} = S_{3c}\,. \tag{39b}$$

It is easy to show that the condition for a "high-quality" QND measurement procedure for the S_{3c} Stokes parameter considered here is the same as for the S_{1b} parameter measurement of (35) and (36) (see also [16]). Thus, the conditions in (31) and (32) for the QND measurement of the S_{3c} Stokes parameter are dependent upon the linear transformation coefficient g arising in the system `L2` and also on the redistribution of fluctuations in an anisotropic medium of cubic nonlinearity.

Finally, we discuss an opportunity to carry out the QND measurement for the S_2 Stokes parameter. Such a measurement has to be obtained within the framework of the Fig. 5 scheme, but using a property of symmetry for the Stokes parameters S_2 and S_3 at their rotation governed by the phase parameter θ (see (2)).

5 The QND measurement of the phase difference of two modes

In the past few years, the problem of phase measurement in quantum optics has been the subject of intensive study of many authors [29]. But different aspects of the problem [30–33] can be solved by physical interpretation of the measurement procedure only. One general concept in that direction has been recently discussed by Braginsky et al. [17]; such an approach is preferable for many cases in comparison with other methods [34, 35]. On the other hand, the theoretical and experimental results obtained by Mandel et al. [36] are

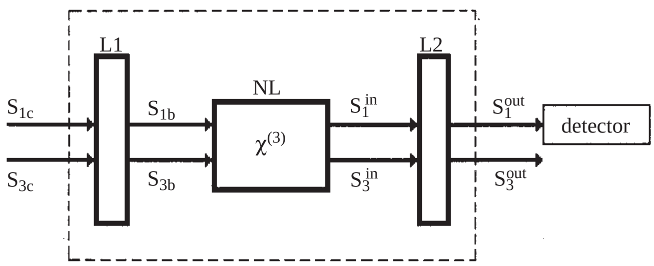

Fig. 5. Scheme for QND measurement of the S_{3c} Stokes parameter. S_{jc} ($j = 1, 3$) is a value of the Stokes parameters at the input of the QND apparatus consisting of a nonlinear medium `NL` and two linear systems `L1` and `L2`. The S_j^{out} are the Stokes parameters at the output of the linear system `L2`

more physically appropriate because their approach is based on the phase difference between two quantum modes, which is an observable quantity. The latter approach is principally the same as that considered by us herein for a precise measurement procedure of the Stokes parameters. So, our analysis could be useful to carry out also phase-sensitive measurements in quantum optics. We will discuss the procedure for that below.

Let us consider the Susskind–Glogower (SG) cosine and sine phase operators for two orthogonally polarized modes (in particular, along the x- and y-axis respectively):

$$\sin(\Phi_{x,y}) = 0.5\mathrm{i}\left(a^{+}_{x,y}\frac{1}{\sqrt{n_{x,y}+1}} - \frac{1}{\sqrt{n_{x,y}+1}}a_{x,y}\right), \text{ and} \tag{40a}$$

$$\cos(\Phi_{x,y}) = 0.5\left(a^{+}_{x,y}\frac{1}{\sqrt{n_{x,y}+1}} + \frac{1}{\sqrt{n_{x,y}+1}}a_{x,y}\right). \tag{40b}$$

These obey the standard commutation relations (see e.g. [30]):

$$\left[\cos(\Phi_{ij}); n_{i,j}\right] = \mathrm{i}\delta_{ij}\sin(\Phi_{i,j}), \tag{41a}$$

$$\left[n_{i,j}; \sin(\Phi_{i,j})\right] = \mathrm{i}\delta_{ij}\cos(\Phi_{i,j}), \quad i, j = x, y \tag{41b}$$

In a quasiclassical approximation when the photon numbers

$$\langle n_{x,y}\rangle \gg 1, \tag{42}$$

we can rewrite the operators of the Stokes parameters S_2 and S_3 in the form (see [14, 16]):

$$S_2 \approx 2\sqrt{n_x n_y}\cos(\Phi_-), \text{ and} \tag{43a}$$

$$S_3 \approx 2\sqrt{n_x n_y}\sin(\Phi_-), \tag{43b}$$

where $\cos(\Phi_-) \equiv \cos(\Phi_y - \Phi_x)$ and $\sin(\Phi_-) \equiv \sin(\Phi_y - \Phi_x)$ are the cosine and sine of the operators of the phase difference for two polarized modes. But for such an approximation as (42), we have some difficulties with the SG operators [14, 30]. So, a more accurate formulation should in principle be attempted, and one way to do that has been established by the Pegg–Barnett formalism described in [31]. Nevertheless, we still discuss further below the problem of phase-sensitive measurement in the framework of the SG approach.

In the simplest case when $\Phi_- \ll 1$, only the S_3 operator depends on the phase difference, thus:

$$S_2 \approx 2\sqrt{n_x n_y}, \tag{44a}$$

$$S_3 \approx 2\sqrt{n_x n_y}\Phi_-. \tag{44b}$$

We could obtain the information about the phase difference of two polarized modes by QND measurement of the S_3 parameters. The appropriate procedure for such a parameter nondemolition measurement (we denote it as a $S_{3\mathrm{c}}$ parameter) has been considered by us in Sect. 4.3 above (see also Fig. 5). The main difficulties in obtaining the measurement are connected with an exact correspondence between the detected Stokes parameter S_1^{out} and the measured value of the phase difference Φ_-. That means that the QND apparatus should measure the phase difference only, and not the amplitudes of two polarized modes. Thus, the fluctuations of the Stokes parameter $S_{3\mathrm{c}}$ (see also (39b)) can contribute to fluctuations of the phase difference. The conditions for obtaining such a QND measurement for the phase difference will be written below.

First of all, let us represent both the photon number and the phase difference operators in the form (see [26, 37]):

$$n_{x,y} = \langle n_x\rangle + \Delta n_{x,y}, \text{ and} \tag{45a}$$

$$\Phi_- \equiv \langle\Phi_-\rangle + \Delta\Phi_-, \tag{45b}$$

where $\Delta n_{x,y}$ and $\Delta\Phi_-$ are only the operators. When condition (42) is satisfied ($\Delta n_{x,y} \ll n_{x,y}$), the following expression for the fluctuations of the measured Stokes parameters could be written down thus:

$$\Delta S_{3\mathrm{c}} = 2(\langle n_x\rangle\langle n_y\rangle)^{1/2}\Delta\Phi_- + (\langle n_y\rangle/\langle n_x\rangle)^{1/2}\Delta n_x + (\langle n_x\rangle/\langle n_y\rangle)^{1/2}\Delta n_y. \tag{46}$$

Thus, for the variance of the fluctuations of the measured Stokes parameter given in (43b) (see also Fig. 5), we have:

$$\langle\Delta S^2_{3\mathrm{c}}\rangle = 4\langle n_x\rangle\langle n_y\rangle\langle\Delta\Phi^2_-\rangle + \frac{\langle n_y\rangle}{\langle n_x\rangle}\langle\Delta n^2_x\rangle + \frac{\langle n_x\rangle}{\langle n_y\rangle}\langle\Delta n^2_y\rangle \tag{47}$$

In (47), the first term on the right-hand side corresponds to the QND measurement procedure of the phase difference Φ_-. But the last two terms in (47) describe the quantum fluctuations of the polarized modes at the input of the QND apparatus (although we can ignore them for nonclassical descriptions of light). In fact, in the case where input radiation is present for amplitude-squeezed light, we have

$$\langle\Delta n^2_{x,y}\rangle \ll \langle n_{x,y}\rangle \tag{48}$$

and so these terms are valid in practice. As a result, the phase difference of two modes is measured by a "pure" method.

Thus, the appropriate scheme for QND measurement of the $S_{3\mathrm{c}}$ Stokes parameter can be proposed as a phase-difference measurement procedure for two orthogonally polarized modes, but only for the case where the amplitude-squeezed light (with sub-Poissonian photon statistics) has been prepared before the measurement procedure [14]. In the classical approximation for one of two modes, a above-described procedure for QND measurement corresponds to the phase measurement (cf. [17]) in terms of the Hermitian quadratures (see also (10)).

6 The QND measurement of the angular momentum of atomic systems

Rapid progress has recently been achieved in the investigation of the nonclassical characteristics of atomic systems [38–40]. The subject of a recent major study has been the interaction of two-level boson-like atoms with an electromagnetic field [5, 6]. For these systems, the QND detection of atomic states [41], the coherent effects [6, 42], and the formation of atomic squeezed states [43] have all been under intensive study. One significant problem encountered has been the experimental observation of predicted nonclassical effects.

We now consider the problem of QND measurement of the angular momentum in atomic systems. In general, the operators of the angular momentum of atomic systems obey the SU(2) algebra commutation relations in (4) and so the above-considered QND measurement procedure can still apply in this case.

We define, for example, the parameter J_z as a nondemolition-measured angular momentum component, in contrast to the parameter J_x as a measuring (probe) component. According to general principles of the QND measurement procedure (see Sect. 4), we suppose that a measured component J_z interacts with a probe component in some physical measuring device (a QND apparatus) and that linear coupling between these components takes place. For such a case let us consider the Hamiltonian of the interaction of the atomic system with classical magnetic field (directed along the y-axis) in the form:

$$H = -\overline{\chi} J_y H_y \tag{49}$$

where coefficient $\overline{\chi}$ describes the magnetic momentum of atomic system and H_y is the classical magnetic field component. The corresponding Heisenberg equations are:

$$\mathrm{d}J_{x,z}/\mathrm{d}t = {}^{-}_{+}(\overline{\chi} H_y/\hbar) J_{z,x}\,, \text{ and} \tag{50a}$$

$$\mathrm{d}J_y/\mathrm{d}t = 0\,. \tag{50b}$$

The following solutions of (50a) and (50b) for the angular momentum components could be written in the form:

$$J_x^{\mathrm{out}} = J_x^{\mathrm{in}}\lambda_1 - J_z^{\mathrm{in}}\lambda_2\,, \tag{51a}$$

$$J_z^{\mathrm{out}} = J_z^{\mathrm{in}}\lambda_1 + J_x^{\mathrm{in}}\lambda_2\,, \text{ and} \tag{51b}$$

$$J_y^{\mathrm{out}} = J_y^{\mathrm{in}}\,, \tag{51c}$$

where $\lambda_1 = \cos(g)$, $\lambda_2 = \sin(g)$ and $g \equiv \overline{\chi} H_y t/\hbar$, but J_i^{in} and J_i^{out} are the i-th component of the angular momentum before (at $t = 0$) and after ($t > 0$) the measurement procedure, respectively.

Here we can discuss the analogy between the QND measurement of the angular momentum of an atomic system and the corresponding measurement of the S_1 Stokes parameter (see (33)). In fact, we declare the following replacement between parameters:

$$S_1 \to J_z\,,\ S_2 \to J_y\,, \text{ and } S_3 \to J_x\,. \tag{52}$$

According to previous work in this study, the inequalities

$$\langle(\Delta J_x^{\mathrm{in}})^2\rangle \ll (\lambda_2^2/\lambda_1^2)\langle(\Delta J_z^{\mathrm{in}})^2\rangle \approx g^2\langle(\Delta J_z^{\mathrm{in}})^2\rangle\,, \tag{53a}$$

$$r_{13}^2 \ll 1\,, \text{ and} \tag{53b}$$

$$r_{12}^2 \ll 1 \tag{53c}$$

should be fulfilled (cf. (35)), where r_{13} and r_{12} are the correlation coefficients of the angular momentum components before measurement.

The first condition (53a) requires that the atomic system has to be especially prepared as a system being in atomic squeezed state, i.e. the fluctuations of the probe angular momentum component J_x^{in} before the QND measurement procedure should be smaller than the measured one. Recently, such a type of quantum state has been theoretically proposed by Agarwal, Schleich et al. in [43]. The conclusion is that the atomic squeezed states $|\varsigma, m\rangle$, can be generated from the so-called coherent Dicke states $|j, m\rangle$ (see for example [42]) or the Dicke ground state $|j, m\rangle = |j, -j\rangle$ (where $m = -j$) by the following transformation of the wave function:

$$|\varsigma, m\rangle = A_m \exp(\vartheta J_z) \exp(-\mathrm{i}\pi J_y/2)|j, m\rangle\,, \tag{54}$$

where A_m is the normalization constant, and the parameter ϑ determines the squeezing effect in the system (see below). The variances of the fluctuations of the angular momentum components before the measurement are these:

$$\langle\varsigma, m|(\Delta J_x^{\mathrm{in}})^2|\varsigma, m\rangle = 0.5\tanh(\vartheta)\langle\varsigma, m|J_z^{\mathrm{in}}|\varsigma, m\rangle\,, \tag{55a}$$

and

$$\langle\varsigma, m|(\Delta J_y^{\mathrm{in}})^2|\varsigma, m\rangle = 0.5\coth(\vartheta)\langle\varsigma, m|J_z^{\mathrm{in}}|\varsigma, m\rangle\,. \tag{55b}$$

We can see that the variance of the probe angular momentum fluctuations is smaller than a standard quantum limit determined by the level $\langle J_z^{\mathrm{in}}\rangle/2$ (see also (9)), in contrast with the variance for the component of angular momentum $\langle(\Delta J_y^{\mathrm{in}})^2\rangle$ and it is important that the last component (J_y^{in}) is isolated from the measuring procedure (see equation (51c)). It is clear from (55a) and (55b) that in this case we have an ideal squeezing effect for the angular momentum probe component before measurement, in other words

$$\langle(\Delta J_x^{\mathrm{in}})^2\rangle\langle(\Delta J_y^{\mathrm{in}})^2\rangle = \langle J_z^{\mathrm{in}}\rangle^2/4\,. \tag{56}$$

This results in the QND measurement of the J_z^{in} angular momentum component being obtained from the squeezed probe component J_x^{in}. But the fluctuations of another angular momentum $\langle(\Delta J_y^{\mathrm{in}}\rangle$ increases as expected – a similar situation occurs in quantum optics when a Stokes parameter of light is measured by another one using the polarization-squeezed states.

The basic scheme for QND measurement of the angular momentum component J_z^{in} is shown in Fig. 6. In general, the QND apparatus contains the squeezed state $|\varsigma, m\rangle$ preparator (SP) and a "measuring box" B1, where the atomic beam interacts with a classical magnetic field, and then the probe (J_x^{out}) component is detected. Experimentally, the squeezed-state preparation can be obtained by the N two-level atomic system interacting with the broadband squeezed-photon ensemble that can be presented as a cavity in a degenerate two-photon down-conversion effect [43, 44]. In this case, the squeezing parameter ϑ characterizes the squeezed bath (i.e. the average photon number).

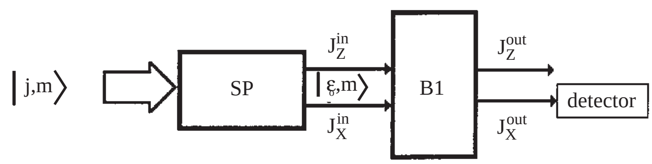

Fig. 6. Scheme for QND measurement of the J_z component of angular momentum of an atomic system. $J_{z,x}^{\mathrm{in}}$ and $J_{z,x}^{\mathrm{out}}$ are the angular momentum components at the input and the ouput respectively, of the QND apparatus. The box SP denotes the state $|\varsigma, m\rangle$ preparator before the measurement procedure

The special nature of these results encourages the study of the quantum nature of an atomic system that is coupled with spin QND measurement. In general, some information about the spin momentum can be obtained for a quantum system being in the state describing by an orbital quantum number $\ell = 0$. Let us consider the scheme (Fig. 6) for angular momentum QND measurement when the input value for $j = 1/2$. It is well known that for this case we deal with two states, $|1/2, 1/2\rangle$ and $|1/2, -1/2\rangle$, of the atomic system (where the latter is precisely the Dicke ground state) when an external magnetic field is applied to an atomic beam to split it into two parts with $m = \pm 1/2$. According to the foregoing procedure, we can carry out QND measurement of the component of angular momentum of the atoms (in this case the initial state, before the state preparator, is a Dicke ground state) when the requirement for additional interaction between the atomic beam and the classical magnetic field is satisfied. When this occurs, the Dicke ground state is formed at the entrance of the squeezed-state preparator SP. Then, after the generation of squeezing in the SP device, the atomic beam is detected by the interaction procedure with a second magnetic field. The QND measurement is achieved by detection of an angular momentum component for the atomic system.

It is important to note, however, that the proposed method of measurement without the squeezed-state preparator is identical to the well known Stern–Gerlach experiment [45]. In quantum optics such an experiment corresponds to the determination of the light polarization component by a linear system, i.e. by polarizer and/or analyzer. But as discussed above, the QND measurement procedure for the Stokes parameters of light adds a new optical element (namely an anisotropic medium of cubic nonlinearity) placed between two linear systems (polarizer and analyzer) in this setup – see Sect. 4. Thus, the quantum properties of the spin in an atomic system could be obtained by modification of the Stern–Gerlach experiment by adding the squeezed-atomic state preparator. Alternatively, the optical Stern–Gerlach effect has recently been proposed in [41] to detect the atomic state by the QND procedure.

The analysis carried out above has established a useful analogy between quantum and atomic optics and has shown that the method proposed by us for the Stokes parameter QND measurement (see Fig. 5) corresponds to a modified Stern–Gerlach experiment for the measurement of the spin states in an atomic system.

In addition we should emphasize that polarization-squeezed light could be useful for the measurement of the P-odd rotation of atomic spin [46, 47]. In fact, the effect of rotation of the atomic spin vector $\boldsymbol{F}$ around the wave vector $\boldsymbol{k}$ in an optical field arises when the quantized left- and right-hand polarized components of the optical field interact with hyperfine components of M1-transition, for example ${}^3P_0 - {}^3P_1$ for the Pb atoms placed in a resonator. In such a case, the angle of the spin rotation depends on the combination of two operators, namely the photon creation operator and the photon annihilation operator. For two orthogonally polarized modes, such a combination directly corresponds to one of the Stokes parameter, i.e. S_2 (see (2c)). Thus, precise measurement of the spin rotation angle for an atomic system becomes possible when fluctuations of the Stokes parameters are suppressed. This fact means that the quantum states of atoms can be controlled by polarization-squeezed light.

7 Conclusion

In the present paper, an opportunity to generate the polarization-squeezed state of light in a spatio-inhomogeneous nonlinear medium with high efficiency of the energy exchange between two linearly polarized modes has been discussed. The expressions obtained for the variances in the Stokes parameters of light have shown an ability to redistribute the quantum fluctuations between different polarization components of the optical field. We also presented two methods for QND measurement of the Stokes parameters: One of them can be applied to QND measurement for the phase difference of a two-mode field in quantum optics; the other gives a general approach to the SU(2) algebra observables measurement developed by us, and has resulted in a procedure for QND measurement of the angular and/or spin momentum in atomic systems.

Finally, we will discuss the main directions of possible application of the results we have obtained. First, the QND measurements under consideration could have a wide-ranging effect, especially using a polarization peculiarity of light fields, where we are dealing with precise measurements of extremely high accuracy for fundamental physical processes in order to obtain unique information about the objects under study. For example, measurements of the Stokes parameters can be useful for observations related to such major topics in general physics as the quantum polarization instabilities and chaos of light – a specific method for that could probably be associated with tunnelly-coupled or twisted birefringent optical fibers (see [27, 37]).

Secondly, the polarization QND measurements are very relevant for quantum computers (using non-classical logic elements (see for instance [48])) and for optical data processing and pattern-recognition systems [26]. In forthcoming papers, we will show how the information could be stored by the Stokes parameters of light so, that a truly logical table could be constructed.

Thirdly, the QND measurements are interesting within the quantum cryptography problem [49]. For example, the introduction of an additional channel (using the procedure for the polarization QND measurement) in common communication channels which connect two correspondents can result in the possibility of third person assembling information without its destruction.

The physical basis of such an approach is established on the fact that a quantum-mechanical measurement of one variable should change the state of the system and should introduce uncertainty into the value of other variables [50]. So the problem is directly related to the subject under discussion in our paper. Moreover, the first demonstrations [49–51] of quantum cryptography have been carried out directly using polarization approach, in other words the system has provided secure communications between two correspondents using a sequence of linearly polarized photons. Complete analysis of such a polarization approach has not yet been carried out, although at present there seems to be no doubt that quantum cryptographic systems (and especially the polarization ones) will soon be capable of reliable operation in practical use. Specific schemes for the systems under discussion are based on different kinds of interferometric approach with optical fibers (namely two-photon interferometry, a series of interferometers etc) and have been investigated repeatedly (see for example [52]).

Fourthly, let us discuss a specific power requirement for use with polarization-squeezed light in the devices considered herein. According to the problem, it seems to be that a fiber system of a special type is the best candidate, and the question is exactly the same as that which applies for quadrature-squeezed light (see Introduction). There is a requirement to generate a nonclassical light by means of an extremely low-powerd laser, and this has become possible through a special type of optical fiber fabricated recently (we discussed some of them in [3, 28]). In fact, among nonlinear devices, the chalgogenide-glass fibers (as third-order nonlinear devices) and the organic-crystal fibers (as second-order nonlinear devices) are expected to have high potential for actual applications [53]. With respect to the subject of the present paper, DFB-fibers of spatial grating including twisted birefringent optical fibers [27] and/or the dual-core tunnel-coupled fibers fabricated on the basis of such a material are very relevant for experimental verification of the quantum phenomena discussed herein. The possibility of using a short–long (about a few centimetres long) device is very important and gives a good advantage to suppress a number of nonlinear optical effects (for instance stimulated scatterings, etc) which compete with the effects considered by us in this paper. More detailed discussion of a valid experimental setup with some mathematical calculations and simulations need to be carried out separately, but to conclude our brief review let us discuss qualitatively an expected efficiency of the effects in such systems.

As to the fibers on the basis of chalgogenide glasses, for example, the high nonlinear absorption has been obtained for a laser intensity of about $10\,\mathrm{W/cm^2}$ for a fiber length of 20 cm, and this results in a remarkable nonlinear phase modulation for traveling light in such an optical fiber under He–Ne ($\lambda = 0.63\,\mu$m) laser (the diameter of the fiber core being 100–250 μm) [54]. This nonlinear effect should give rise to the development of the phenomena discussed in a amount paper.

Besides the high nonlinearity of the material itself, the dual core optical fibers have a principal advantage due to a special regime of switching (self-switching) existing two-coupled modes of orthogonal polarizations in birefringent (screwed) waveguides [28]. The effect results in the possibility of inducing a gigantic change of the intensity of the output mode by a very weak variation of input intensity under specific conditions (the so-called "optical transistor effect"). For example, for a multilayer optical fiber core fabricated on the basis of MQW structure (i.e. GaAs $\mathrm{Ga_{0.3}Al_{0.7}As}$) with a cubic nonlinear coefficient $\sim 10^{-4}$ esu, the necessary input light intensity is about 1 mW ($\lambda \sim 0.9\,\mu$m) for the fiber length ~ 1.5 mm and for core diameter $\sim 2\,\mu$m [28]. Naturally, such an effect results in dramatical change of the fluctuation behavior in the system as well. In addition, we have to mentioned that the setup discussed above is a very sensitive interferometric scheme but, in contrast to ordinary fibers, the dual-core optical fiber is not sensitive to external destabilizing factors and this is very important for such delicate states of light as those relating to quantum polarization.

The theory of quantum polarization light presented by us can be also applied to solve the problem for N identical two-level Boson-like atoms that interact with an electromagnetic field [6]. In this case, the Stokes parameters of light describe the polarization states of an atomic system. At the same time, the QND measurement of angular momentum could be useful for observation of nonclassical atomic states, including squeezed states [43]. The rapid progress that recently has been achieved towards the manipulation of quantum states of cooled atoms makes this problem soluble.

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Appendix

Here we consider the method of preparation of an optical field for QND measurement of the S_1^{in} Stokes parameter of light by the procedure discussed in [10]. Let us suppose that an optical field propagates through an anisotropic medium with third-order nonlinearity (a Kerr-like medium), as indicated in Fig. 3. The two orthogonally polarized modes, described by the operators b_x and b_y, are transformed as:

$$a_x^{\mathrm{in}} = \mathrm{e}^{\mathrm{i}(\gamma_1 b_x^+ b_x + \gamma b_y^+ b_y)} b_x \text{ and}$$
$$a_y^{\mathrm{in}} = \mathrm{e}^{\mathrm{i}(\gamma_2 b_y^+ b_y + \gamma b_x^+ b_x)} b_y \,, \tag{A.1}$$

where the coefficients $\gamma_{1,2}$ and γ are proportional to the components of nonlinear susceptibility and to the length of the medium. It can be shown that the mode photon number remains constant, i.e. $(a_{x,y}^{\mathrm{in}})^+ a_{x,y}^{\mathrm{in}} = b_{x,y}^+ b_{x,y}$ and that therefore the condition $S_{1b} = S_1^{\mathrm{in}}$ fulfilled (see Sect. 4.2).

As we have mentioned in Sect. 4, it is necessary to satisfy two conditions for the QND measurement of the Stokes parameter S_1^{in}. First, the S_3 parameter fluctuations must be suppressed, that is, PS light has to be generated. Secondly, the absence of correlations between the Stokes parameters is also required ($r_{13} = r_{12} = 0$). Taking into account the formula (6) and (A.1) for the correlation coefficients r_{13} and r_{12}, we obtain:

$$r_{13} \approx \frac{2\mathrm{e}^{-W}\langle n\rangle \sin\Psi \cos\phi}{\left\{\langle \Delta(S_3^{\mathrm{in}})^2\rangle/2\langle n\rangle\right\}^{1/2}}\,, \text{ and} \tag{A.2}$$

$$r_{12} \approx -\frac{2\mathrm{e}^{-W}\langle n\rangle \sin\Psi \sin\phi}{\left\{\langle \Delta(S_2^{\mathrm{in}})^2\rangle/2\langle n\rangle\right\}^{1/2}}\,, \tag{A.3}$$

where the variance $\langle \Delta(S_3^{\mathrm{in}})^2\rangle$ is given by the expression (see [10]):

$$\langle \Delta(S_3^{\mathrm{in}})^2\rangle \cong 2\langle n\rangle \left\{1 + \langle n\rangle(4W\cos^2\phi - \Delta\gamma \sin 2\phi)\right\} \tag{A.4}$$

Here we denote $\langle n\rangle \equiv \langle (b_x^{\mathrm{in}})^+ b_x^{\mathrm{in}}\rangle = \langle (b_y^{\mathrm{in}})^+ b_y^{\mathrm{in}}\rangle$, $\Psi = \gamma - 0.5(\gamma_1+\gamma_2)$, $W = 0.5\langle n\rangle \left\{(\gamma-\gamma_1)^2 + (\gamma-\gamma_2)^2\right\}$, and $\phi = \varphi + \Psi_{\mathrm{n1}}$, $\Delta\gamma = \gamma_2 - \gamma_1$, where $\Psi_{\mathrm{n1}} \equiv \langle n\rangle\Delta\gamma$ is the effective nonlinear phase shift, and φ is the phase-mode difference at the input of the medium. The expressions (A.2) and (A.3) are obtained under approximation, with $\gamma_{1,2}$, γ, $\langle n\rangle\gamma_{1,2}^2$ and $\langle n\rangle\gamma^2$ all $\ll 1$.

It follows from (A.2) and (A.3) that $r_{13} = r_{12} = 0$ for the case $\phi = \pi/2$ and $\phi = \pi m$ $(m = 0, 1, 2...)$ or $\Psi = 0$, i.e. $\gamma = (\gamma_1+\gamma_2)/2$. The latter condition is more preferable because we can regulate the level of quantum fluctuations,

$\langle \Delta(S_3^{in})^2 \rangle$, by adjusting phase ϕ and φ. For example, from (A.4) we get a minimum value of $\langle \Delta(S_3^{in})^2 \rangle$ for $\gamma = (\gamma_1 + \gamma_2)/2$ and the limiting phase value $\phi = -0.5 \arctan(2/\Psi_{n1})$, so that

$$\langle \Delta(S_3^{in})^2 \rangle = 2\langle n \rangle \left[1 - 0.5\Psi_{n1} \left\{ (4 + \Psi_{n1}^2)^{1/2} - \Psi_{n1} \right\} \right] \quad (A.5)$$

The expression (A.5) shows the possibility of suppression of the Stokes parameter fluctuations, which may be lower than unity for a coherent state.

It seems essential to emphasize that the value of $\langle \Delta(S_3^{in})^2 \rangle$ from (A.4) corresponds to the case of an ideal squeezing, when the uncertainty relation (5) is minimal and the correlation coefficient $r_{13} = 0$. Thus, under certain conditions for the parameters $\gamma_{1,2}$ and γ there is no correlation for the Stokes parameters, i.e. $r_{13} = r_{12} = 0$, at the output of an anisotropic cubic-nonlinearity medium. At the same time the ideal PS light is formed.

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