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Second-order coherence $g^{(2)}(\tau)$ and its frequency-dependent characteristics of a two-longitudinal-mode laser

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Abstract. The degree of second-order coherence $g^{(2)}(\tau)$ and its frequency-dependent formula of a two-longitudinal-mode laser (simply called two-mode laser) are derived from the quantum theory of light. The quasi-periodicity, frequency-dependent characteristics and photon statistics properties of $g^{(2)}(\tau)$ are analysed theoretically. It is found that for a two-mode laser field there exists the effect of photon anticorrelation and the $g^{(2)}(\tau)$ can be measured by the optical interference method. The frequency-tuning characteristics of $g^{(2)}(\tau)$ in the two-mode laser field is also analysed. It is shown that the frequency-dependent characteristics of $g^{(2)}(\tau)$ can be applied to frequency and power stabilization of the two-mode laser and the measurement of two-mode laser linewidth $\Delta\nu_D$ and the frequency width $\delta\nu_h$ of each longitudinal-mode, respectively.

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Since two-mode lasers have been widely used in frequency stabilization [1–6], interferometer of two-mode laser [7], high-resolution laser spectroscopy and optical frequency standards, two-mode squeezed state [8] and two-mode correlated spontaneous emission, etc., it is very important to investigate the properties of photon statistics and frequency dependence of a two-mode laser field, while the statistical properties of light are determined by the degree of second-order coherence $g^{(2)}(\tau)$.

Recently, the first-order temporal coherence $g^{(1)}(\tau)$ of a two-mode and multimode laser field at steady state was analysed in [9–12] using classical theory. The frequency-dependent properties of first-order temporal coherence $g^{(1)}(\tau)$ in a freely operated two-mode laser were studied in [13]. The methods of frequency and power stabilization of two-mode lasers were introduced in [1–6].

In this paper, second-order coherence and its frequency-dependent characteristics, and the photon statistical properties of the two-mode laser field are investigated. In Sect. 1, the degree of second-order coherence $g^{(2)}(\tau)$ of the two-mode laser field at the steady-state and the frequency-dependent formula of $g^{(2)}(\tau)$ for freely operated two-mode laser are derived from the quantum theory of light. In Sect. 2, the frequency-dependent characteristics and its quasi-periodicity of $g^{(2)}(\tau)$ of a two-mode laser field are calculated and analysed theoretically. In Sect. 3, the photon statistical properties of a two-mode laser, such as photon correlation and anticorrelation effects and the condition of photon non-correlation, are discussed. The frequency-tuning properties of $g^{(2)}(\tau)$ of a two-mode He–Ne laser are analysed. The possible applications of the frequency-dependent characteristics of $g^{(2)}(\tau)$ to the frequency and power stabilization and the measurement of the linewidth $\Delta\nu_D$ and frequency width $\delta\nu_h$ of the longitudinal-mode in a two-mode laser are discussed.

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1 Frequency-dependent Formula of $g^{(2)}(\tau)$

If the two-mode annihilation operators are $\hat{a}_1$ and $\hat{a}_2$, the wave functions with space variables are $u_1(\mathbf{x})$ and $u_2(\mathbf{x})$, and the mode volume is $V_1 = V_2 = V$, the electric field operator of a two-mode laser with frequencies ω_1 and ω_2 follows:

$$\hat{E}(\mathbf{x},t) = \hat{E}^{(+)}(\mathbf{x},t) + \hat{E}^{(-)}(\mathbf{x},t), \tag{1.1}$$

where

$$\hat{E}^{(+)}(\mathbf{x},t) = \sum_{k=1}^{2} u_k(\mathbf{x})\hat{a}_k \exp(-\mathrm{j}\omega_k t),$$

$$\hat{E}^{(-)}(\mathbf{x},t) = \sum_{k=1}^{2} u_{\mathbf{k}}^*(\mathbf{x})\hat{a}_k^{(+)} \exp(\mathrm{j}\omega_k t) \tag{1.2}$$

and

$$u_k(\mathbf{x}) = \mathrm{j}(\hbar\omega_k/2\varepsilon_0 V)^{1/2} \exp(\mathrm{j}\mathbf{k}_k\cdot\mathbf{x}), \quad (k = 1, 2). \tag{1.3}$$

For an ideal multimode laser field, the density matrix can be written as a coherent state with random phase

$$\rho = \prod_{k=1}^{N} \rho_k \otimes \prod_{k=N+1} |O_k \times O_k|, \tag{1.4}$$

where

$$\rho_k = |\beta_k \times \beta_k| = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\Phi_k\, |\beta_k \times \beta_k|, \tag{1.5}$$

with

$$\beta_k = |\beta_k| \exp(\mathrm{j}\Phi_k). \tag{1.6}$$

Here $|\beta_k|^2 = \langle \hat{n}_k \rangle$ is the average photon number of the kth mode, Φ_k is the random phase, $|O_k\rangle$ is the state vector of the kth mode of vacuum state. The creation operator $\hat{a}_k^{(+)}$ and the annihilation operator $\hat{a}_{k'}$ satisfy

$$\langle \hat{a}_k^{(+)} \hat{a}_{k'} \rangle = \mathrm{Tr}\{\rho \hat{a}_k^{(+)} \hat{a}_{k'}\} = \langle \hat{n}_k \rangle \delta_{kk'}. \tag{1.7}$$

For a two-mode laser field, $N = 2$, $k = 1, 2$. From the definition of the second-order quantum correlation function of the light field

$$G^{(2)}(\tau) = \langle : \hat{I}(t)\hat{I}(t+\tau) : \rangle = \langle \hat{E}^{(-)}(t)\hat{E}^{(-)}(t+\tau)\hat{E}^{(+)}(t+\tau)\hat{E}^{(+)}(t) \rangle, \tag{1.8}$$

one has

$$G^{(2)}(\tau) = [|u_1|^2\langle \hat{n}_1 \rangle + |u_2|^2 \langle \hat{n}_2 \rangle]^2 + 2|u_1|^2|u_2|^2\langle \hat{n}_1 \rangle \langle \hat{n}_2 \rangle \cos(\Delta\omega_{\mathrm{q}}\tau). \tag{1.9}$$

Since the first-order quantum correlation function of a two-mode laser field is given by

$$G^{(1)}(\tau) = \langle \hat{E}^{(-)}(t)\hat{E}^{(+)}(t+\tau) \rangle = |u_1|^2\langle \hat{n}_1 \rangle \exp(-\mathrm{j}\omega_1\tau) + |u_2|^2\langle \hat{n}_2 \rangle \exp(-\mathrm{j}\omega_2\tau) \tag{1.10}$$

Equation (1.9) can be rewritten as

$$G^{(2)}(\tau) = [|u_1|^2\langle \hat{n}_1 \rangle + |u_2|^2\langle \hat{n}_2 \rangle]^2 - [|u_1|^4\langle \hat{n}_1 \rangle^2 + |u_2|^4\langle \hat{n}_2 \rangle^2] + |G^{(1)}(\tau)|^2. \tag{1.11}$$

From the definition of the degree of second-order coherence

$$g^{(2)}(\tau) = G^{(2)}(\tau)/[G^{(1)}(0)G^{(1)}(0)] = \frac{\langle : \hat{I}(t)\hat{I}(t+\tau) : \rangle}{\langle \hat{I}(t) \rangle^2}, \tag{1.12}$$

the general formula of the degree of second-order coherence of a two-mode laser field at the steady state is given by

$$g^{(2)}(\tau) = 1 - \frac{\langle \hat{I}_1 \rangle^2 + \langle \hat{I}_2 \rangle^2}{[\langle \hat{I}_1 \rangle + \langle \hat{I}_2 \rangle]^2} + |g^{(1)}(\tau)|^2, \tag{1.13a}$$

where $\langle \hat{I}_1 \rangle = \hbar\omega_1\langle \hat{n}_1 \rangle$, $\langle \hat{I}_2 \rangle = \hbar\omega_2\langle \hat{n}_2 \rangle$ are the output intensities of a two-mode laser at the steady state.

If $k = \langle \hat{I}_1 \rangle / \langle \hat{I}_2 \rangle$ is the relative intensity of the two-mode output, (1.13a) can be rewritten as

$$g^{(2)}(\tau) = \frac{2k}{(1+k)^2} + |g^{(1)}(\tau)|^2, \tag{1.13b}$$

where $g^{(1)}(\tau)$ is the degree of first-order coherence of the light. In order to discuss the influence of the frequency width $\delta\nu_{\mathrm{H}}$ of longitudinal mode upon the $g^{(2)}(\tau)$, we employ the semi-classical theory of light to derive the degree of first-order coherence $g^{(1)}(\tau)$ [11, 13]. The form of $g^{(1)}(\tau)$ is given by

$$g^{(1)}(\tau) = \frac{\exp(-\pi\delta\nu_{\mathrm{H}}\tau)}{\langle \hat{I}_1 \rangle + \langle \hat{I}_2 \rangle} |\langle \hat{I}_1 \rangle^2 + \langle \hat{I}_2 \rangle^2 + 2\langle \hat{I}_1 \rangle \langle \hat{I}_2 \rangle \times \cos(2\pi\Delta\nu_{\mathrm{q}}\tau)|^{1/2} = \frac{\exp(-\pi\delta\nu_{\mathrm{H}}\tau)}{1+k} |1 + k^2 + 2k\cos(2\pi\Delta\nu_{\mathrm{q}}\tau)|^{1/2}, \tag{1.14}$$

where $\delta\nu_{\mathrm{H}}$ is the frequency width of the longitudinal modes. When $\delta\nu_{\mathrm{H}} \to 0$, or the delay time τ (i.e., optical path difference $\Delta l = C\tau$) is very small, (1.13) can be simplified as

$$g^{(2)}(\tau) = 1 + \frac{2\langle \hat{I}_1 \rangle \langle \hat{I}_2 \rangle}{[\langle \hat{I}_1 \rangle + \langle \hat{I}_2 \rangle]^2} \cos(2\pi\Delta\nu_{\mathrm{q}}\tau) = 1 + \frac{2k}{(1+k)^2}\cos(2\pi\Delta\nu_{\mathrm{q}}\tau). \tag{1.15}$$

If the output intensities of the two modes are the same, i.e., $\langle \hat{I}_1 \rangle = \langle \hat{I}_2 \rangle$, or $k = 1$, with $\delta\nu_{\mathrm{H}} \to 0$, (1.13a) or (1.15) can be simplified further

$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2 = 1 + \tfrac{1}{2}\cos(2\pi\Delta\nu_{\mathrm{q}}\tau). \tag{1.16}$$

It is clear that the range of the degree of second-order coherence $g^{(2)}(\tau)$ of a two-mode laser field is: $\frac{1}{2} \le g^{(2)}(\tau) \le \frac{3}{2}$. In addition, (1.16) shows that the degree of second-order coherence $g^{(2)}(\tau)$ (or second-order correlation function) of a two-mode laser field can be determined by the measurement of the degree of first-order coherence, i.e., the second-order coherence and statistical properties of an electromagnetic field can be studied by the optical interference method. Of course, this is a method of indirect-measurement on $g^{(2)}(\tau)$.

If the influence of the drift effect $\Delta\nu$ of longitudinal-mode frequency on the two-mode output intensities in a freely operated laser is considered, the general formula (1.13) of the frequency-dependent $g^{(2)}(\tau)$ in a two-mode laser field can be modified as

$$g^{(2)}(\Delta\nu, \tau) = 1 - \frac{\hat{I}_1^2(\Delta\nu) + \hat{I}_2^2(\Delta\nu)}{[\hat{I}_1(\Delta\nu) + \hat{I}_2(\Delta\nu)]^2} + |g^{(1)}(\Delta\nu, \tau)|^2 = \frac{2k(\Delta\nu)}{[1 + k(\Delta\nu)]^2} + |g^{(1)}(\Delta\nu, \tau)|^2, \tag{1.17}$$

where $\Delta\nu = \nu_1 - (\nu_0 - \Delta\nu_{\mathrm{q}}/2) = \nu_2 - (\nu_0 + \Delta\nu_{\mathrm{q}}/2)$. The corresponding $g^{(1)}(\Delta\nu, \tau)$ is given by [13]

$$g^{(1)}(\Delta\nu, \tau) = \frac{\exp(-\pi\delta\nu_{\mathrm{H}}\tau)}{\hat{I}_1(\Delta\nu) + \hat{I}_2(\Delta\nu)} |\hat{I}_1(\Delta\nu)^2 + \hat{I}_2(\Delta\nu)^2 + 2\hat{I}_1(\Delta\nu)\hat{I}_2(\Delta\nu)\cos(2\pi\Delta\nu_{\mathrm{q}}\tau)|^{1/2} = \frac{\exp(-\pi\delta\nu_{\mathrm{H}}\tau)}{1 + k(\Delta\nu)} |1 + k(\Delta\nu)^2 + 2k(\Delta\nu) \times \cos(2\pi\Delta\nu_{\mathrm{q}}\tau)|^{1/2}, \tag{1.18}$$

where $k(\Delta\nu) = \hat{I}_1(\Delta\nu)/\hat{I}_2(\Delta\nu)$ is the instantaneous intensity ratio of the two-mode output at the drift amount $\Delta\nu$ of longitudinal-mode frequency. When $\delta\nu_H$ can be neglected, or τ (i.e., Δl) is very small, (1.17) can be simplified as

$$g^{(2)}(\Delta\nu,\tau) = 1 + \frac{2\hat{I}_1(\Delta\nu)\hat{I}_2(\Delta\nu)}{[\hat{I}_1(\Delta\nu)+\hat{I}_2(\Delta\nu)]^2}\cos(2\pi\Delta\nu_q\tau)$$
$$= 1 + \frac{2k(\Delta\nu)}{[1+k(\Delta\nu)]^2}\cos(2\pi\Delta\nu_q\tau). \quad (1.19)$$

For a two-mode He–Ne laser working at steady state, if the two-mode intensities are equal, i.e., $k = 1$, the degree of second-order coherence $g^{(2)}(\tau)$ is given by

$$g^{(2)}(\tau) = \tfrac{1}{2} + \exp(-2\pi\delta\nu_H\tau)\cos^2(\pi\Delta\nu_q\tau). \quad (1.20)$$

However, for a freely operated two-mode He–Ne laser, the ratio of the two-mode laser intensities at $\Delta\nu$ is given by

$$k(\Delta\nu) = \hat{I}_1(\Delta\nu)/\hat{I}_2(\Delta\nu) = \exp\left[\frac{8\ln 2}{\Delta\nu_D^2}\cdot\Delta\nu_q\cdot\Delta\nu\right], \quad (1.21)$$

where $\Delta\nu_D$ and $\Delta\nu_q$ are the laser linewidth and the space of longitudinal modes, $\Delta\nu$ is the amount of the frequency drift of the two-mode frequency ν_1, ν_2 relative to the frequency-symmetric point $(\nu_0 \pm \Delta\nu_q/2)$. Substituting (1.21) into (1.17) and (1.18), one has the general formula of the frequency-dependent $g^{(2)}(\tau)$

$$g^{(2)}(\Delta\nu,\tau) = \frac{1+\exp(-2\pi\delta\nu_H\tau)[\cos(2\pi\Delta\nu_q\tau)+\cosh(8\ln 2\cdot\Delta\nu_q\cdot\Delta\nu/\Delta\nu_D^2)]}{1+\cosh(8\ln 2\cdot\Delta\nu_q\cdot\Delta\nu/\Delta\nu_D^2)}. \quad (1.22)$$

If $\Delta\nu$ does not vary with the time t, (1.22) becomes the general formula of $g^{(2)}(\tau)$ of the two-mode He–Ne laser field at the steady state. When $\delta\nu_H \to 0$, or τ(i.e., Δl) is very small, (1.22) reduces to

$$g^{(2)}(\Delta\nu,\tau) = 1 + \frac{\cos(2\pi\Delta\nu_q\tau)}{1+\cosh(8\ln 2\Delta\nu_q\Delta\nu/\Delta\nu_D^2)}, \quad (1.23)$$

where $\Delta\nu_q = C/2nL \approx C/2L$, and L is the length of the laser cavity.

2 Frequency-dependent Characteristics of $g^{(2)}(\tau)$

Assuming $\Delta l = 2mL (m = 0, \pm 1, \pm 2, \dots)$, i.e., the optical path difference Δl is even multiples of the laser cavity length L, from (1.17) and (1.18), one obtains the frequency-dependent $g^{(2)}(\tau)$ of the two-mode laser field

$$g^{(2)}(\Delta\nu, 2mL) = \exp(-4m\pi\delta\nu_H L/c) + \frac{2k(\Delta\nu)}{[1+k(\Delta\nu)]^2} \quad (2.1)$$

and

$$g^{(2)}(\Delta\nu, 2mL) = 1 + \frac{2k(\Delta\nu)}{[1+k(\Delta\nu)]^2}. \quad (2.2)$$

Similarly, if $\Delta l = (2m+1)L (m = 0, \pm 1, \pm 2, \dots)$, i.e., the optical path difference is odd multiples of the laser cavity length L, the frequency-dependent $g^{(2)}(\tau)$ of the two-mode laser field is given by

$$g^{(2)}(\Delta\nu,(2m+1)L) = \frac{2k(\Delta\nu) + [1-k(\Delta\nu)]^2\exp[-2(2m+1)\pi\delta_H L/C]}{[1+k(\Delta\nu)]^2} \quad (2.3)$$

and

$$g^{(2)}(\Delta\nu,(2m+1)L) = 1 + \frac{2k(\Delta\nu)}{[1+k(\Delta\nu)]^2}. \quad (2.4)$$

Obviously, at $\Delta l = 2mL$, or $\Delta l = (2m+1)L$, the degree of second-order coherence $g^{(2)}(\tau)$ is related to the relative intensity $k(\Delta\nu)$, i.e., to the frequency-drift effect $\Delta\nu$ of the longitudinal modes.

Moreover, when $\Delta l = (2m+1)L/2$, $(m = 0, \pm 1, \pm 2, \dots)$, i.e., the optical path difference is odd multiples of $L/2$, the frequency-dependent $g^{(2)}(\tau)$ of the two-mode laser field is given by

$$g^{(2)}[\Delta\nu,(2m+1)L/2] = 1 \quad (2.5)$$

if $\delta\nu_H \to 0$.

When the longitudinal mode is drifted and the relative intensity ratio $k(\Delta\nu)$ varies from 0.1 to 10.0, the frequency-dependent curve of $g^{(2)}(\tau)$ is calculated from (1.17) and (1.18) for the optical path diference $\Delta l = 0, \frac{L}{2}, L, \frac{3}{2}L$, and $2L$, or for $k(\Delta\nu) = 0.1, 0.5, 1.0, 5.0$ and 10.0. The variation of $g^{(2)}(\tau)$ against the relative intensity ratio $k(\Delta\nu)$, or the optical path difference Δl for $L = 25$ cm ($\Delta\nu_q = 600$ MHz), $\delta\nu_H = 30$ MHz is shown in Figs. 1a and 2a, respectively. When $\delta\nu_H = 0$, the relative results are shown in Figs. 1b and 2b. If we compare Fig. 1a with Fig. 1b, or compare Fig. 2a with Fig. 2b, it can be found that $g^{(2)}(\tau)$ of the two mode laser is intensely depend on the frequency width $\delta\nu_H$ of longitudinal mode, and the periodicity of $g^{(2)}(\tau)$ becomes the quasi-periodicity when $\delta\nu_H \neq 0$. The maximum values of $g^{(2)}(\Delta\nu,\tau)$ are $\exp(-2\pi\delta\nu_H\Delta l/c) + 2k(\Delta\nu)/[1+k(\Delta\nu)]^2$ at $\Delta l = 2mL$, the minimum values of

$$g^{(2)}(\Delta\nu,\tau) \text{ are } 2k(\Delta\nu)/[1+k(\Delta\nu)]^2 + \exp(-2\pi\delta\nu_H\Delta l/c)$$
$$\times\left[\frac{1-k(\Delta\nu)}{1+k(\Delta\nu)}\right]^2 \text{ at } \Delta l = (2m+1)L,$$

the relative variation range is from 0 to $\frac{3}{2}$. When $\Delta\nu$ is equal to zero, the maximum values of $g^{(2)}(\Delta\nu,\tau)$ are $\frac{1}{2} + \exp(-2\pi\delta\nu_H\Delta l/c)$ at $\Delta l = 2mL$, the minimum values of $g^{(2)}(\Delta\nu,\tau)$ are constantly $\frac{1}{2}$ at $\Delta l = (2m+1)L$. The corresponding period is $2L$.

For a two-mode He–Ne laser, if $L = 25$ cm, $\delta\nu_H = 30$ MHz, $\Delta\nu_D = 1000$ MHz, and the longitudinal-mode drift amount $\Delta\nu$ drifts from $-\Delta\nu_q/2$ to $\Delta\nu_q/2$ in one direction, the frequency-dependent curve of $g^{(2)}(\tau)$ can be calculated from (1.22) for $\Delta l = 0, \frac{L}{2}, L, \frac{3}{2}L$, and $2L$, or for

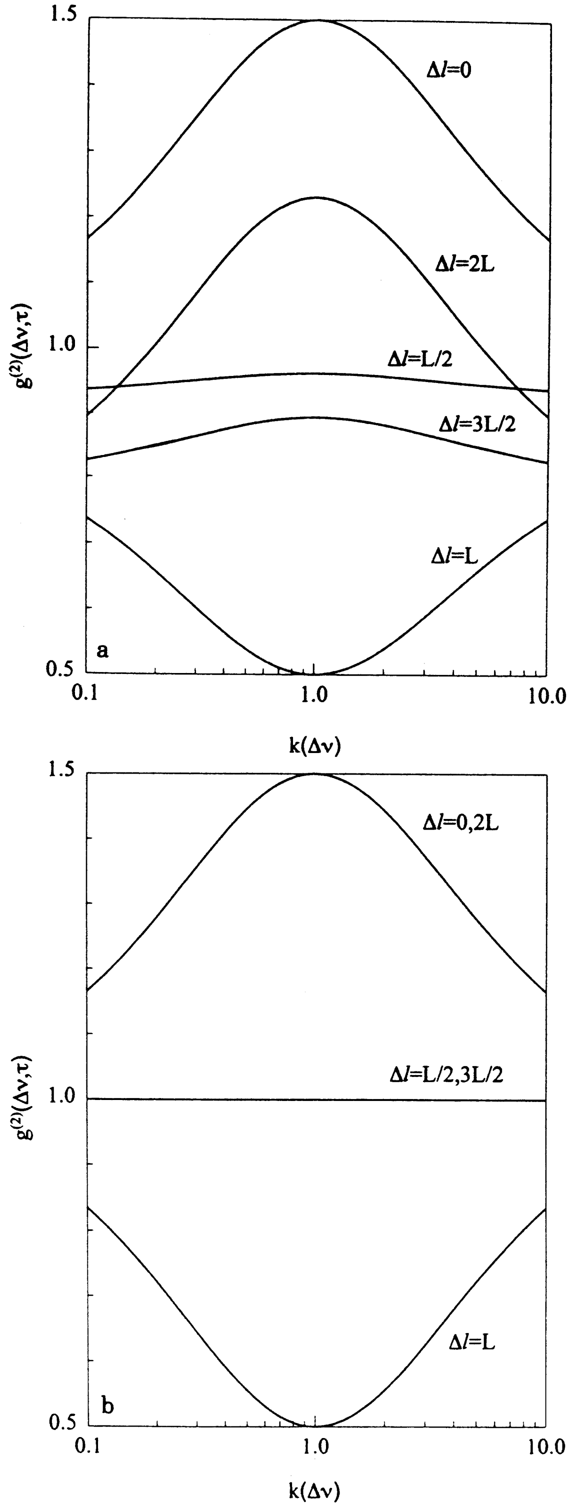

Fig. 1. The frequency-dependent curve of the variation of $g^{(2)}(\tau)$ against the relative intensity ratio $k(\Delta\nu)$ for a general two-mode laser. In Fig. 1: (a) $\delta\nu_H = 30$ MHz; (b) $\delta\nu_H = 0$; Other calculated parameters: $L = 25$ cm, and $\Delta\nu_q = 600$ MHz

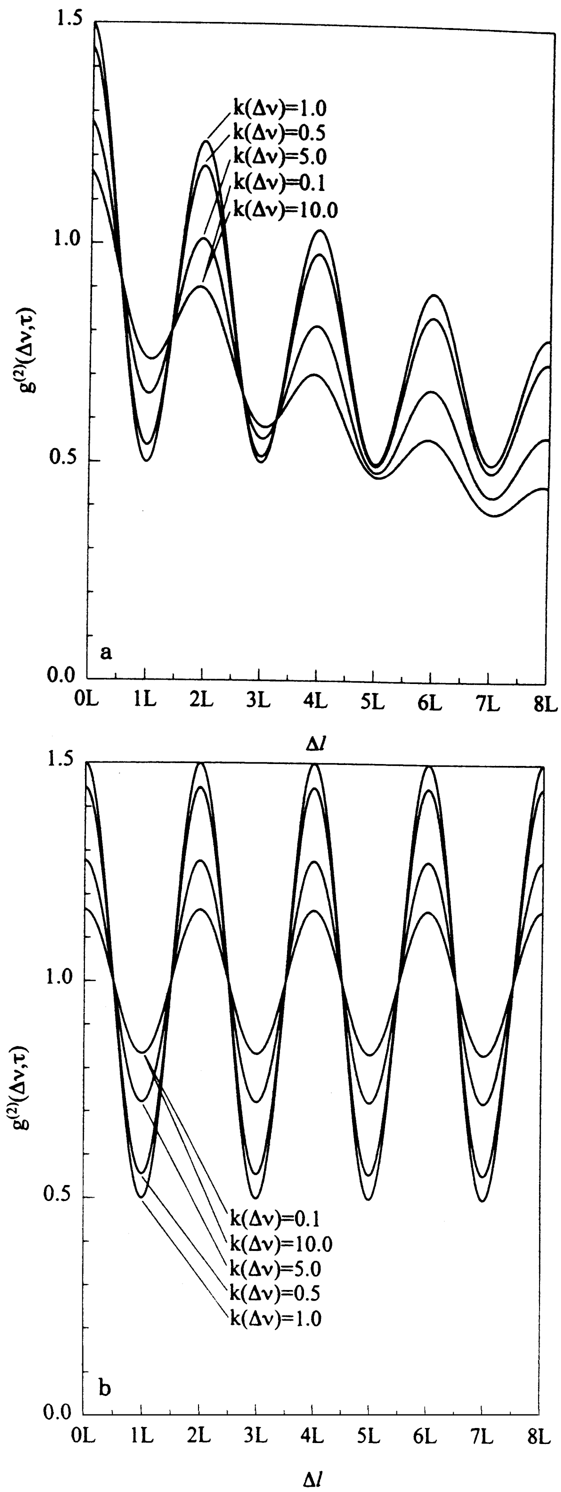

Fig. 2. The frequency-dependent curve of the variation of $g^{(2)}(\tau)$ against the optical path difference Δl for a general two-mode laser. In Fig. 2: (a) $\delta\nu_H = 30$ MHz; (b) $\delta\nu_H = 0$; Other calculated parameters: $L = 25$ cm, and $\Delta\nu_q = 600$ MHz

$\Delta\nu = -\frac{1}{2}\Delta\nu_q$, $-\frac{1}{4}\Delta\nu_q, 0, \frac{1}{4}\Delta\nu_q$ and $\frac{1}{2}\Delta\nu_q$. The variation of $g^{(2)}(\tau)$ against the relative intensity ratio $k(\Delta\nu)$, or the optical path difference Δl is shown in Figs. 3a and 4a, separately. When $\delta\nu_H = 0$, the relative results are shown in Figs. 3b and 4b. If $k(\Delta\nu)$ and $\Delta\nu$ are substituted by the steady state k and $\Delta\nu$, Figs. 1–4 are the curves of $g^{(2)}(\tau)$ of the two-mode laser field at the steady state calculated from (1.17), (1.18) and (1.22).

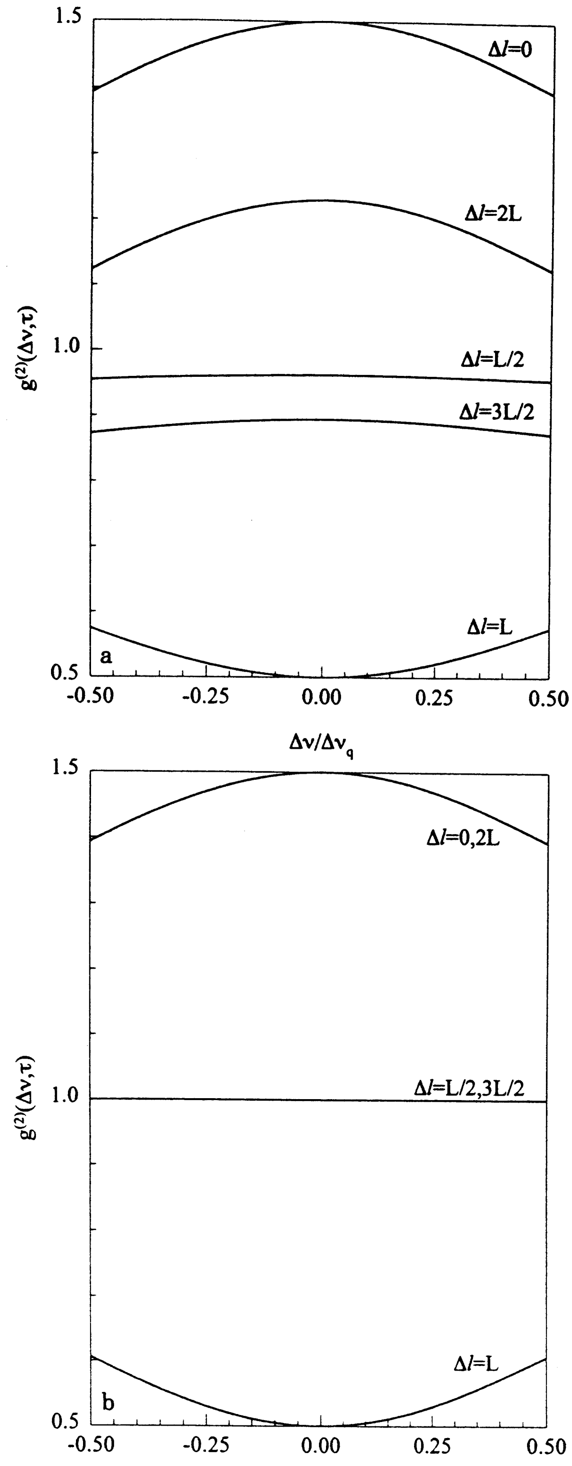

Fig. 3. The frequency-dependent curve of the variation of $g^{(2)}(\tau)$ against the relative intensity ratio $k(\Delta\nu)$ for a two-mode He–Ne laser. In Fig. 3: (a) $\delta\nu_H = 30$ MHz; (b) $\delta\nu_H = 0$; Other calculated parameters: $L = 25$ cm, $\Delta\nu_q = 600$ MHz, and $\Delta\nu_D = 1000$ MHz

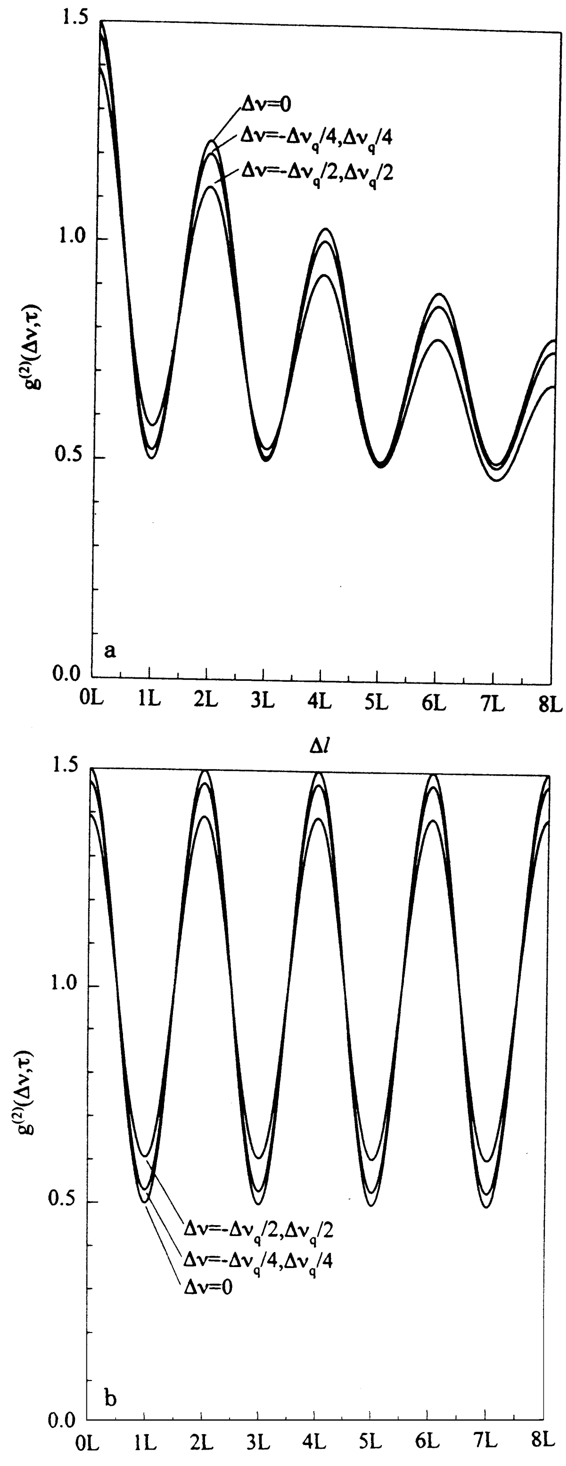

Fig. 4. The frequency-dependent curve of the variation of $g^{(2)}(\tau)$ against the optical path difference Δl for a two-mode He–Ne laser. In Fig. 4: (a) $\delta\nu_H = 30$ MHz; (b) $\delta\nu_H = 0$; Other calculated parameters: $L = 25$ cm, $\Delta\nu_q = 600$ MHz; and $\Delta\nu_D = 1000$ MHz

From the above theoretical analysis, it is seen that the $g^{(2)}(\tau)$ of the two-mode laser field has the following frequency-dependent properties.

(1) The degree of second-order coherence $g^{(2)}(\tau)$ of the freely operated two-mode laser field is not only related to the optical path difference Δl but also related to the longitudinal mode drift effect $\Delta\nu$. If the influence of the longitudinal-mode frequency width $\Delta\nu_H$ is considered, $g^{(2)}(\Delta\nu, \Delta l)$ decays with quasi-periodic oscillations as a function of the optical path difference Δl. The period is $2L$.

(2) When $\Delta l = 2mL$, $g^{(2)}(\Delta\nu, 2ml)$ of the two-mode laser field is not only related to the laser linewidth $\Delta\nu_D$ and longitudinal-mode linewidth $\delta\nu_H$, but also related to the longitudinal-mode drift amount $\Delta\nu$. If the drift amount $\Delta\nu$ is small, the value of $g^{(2)}(\Delta\nu, 2mL)$ becomes large. The curve of the frequency-dependent characteristics presents a shape of inverse "V". This is quite different from the frequency-dependent characteristics of $g^{(1)}(\tau)$, which is $g^{(1)}(\Delta\nu, 2mL) = 1$ [13].

(3) When $\Delta l = (2m+1)L$, the frequency-dependent characteristics curve of $g^{(2)}(\Delta\nu, (2m+1)L)$ is opposite to that of $g^{(2)}(\Delta\nu, 2mL)$ at $\Delta l = 2mL$. It appears as a "V" shape curve and symmetric about the direct line $\Delta\nu = 0$.

(4) When $\Delta l = (2m+1)L/2$ and when $\delta\nu_H = 0$, or Δl is very small, the $g^{(2)}(\tau)$ of the two-mode laser field depends only on the longitudinal-mode frequency width $\delta\nu_H$ and independent of the longitudinal-mode drift effect $\Delta\nu$. These are only time–space points that are independent of $\Delta\nu$ in $g^{(2)}(\tau)$ of the two-mode laser field.

(5) When the influence of $\delta\nu_H$ is considered at $\Delta l = 0, 2L$(or L), the variation of $g^{(2)}(\Delta\nu, \tau)$ of the two-mode laser field as a function of the longitudinal-mode drift $\Delta\nu$ is the largest. The variation range is about from $\frac{1}{2}$ to $\frac{3}{2}$. This is very suitable for the measurement of laser linewidth $\Delta\nu_D$ and the frequency and power stabilization of the two-mode laser.

3 Analyses and discussions

3.1 Photon statistical properties

The degree of second-order coherence $g^{(2)}(\tau)$ of a light field reflects the correlations between photons at time t and time $t+\tau$ in a time–space point of the light field. To describe the properties of photon correlation in light field, it is usually called $g^{(2)}(\tau) > 1$ photon correlation effect while $g^{(2)}(\tau) < 1$ photon anticorrelation effect (in a two-mode laser field, this definition is equivalent to the standard definitions $g_{12}^{(2)}(\tau) < 1$, or $C_{12}(\tau) < 0$). When $g^{(2)}(\tau) = 1$, it is called photon non-correlation effect [14]. For a two-mode laser field at steady state, if $\delta\nu_H \to 0$, $g^{(2)}(\tau)$ of (1.15) shows variation from $\frac{1}{2}$ to $\frac{3}{2}$ periodically. It varies as a function of the time delay τ, or the optical path difference Δl. Some time–space points in the two-mode laser field show periodical photon correlation and anticorrelation effect. At the points $\Delta l = (2m+1)L/2$, there are photon non-correlation effects. If the influence of $\delta\nu_H$ is considered, the two-mode laser field shows non-correlation effect $(g^{(2)}(\tau_0) = g^{(2)}(\Delta l_0/C) = 1)$ at following time–space points Δl_0

$$1 - \exp(2\pi\delta\nu_H \Delta l_0/C) = \frac{2k}{1+k^2}\cos(2\pi\Delta\nu_q \Delta l_0/C). \qquad (3.1)$$

For a freely running two-mode laser field, it is seen from Figs. 2 and 4 that the variation range of $g^{(2)}(\tau)$ is from 0 to 1.5 when $\Delta\nu \neq 0$, while when $\Delta\nu = 0$ the variation range of $g^{(2)}(\tau)$ is from 0.5 to 1.5. In other words, Figs. 1, 2, and 4 show that photon correlation and anti-correlation effects vary periodically not only with time delay τ (or Δl) but also with the drift effect $k(\Delta\nu)$ (or $\Delta\nu$) of longitudinal-mode frequency. At $\Delta l = 2mL$, the light field shows photon correlation effect and at $\Delta l = (2m+1)L/2$, it shows photon non-correlation effect. If the influence of $\delta\nu_H$ is considered, the photon correlation effect of a two-mode laser field decreases while photon anticorrelation effect increases when $\Delta\nu \neq 0$. The amount of decrease or increase depends on the magnitude of $\delta\nu_H$, $\Delta\nu$ and Δl. While when $\Delta\nu = 0$ the frequency width $\delta\nu_H$ reduces only the photon correlation effect, and does not affect the photon anticorrelation effect at $\Delta l = (2m+1)L$. Since $g^{(2)}(0) \geq g^{(2)}(\tau)$, the photon anticorrelation effect in the two-mode laser is not a non-classical effect. The two-mode laser field is an only example that both photon anticorrelation and photon bunching effects appear.

3.2 Frequency-tuning properties of $g^{(2)}(\tau)$

For a two-mode He–Ne laser with frequency tuning $\Delta\nu$, optical path difference $\Delta l = L$, from (1.22), one has

$$g^{(2)}(\Delta\nu, L) = \frac{\cosh(8\ln 2\cdot\Delta\nu_q\cdot\Delta\nu/\Delta\nu_D^2)}{1+\cosh(8\ln 2\cdot\Delta\nu_q\cdot\Delta\nu/\Delta\nu_D^2)} \qquad (3.2)$$

This is the tuning equation of $g^{(2)}(\tau)$ for a two-mode He–Ne laser field at $\Delta l = L$. Similarly, when $\Delta l = 0$ or $2L$, (1.22) gives

$$g^{(2)}(\Delta\nu, 0) = g^{(2)}(\Delta\nu, 2L) = 1 + \frac{1}{1+\cosh(8\ln 2\cdot\Delta\nu_q\cdot\Delta\nu/\Delta\nu_D^2)}. \qquad (3.3)$$

Assuming $L = 25$ cm ($\Delta\nu_q = 600$ MHz), $\Delta\nu_D = 800$, 1000, and 1200 MHz, when the tuning amount $\Delta\nu = \pm 600$ MHz, the frequency-tuning curves of $g^{(2)}(\tau)$ calculated from (3.2) and (3.3) are shown in Figs. 5a and 5b.

From Fig. 5, it is seen: (1) At $\Delta l = L$ and $\Delta l = 0$, $2L$, the frequency-tuning property of $g^{(2)}(\tau)$ of the two-mode laser field is of the same shape and opposite opening. This shows that the frequency-dependent method of $g^{(2)}(\tau)$ on frequency and power stabilization of two-mode He–Ne laser can be achieved either at $\Delta l = L$, or $\Delta l = 0$ and $2L$. This is different from the method of the frequency-dependent $g^{(1)}(\tau)$ for frequency and power stabilization of two-mode He–Ne laser [13].

(2) The frequency-tuning characteristics of $g^{(2)}(\tau)$ shows "V" shape curve and is symmetric about the line $\Delta\nu = 0$. When $\Delta\nu = 0$, $g^{(2)}(0, L) = \frac{1}{2}$ is the minimum. The photon anticorrelation effect is the strongest. The output intensities of the two mode are equal (i.e., $k = 1$) and the polarizations of the two modes are perpendicular for the intracavity two-mode laser.

(3) When the laser linewidth $\Delta\nu_D$ becomes smaller, the variation of frequency-tuning curve of $g^{(2)}(\Delta\nu, L)$ becomes steeper. It is necessary to choose two-mode He–Ne laser with small $\Delta\nu_D$ to improve the precision of the frequency and power stabilization.

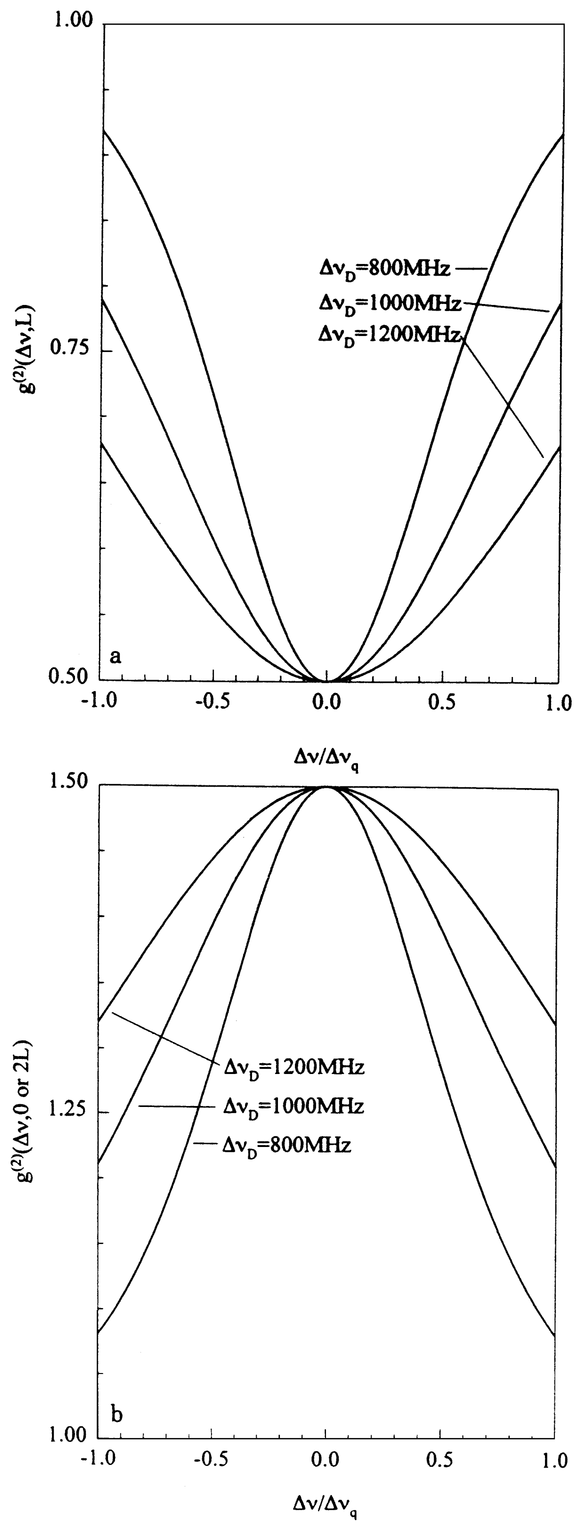

Fig. 5. The frequency-tuning curve of $g^{(2)}(\tau)$ for a two-mode He–Ne laser. In Fig. 5; (a) the variation of $g^{(2)}(\tau)$ against the relative tuning amount $\Delta v/\Delta v_q$ at $\Delta l = L$; (b) the variation of $g^{(2)}(\tau)$ against the relative tuning amount $\Delta v/\Delta v_q$ at $\Delta l = 0.2L$ ($L = 25$ cm, i.e., $\Delta v_q = 600$ MHz)

3.3 Possible applications

From the theoretical analysis of the frequency-dependent and frequency-tuning properties of $g^{(2)}(\tau)$, it is seen that if $\Delta v = 0$ is the frequency-locking point with $\Delta l = L$ or $\Delta l = 0, 2L$, the frequency and power stabilization of the two-mode He–Ne laser can be achieved by using the frequency-dependent signal of $g^{(2)}(\Delta v, L)$ or $g^{(2)}(\Delta v, 0/2L)$ as an error signal to control the cavity length L of the two-mode laser. This principle and method are similar to that of the frequency-dependent method of $g^{(1)}(\tau)$ for the frequency and power stabilization [13].

Since the slope of the tuning curve of $g^{(2)}(\tau)$ near $\Delta v = 0$ is not very large, the degree of frequency stabilization cannot be very high if $\Delta v = 0$ is chosen as the frequency-locking point. This is also different from the frequency-dependent method of $g^{(1)}(\tau)$ for frequency stabilization. However, it is seen from Fig. 5a that if a slope discriminator is used [15] and the frequency-locking point v_{lock} is chosen in the middle at the linear part of one side of the "V" shape curve, it will be very suitable to achieve frequency and power stabilization of the two-mode He–Ne laser using the frequency-dependent characteristics of $g^{(2)}(\Delta v, L)$. This is similar to the frequency-stabilization method of transverse Zeeman laser [16].

Differentiation of (3.2) gives the slope of $g^{(2)}(\Delta v, L)$ tuning curve at some amount Δv of frequency tuning:

$$k(\Delta v) = \frac{\mathrm{d}}{\mathrm{d}(\Delta v)} g^{(2)}(\Delta v, L) = \frac{a \sinh(a\Delta v)}{[1 + \cosh(a\Delta v)]^2}, \tag{3.4}$$

where

$$a = 8 \ln 2\Delta v_q/\Delta v_D^2. \tag{3.5}$$

If $L = 25$ cm ($\Delta v_q = 600$ MHz) and $\Delta v_D = 800$ MHz, from (3.5), $a = 5.3 \times 10^{-9}$. It is seen from Fig. 5a that the coordinates of the middle points on the linear part of either side of $g^{(2)}(\Delta v, L)$ are approximately $\Delta v_0 = \pm 250$ MHz and $g_0^{(2)}$ (± 250 MHz, $L) = 0.6634$. From (3.4), the slope of $g^{(2)}(\Delta v, L)$ at $\Delta v_0 = 250$ MHz is about $k_0 = 1.02 \times 10^{-9}$(1/Hz). The degree of second-order coherence $g^{(2)}(\tau)$ at the middle position $\Delta v_0 = 250$ MHz in one branch of the linear parts of $g^{(2)}(\Delta v, L)$ tuning curve is given by

$$g^{(2)}(\Delta v, L) \approx k_0 \, |\Delta v| + b, \tag{3.6}$$

where b is the distance of section of the straight line (3.6) on $g^{(2)}(\Delta v, L)$ axis. The corresponding degree of the frequency stabilization is about

$$\frac{\delta v}{v_0} \approx \frac{1}{k_0 v_0} \cdot \Delta g^{(2)}(\Delta v, L), \tag{3.7}$$

where v_0 is the central frequency of the laser and $\Delta g^{(2)}(\Delta v, L)$ the amplitude distinguishing ability of the measurement of $g^{(2)}(\tau)$. For a He–Ne laser, $\lambda_0 = 632.8$ nm, $v_0 = 4.74 \times 10^{14}$ Hz, with $\Delta g^{(2)}(\Delta v, L) = 10^{-3}$–$10^{-4}$, the degree of frequency-stabilization calculated from (3.7) is about 2.1×10^{-9}–2.1×10^{-10}.

From (3.2) and (3.3), or Figs. 5a and 5b, it is seen that the frequency-dependent characteristics of $g^{(2)}(t, L)$ can also be used in the measurement of the linewidth Δv_D of the two-mode laser.

4 Conclusion

The general formulae of the degree of second-order coherence $g^{(2)}(\tau)$ and of the frequency-dependent relationship are derived from the quantum theory of the light. The second-order quantum coherence, its frequency-dependent and photon statistical properties of the steady state and of the freely operated two-mode laser field are analysed and discussed. On the basis of investigation of $g^{(2)}(\tau)$ frequency-tuning property, the possible applications of $g^{(2)}(\tau)$ frequency-dependent characteristics in the measurement of the linewidth $\Delta\nu_D$ and the longitudinal-mode frequency width $\delta\nu_H$ of two-mode laser and the frequency and power stabilization of two-mode laser are discussed. Theoretical investigations show:

(1) The second-order quantum coherence and its frequency-dependence of the two-mode laser vary periodically with the time delay τ and the frequency drift $\Delta\nu$. The maximum value is $\frac{3}{2}$ and the minimum value is $\frac{1}{2}$ with corresponding period $2L$ and $2\Delta\nu_q$.

(2) When two longitudinal modes have the same output intensities (i.e., $\langle\hat{I}_1\rangle = \langle\hat{I}_2\rangle$), the degree of second-order coherence can be observed experimentally by the optical interference method in Michelson interferometer.

(3) At $\Delta l = 2mL$ or $(2m+1)L(m = 0, \pm 1, \pm 2, \dots)$ the frequency-dependent $g^{(2)}(\tau)$ of the two-mode laser varies most remarkably with $\Delta\nu$. It is very useful for the frequency and power stabilization of the two-mode laser.

(4) There is photon anticorrelation effect in the two-mode laser field, but which is not a non-classical effect. According to the variation of the time delay τ (or Δl) and the amount $\Delta\nu$ of frequency drift, the photon correlations of the two-mode laser show periodic changes from photon correlation to photon anticorrelation with period $2L$. Only at $\Delta l = (2m+1)L/2$, there is photon non-correlation effect.

(5) The frequency-dependent characteristics of $g^{(2)}(\Delta\nu, \tau)$ of the two-mode laser field is similar to that of $g^{(1)}(\Delta\nu, \tau)$ [13] which can also be applied in the measurement of the linewidth $\Delta\nu_D$ and the frequency and power stabilization of the two-mode laser. The theoretical precision of frequency stabilization is of the order of 2.1×10^{-9}–2.1×10^{-10}.

The frequency stabilization method of $g^{(2)}(\tau)$ frequency-dependence described in this paper is similar to that of $g^{(2)}(\tau)$ [13]. It essentially overcomes some disadvantages in the traditional method of frequency stabilization which uses the two-mode polarization properties of the two-mode intracavity laser [1–4]. The precision of the frequency and power stabilization can be improved further. This method is also suitable to two-mode intracavity He–Ne laser with output of random polarization property and two mode half-extracavity with parallel-linear polarization output. Compared to the frequency-stabilization method of $g^{(1)}(\tau)$ with frequency-dependence, this method has some advantages, such as simple optical system, strong anti-interfere ability, and good dynamic property and so on because the correlator (i.e., the intensity interferometer) to measure the degree of second-order coherence $g^{(2)}(\tau)$ is almost a pure electronic device, while is not an optical interferometer. Then it is not sensitive to the influence of on-the spot working conditions or circumstances, such as: the mechanical vibration, temperature and refractive-index fluctuation and atmospheric disturbance, etc.

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