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QND-measurement of twin beams by phase sensitive preamplification

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Abstract. We propose a QND-measurement of twin beams to improve the noise suppression achieved with squeezed states, which is currently limited by the finite sensitivity of the photo detectors. The scheme can be implemented with existing fiber technology.

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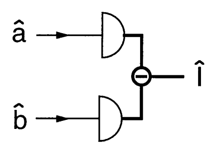

Fig. 1. Direct twin beam detection with 100% detector efficiency

Quantum theory allows for a noise-free measurement of a quantum observable. An interferometric measurement of phase imbalance in a Mach-Zehnder interferometer could be achieved, in principle, with no uncertainty (noise) using balanced homodyne detection. Balanced homodyne detection is an example of noise-free amplification of one quadrature component of the electromagnetic field, if ideal detectors are employed. The non-unity quantum efficiency of the detectors, however, prevents the achievement of noise-free amplification. For a detector quantum efficiency of 90%, the detection noise level can be only 10 dB below shot noise, under otherwise ideal conditions. Ideally, phase sensitive amplification of one phase component of the electromagnetic field is another example of noise-free amplification. If such an amplification can be realized, approaching the ideal limit more closely than balanced homodyne detection with nonideal detectors, then phase sensitive preamplification can improve the sensitivity. This has been shown by Caves for the case of dark fringe interferometry using a degenerate parametric amplifier as a phase sensitive preamplifier before detection [1]. In that case the preamplification is based on second order nonlinearities. Here, we propose a QND-measurement of twin beams by a phase-sensitive preamplification that uses third order optical nonlinearities as they occur in an optical fiber. Such a scheme can be used to improve balanced homodyne detection if detectors with non-unity quantum efficiency are employed.

The usual arrangement for detection is schematically shown in Fig.1. The twin beams $\hat{a}$ and $\hat{b}$ incident upon the balanced detectors are assumed to be correlated. Section 1 recapitulates the analysis of this case. One may, however, couple each of the twin beams to a nonlinear Mach-Zehnder QND-measurement apparatus and detect the probe beams of the measurement apparatus instead. This scheme introduces gain into the measurement. If the gain were phase insensitive, noise would be added. However, the gain achieved in a QND-measurement is phase sensitive and need not be accompanied by noise. In this way one may overcome the effect of a non-ideal quantum efficiency of the detectors. In Sect. 2 we analyze the case with a doubly resonant Kerr medium in which self-phase modulation can be suppressed. Section 3 deals with a more realistic Kerr medium, such as realized with a fiber, that includes the self-phase modulation. It is shown that even in the presence of self-phase modulation improvements in sensitivity can be achieved. In Sect. 4 we apply the concept to homodyne detection of squeezed states and in Sect. 5 we discuss its experimental implementation with existing fiber technology.

1 Measurement of photon number difference in twin beams by direct photo detection

The term ”twin beam” has connotations of quantum entanglement, such as occurs in the generation of photon pairs in parametric amplification. In this discussion we consider the general case that may or may not involve entangled states [2]. A simple model for a measurement of the difference in photon number between a pair of two monomode optical beams is direct detection with two photo detectors with 100% detection efficiency (see Fig.1).

The difference in photon number is given by

$$\hat{I} = \hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b} = \hat{n}_a - \hat{n}_b. \tag{1}$$

If the input beams are in photon number states, the mean square fluctuations in excess of the signal fluctuations vanish.

$$\begin{aligned} < \hat{I}^2 > - < \hat{I} >^2 &= < (\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b})^2 > + < \hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b} >^2 \\ &= 0\ . \end{aligned} \tag{2}$$

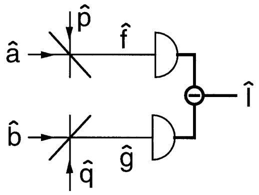

Fig. 2. Direct twin beam detection with non ideal detectors

Thus, the signal to noise ratio of the difference photon number measurement is infinite. Usually 100% detection efficiency is not feasible. We can model a photo detector with less than 100% efficiency by an ideal photo detector behind a beam splitter which taps off a fraction $1-\eta$ of the signal power before detection, where η denotes the finite detector efficiency [3] (see Fig. 2). We denote the creation operators for the unexcited ports of the beam splitters by $\hat{p}$ and $\hat{q}$. In the subsequent analysis we shall assume that they are in the vacuum state and shall call these ports the vacuum ports. The modes detected after the beam splitter are related to the input modes via

$$\hat{f} = \sqrt{\eta}\,\hat{a} + \sqrt{1-\eta}\,\hat{p} \tag{3}$$

$$\hat{g} = -\sqrt{\eta}\,\hat{b} + \sqrt{1-\eta}\,\hat{q} \tag{4}$$

Then the operator measured with such a device is given by

$$\hat{I} = \hat{f}^\dagger \hat{f} - \hat{g}^\dagger \hat{g}. \tag{5}$$

The detector current is decomposed into a singal part (the photon number difference) and a noise part

$$\hat{I} = \eta \Delta \hat{n}_s + \hat{i}_n. \tag{6}$$

The signal is represented by the photon number difference

$$\Delta \hat{n}_s = \hat{n}_a - \hat{n}_b \tag{7}$$

and the noise current part $\hat{i}_n$ is an operator consisting of the beat between the signal, as a local oscillator, and the vacuum port modes, and the noise operator from the vacuum port, of zero expectation value.

$$\hat{i}_n = \sqrt{\eta(1-\eta)}\left(\hat{a}^\dagger \hat{p} + \hat{p}^\dagger \hat{a} + \hat{b}^\dagger \hat{q} + \hat{q}^\dagger \hat{b}\right) + (1-\eta)(\hat{n}_p - \hat{n}_q). \tag{8}$$

Assuming that the beams $\hat{a}$ and $\hat{b}$ are in photon number states $|n_a>$ and $|n_b>$ with $n_a - n_b \equiv \Delta n_s$, the signal photon number, we obtain

$$<\hat{I}> = <\eta\Delta\hat{n}_s> = \eta(n_a - n_b) = \eta\Delta n_s \tag{9}$$

$$<\Delta\hat{n}_s \cdot \hat{i}_n> = <\hat{i}_n> = 0 \tag{10}$$

$$<(\hat{i}_n)^2> = \eta(1-\eta)(n_a + n_b) \tag{11}$$

Then the signal to noise ratio for direct detection of the photon number difference in the twin beams is

$$\frac{S}{N}|_{\mathrm{DD}} = \frac{<\eta\Delta\hat{n}_s>}{\sqrt{<\hat{i}_n^2>}} = \frac{\eta\Delta n_s}{\sqrt{\eta(1-\eta)(n_a+n_b)}} \tag{12}$$

For a signal to noise ratio greater than unity we must have

$$\Delta n_s > \sqrt{\frac{1-\eta}{\eta}}\sqrt{n_a + n_b}. \tag{13}$$

Thus the minimum detectable photon number difference strongly depends on the detector efficiency η.

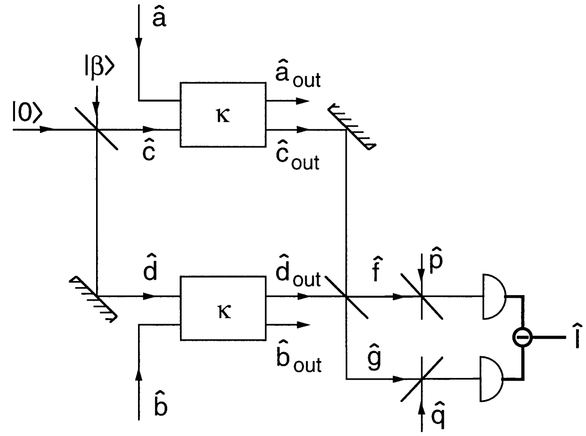

Fig. 3. QND-measurement of twin beams

2 QND-measurement of photon number difference in twin beams using cross phase modulation only

Figure 3 shows the balanced detector with phase sensitive preamplification. The system consists of a Mach-Zehnder Interferometer with Kerr media in each of its arms. The probe is imbalanced by the signal beams $\hat{a}$ and $\hat{b}$, coupled into the Kerr media via dichroic mirrors in each of the two arms. This arrangement transfers the excitation of the $\hat{a}$ and $\hat{b}$ beams onto the two probe beams. In a sense this is parametric amplification, however, the reader should be aware that it is not linear. The operators exiting the Kerr media have become nonlinear operators of the input signals that result in outputs $\hat{f}$ and $\hat{g}$ such that the difference current of the detectors is an amplified version of the input signal. If $\hat{c}$ and $\hat{d}$ are the probe beams, the Hamiltonian describing the field dynamics for the resonant Kerr media with cross-phase modulation only [4] is then given by

$$\hat{H} = \hbar K\left(\hat{a}^\dagger \hat{a}\,\hat{c}^\dagger \hat{c} + \hat{b}^\dagger \hat{b}\,\hat{d}^\dagger \hat{d}\right) \tag{14}$$

The creation operators of the signal and probe pulses leaving the interaction region are given by

$$\hat{a}_{\mathrm{out}} = e^{i\kappa\hat{c}^\dagger\hat{c}}\hat{a}_{\mathrm{in}}; \qquad \hat{b}_{\mathrm{out}} = e^{i\kappa\hat{d}^\dagger\hat{d}}\hat{b}_{\mathrm{in}} \tag{15}$$

$$\hat{c}_{\mathrm{out}} = e^{i\kappa\hat{a}^\dagger\hat{a}}\hat{c}_{\mathrm{in}}; \qquad \hat{d}_{\mathrm{out}} = e^{i\kappa\hat{b}^\dagger\hat{b}}\hat{d}_{\mathrm{in}} \tag{16}$$

with $\kappa = K\ell/v_g$, where ℓ denotes the propagation distance in the Kerr medium and v_g is the group velocity. The photon numbers $\hat{a}^\dagger\hat{a}$, $\hat{b}^\dagger\hat{b}$, $\hat{c}^\dagger\hat{c}$ and $\hat{d}^\dagger\hat{d}$ are invariants and thus we may drop the subscript "in". This is the QND nature of the measurement equipment, i.e. we do not change the photon number in each arm and therefore, we also do not change the photon number difference in the arms. With the beam splitter transformations at the output port of the NLMZ we obtain

$$\hat{f} = \frac{i}{\sqrt{2}}\left(\hat{c}_{\mathrm{out}} + \hat{d}_{\mathrm{out}}\right) \tag{17}$$

$$\hat{g} = \frac{1}{\sqrt{2}}\left(-\hat{c}_{\mathrm{out}} + \hat{d}_{\mathrm{out}}\right). \tag{18}$$

Again, we model the nonideal detectors by insertion of beam splitters, that couple the modes $\hat{f}$ and $\hat{g}$ to the vacuum ports

$\hat{p}$ and $\hat{q}$ as in Sect. 1. Hence, the detector current expressed as a function of the probe beam operators is given by

$$\hat{I} = \eta\left(\hat{c}^{\dagger}\hat{d}e^{i\kappa\Delta\hat{n}_s} + \hat{d}^{\dagger}\hat{c}e^{-i\kappa\Delta\hat{n}_s}\right) + \sqrt{\eta(1-\eta)}\left(\hat{f}^{\dagger}\hat{p} + \hat{p}^{\dagger}\hat{f} + \hat{g}^{\dagger}\hat{q} + \hat{q}^{\dagger}\hat{g}\right) + (1-\eta)(\hat{n}_q - \hat{n}_p) \tag{19}$$

with the difference photon number operator

$$\Delta\hat{n}_s = \hat{b}^{\dagger}\hat{b} - \hat{a}^{\dagger}\hat{a}. \tag{20}$$

We assume small differences in the phase shifts of the Kerr media and expand the exponentials to first order in $\Delta\hat{n}_s$. The difference current is further decomposed into a signal and noise part that results in

$$\hat{I} = -i\eta\left(\hat{c}^{\dagger}\hat{d} - \hat{d}^{\dagger}\hat{c}\right)\kappa\Delta\hat{n}_s + \hat{i}_n \tag{21}$$

with the noise source

$$\hat{i}_n = \eta\left(\hat{c}^{\dagger}\hat{d} + \hat{d}^{\dagger}\hat{c}\right) + \sqrt{\eta(1-\eta)}\left(\hat{f}^{\dagger}\hat{p} + \hat{p}^{\dagger}\hat{f} - \hat{g}^{\dagger}\hat{q} - \hat{q}^{\dagger}\hat{g}\right) + (1-\eta)(\hat{n}_q - \hat{n}_p). \tag{22}$$

The input fields of the probe beams are coherent states with amplitude $\frac{\beta}{\sqrt{2}}$ and $\frac{i\beta}{\sqrt{2}}$. Thus we obtain for the expectation value of the photo current

$$<\hat{I}>= \eta|\beta|^2\kappa\Delta n_s\ . \tag{23}$$

The noise for equal signal photon numbers, i.e. $\Delta n_s = 0$, is the shot noise

$$<(\hat{i}_n)^2>= \eta^2|\beta|^2 + \eta(1-\eta)|\beta|^2 = \eta|\beta|^2. \tag{24}$$

The signal to noise ratio for the QND twin beam detection using non ideal detectors is then

$$\frac{S}{N}|_{\text{QND}} = \frac{<\hat{I}>}{\sqrt{(\Delta\hat{I})^2}} = \sqrt{\eta}|\beta|\kappa\Delta n_s\ . \tag{25}$$

Now, let us assume a quantum efficiency η_{DD} in the direct-detection setup, and a quantum efficiency η_{QND} for the QND-scheme. Then, we obtain from Eqs. (13) and (25) for the ratio S between the two signal-to-noise ratios

$$S = \frac{\frac{S}{N}|_{\text{QND}}}{\frac{S}{N}|_{\text{DD}}} = \sqrt{\frac{\eta_{\text{QND}}}{\eta_{\text{DD}}}}\sqrt{1-\eta_{\text{DD}}}\Phi_{\text{CPM}} \tag{26}$$

with the cross phase modulation angele Φ_{CPM} given by

$$\Phi_{\text{CPM}} = \kappa|\beta|\sqrt{n_a + n_b}\ . \tag{27}$$

Eq.(26) implies that we need a large nonlinear cross phase shift Φ_{CPM} to obtain an improvement in the signal-to-noise ratio. Suppose we want to equal the signal-to-noise ratio of a 99% efficient detector in direct detection with an available detector of only 90% but using the QND scheme. Then we need a cross phase modulation angle of 10.

3 QND-Measurement of photon number difference in twin beams in Kerr media with cross and self-phase modulation

Thus far we have considered a resonant Kerr medium that avoids self-phase modulation. An optical fiber, the obvious candidate for the construction of a nonlinear Mach-Zehnder interferometer, produces both self and cross-phase modulation. In this case the Hamiltonian describing the field dynamics in the interaction zones is given by

$$\hat{H} = \hbar K\left(\frac{1}{4}\left(\hat{a}^{\dagger}\hat{a}^{\dagger}\hat{a}\hat{a}\right) + \hat{a}^{\dagger}\hat{a}\,\hat{c}^{\dagger}\hat{c} + \frac{1}{4}\left(\hat{c}^{\dagger}\hat{c}^{\dagger}\hat{c}\hat{c}\right)\right) + \hbar K\left(\frac{1}{4}\left(\hat{b}^{\dagger}\hat{b}^{\dagger}\hat{b}\hat{b}\right) + \hat{b}^{\dagger}\hat{b}\,\hat{d}^{\dagger}\hat{d} + \frac{1}{4}\left(\hat{d}^{\dagger}\hat{d}^{\dagger}\hat{d}\hat{d}\right)\right) \tag{28}$$

and therefore the creation operators of the signal and probe pulses leaving the interaction region are given by

$$\hat{a}_{\text{out}} = e^{i\kappa\left(\frac{1}{2}a^{\dagger}\hat{a}+\hat{c}^{\dagger}\hat{c}\right)}\hat{a}_{\text{in}}, \tag{29}$$

$$\hat{b}_{\text{out}} = e^{i\kappa\left(\frac{1}{2}\hat{b}^{\dagger}\hat{b}+\hat{d}^{\dagger}\hat{d}\right)}\hat{b}_{\text{in}}, \tag{30}$$

$$\hat{c}_{\text{out}} = e^{i\kappa\left(\frac{1}{2}\hat{c}^{\dagger}\hat{c}+\hat{a}^{\dagger}\hat{a}\right)}\hat{c}_{\text{in}}, \tag{31}$$

$$\hat{d}_{\text{out}} = e^{i\kappa\left(\frac{1}{2}\hat{d}^{\dagger}\hat{d}+\hat{b}^{\dagger}\hat{b}\right)}\hat{d}_{\text{in}}, \tag{32}$$

In the case of photon number difference detection as shown in Fig. 3 in the presence of cross and self phase modulation, the output operators aquire an additional self-phase modulation. We can implement the additional effect of self-phase modulation in (19) to (21) by introducing the self-phase shift into the output operators with the replacement

$$\hat{c} \rightarrow e^{i\frac{\kappa}{2}c^{\dagger}\hat{c}}\hat{c}, \tag{33}$$

$$\hat{d} \rightarrow e^{i\frac{\kappa}{2}d^{\dagger}\hat{d}}\hat{d}. \tag{34}$$

Thus the lower order moments of the self-phase modulated probe beams are given by

$$<\frac{\beta}{\sqrt{2}}|\hat{c}|\frac{\beta}{\sqrt{2}}> = \frac{\beta}{\sqrt{2}}\exp\left[\left(\exp\left(i\kappa/2\right)-1\right)|\beta|^2/2\right] \tag{35}$$

$$<i\frac{\beta}{\sqrt{2}}|\hat{d}|i\frac{\beta}{\sqrt{2}}> = i\frac{\beta}{\sqrt{2}}\exp\left[\left(\exp\left(i\kappa/2\right)-1\right)|\beta|^2/2\right] \tag{36}$$

$$<\frac{\beta}{\sqrt{2}}|\hat{c}^2|\frac{\beta}{\sqrt{2}}> = -<i\frac{\beta}{\sqrt{2}}|\hat{d}^2|i\frac{\beta}{\sqrt{2}}>. \tag{37}$$

Then we obtain for the average value and the fluctuations of the detector current

$$<\hat{I}> = \eta|\beta|^2e^{-|\beta|^2(1-\cos(\kappa/2))}\kappa\Delta n_s, \tag{38}$$

$$<(\hat{i}_n)^2> = \eta|\beta|^2 + \eta^2|\beta|^4\left(1-e^{-|\beta|^2(1-\cos(\kappa))}\right). \tag{39}$$

The phase shift per photon is assumed to be much smaller than one, i.e. $\kappa \ll 1$ which simplifies Eqs. (38), (39) to

$$<\hat{I}> = \eta|\beta|^2e^{-|\kappa\beta|^2/8}\kappa\Delta n_s \tag{40}$$

$$<(\hat{i}_n)^2> = \eta|\beta|^2 + \eta^2|\beta|^4\left(1-e^{-|\kappa\beta|^2/2}\right). \tag{41}$$

As we see from (40) we only obtain a large average signal if $|\beta\kappa| \ll 1$. In this limit we obtain

$$<\hat{I}> = \eta\kappa|\beta|^2\Delta n_s \tag{42}$$

$$<(\hat{i}_n)^2> = \eta|\beta|^2\left(1+8\eta\phi_\beta^2\right) \tag{43}$$

where $\phi_\beta = \kappa|\beta|^2/4$ is the nonlinear phase shift of the probe beam due to self-phase modulation. This shows that the signal to noise ratio for the QND-measurement with SPM is reduced to

$$\frac{S}{N}|_{\text{QND,SPM}} = \frac{\sqrt{\eta}|\beta|\kappa\Delta n_s}{\sqrt{1+8\eta\phi_\beta^2}}. \tag{44}$$

Comparison of the signal-to-noise ratio (44) with the corresponding result for cross phase modulation only, (25), shows that we get a reduction by a factor $\sqrt{1+8\eta\phi_\beta^2}$ due to self-phase modulation and that we are limited to $|\kappa\beta| \ll 1$. However, the last restriction leaves enough room for improvement since fiber experiments with $|\kappa\beta| > 1$ correspond to unrealistically high power levels [5, 6] where we are already able to detect single photons.

4 Application: Homodyne detection of quadrature squeezing

4.1 Direct homodyne detection of quadrature squeezing

Figure 4 shows the direct detection of the quadrature component of a squeezed state with a local oscillator, $\hat{c}$. We describe the squeezed state $\hat{b}$ at the input port by a Bogoliubov transformation of a vacuum mode $\hat{a}$, namely $\mu\hat{a} + \nu\hat{a}^\dagger$ where $\mu^2 - \nu^2 = 1$. The local oscillator input port $\hat{c}$ is in the coherent state $|\gamma>$. Then we obtain for the detector current

$$\hat{I} = -i\left[\left(\mu^*\hat{a}^\dagger + \nu^*\hat{a}\right)\hat{c}e^{i\theta} - \left(\mu\hat{a} + \nu\hat{a}^\dagger\right)\hat{c}^\dagger e^{-i\theta}\right] \tag{45}$$

and for the mean value and the fluctuations

$$\begin{aligned} &< \hat{I} > = 0 \\ &< \hat{I}^2 > = |\gamma|^2[\left(\mu^2 + \nu^2\right) - 2\mu\nu\cos(2\theta)] + \nu^2. \end{aligned} \tag{46}$$

Thus we obtain for the measured minimum normalized quadrature fluctuations at a detection phase angle of $\theta = 0$

$$\begin{aligned} S_{\min}^{\text{DD}} &= \frac{< \hat{I}^2 >}{|\gamma|^2} = (\mu-\nu)^2 + \frac{|\nu|^2}{|\gamma|^2} \to (\mu-\nu)^2 \\ &= S_{\min} \quad \text{for} \quad |\gamma| \to \infty \end{aligned} \tag{47}$$

In the limit of a large local oscillator these are the precise quadrature fluctuations of the squeezed state. However, in the case of a non ideal detector with quantum efficiency η we obtain with the detector model shown in Fig. 2

$$S_{\min}^{\text{DD}\eta} = S_{\min}^{\text{DD}} + \frac{(1-\eta)}{\eta} \tag{48}$$

Thus the measured squeezing is limited by a finite quantum efficiency of the detector.

4.2 QND homodyne detection of quadrature squeezing

Figure 5 shows the balanced homodyne detection of quadrature squeezing using the QND scheme for twin beam detection. We obtain for the measured fluctuations in the difference detector current for the case of finite quantum efficiency of the detectors, cross and self-phase modulation in the Kerr media (the derivation is analogous to Sect. 3)

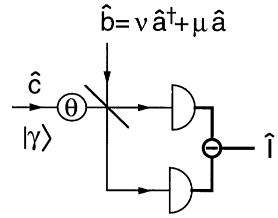

Fig. 4. Homodyne detection of quadrature squeezed state

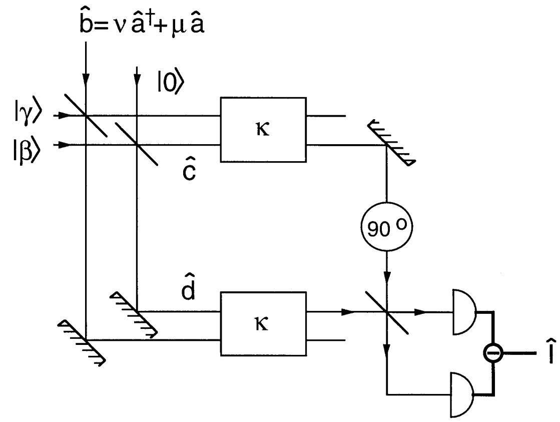

Fig. 5. QND-measurement of quadrature squeezing

$$\begin{aligned} < \hat{I}^2 > &= \eta^2|\beta|^2\left(1 + \frac{|\beta|^2}{2}e^{-|\beta|^2\left(1-\cos(\kappa/2)\right)}\right)\kappa^2|\gamma|^2 S_{\min} \\ &\quad + \eta|\beta|^2 + \eta^2|\beta|^4\left(1 - e^{-|\beta|^2(1-\cos(\kappa))}\right). \end{aligned} \tag{49}$$

We introduce the self-phase shifts $\phi_\beta = \kappa|\beta|^2/4$ and $\phi_\gamma = \kappa|\gamma|^2/4$ that the probe and signal beam undergo during propagation in the Kerr medium. For $|\kappa\beta| \ll 1$ and $|\beta|^2 \gg 1$, the normalized measured quadrature fluctuations can be written approximately as

$$S_{\min}^{\text{QND}} = \frac{2 < \hat{I}^2 >}{\eta^2|\beta|^4\kappa^2|\gamma|^2} \approx S_{\min} + \frac{\phi_\beta}{\phi_\gamma} + \frac{1}{8\eta\phi_\beta\phi_\gamma}\,. \tag{50}$$

The additional two terms in (49) are minimized for $\phi_\beta = \sqrt{1/8\eta}$. Substitution of this value into Eq. (49) leads to

$$S_{\min}^{\text{QND}} \approx S_{\min} + \frac{1}{\sqrt{2\eta}\phi_\gamma}\,. \tag{51}$$

For large enough local oscillator power $|\gamma|^2$, i.e. large nonlinear self-phase shift ϕ_γ we can overcome the limitation due to the finite detector efficiency. For example, if we choose a detector with 90% efficiency and probe beams $|\gamma>$ and $|\beta>$ so intense that we achieve a nonlinear phase shift of $\phi_\beta = \sqrt{\frac{1}{8\eta}} \approx 0.4$. Then a self-phase shift as large as $\phi_\gamma = 75$ due to the local oscillator is necessary to achieve an overall detector sensitivity of 20 dB for quadrature squeezing. Such phase shifts are possible in silica fibers. Silica fibers show a nonlinear refractive index of $n_2 = 3.6 \cdot 10^{-16}\text{cm}^2/\text{W}$. Therefore, pulse propagation in a fiber with an effective core crosssection of $A_{\text{eff}} = 100\mu\text{m}^2$ and a length $L = 50\,\text{m}$ when using pulses with a pulse energy of $W = 1\,\text{nJ}$ and a pulse length $\tau = 1\,\text{ps}$ at a wavelength of $1.3\,\mu\text{m}$ would result in a nonlinear self-phase shift $\phi = \frac{2\pi}{\lambda}\frac{n_2}{A_{\text{eff}}}\frac{W}{\tau} = 85$. Fiber sources delivering pulses with such pulse energies are available [10]. The pulse energies of the signal pulse with 1 nJ and the probe pulse of about 10 pJ correspond to about $|\gamma|^2 = 10^{10}$ and $|\beta|^2 = 10^8$ photons, respectively. Therefore, the condition $|\kappa\beta| = 4\phi_\beta/|\beta| \ll 1$ is fulfilled. From this example we can conclude, that the proposed QND-scheme is in principle possible using silica fibers.

5 Self-stabilized fiber optic realization of the QND-quadrature squeezing measurement

The scheme for a QND-quadrature squeezing measurement is only feasible if the fiber interferometer with an arm length of about 50 m can be set up in a self-stabilized way. A self stabilized setup is the Sagnac-loop introduced by Shirasaki et al. [7] which made pulsed squeezed state generation with fiber technology possible [8, 9]. Such a self-stabilized scheme is shown in Fig. 6. The upper part of the figure shows the Sagnac-loop used as a squeezer, the lower part shows the Sagnac-loop used as a self-stabilzed interferometer for the QND-twin beam detection in the balanced homodyne detector. The sagnac loop acts as a backfolded Nonlinear Mach-Zehnder Interferometer, that is possible since we use pulses. The beam splitter at the input and the output of the interferometer are realized by the some 3dB-coupler. Therefore, a pulse arriving at the coupler of the saganc loop is split into two pulses. Both pulses counterpropagate in the automatically balanced ring and undergo self-phase modulation. After recombination at the coupler the average field is rejected to the entrance port, because both pulses suffered the same phase shift in the loop. However, the squeezed fluctuations leave the other port. Thus the pump light used for the squeezing experiment is fully recovered and can be reused for the subsequent homodyne detection. From the recovered pump the probe light is tapped off. In the sagnac loop for homodyne detection both polarization states of the fiber are used. One for the propagation of the signal and the other for the probe beam. The linear polarization states are transformed to circular polarization states at the entrance of the loop by quarter wave plates, so that the above analysis that has been carried out for circular polarization states is strictly valid. At one end of the loop is a 90° phase shifter. The phase shifter is set electronically after passage of the counter clock wise propagating pulses, so that the ring is phase sensitive for the probe pulses. The difference in photon number of the detected output probe beams is then due to an imbalance in cross phase modulation by the signal beams.

6 Conclusion

Using the QND-measurement scheme for photon number measurements introduced by Imoto et.al. [4] which is based on pure cross phase modulation one can set up a QND-measurement scheme for twin-beam detection. The detection sensitivity is then not limited by the efficiency of the photo detectors. The twin-beam detector can be employed in a homodyne detector for measuring the quadrature fluctuations of squeezed light. It overcomes the limits on detection

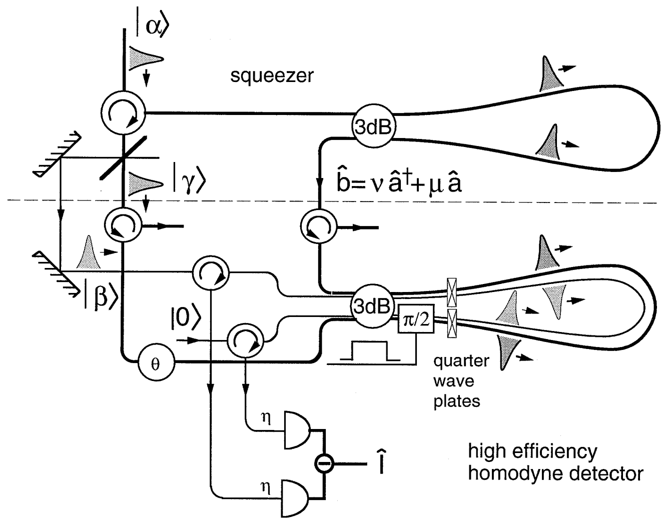

Fig. 6. Self-stabilized fiber optic realization of the quadrature squeezing measurement using QND-photodetection. The thin and thick lines in the second sagnac loop indicate the two polarisations in the fiber

sensitivity imposed by the quantum efficiency of the detectors. We showed that this scheme does not only work with pure cross phase modulation but can even be implemented using the nonlinearity of silica fibers. Using a sagnac loop very similar to the loop currently used in fiber squeezing experiments the QND-scheme is self-stabilized.

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