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Vortex creation in Bose–Einstein condensates by laser-beam vortex guiding

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Abstract. A technique for vortex creation in trapped Bose–Einstein condensates is suggested: vortices can be excited at the edge of a condensate and guided to the center by a laser beam moving along a spiral trajectory. Numerical simulations demonstrate the suggested technique. Parameter ranges for the method are given. Computer animations illustrate the dynamics of the guided vortices.

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The creation of vortices and vortex ensembles in Bose–Einstein condensate (BEC) is presently of interest. Various schemes have been suggested for vortex excitation. Most of them are based on the rotation of the magnetic trap in order to impose an angular momentum on the condensate by external force. This rotation could be achieved by rotating the magnetic fields of an anharmonic trap [1], a technique well known for superfluids. The rotation could be also achieved by stirring rotationally the condensate with a laser beam [2]. The latter technique has recently allowed first realization of vortices in Bose–Einstein condensates: (i) in a two-component condensate [3]; (ii) in a single-component condensate [4]. Another proposal is to use optical beams with appropriate intensity distributions to "imprint" a phase distribution onto the condensate [5].

Here we propose a technique to create vortices, and vortex ensembles, by creation at the edge of the condensate and then guiding them into the condensate, by a laser beam. A blue-detuned narrow laser beam (narrower than the condensate) is known to produce a dip of the density of the condensate [6]. Presumably such a laser beam can be used to pin the vortex. If the laser beam moves slowly through the condensate it can drag the pinned vortex and guide it to a desired location. Our proposal is that a laser beam entering the condensate along a spiral path creates vortex pairs at the boundary of the condensate. The laser beam then pins one of the generated vortices and guides it to a desired location of the condensate.

We show numerically how one vortex can be brought into the condensate and made to remain at the condensate center after the laser beam is switched off. The technique can be generalized bringing in, one after another, or simultaneously, many vortices into the condensate if manipulating with several laser beams.

1 Theory

This study is based on the numerical integration of the two-dimensional Gross–Pitaevskii (GP) equation for the condensate wave-function $V(\boldsymbol{r}, t)$:

$$i\frac{\partial\psi(\boldsymbol{r},t)}{\partial t} = -\nabla^2\psi(\boldsymbol{r},t) + V(\boldsymbol{r},t)\psi(\boldsymbol{r},t) + C\,|\,\psi(\boldsymbol{r},t)\,|^2\psi(\boldsymbol{r},t)\,. \quad (1)$$

Here $V(\boldsymbol{r}, t) = V_\mathrm{t}(\boldsymbol{r}) + V_\mathrm{l}(\boldsymbol{r}, t)$ is the external potential consisting of a stationary part corresponding to the harmonic magnetic trap $V_\mathrm{t}(\boldsymbol{r}) = r^2/4$, and of the nonstationary part corresponding to the potential of the moving laser beam $V_\mathrm{l}(\boldsymbol{r}, t) = V_0 \exp[-|\boldsymbol{r} - \boldsymbol{r}_\mathrm{s}(t)|^2 / r_\mathrm{ls}^2]$, where r_ls is the half width-of the laser beam intensity profile, and $\boldsymbol{r}_s(t)$ the trajectory of its

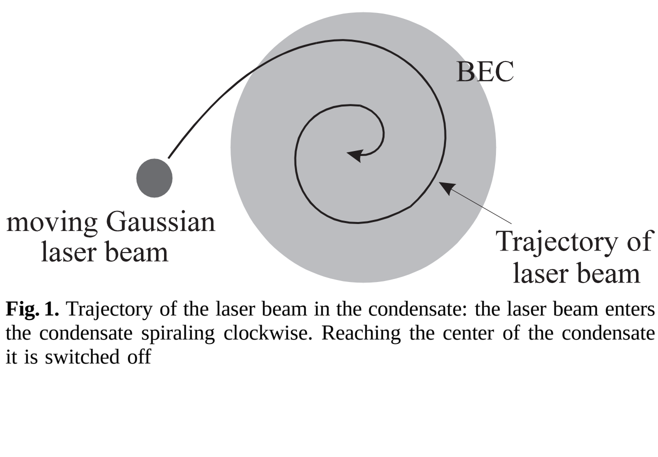

Fig. 1. Trajectory of the laser beam in the condensate: the laser beam enters the condensate spiraling clockwise. Reaching the center of the condensate it is switched off

center. The time in (1) is in units of the inverse frequency of the harmonic magnetic trap ω_t^{-1}, and the spatial coordinates are in units of the size of the noninteracting condensate: $(\hbar/(2m\omega_t))^{1/2}$, where m is the mass of the atoms. The density of the wave-function of the condensate $\varrho(\boldsymbol{r},t) = |\psi(\boldsymbol{r},t)|^2$ is normalized: $\int \varrho(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r} = 1$, and the coefficient C is proportional to the number of atoms in the trap N and their scattering length a: $C = 2Na(2m\omega_t/\hbar)^{1/2}$.

Figure 1 illustrates the motion of the laser beam as simulated: the laser beam enters the condensate spiraling clockwise: ($|\boldsymbol{r}_s(t)| = r_0 - v_r(t)$, $\phi(t) = -\omega t$ in polar coordinates, v_r is the radial and velocity, and ω is the angular frequency) and is switched off when reaching the center of the condensate, by imposing $V_0(t > r_0/v_r) = 0$.

2 Results and discussion

The results of the numerical integration are given in Figs. 2–4, and respectively in Animations 1–3. In all simulations, the parameters of the condensate were chosen as follows: The interaction coefficient was fixed to $C = 180$ in all calculations. This corresponds to $N = 10^5$ of ^{87}Rb atoms ($a = 5.2 \times 10^{-9}$ m) in a magnetic trap of frequency $\omega_t = 20 \times \pi\, rad\, s^{-1}$. This results in a spatial size (diameter at half of the maximum density) of the condensate equal to 8 in adimensional units used, or to 50 µm in physical units (one adimensional unit corresponds to 6.2 µm for the trap described above).

Such choice of parameters results in an adimensional density at the center of condensate $\varrho_0 = 0.03$. A significant parameter for the bulk dynamics is the nonlinearity parameter $C\varrho$, which at the center of condensate is $C\varrho = 5.4$ in the calculated cases. This results in a healing length at the center of the condensate of $r_v = 0.61$ in adimensional units, or 3.8 µm in physical units ($r_v = \sqrt{2/(C\varrho)}$), which is approximately equal to the radius of the vortex core. This also results in an adimensional velocity of sound at the center of the condensate of $v_s = 3.29$ ($v_s = \sqrt{2C\varrho}$), corresponding to 1.28 mm s^{-1} in physical units. As the sound velocity is proportional to the density of condensate, it reduces to zero at the margin of condensate, where the density $\varrho(\boldsymbol{r},t)$ tends to zero.

The parameters for the moving potential induced by the laser beam are: The half-width of the laser beam intensity profile: $r_{ls} = 0.7$, thus is chosen nearly equal to the healing length of the condensate. The potential induced by the laser beam $V_0 = 2$ is comparable with the potential of the magnetic trap at the edge of condensate. The laser beam, if directed to the center of the condensate "burns" a dip of half-width $r_{dip} = 1.2$, and depth of the dip: $\varrho_{dip}/\varrho_0 = 0.3$ (density at the center of the dip compared to the density without the laser beam).

The parameters of the condensate and of the laser beam correspond to those used in other numerical calculations [2]. Comparing with the experiments we simulate a relatively small condensate, with the size of the condensate of only about 15 healing lengths. In experiments, for example in [6], the size of the condensates can reach around 100 healing lengths and more. The vortex guiding in larger condensates is discussed below in conclusions.

The radial velocity v_r of the spiraling beam was fixed in all calculations: $v_r = 0.3$, and the angular velocity was

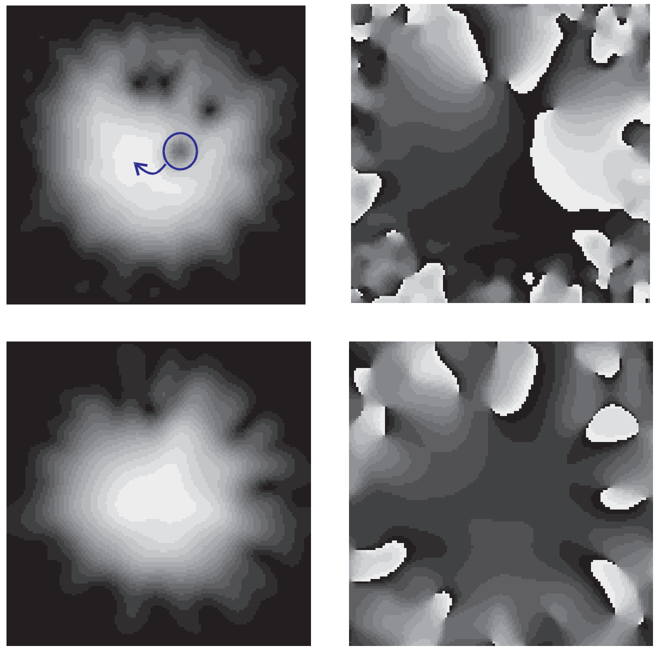

Fig. 2. Dynamics of BEC, as obtained by numerical integration of GP (1) with angular frequency of spiraling laser beam: $\omega = 0.69$. Density of BEC left; phase right. Periodic boundary conditions were used. Size of the integration region (and of the plot) is 13.4 in adimensional units. *Top row*: the transient distribution of the BEC ($t = 14.5$): the laser beam is marked by a *circle*, and its motion by an *arrow*. The laser beam excites three vortices when entering the BEC. *Bottom row*: the final distribution of the BEC ($t = 36$): The laser beam is switched off (at $t = 22$). It excited a number of vortices at the BEC edge. In mode language: modes of the trap of order 10–12 are excited. Dynamics is shown in *Animation 1*

varied. The velocity of the motion of the laser beam was always smaller than the sound velocity at the center of the condensate.

Figure 2 and Animation 1 show the case when the moving laser beam spirals in with an angular frequency $\omega = 0.69$, which is larger than the optimum one. Although, at the boundary of the condensate the laser beam excites vortices, these vortices are in this case not pinned and not guided by the laser beam. The excited vortices (three vortices in this case, all of negative charge, as seen from the phase pictures) circle clockwise near the edge of the BEC, and interact mutually and with the condensate boundary. When the laser beam reaches the center of the BEC, it is switched off abruptly. The vortices remain close to the edge of the BEC. In a mode language this means that besides the fundamental mode of the trap, some higher order modes are excited, which corresponds to an increase of the mean energy of condensate. In the calculated case, modes with $n = 10 - 12$ are visible in the final plots in Fig. 2. The higher order modes are excited due to: (i) perturbation of the condensate by the moving laser beam across the condensate boundary; (ii) perturbation of the condensate by the sudden switching-off of the laser beam at the center of the condensate.

The effects of the sudden switching-off are clearly seen in Animation 1: a sound wave is generated during the switch-off of the laser beam, and propagates outwards.

The motion of the laser beam within the condensate does not excite higher order modes (does not increase the mean energy), because the beam moves much slower than the critical

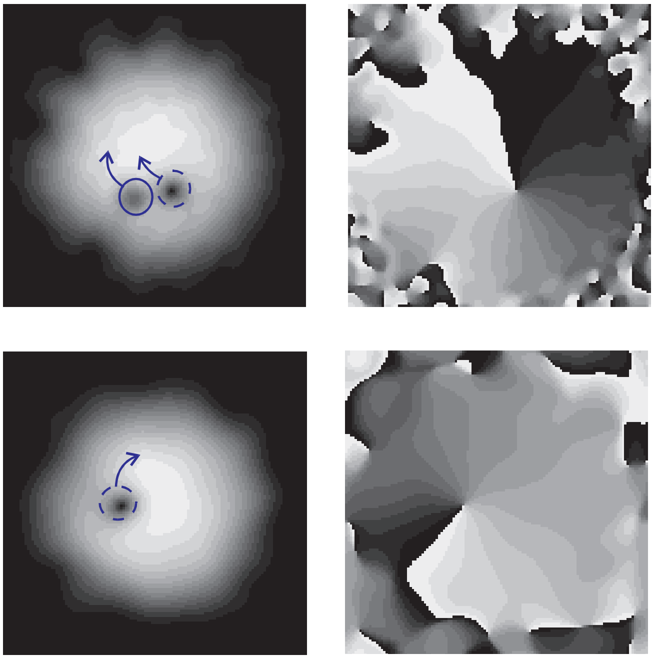

Fig. 3. Dynamics of BEC. As in Fig. 2 except for angular frequency of spiraling laser beam: $\omega = 0.55$. *Top row*: the laser beam excited a vortex (shown by *dashed circle*), but loses it when spiraling to the BEC center. *Bottom row*: the vortex circles around the condensate. Dynamics is shown in *Animation 2*

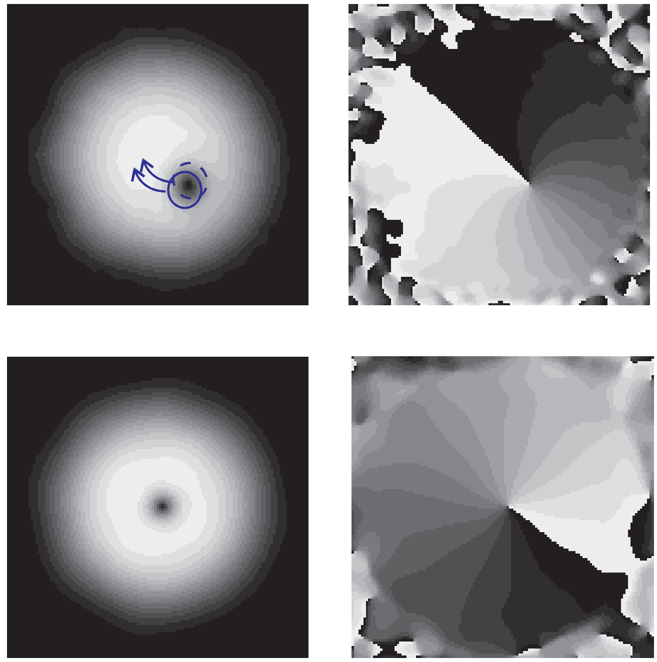

Fig. 4. Dynamics of BEC. As in Fig. 2 except for angular frequency of spiraling laser beam: $\omega = 0.44$. *Top row*: the laser beam is guiding the vortex (somewhat behind and to the right of the beam trajectory). *Bottom row*: the vortex remains close to the center of the BEC. Dynamics is shown in *Animation 3*

Landau velocity for vortex generation (which is of the order of magnitude of the sound velocity). The situation is different at the edge of the condensate: the sound velocity (being proportional to the atom density in the condensate) is small there. Any finite velocity of the laser beam is thus larger than the velocity of sound, close to the edge, thus larger than the critical Landau velocity for vortex generation. Therefore vortices are always excited at the edge of condensate. The question is thus only if one of the vortices can be guided by the laser beam to the interior of the condensate. This is not the case for Fig. 2 and Animation 1.

In the calculations shown in Fig. 3 and Animation 2 the laser beam is spiraling in with a smaller angular velocity: $\omega = 0.55$. The laser beam can pin and guide one of the excited vortices into the interior of the condensate, however, not precisely to the center. The laser beam loses the vortex before reaching the center. The lost vortex continues then to circle around the condensate.

The circling behavior of a vortex in a Bose condensate has been investigated in [7]. The analogous phenomenon of optical vortex circling around a laser beam was shown numerically and experimentally some time ago [8, 9]. In optics the circling optical vortex is interpreted in terms of simultaneous excitation and beating of transverse modes of the laser resonator [8]. These correspond qualitatively to the modes of a magnetic trap. (The modes of a condensate in a magnetic trap are highly nonlinear, whereas in laser fields the resonator modes are usually nearly linear, for which reason the correspondence is only qualitative.) Another interpretation of the optical vortex circling around the laser beam is in terms of the Magnus drift [9]. The vortex, being a “hollow” object, experiences a buoyancy force directed away from the optical axis of the laser (from the condensate center). Due to the angular momentum of a vortex the buoyancy force results in a drift perpendicular to the direction of force (Magnus effect). It results in the circular motion of the vortex. The vortex drift velocity according to [9] is: $v_{\mathrm{drift}} = |\nabla\varrho|/\varrho$, where ϱ is the density, and $\nabla\varrho$ is the gradient of density of the condensate (in absence of the vortex).

Animation 2 shows the subsequent circling of the vortex in the condensate. The numerically calculated circling frequency of the vortex close to the boundary of the condensate is $\omega_{\mathrm{circling}} = 0.41$. It corresponds well to the value calculated according to [9], using a model of buoyancy plus Magnus effect.

The vortex created would circle forever in case of zero temperature as shown in Animation 2. For finite temperatures, the vortex would eventually spiral out, as calculated in [7].

In the case shown in Fig. 4 and Animation 3 the laser beam is spiraling in with a smaller angular velocity: $\omega = 0.44$, which is nearly equal to the natural circling frequency of the vortex in the condensate, as calculated in the previous case (Fig. 3 and Animation 2). In this case, the vortex can be guided close to the center of the condensate, and remains there after the laser beam is switched off.

It follows thus, that a vortex can be guided to the center of the condensate for small angular frequencies of the laser beam. However, for too small frequency, the vortex will also not be trapped by the guiding beam and will remain circling near the edge of the condensate. As the numerical calculations show, the vortex is not guided for angular velocities $\omega < 0.34$.

From the calculations above it follows that vortices can be indeed guided to desired locations of the condensate by laser beams spiraling in with a proper angular frequency. In our calculations the frequency interval for guiding the vortices to the center of the condensate was $0.34 < \omega < 0.55$. This interval is related with the natural frequency of a vortex circling freely in the trap. In the calculations this frequency (see Animation 2 for a vortex circling freely in the trap) is $\omega_{\text{circling}} = 0.41$. The optimum angular frequency of the laser beam would be equal to the natural frequency of a vortex. These frequencies must however not necessarily coincide precisely: a particular locking interval appears, in which the vortex remains pinned to the guiding laser beam.

Related to this is a separate problem: what is the maximum velocity of a laser beam capable of guiding a vortex on a homogeneous background (limit of infinitely large traps)? In order to evaluate the maximum guiding velocity we outline the following model for vortex guiding. We assume a homogeneous condensate (infinitely large trap) and study first the dynamics of a vortex in the presence of a stationary "dip" in the condensate produced by the repulsive potential of the laser beam. One may expect, that due to the dip the vortex is attracted to the center of the laser beam. This is however not true for purely conservative (zero temperature) condensates described by the GP (1). The vortex in general moves around the corresponding dip of the condensate, and is stationary only in the case when its core and the center of the stationary dip coincide. The velocity of the vortex rotation around the dip depends on the radius of the circular vortex trajectory (the distance between the vortex core and the center of the dip) and can be calculated using the model of Magnus drift, according to [9]: it has been shown in [9], that the optical vortices drift in a direction perpendicular to the applied force, or equivalently perpendicular to the gradient of the pressure or of the density of the background condensate, with the velocity modulus: $v_{\text{drift}} = |\nabla\varrho|/\varrho$. Assuming, that the dip produced by the stationary laser beam is Gaussian: $\varrho(\boldsymbol{r}) = \varrho_0 - (\varrho_{\text{dip}} - \varrho_0)\exp[-r^2/r_{\text{dip}}^2]$, the velocity of vortex circling is: $v_{\text{drift}} = (\varrho_{\text{dip}} - \varrho_0)/\varrho_0 \times 2|r|/r_{\text{dip}} \exp[-r^2/r_{\text{dip}}^2]$. The vortex then circles with the maximum velocity at the distance from the dip center $r_{\max} = r_{\text{dip}}/\sqrt{2}$, and the value of this maximum velocity is.

$$v_{\max} = \frac{\varrho_{\text{dip}} - \varrho_0}{\varrho_0} \frac{\sqrt{2}e^{-1/2}}{\boldsymbol{r}_{\text{dip}}} . \tag{2}$$

Here $(\varrho_0 - \varrho_{\text{dip}})/\varrho_0$ is the contrast of the dip which is equal to 0.3 in the numerical integration.

In the case of a moving laser beam the trajectory of the vortex is in general complicated. Obviously, the vortex can not be guided by the laser beam moving faster than $v_{\max}$ given by (2). A relatively slow moving laser beam can guide the vortex if the vortex is positioned in a particular area relative to the center of the guiding laser beam. In particular one can find two positions of the vortex relative to the laser beam center resulting in a straight motion of the guided vortex. In general the trajectory of the guided vortex is complicated and requires a more detailed study. However the expression (2) allows us to evaluate the maximum velocity of vortex guiding $v_{\max}$. Normalizing this velocity (2) to the velocity of sound in the condensate $v_{\text{s}} = \sqrt{2C\varrho} = 2/r_{\text{v}}$ one obtains:

$$\frac{v_{\max}}{v_{\text{s}}} = \frac{e^{-1/2}}{\sqrt{2}} \frac{\varrho_0 - \varrho_{\text{dip}}}{\varrho_0} \frac{r_{\text{v}}}{r_{\text{dip}}} . \tag{3}$$

Here $r_{\text{v}}/r_{\text{dip}}$ is the ratio between the vortex radius (healing length) and the radius of the laser-beam-induced dip. The expression (3), also our preliminary numerical calculations for unbounded condensates and for the above parameters of the laser beam, show that the moving laser beam can guide a vortex for velocities smaller than 0.15 of the sound velocity. For comparison, we find the critical Landau velocity (the velocity for vortex pair generation) equal to 0.64 of the sound velocity in this case (for the above values of the contrast and width of the laser-beam-induced dip of the condensate).

These calculations for an unbounded condensate allow us to expect that the vortex guiding in a bounded condensate is possible if the velocity of the laser beam is by ≈ 0.15 of the sound velocity larger or smaller than the eigen-velocity of the vortex. (The locking effects of the laser beam can indeed increase or reduce the velocity of the vortex motion with respect to its eigen-velocity.). The numerical simulations of the finite-size condensate agree well with the results following from the analysis of vortex dynamics in the unbounded condensate.

The guided vortices are always of negative (positive) topological charge for clockwise (anti-clockwise) spiraling of the laser beam.

The calculations have been performed by simulating very narrow laser beams, with width of the order of the healing length of the condensate. A question is whether vortex excitation and guiding is also possible by broader laser beams.

Numerical calculations show that broader laser beams can also excite and guide vortices in BECs. Up to $r_{\text{ls}} \approx 3$ is possible for the particular parameters of the condensate used, which means that the size of the laser beam may be almost of the size of the condensate in the trap, and thus may substantially exceed the healing length in the condensate. However, the locking interval for the vortex pinning then becomes smaller, in accordance with the evaluation (3). Therefore the vortex locking range decreases with increasing width of the dip, i.e. the vortex guiding becomes more sensitive to the angular frequency of the spiraling beam.

As noted in the introduction, it is possible to extend the suggested technique for excitation of several vortices, by manipulating with several laser beams. Animation 4 shows the case where two laser beams spiral in clockwise. As expected, two vortices, both of negative topological charge, are created and guided close to the center of the condensate. This example demonstrates on the one hand the possibility of exciting more complex vortex ensembles. On the other hand, it demonstrates, that the suggested technique is not a simple modification of the condensate stirring suggested in [2], but rather a "vortex guiding" by the laser beam.

And finally, concerning the influence of the size of the condensate on the laser beam guiding effects: the above calculations were performed for relatively small condensates, where the size of the condensate is ≈ 15 healing lengths. What differences could be expected for larger condensates, such as obtained experimentally in [6], where the size of the condensate is 100 and more healing lengths? According to the expression (2) the locking range for the vortex guiding does not depend on the size of the condensate. In order to check that we performed numerical calculation with larger values of parameter C in (1), up to 1800, which correspond to $N = 10^6$ of ^{87}Rb atoms. This resulted in larger condensates and smaller healing lengths (the size of the condensate is ≈ 150 healing length for $C = 1800$). This however did not influence the locking range for the vortex guiding by laser beam, as follows from our numerical calculations, and allows us to conclude that the suggested technique of vortex guiding can be realized for the condensates of different sizes.

3 Animations (see http://link.springer.de/journals/apb)

Animation 1. Dynamics of BEC, as obtained by numerical integration of GP equation (1) with the angular frequency of spiraling laser beam equal to: $\omega = 0.69$. Density of BEC left. Phase right. The position of the guiding laser beam corresponds to a dip in condensate density. The positions of vortices correspond to the density zeroes (left), and phase branching points (right).

The duration of animation is $t = 36$. The laser beam reaches the center of the condensate and is switched off at $t = 22$. A sound wave appearing due to switch-off of the laser beam is visible in the animation. The animation corresponds to Fig. 2.

Animation 2. Dynamics of BEC. As in Animation 1 except for angular frequency of spiraling laser beam: $\omega = 0.55$. Animation corresponds to Fig. 3.

Animation 3. Dynamics of BEC. As in Animation 1 except for angular frequency of spiraling laser beam: $\omega = 0.44$. Animation corresponds to Fig. 4.

Animation 4. Dynamics of BEC for two spiraling laser beams. The angular frequency of spiraling laser beams is $\omega = 0.44$. Parameters correspond to Animation 3. Eventually two vortices are created which circle around the center of the condensate.

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