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Unusual bound modes in asymmetrically graded planar waveguides

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Abstract. We show analytically that for certain asymmetrically graded-index planar waveguides at the cutoff point where the effective refractive index of a mode coincides with the index of the substrate, there exist unusual bound modes. We have calculated the refractive-index profile that makes these modes possible.

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Most thin film waveguides used in integrated optics are not symmetrical in nature and are characterized by more or less varying refractive-index profile over the cross-section.

Planar optical waveguides having a graded-index fabricated by various diffusion processes like the diffusion of metal ions into $LiNbO_3$ and $LiTaO_3$ or ion implantation techniques [1–3] are of considerable interest for the construction of electrooptic modulators and switches. In such waveguides, the index of refraction is a smoothly varying function of depth and is higher at or near the surface than in the bulk substrate material. The essential task of the designer of these waveguides is the calculation of the modelfield distribution for a given index profile.

Here we show analytically that there exist localized modes for a certain class of asymmetrically graded-index planar guides at the cutoff point where the effective index coincides with the bulk index at infinity. We determine the refractive index profile that makes these modes possible.

1 Mode-field distribution

Let us consider that the dielectic waveguide material occupy the half-space $x > 0$. For $x < 0$ the medium is taken to be air. Thus the square of the index of refraction is defined everywhere by the following expressions:

$$n^2(x) = n_s^2 + 2n_s \Delta n\, g(x) \quad \text{for } x > 0, \tag{1a}$$

$$n^2(x) = 1 \quad \text{for } x < 0, \tag{1b}$$

where $n_s (n_s > 1)$ is the unperturbed refractive index of the substrate, the quantity $(n_s^2 + 2n_s \Delta n)^{1/2}$ is the maximum index at $x = 0$ and the shape function $g(x)$ is real, analytical function describing the refractive-index profile in the half-space $x \geqslant 0$. We assume that $g(x) \to 0$ as $x \to +\infty$, $g(x) \leqslant 1$ and that $g(0) = 1$.

The electric field for TE modes propagating in the z direction is given by $E_y = f(x)\exp[\mathrm{i}(\beta z - \omega t)]$, where β is the propagation constant determined from the solution of the Helmholtz equation for the modal function $f(x)$:

$$\mathrm{d}^2 f(x)/\mathrm{d}x^2 + (k_0^2 n^2(x) - \beta^2) f(x) = 0, \tag{2}$$

where $k_0 = 2\pi/\lambda$ is the space wave number. For $x < 0, f(x)$ is given by the exponential function

$$f(x) = B\exp[(\beta^2 - k_0^2)^{1/2} x], \tag{3}$$

where $B = f(0)$. According to the conventional waveguide theory [3], the graded-index waveguide we are considering exibit discrete bound modes (guided modes) associated to an effective index $n_e = \beta/k_0$ for $n_s < n_e < n_s + \Delta n$. It is well known that for the above-cutoff guided modes the field $f(x)$ decay exponentially in the substrate region $(x > 0)$ as $f(x) \approx \exp(-\sqrt{n_e^2 - n_s^2}\, x)$. Thus at the cutoff point where the effective index n_e coincides with the bulk index n_s of the substrate at infinity, the field spreads over the infinity region without decay. The cutoff point $(\beta/k_0 = n_e = n_s)$ defines the transition between the guided and the radiation modes of the waveguide.

Recently, however, analytically the existence of unusual bound modes at the cutoff point for certain graded-index planar waveguides with reflection symmetry across the center $(n^2(x) = n^2(-x))$ has been shown [4]. Indeed it has been found that the modal function describing the field is given by $f(x) = (1 + \alpha x)^{-1/m}$ where α and m are arbitrary positive numbers. Thus the modes are more weakly localized than those of the above cutoff because they undergo an algebric (power-law) decay rather than an exponential decay.

In this Letter, we further discuss this concept by investigating if localized modes can be supported at the cutoff point in the case of an asymmetric planar waveguide whose refractive index distribution is given by (1a) and (1b).

We find that for a choice of the shape function $g(x)$, the example waveguide we are considering supports bound modes at the cutoff given by

$$f(x) = A(1+\beta x)\exp[-(1+\alpha x)^{1-m}] \tag{4}$$

where $0 < m < 1$, α and β are positive numbers. In order to show that (4) is a localized solution of (2), let us consider that (2) at the cutoff point becomes

$$\mathrm{d}^2 f(x)/\mathrm{d}x^2 + 2n_s \Delta n\, g(x) f(x) = 0. \tag{5}$$

In (5) the spatial coordinate is normalized by k_0 as $x \to k_0 x$. Equation (4) is an eigensolution of (5) corresponding to the shape function

$$g(x) = \frac{(1-m)}{2n_s \Delta n(1+\beta x)(1+\alpha x)^m} \times [2\alpha\beta - m\alpha^2(1+\beta x)(1+\alpha x)^{-1} - (1-m)\alpha^2(1+\beta x)(1+\alpha x)^{-m}] \tag{6}$$

as can be verified by substituting (4) into (5). From (6) we easily verify that $g(+\infty) = 0$. Furthermore the condition $g(0) = 1$ implies that

$$\gamma = 2\alpha\beta - \alpha^2, \tag{7}$$

where $\gamma = 2n_s \Delta n/(1-m)$. The boundary condition requires that the field and its derivative be continuous at $x = 0$. Thus, from (3) and (5) we obtain $f(0) = A/e = B$ and

$$\beta = (1-m)\alpha + (n_s^2 - 1)^{1/2}. \tag{8}$$

Equations (7) and (8) can be used to determine the positive constant α as a function of the bulk index of the substrate n_s and Δn, namely

$$\alpha = \frac{[n_s^2 - 1 + \gamma(1-2m)]^{1/2} - (n_s^2-1)^{1/2}}{1-2m}. \tag{9}$$

The values of α and m govern the decay speed of the model field distribution $f(x)$ [cf. (4)]. The decay speed gets slower with the increase in the value of m or for values of $\alpha \to 0$, while in the limit of $m \to 0$, the mode field (4) undergoes an exponential decay.

Figure 1 shows the decaying behavior of the mode-field distribution for $m = 0.4$. The modal-field [(4) with $A = 1$] as a function of the normalized transverse x coordinate has been computed for $\Delta n = 0.05$, and for two values of the substrate index, namely, $n_s = 2$ and 2.4. Figure 2 shows the plot of the shape function $g(x)$ [(6)] as a function of the normalized transverse x coordinate for the same values of n_s, Δn and for $m = 0.4$.

In conclusion we have shown analytically that for certain asymmetrically graded-index planar waveguides at the cutoff point where the effective refractive index of a mode coincides with the index of the substrate, there exist unusual bound modes with a decay behavior $f(x) = A\exp(-\alpha x)^{1-m}$ in the limit of $x \to \infty$, where $\alpha > 0$ and $0 < m < 1$.

These modes are more or less localized depending on the values the bulk index of the substrate and the maximum value of the refractive-index difference. We have found that the family of refractive-index profiles which makes possible these modes is characterized by an algebraic decaying behavior in the limit of $x \to \infty$.

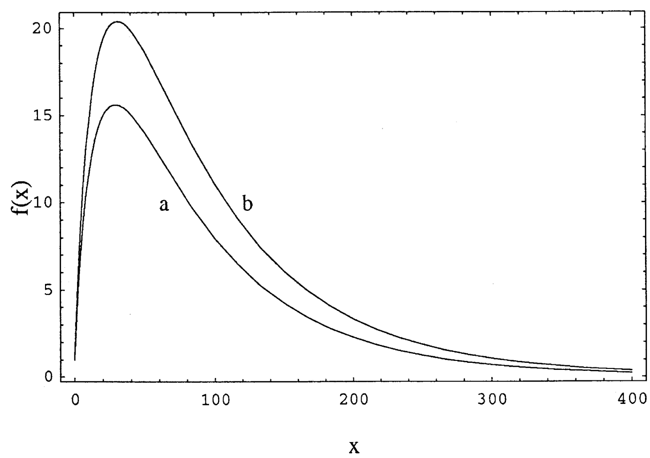

Fig. 1a, b. The modal-field distributions as a function of the normalized transverse coordinate for $\Delta n = 0.05$, $m = 0.4$; **a** $n_s = 2$, **b** $n_s = 2.4$

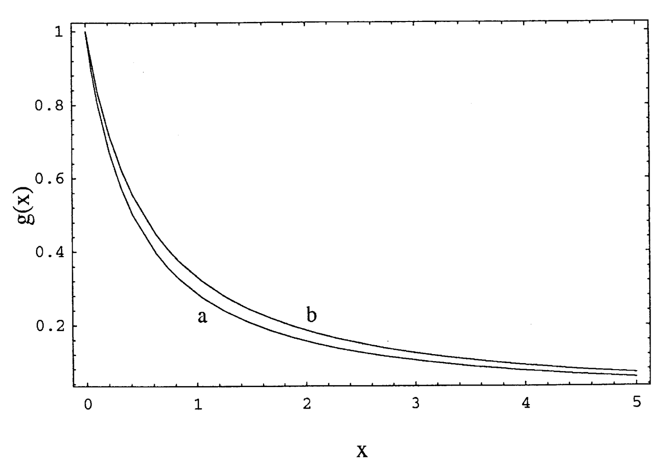

Fig. 2a, b. The shape function $g(x)$ as a function of the normalized transverse coordinate for $\Delta n = 0.05$, $m = 0.4$; **a** $n_s = 2$, **b** $n_s = 2.4$

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