# Flashlamp-pumped Nd: KGW laser at repetition rates up to 50 Hz

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**Abstract.** The laser performance of a Nd:KGW rod has been studied in a double, then in a single flashlamp cavity in the free running mode at input electrical energies ranging from 1-41 J and at a pulse repetition rate from 1 to 50 Hz. A maximum average output power of 61 W was achieved with a total efficiency better than 6%. We also tested the influence of the radius of curvature of the cavity-end mirror on the output energy at high repetition rates and on the stability of the cavity. Results in Q-switched mode are also presented, obtained either with an electro-optic switch or with a Cr<sup>4+</sup>:YAG passive element. Energies up to 1.1 J and 550 mJ, respectively, are obtained with a total efficiency close to 3%.

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Laser emission has been observed in a variety of Nd host crystals such as garnet (YAG, GGG, GSGG) or yttrium composite (YLF, YVO<sub>4</sub>). Until now, the best material for flashlamp pumping has always been Nd:YAG, followed by more recently studied  $Cr^{3+}$  co-doped crystals. The problem with YAG is that a high concentration of Nd causes local distorsion of the host crystal: the ionic radius is 1.05 Å for  $Y^{3+}$  and 1.12 Å for Nd<sup>3+</sup>. The mismatch in ionic radii between these two ions may be minimized by replacing  $Y^{3+}$  with Gd<sup>3+</sup>. The size of Gd<sup>3+</sup> ions is close to that of Nd<sup>3+</sup> and the concentration in active ions can therefore be enhanced. Nd concentration as high as 3–7 at.% has been obtained in a neodymium-doped potassium gadolinium tungstate KGd(WO<sub>4</sub>)<sub>2</sub> or KGW. This relatively new crystal was tested for the first time under flashlamp pumping by Kaminskii et al. [1] and better results were recently obtained in both free-running and Q-switched modes [2-4]. The authors showed that it is possible to produce higher pulsed energies at lower threshold from a flashlamp-pumped Nd:KGW rod than from a Nd:YAG rod of identical dimensions. However, all these experiments were performed at low repetition rates (single shot or 10 Hz max) with weak thermal loading of the rod. We have tested several Nd:KGW rods with a 3 at.% Nd concentration at different pumping energies and repetition rates from single shot up to 50 Hz, which allows us to

see the effects of thermal lensing. The thermal conductivity of Nd-KGW is about one third of that for Nd:YAG and subsequently the induced thermal lensing is expected to be larger and this is easily verified. In order to compensate this lensing effect we used end-cavity mirrors with different radii of curvature from infinity to -1 m coupled if necessary with an intra-cavity convergent lens. We also present results obtained with different pumping configurations: one or two flashlamps with different filling gases, Xe or Kr.

The thermo-mechanical properties of Nd:KGW are not so good as those from Nd:YAG and we will present the results obtained for the determination of the dioptric power of this crystal through the criterion of resonator instability. The Findlay-Clay analysis will give us the round-trip losses and the parameter K, characteristic of the laser medium and of the pumping. The lasing performances are given in the freelasing and Q-switched regimes. Thanks to the use of good pumping heads and a special design of the cavity, it was possible to obtain in free-lasing mode up to 61 W of average output power at repetition rates of 40 Hz with a total efficiency close to 6% and a differential efficiency of about 7%. The treshold was very low, much lower than a comparable Nd:YAG rod under the same conditions. This opens new possibilities for the use of Nd:KGW at low pumping energies and high repetition rates. Other experiments under Q-switched mode are carried out either with an active Qswitch (Pockels cell), allowing us to obtain an energy up to 1.1 J, or with a passive one ( $Cr^{4+}$ :YAG crystal). The results obtained show a very good efficiency (more than 2.9%).

## **1** Electrical power supply

The electrical part of the laser is made of a commercial high power capacitor charging unit (Model RCS 1500 from Converter Power Inc.) with a maximum output voltage of 2 kV and maximum charging power of 1500 J/s at a repetition rate up to 100 Hz. The charge and discharge supply is a labmade specially designed pulse-forming network (PFN) made of 8 identical cells, each one with  $C = 4 \mu F$  and  $L = 14 \mu H$ . These cells can be connected in series or in parallel to ob-

tain the desired pulse duration (between 60 and 120 µs). The longer duration is close to the fluorescence lifetime of the material (about 120 µs) and the shorter durations are more suitable for Q-switched operation as we will see later. In order to increase the flashlamp lifetime, minimize the parasitic emission, and improve laser energy stability and efficiency, we use a low d.c. simmer current of about 50 to 60 m A. The maximum stored energy in the PFN is 64 J and the maximum repetition rate is 50 Hz in our experiments. Initial lamp ignition is obtained through a parallel high-voltage discharge into a transformer coupled with an automatic detection of lamp conduction. The main capacitor discharge is obtained through a high-voltage-high-current thyristor (Semikron model SKT 520-24E) and gives a rectangular and very reproducible shape for the current. The major limitation in the performances of our laser are due, as we will see later, to the maximum power of 1500 J/s of our power supply.

## 2 Laser heads and cavity

The schematic diagram of the experimental set-up is shown in Fig 1. It consists for the first experiments of a lab-made bielliptical reflective cavity (molded pyrex tube coated outside with vacuum-evaporated silver, protected by a copper layer). The laser head [5] contains two flashlamps in series with a discharge length of 3 inches and an inner and outer diameter of, respectively, 4 and 6 mm. We tested several envelope materials but the best results were obtained with a cerium doped silica envelope. The lamps are filled with xenon or krypton gas at various pressures. The pumping chamber without any flow-tube is completely filled with deionized flowing water, which also contributes to UV filtering.

The second group of experiments were made with the help of a commercial single-lamp laser head (Kigre Inc. model FE 254KK) with a Xe-filled lamp with a pumping length of 3". This allowed us to obtain better efficiency than with the double-lamp head thanks to the better coupling between the lamp and the rod with an efficient diffuser around the doublebore close-coupled element. Moreover, this allows for good filtration of the detrimental UV emission of the lamp.

The Nd:KGW, 6.35 mm × 75 mm laser rod (maximum available size) is flat ended, parallel, polished, and antireflection coated at 1.067  $\mu$ m, and was provided by K. Weber Laserkomponenten (Germany). The Nd concentration is 3 at.%. About 68 mm of the rod is effectively pumped by the lamps.The laser cavity is about 50 cm long. We tested different radii of curvature for the highly reflective end mirror (-10 m to -1 m) and we monitored the influence of this value on the output energy at high repetition rates in order to stay in the stable part of the resonator during the changes in the thermal loading of the rod. The output mirror was flat with reflectivities of 40, 50, 60, 70, 80, and 90% at  $1.06 \,\mu\text{m}$ .

In the last experiments we used an intra-cavity convergent lens with a focal length of 1 m set just in front of the concave end mirror in order to compensate the high thermal lensing of the rod and to stay in the stable regime of the cavity. This allowed us to obtain more than 60 W of average power. The laser pulse energy was measured with the help of a recently calibrated energy meter (Ophir, model P3OA) and suitable calibrated beamsplitters for the powers above 30 W.

# **3** Experimental results

#### 3.1 Free-lasing emission

First we tested the double-lamp laser head and the the influence of different gas-filled flashlamps. The different lamps were: an ILC cerium-doped flashlamp filled with xenon at a 0.5 atm pressure and Verre et Quartz flashlamps filled with krypton at 0.9 atm. There was no significant difference between the xenon and krypton lamps: the emission spectrum of the two lamps seems to match well with the pumping bands of the rod. No UV solarization was noted even at high pumping energy and fast repetition rate. Slightly better results were achieved with a krypton lamp with a high pump energy (25.8 W at 30 Hz with 27-J input).

In the study of the effects of gas-filling of the lamps it was possible to use one lamp alone in the pump cavity. As mentioned before, we did not see any noticeable difference between the two gases but we measured a better efficiency by using one lamp alone instead of two lamps in series (at the same pumping energy) although we used a double elliptical reflective cavity. We obtained an output power of 26 W at a 30-Hz repetition rate and a 27-J pump energy: the total efficiency is then about 3.2%. Two explanations can be given for this improved efficiency: first there is much less reabsorption in the lamps when only one is used; second as the same energy is delivered only in one lamp, leading to the current density being twice as large, and the lamp emission may match better the absorption bands of the rod.

Figure 2 presents the results obtained with a -1-m mirror and different repetition rates, with two flashlamps in series in the double-lamp head. We can observe the very good linearity of all the curves, indicating that we are far from saturation of the laser medium. We obtain the best output power of 26 W at 30 Hz with a pumping energy of 27 J, an output mirror of 60% reflectivity, and a -1-m end mirror. This result corresponds to an output energy of about 900 mJ per pulse at 30 Hz; the pulse length was 120 µs with our PFN made of the 8 cells in series. The threshold with a 60% output mirror was 3 J; the best efficiencies we achieved was close to 3% at 27 J



Fig.1. Experimental set-up (laser head and resonant cavity)



Fig. 2. Nd:KGW laser energy vs. pump energy with the double lamp head

and 30 Hz and 3.45% for the differential efficiency. We note the good stability of the laser efficiency at different repetition rates and at the same pump energies. By increasing the average pump power above 800 W (by increasing either the pump energy or the repetition rate) we observe a strong decrease in the output energy of the laser, indicating that we are reaching the unstable regime of the cavity by thermal lensing of the laser rod.

Then we put the same crystal into a single-lamp pumping cavity (Model FE 254 KK from Kigre Inc.). In this case the optical coupling between the lamp and the rod was much more efficient than with our previous reflective cavity and the results in terms of output power and efficiency were approximately multiplied by a factor of two. We tested, as before, different end mirrors with radii of curvature of -3, -2, and -1 m. To extend the range of cavity stability we used an endof-cavity system made of a concave  $R_{\text{max}}$  mirror with a -2-m radius and a plano-convex lens, AR coated at 1.06 µm, with a focal length of 1 m placed just before the mirror. We tested the same flat output mirrors with reflectivities of 40, 50, 60, 70, 80, and 90%. The best results were obtained with a 50% reflectivity and are presented in Fig. 3 for repetition rates up to 50 Hz. The maximum output power of 61 W was obtained for a pump energy of 31 J and a repetition rate



Fig. 3. Average output power vs. input energy for single flash operation (Kigre head) at different repetition rates

of 40 Hz, leading to a theoretical pump power of 1200 W. The compensation of the thermal effect is dramatically improved by using the lens-mirror system as compared to the -1-m mirror alone. In this last configuration the laser emission was not stable in energy and the cavity became unstable for a pump power of 700 W, leading to a maximum average output power of less than 35 W. With the lens-mirror system, the laser emission became stable in all cases from 5 to 50 Hz and it was not possible to destabalize the cavity even for the maximum available pump power of 1200 W, leading to an output power of 61 W. We can note on the curve (Fig. 3) that there appears to be a saturation of the output power above 31 J for 40 Hz, but this is due to the capacitor charging power supply which saturates (current limitation) at about 1200 J/s in our case. The maximum efficiency of the supply, (and therefore the maximum power of 1500 J/s) is obtained above 1.8 kV, but we work at 1.5 kV maximum. No effect of cavity instability was noted in this configuration (lens plus mirror) up to 1200 W of pump power and the output energy was very stable. The conditions for the maximum output power correspond to an output energy better than 1.5 J at 40 Hz.

Figure 4 presents the output energy as a function of the input energy at a repetition rate of 30 Hz. The change in the slope above 30 J indicates the beginning of power supply saturation. The maximum energy per pulse is better than 2 J at this 30-Hz repetition rate and we have obtained a maximum of 2.5 J with a pump energy of 46 J at a 20-Hz repetition rate. If we look at the total efficiency (Fig. 5), we obtain about 6% up to 20 J of pump energy and more than 5% in every case. The slope efficiency is between 6 and 7%. The laser threshold varies from 1.3 J for a mirror with  $R_{out} = 40\%$  to only 0.3 J for  $R_{out} = 90\%$ . This threshold is very low as compared with a similar Nd:YAG rod tested in the same conditions, which lies from 2 to 5 J. This standard Nd:YAG rod with the same geometry and a Nd doping of 1 at.%, presents, with the same laser head and cavity, a total efficiency of about 4%. This very low threshold for Nd:KGW suggests the possibility of realizing efficient and compact lasers with small-sized rods and pumped at high repetition rates of several hundreds of Hz.



Fig. 4. Output energy vs. input energy at a 30 Hz repetition rate



Fig. 5. Total efficiency of Nd:KGW for different pump energies

#### 3.2 Q-switched mode

The first experiments are made by inserting into the laser cavity a Pockels cell (Gsänger, model LM10 IM) with an antireflection (AR) coating for  $1.06 \,\mu\text{m}$ . The insertion losses are quite low because of the good quality of the KDP crystal. Although the Nd:KGW gives a strongly polarized beam, it is necessary to insert polarizing elements in the cavity to obtain a stable emission of the laser. For that we use a set of three parallel silica plates set at Brewster incidence. These plates have a diameter of 25 mm and a thickness of 2 mm. The results are presented on Fig. 6 and show a maximum output energy of 1.1 J at a pump energy of 41 J and a repetition rate of 20 Hz, leading to an average power of 22 W of Q-switched emission. The best results are not obtained for a 120-µs pump duration but at a shorter duration. This occurs where we see that the optimum pulse duration is about 75  $\mu$ s. This duration is obtained by varying the number of L-C cells of the PFN (in this case 5 cells in series are used). The total efficiency of the laser under these conditions is 2.9% and the pulse duration varies between 25 and 30 ns. The laser emission in this case is made of several wavelengths in the near IR owing to the strong internal Raman conversion in the Nd:KGW rod [4]. No attempt was made to separate and measure the output en-



Fig. 6. Output energy in Q-switched mode with a Pockels cell



Fig. 7. Influence of the pump duration on the output energy in the Q-switch mode

ergies delivered on the fundamental at  $1.06 \,\mu\text{m}$  and on the first and second Stokes lines at 1.18 and  $1.32 \,\mu\text{m}$ . The average power given here is the sum of the contributions of the different lines.

The second experiments were made by inserting into the cavity a passive Q-switch made of a plate of Cr<sup>4+</sup>:YAG crystal which acts as a bleachable element. The diameter of this crystal is 9.5 mm, the thickness is about 3 mm, and its optical density is about 0.3 under low illumination. This crystal is used at a weak angle of incidence and as no AR coating is deposited on the faces it is necessary to pay attention to the reflections (eventually focused) from the crystal faces which can be dangerous for the end face of the laser rod. Passive Qswitching is easily obtained but it is difficult to obtain only one laser pulse during the time of lamp excitation; generally two spikes are emitted. The results obtained are presented in Fig. 8, showing a maximum output energy of 550 mJ for a pump energy of 27 J at a 10-Hz repetition rate and with a R = 50% output mirror. No attempt was made to increase the pump energy because of the high risks of damage to the



Fig. 8. Output energy in Q-switched mode with a Cr<sup>4+</sup>:YAG crystal

rod by reflection on the bleachable crystal. The pulse duration is then slightly longer than previously: about 40 ns. The insertion loss due to the crystal is relatively high and the output energy is then lower than in the case of a Pockels cell Q-switch. A great part of the losses is due to the Fresnel reflection on the crystal, which has a high index of refraction, so it could be interesting to use an AR-coated crystal or a larger crystal set at Brewster incidence [6].

#### 4 Resonator losses

The resonator losses can be determined by measuring the threshold input energy for lasing by using output mirror with different reflectivities (Findlay–Clay method) [7].

We can write:

$$-\ln R = 2K \times E_{\rm th} - L - \ln R = 2K \times E_{\rm th} - L \,,$$

where *R* is the reflectivity of the output mirror and  $E_{th}$  is the input energy at threshold. Extrapolation of the straight-line plot of  $-\ln R$  versus  $E_{th}$ , at  $E_{th} = 0$ , yields the round-trip resonator loss *L*. The slope of the straight line is 2*K*. With *K* measured, we can then calculate the small-signal single-pass gain  $g_0 l = K E_{in}$  and the gain  $G_0 = \exp(g_0 l)$  for a given pump energy  $E_{in}$ . Figure 9 presents the results obtained. The resonator loss *L* is 12.5% and the *K* parameter is 0.39 J<sup>-1</sup>. For an input energy of 30 J we have a small-signal gain  $g_0 l$  of 11.5 and a gain  $G_0$  of 10<sup>4</sup>. We can express the laser output in terms of the input pump energy:

$$E_{\rm out} = \sigma_{\rm s} \left( E_{\rm in} - E_{\rm th} \right)$$

We can establish a simple relationship between the overall system efficiency  $\eta_{sys}$ , the slope efficiency, the gain and loss parameters, and the saturation energy density [8]:

$$\eta_{\rm sys} = \left(\frac{T+L}{T}\right)\sigma_{\rm s}$$
.

The measured overall system efficiency is 7.4%.

Our goal was also to test the thermal stability of the laser rod



Fig. 9. Findlay and Clay analysis of the resonator

at high pump powers, and to compare the thermal lensing effect in KGW in comparison with the known values for YAG. First, we tested the rod with a flat end mirror. We observed a high instability with increased pump power even at low repetition rates. We also tested several concave high-reflectivity mirrors. With -5, -3, and -2 m end mirrors one obtains a decrease in the slope efficiency connected to the repetition rate (i.e. the thermal loading). This effect can be explained by the appearance of a fast thermal gradient in the laser crystal leading to a strong thermal lensing of the rod and consequently giving a high thermal focal length for the rod and an unstable regime in the laser cavity.

To determine the length of the internal thermal lens, we use the theory of a laser resonator [9]. The stability criterion of this resonator with a lens of focal length  $\infty$  is:

$$0 < g_1 g_2 < 1$$
  
$$0 < \left(1 - \frac{L_1}{R_1}\right) \left(1 - \frac{L_2}{R_2}\right) < 1$$

with an internal lens of focal length  $f_{\rm th}$  this inequation becomes

$$0 < g_{1}^{*}g_{2}^{*} < 1$$
  
,  $g_{i}^{*} = g^{\Delta} - \frac{L_{j}}{f_{\text{th}}}g_{i}^{'}$  with  $i \neq j$   
,  $g_{i}^{\Delta} = 1 - \frac{L_{1} + L_{2}}{R_{i}}$   
,  $g_{i}^{'} = 1 - \frac{L_{1}}{R_{1}}$ .

 $L_1$ ,  $L_2$  are distance between the mirror *i* (*i* = 1 or 2) and the principal plane *i* of the lens. In our configuration, we take:  $L_1 = 110 \text{ mm}$  and  $L_2 = 230 \text{ mm}$ . For each radius of curvature of the  $R_{\text{max}}$  mirror (-5 m, -3 m, -2 m, and -1 m) we note the input power  $P_{\text{in}}$  for which the optical resonator becomes unstable. With the resolution of the inequation and knowledge of the stability diagram of the resonator, we obtain  $D_{\text{th}} = f_{\text{th}}^{-1}/P(kW)$ . Figure 10 presents the focal length



Fig. 10. Thermal lens vs input power for Nd:KGW

#### Table 1. Thermal coefficients for Nd:KGW and Nd:YAG

Parameters	Nd:YAG	Nd:KGW
Thermal conductivity [W/mK]	9.76, 12.9	(100) = 2.8, (010) = 2.2, (001) = 3.5
Thermal expansion	(100) = 8.2, (110) = 7.7,	(100) = 4.0, (010) = 3.6,
$[10^{-6} \text{ K}^{-1}]$	(111) = 7.8	(001) = 8.5
Specific heat $[Jkg^{-1} \cdot K^{-1}]$	590	500
Thermal diffusivity [cm <sup>2</sup> s <sup>-1</sup> ]	0.046	
Life time	230	110
[µs]		

Table 2. Thermal dependance of *n* for Nd:KGW

Orientation	Orientation of the polarisation		1
of the heating –	$\delta n_{\rm p}/\delta T$	$\delta n_{\rm m}/\delta T$	$\delta n_{\rm g}/\delta T$
along $n_{\rm p}$ along $n_{\rm m}$ along $n_{\rm g}$	_3+1	$+43 \pm 9$	$+17 \pm 4$ -19 $\pm 4$
	$-55 \pm 11$	$+8 \pm 2$	-1) _ +

obtained for different pump powers. We have measured an optical power of  $-1.54 \delta/kW$  for the Nd:KGW. This value can be compared to the theorical value of Nd:YAG:

$$f_{\rm th} = \frac{1}{k} \left( \frac{1}{2} \frac{\mathrm{d}n}{\mathrm{d}T} + \alpha n_0^3 C \right) \frac{P_{\rm a}}{\pi R^2},$$

which is  $0.5 \delta/kW$  for the same rod size. We conclude that

$$D_{\rm th}(\rm KGW) = 3 \times D_{\rm th}(\rm YAG)$$
.

The value of dn/dT (Table 2) and the unknown coefficient C explain the difficult calculation of the theoretical value of  $f_{\rm th}$  for the KGW. Table 1 shows the other various coefficients for Nd:YAG and Nd:KGW. We understand easily that to compensate the less favourable values for Nd:KGW it is necessary to use an end mirror with a short radius of curvature such as -1 m, or better to use a combination of a -2-m end mirror with a convergent lens of focal length of 1 m as was verified experimentally.

# **5** Conclusion

We have studied the laser performance of a Nd:KGW rod at a 3 at.% Nd concentration pumped either in a double elliptical reflective cavity or in a commercial single lamp diffusing cavity at different repetition rates. We have compensated the thermal lensing effect of the rod at high repetition rates by using relatively low radius of curvature end mirrors or a combination of a convergent lens and a concave mirror in order to stay in the stable regime of the cavity. In free-running mode, we have obtained more than 6% for the total efficiency, this value staying nearly independent of the repetition rate up to 50 Hz. At this rate we have obtained in the single-lamp cavity, an average output power of 61 W (1.5 J at 40 Hz), which is our knowledge far above the highest values obtained with a flashlamp-pumped Nd:KGW rod; moreover the output power is actually limited by the saturation of our power supply.

Future tests will be made with a 2000 J/s capacitor charging supply working at a maximum voltage of 1.5 kV. In the O-switched mode we have also obtained good efficiencies, either with a passive bleachable intra-cavity Cr<sup>4+</sup>:YAG crystal or with an active Pockels cell. In this last case, the maximum output energy is 1.1 J at a repetition rate of 20 Hz, giving a total efficiency of 2.9%. We will soon test end mirrors with shorter radii of curvature in place of the lens-mirror system. It would be also interesting to test rods with a higher Nd doping, the results presented by Kushawaha et al. [4] giving an optimum doping of 5 at.%. A power supply more suited for our pumping system should allow us to avoid the power limitation and to obtain an average power in free-lasing mode of about 100 W and in Q-switched mode of about 40 W.

We have also determined a certain number of parameters of Nd:KGW, such as the small signal gain and  $G_0$ . The calculated overall efficiency of our system is 7.4%, which makes this material one of the best solid-state laser crystals. The thermal lensing of this rod is unfortunately about 3 times more important than for Nd:YAG. Nevertheless, Nd:KGW presents some advantages over Nd:YAG with much higher efficiency at low and moderate pump energies. Some applications in free-lasing mode with average powers under 100 W (micro welding, drilling, or marking) can be viewed with Nd:KGW in place of Nd:YAG. The results obtained in the O-switched regime can be compared very favourably to those obtained with YAG and as the pulse is longer, some interesting possibilities of transmission through optical fibres are offered. Another interesting development can be for low-energy, high-repetition-rate lasers in free-lasing or Qswitched mode at a cost well under that of a diode-pumped system. For average powers above 100 W it seems that the good thermal properties of Nd:YAG ensure it has many advantages over KGW.

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