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RESEARCH

# Faraday isolator with compensation depolarization caused by Verdet constant temperature dependence

**Ilya Snetkov**[1,2]

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**Abstract**
Thermally induced depolarization is known to be the principal factor limiting the usage of Faraday devices in laser radiation with high average power. In a number of practically important cases, the major contribution to the depolarization in Faraday devices is inhomogeneous Faraday rotation due to the temperature dependence of the Verdet constant. At the moment there are no satisfactory solutions to this problem. In this work a new scheme of a Faraday isolator with compensation of the contributions to thermally induced depolarization from the temperature dependence of the Verdet constant and thermally induced birefringence was proposed. The efficiency of using the proposed scheme and comparison with known schemes is analyzed analytically and numerically on an example of two magneto-optical crystals: TGG at cryogenic temperature and $EuF_2$ in critical orientation at room temperature in which the contribution to thermally induced depolarization from non-uniform Faraday rotation due to the temperature dependence of the Verdet constant is major.

## 1 Introduction

Laser radiation with a high average power is used in many areas of human activity, ranging from industrial applications for cutting, welding and drilling to space debris removal. Faraday devices are the key elements of most high-power laser systems needed for organizing multipass schemes of laser amplifiers, compensating for thermally induced depolarization in the active elements of lasers (Faraday mirror), as well as for optical isolation of individual parts of the laser from each other or the laser from unwanted back-reflections [1]. When Faraday devices operate in high-average power (with high rep-rate or CW) laser radiation, the thermally induced depolarization arising in the magneto-optical elements (MOE) of these devices and growing with increasing power significantly limits their performance [2, 3].

An increase in power leads to an increase in heat release in the MOE bulk due to absorption. This results in an increase in MOE average temperature and in the appearance of a temperature gradient. The temperature gradient, in turn, (i) leads to Faraday rotation angle dependence due to the temperature dependence of the Verdet constant, (ii) leads to the temperature stress and strain and to thermally induced linear birefringence due to the photoelastic effect. The direction of eigenpolarizations (angle $\Psi$) and the phase difference between them ($\delta_l$) depend on the transverse coordinates and on the power of heat generation. Let us consider the main reasons for reducing the isolation ratio during the operation of a Faraday isolator (FI) in high-power laser radiation:

(1) Homogeneous-over-cross-section changes of the angle of Faraday rotation due to the temperature dependence of the Verdet constant as a result of increasing average temperature ($T_{av}$) leads to a decrease of the angle of Faraday rotation $\theta_{F0} = \delta_c(T_{av})/2 = \delta_{c0}/2$, where $\delta_c$ is circular birefringence. In experiment, it looks like a beam trace, which can be eliminated by turning the output polarizer Fig. 1a.

(2) Changes (inhomogeneous over cross section) in the polarization state of laser radiation due to thermally induced linear birefringence, as a result of which the thermally loaded MOE becomes equivalent to a phase plate inhomogeneous over cross section, with the parameters dependent on the laser radiation power. In experiment, it looks like a "Maltese cross", which can't be eliminated by tuning the output polarizer Fig. 1b.

3) Inhomogeneous-over-cross-section changes of the angle of Faraday rotation due to the temperature dependence

✉ Ilya Snetkov
snetkov@appl.sci-nnov.ru; snetkov@ipfran.ru

[1] Federal Research Center A.V. Gaponov-Grekhov Institute of Applied Physics of the Russian Academy of Sciences, 46 Ulyanov St, 603950 Nizhny Novgorod, Russia

[2] Lobachevsky State University of Nizhny Novgorod, 23 Gagarin Ave., 603950 Nizhny Novgorod, Russia

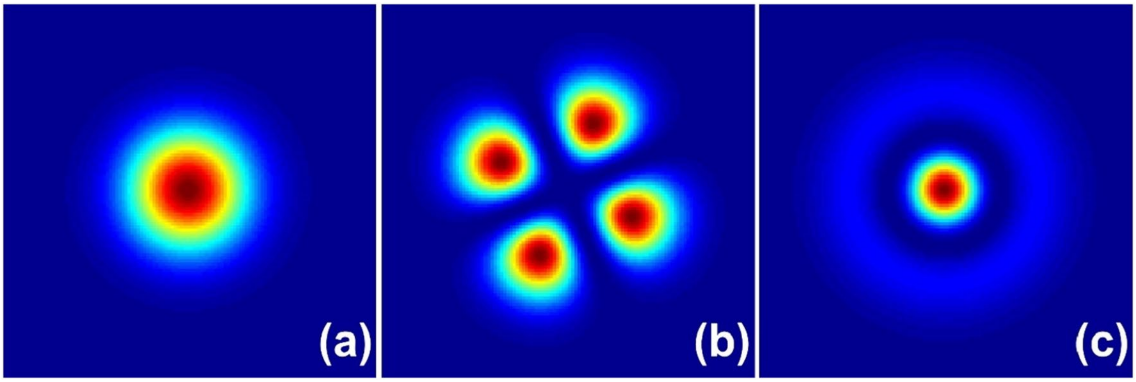


**Fig. 1** Most common intensity distributions of a depolarized field component that appear in FI MOEs: **a** homogeneous-over-cross-section changes of the angle of Faraday rotation, **b** depolarization due to thermally induced linear birefringence, **c** depolarization due to inhomogeneous Faraday rotation caused by the Verdet constant temperature dependence

of the Verdet constant, linear expansion of MOE and the temperature gradient $\Delta\theta_F = (\delta_c(T) - \delta_{c0})/2 = \delta_c^*(r)/2$, as a result of which the thermally loaded MOE becomes equivalent to an element with inhomogeneous over cross section optical activity. In experiment, it looks like a "donut" (or "sombrero"), which cannot be eliminated by tuning the output polarizer Fig. 1c.

In most cases and for most widely used magneto-optical materials, the contributions to the depolarization of radiation from the first two effects are significantly greater than from the third. Therefore, a large number of works have been devoted to the study of these effects, methods for its weakening and compensation [2–5]. However, the radiation parameters of modern laser systems have increased so much that to suppress thermally induced linear birefringence there is increasingly a need to use new materials, stronger magnetic fields and cryogenic cooling when creating Faraday isolator that provide the required isolation degree [1]. In these cases, the main contribution to the depolarization of radiation in Faraday devices comes from inhomogeneous Faraday rotation due to the temperature dependence of the Verdet constant ($\delta_c^*(r)$). Let's consider when $\delta_c^*(r)$ makes a significant contribution to the isolation ratio. At cryogenic temperatures, the thermo-optical parameters determining thermally induced birefringence decrease, the Verdet constant of paramagnet material markedly increases, so the MOE length needed for rotation by 45° decreases, and the depolarization caused by $\delta_c^*(r)$ increases substantially [6]. The depolarization caused by $\delta_c^*(r)$ increases for materials with a small value of the thermo-optical constant $Q$ [7], with a high Verdet constant even at room temperature [8], or when using a magnetic system with a high magnetic field value [9]. The depolarization caused by $\delta_c^*(r)$ becomes significant when part caused by thermally induced linear birefringence is suppressed by choosing the critical crystallographic axis orientation [C] in a material with piezo-optical anisotropy parameter $\xi < 0$ [10] and is maximal as $\xi$ approaches -0.5, when $\delta_l$ equals zero. In such cases, it is necessary to weaken or compensate part of depolarization made by the dependence $\delta_c^*(r)$, while trying not to increase the other part from thermally induced linear birefringence.

Figure 2 shows the known optical schemes of Faraday isolators consisting of MOEs (1) placed in the field of a magnetic system (2) located between two polarizers (3). The output polarizer is rotated by an angle equal to the polarization plane rotation in the MOE and the reciprocal rotator (if present). The simplest is the traditional scheme Fig. 2a. No one of the self-induced thermal effects can be compensated and only can be reduced, for example, by choosing the orientation of the crystallographic axes.

Schemes with compensation of depolarization caused by thermally induced linear birefringence with reciprocal rotator (RR) [11, 12] Fig. 2b and with absorbing element (AE) [13] Fig. 2c. The main idea is to use two optical elements in which the beam distortions compensate each other—they add up in one and are subtracted in the other. The quartz rotator and second optical element (one of the MOEs or absorbing element) act as a compensating inhomogeneous phase plate. In the scheme with AE this compensating inhomogeneous phase plate due to the possibility of using another material, additionally could compensate for the thermal lens. Thermally induced polarization distortions caused by $\delta_c^*(r)$ in these two schemes equal to the same distortions in the traditional scheme. So, the use of these schemes cannot help in any way with this kind of distortions.

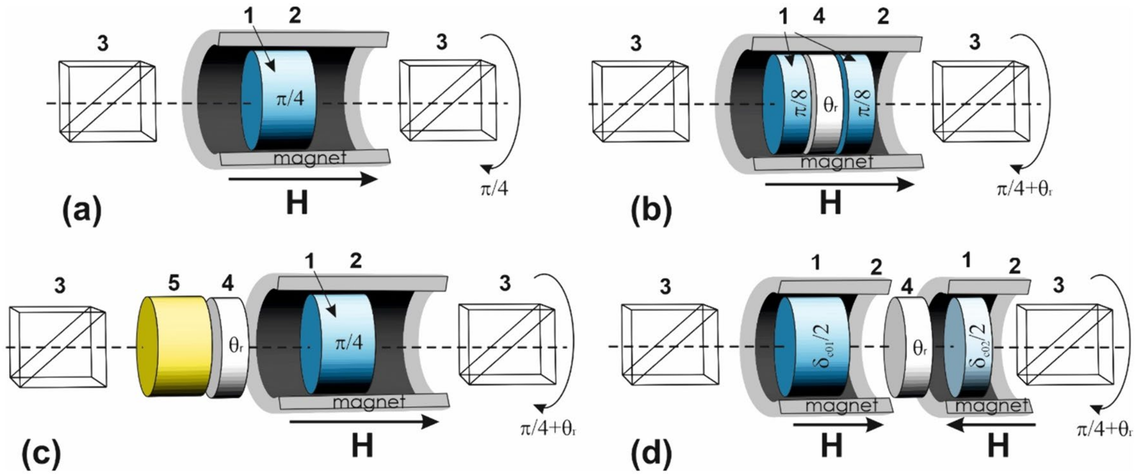


**Fig. 2** Optical schemes of Faraday isolators: **a** traditional; **b** with reciprocal rotator (RR); **c** with absorbing element (AE); **d** with compensation distortions from inhomogeneous Faraday rotation due to the temperature dependence of the Verdet constant. 1—magneto optical element; 2—magnet system; 3—polarizer; 4—reciprocal rotator of polarization (crystalline quartz); 5—optical absorbing element

First of all, let's consider what methods of eliminating or reducing polarization distortion from $\delta_c^*(r)$ are known. This distortion (1) does not occur in diamagnetic materials and in materials with $\beta_T = 1/V \cdot dV/dT = 0$ [14–16]. (2) may be reduced by profiling the magnetic field [17]. However, magnetic field profiling is non-adaptive and this method is effective only for a definite laser radiation power. (3) by using the schemes with compensation of temperature changes of the Verdet constant [18] that consist of MOE ensuring Faraday rotation by – 45°, λ/12 waveplate, MOE ensuring Faraday rotation by 90° and another λ/12 waveplate. Note, this scheme partially compensates distortion from $\delta_c^*(r)$ and demands two MOEs, which is equivalent to a three-fold increase length compared to the MOE of a traditional FI (provided that the material and magnetic field value are identical). This will inevitably degrade the isolation ratio due to increased thermally induced depolarization from both linear birefringence ($\delta_l$) and non-uniform Faraday rotation $\delta_c^*(r)$.

In this work a theoretical analysis of the new scheme with compensation of the contribution to thermally induced depolarization from $\delta_c^*(r)$ was performed. Values of all parameters of optical elements needed for the effective compensation were determined for two cases: cryogenic FI and FI base on crystalline material in critical orientation [C]. Analytical equation for thermally induced depolarization for the proposed compensation scheme were obtained. The effectiveness of the proposed scheme for a number of perspective magneto-optical materials was analyzed and compared with other known FI schemes.

## 2 Thermally induced depolarization in Faraday isolators with compensation

The thermally induced depolarization of linearly polarized radiation arising after passage of a system of two thermally loaded MOEs separated by a quartz rotator was considered in [19, 20]. The MOEs were made of arbitrary materials belonging to the cubic class with 432, $\bar{4}$3m and m3m crystal lattice symmetry and possessed arbitrary circular birefringence (responsible for Faraday rotation). Using the Jones matrix formalism, the $\Gamma$ and $\gamma$—the local and integral thermally induced depolarizations, respectively, can be written as

$$\gamma = \frac{\int_0^{2\pi} d\varphi \int_0^{R_0} \Gamma\, I(r) r \mathrm{d}r}{\int_0^{2\pi} d\varphi \int_0^{R_0} I(r) r \mathrm{d}r},$$
$$I_d = \Gamma I(r),\ \Gamma = \left|T_{21}\right|^2 = \left[\mathrm{Re}^2\left(T_{21}\right) + \mathrm{Im}^2\left(T_{21}\right)\right],$$
$$E_{\mathrm{out}} = \mathbf{T}E_{\mathrm{in}} = \mathbf{R}\left(-\theta_r - \frac{\delta_{c01}}{2} - \frac{\delta_{c02}}{2}\right)$$
$$\mathbf{M}\left(\delta_{c2}, \delta_{l2}, \Psi_2\right)\mathbf{R}\left(\theta_r\right)\mathbf{M}\left(\delta_{c1}, \delta_{l1}, \Psi_1\right)E_{\mathrm{in}}. \tag{1}$$

Here, without loss of generality $E_{in} = E_0(1; 0)$ is the field linearly polarized in the x-direction; **M** is the Jones matrix for MOE [20, 21] (in the absence of thermal effects equal to $\mathbf{R}(\delta_{c0i}/2)$); **R** is the rotation matrix; $\theta_r$ is the angle of rotation of the polarization plane in the reciprocal quartz rotator; $I_d$ is the intensity distribution of a depolarized field component; $T_{21}$ is the element of the Jones matrix describing a system of two thermally loaded MOEs separated by a quartz rotator (the last rotation by angle $-(\theta_r+\delta_{c01}/2+\delta_{c02}/2)$ needed to return the direction of the main polarization parallel to the x axis and then the depolarized field is completely determined by the matrix element $T_{21}$); $I(r)$ is the intensity of incident radiation ($I(r)=I_0 F_h(r)$; $F_h$ is the normalized transverse shape of the radiation ($F_h(r) = I(r)\Big/\int_0^{R_0} I(r)rdr$), $R_0$ is the radius of the crystal).

To obtain analytical solutions for thermally induced depolarization, we will assume that the problem is axially symmetric (the MOE has the rod-shape, the radiation propagates along the symmetry axis of the rod and has an axisymmetric intensity distribution), the MOE material has isotropic thermal and elastic properties, heat is removed from the lateral surface of the cylinder. Then in the stationary case, in the plane strain approximation in the polar coordinate system, the temperature depends only on the polar radius $r$, the stress tensor has only diagonal elements, and the Jones matrix can be analytically obtained for an arbitrarily oriented single-crystal MOE [22]. These approximations work well in a case of Faraday isolators. At weak birefringence approximation ($\delta_l << 1$) the normalized heat generation power $p$ is a small parameter and it is possible to make use of the perturbation method by expanding (1) in the order of smallness, where the thermally induced depolarization will depend on the derivatives of real and imaginary parts of the coefficient $T_{21}$ taken at $p=0$:

$$p = -QP_h/\lambda\kappa \approx -\alpha_0 LQP_{las}/\lambda\kappa,$$
$$\Gamma = \left[\left(\mathrm{Re}(T_{21})'\Big|_{p=0}\right)^2 + \left(\mathrm{Im}(T_{21})'\Big|_{p=0}\right)^2\right]p^2 +$$
$$+\left[\mathrm{Re}(T_{21})'\Big|_{p=0}\mathrm{Re}(T_{21})''\Big|_{p=0} + \mathrm{Im}(T_{21})'\Big|_{p=0}\mathrm{Im}(T_{21})''\Big|_{p=0}\right]p^3 +$$
$$+\left[4\,\mathrm{Re}(T_{21})'\Big|_{p=0}\mathrm{Re}(T_{21})'''\Big|_{p=0} + 4\,\mathrm{Im}(T_{21})'\Big|_{p=0}\mathrm{Im}(T_{21})'''\Big|_{p=0} +\right.$$
$$\left.+3\left(\mathrm{Re}(T_{21})''\Big|_{p=0}\right)^2 + 3\left(\mathrm{Im}(T_{21})''\Big|_{p=0}\right)^2\right]p^4/12,$$
$$\gamma_V = p^2\frac{\int_0^{2\pi} d\varphi \int_0^{R_0}\left(\mathrm{Re}(T_{21})'\Big|_{p=0}\right)^2 I(r)rdr}{\int_0^{2\pi} d\varphi \int_0^{R_0} I(r)rdr},$$
$$\gamma_P = p^2\frac{\int_0^{2\pi} d\varphi \int_0^{R_0}\left(\mathrm{Im}(T_{21})'\Big|_{p=0}\right)^2 I(r)rdr}{\int_0^{2\pi} d\varphi \int_0^{R_0} I(r)rdr}. \tag{2}$$

Here we only used what $\mathrm{Re}(T_{21})|_{p=0}=\mathrm{Im}(T_{21})|_{p=0}=0$ (depolarization is absent without heating radiation). There $Q$ is the thermo-optical constant [23]; $\lambda$ is the radiation wavelength; $\kappa$ is the thermal conductivity coefficient; $P_h$ is the total heat generation power in the bulk element (at weak absorption $\alpha_0 << 1/L$, $P_h \approx \alpha_0 LP_{las}$); $\alpha_0$ is the absorption coefficient; $L$ is the MOE length; and $P_{las}$ is the laser power; $\gamma_P$ and $\gamma_V$ are the major parts of depolarization due to the thermally induced linear birefringence and non-uniform Faraday rotation $\delta_c^*(r)$ respectively. We also obtained expressions for the first derivatives of $\mathrm{Re}(T_{21})$ and $\mathrm{Im}(T_{21})$ taken at $p=0$ [20]:

$$\mathrm{Re}(T_{21})'\Big|_{p=o} = \frac{W}{2}\left(\delta_{c01}\eta_1 + D\delta_{c02}\eta_2\right),$$
$$\mathrm{Im}(T_{21})'\Big|_{p=o} = -\frac{\sin(\delta_{c01}/2)}{\delta_{c01}}\left[A_1\cos\left(\frac{\delta_{c01}}{2}\right)\right.$$
$$\left.-B_1\sin\left(\frac{\delta_{c01}}{2}\right)\right] - -D\frac{\sin(\delta_{c02}/2)}{\delta_{c02}}$$
$$\left[A_2\cos\left(2\theta_r + \delta_{c01} + \frac{\delta_{c02}}{2}\right) - B_2\sin\left(2\theta_r + \delta_{c01} + \frac{\delta_{c02}}{2}\right)\right], \tag{3}$$

where $D=p_2/p_1$ is the ratio of the normalized powers in the first and second MOEs. $A_i$ and $B_i$ are determined by the crystal orientation, piezo-optical anisotropy parameter and shape of heating radiation, and $W$ is determined only by the transverse shape of heating radiation (the corresponding expressions and expression of next derivatives can be found in [20]).

The temperature dependence of the circular birefringence is described [19, 20]

$$\delta_{ci}(u) = \delta_{c0i}\left\{1 + p_i\eta_i W\right\}, \quad \eta_i = -\frac{\lambda}{4\pi L_i Q_i}\left(\alpha_{T\,i} + \beta_{T\,i}\right), \tag{4}$$

where $\alpha_{Ti}$ is the coefficient of linear expansion of the $i$-th MOE material. The goal of our present study is to find a solution at which $\gamma_V$ would be compensated ($\mathrm{Re}(T_{21})'|_{p=0}=0$) and, at the same time, the $\gamma_P$ under the new conditions would be minimized or compensated for, which would lead to an increase in the isolation ratio and $P_{max}$ value (power above which their key characteristics will cease to meet the imposed requirements e.g., isolation > 30 dB).

The $\mathrm{Re}(T_{21})'|_{p=0}=0$ can be satisfied in the following cases: (1) different signs of $\beta_T$ (the signs of $\eta_1$ and $\eta_2$ are different) in the used materials or $\eta_i=0$ (is performed for diamagnetic materials to an accuracy of neglecting thermal expansion); (2) opposite directions of Faraday rotation of the plane of polarization in the MOEs (due to different signs of the Verdet constant or due to different directions of the magnetic field). The materials with a positive $\beta_T$ sign are unknown; therefore, will consider the second case. From the requirements to the nonreciprocal rotation by 45° and $\mathrm{Re}(T_{21})'|_{p=0}=0$ we obtain:

$$\begin{cases} \delta_{c01} + \delta_{c02} = \pi/2, \\ p_1\delta_{c01}\eta_1 + p_2\delta_{c02}\eta_2 = 0. \end{cases} \tag{5}$$

The substitution of the expression for $p_i$ from (2) and $\eta_i$ from (4) into (5) you can find expressions for $\delta_{c01}$ and $\delta_{c02}$ separately and expression for their ratio

$$\frac{\delta_{c01}}{2} = \frac{\pi}{4}\left[\frac{\alpha_{02}(\alpha_{T2}+\beta_{T2})\kappa_1}{\alpha_{02}(\alpha_{T2}+\beta_{T2})\kappa_1 - \alpha_{01}(\alpha_{T1}+\beta_{T1})\kappa_2}\right] = V_1H_1L_1,$$
$$\frac{\delta_{c02}}{2} = -\frac{\pi}{4}\left[\frac{\alpha_{01}(\alpha_{T1}+\beta_{T1})\kappa_2}{\alpha_{02}(\alpha_{T2}+\beta_{T2})\kappa_1 - \alpha_{01}(\alpha_{T1}+\beta_{T1})\kappa_2}\right] = V_2H_2L_2,$$
$$\frac{\delta_{c02}}{\delta_{c01}} = -\frac{\alpha_{01}(\alpha_{T1}+\beta_{T1})\kappa_2}{\kappa_1\alpha_{02}(\alpha_{T2}+\beta_{T2})}. \tag{6}$$

The value $D = p_2/p_1$ will be determined by the ratio $D = \alpha_{02}Q_2L_2\kappa_1/\alpha_{01}Q_1L_1\kappa_2$. It can be noted that for $\mathrm{Re}(T_{21})'|_{\mathrm{p=0}} = 0$ to be satisfied, the values of the Faraday rotation angle are fully determined by four material constants ($\alpha_0$, $\alpha_T$, $\beta_T$, $\kappa$), which results in rigid restriction on the MOEs length. When $\mathrm{Re}(T_{21})'|_{\mathrm{p=0}} = 0$ is fulfilled, the values of $\delta_{c01}$, $\delta_{c02}$, and $D$ are fixed. From (6) it also follows that with the use of completely identical materials ($\alpha_{01} = \alpha_{02}$, $\alpha_{T1} = \alpha_{T2}$, $\beta_{T1} = \beta_{T2}$, $\kappa_1 = \kappa_2$), system (5) will not be satisfied. Therefore, the materials must either be different, or have some differing properties. Thus, if we assume that the maximum magnetic field value in the Faraday isolator magnet system is $H^*$, then from (6) it can be found that the total length of MOEs in the proposed scheme with compensation $L_1 + L_2$ will be larger than the MOE length in a traditional FI $L^* = \pi/(4V_1H^*)$. Such an increase in length will increase the $\gamma_P$ and reduce the value of $P_{max}$. To obtain compensation of thermally induced depolarization from linear birefringence, MOEs should be separated by quartz rotator and to achieve maximum efficient compensation MOEs should be manufactured from the material with $\xi_1 = \xi_2$ [2, 20]. But for compensating the $\gamma_V$ their material characteristics should be differ. According to the Eq. (6), the most easily varying material characteristic is the absorption coefficient $\alpha_0$. It is known that material absorption coefficient $\alpha_0$ even of one manufacturer may vary significantly [2], other material characteristics being unchanged. Another way is change value of the thermal conductivity coefficient $\kappa$ of material by doping [24]. However doping do not lead to change in material properties significantly, moreover it may result in change other material parameters (for example $\xi$ [16]).

Let us consider the case of using identical material for both MOEs ($V_1 = V_2$, $\alpha_{T1} = \alpha_{T2}$, $\beta_{T1} = \beta_{T2}$, $\kappa_1 = \kappa_2$, $\xi_1 = \xi_2$) with differing absorption coefficients $\alpha_{01} \neq \alpha_{02}$ (Fig. 2d). Assume that $\alpha_{01} < \alpha_{02}$, then according to (6) $\delta_{c01} = \pi/2\cdot[\alpha_{02}/(\alpha_{02}-\alpha_{01})] = 2VH^*L_1$, $\delta_{c02} = -\pi/2\cdot[\alpha_{01}/(\alpha_{02}-\alpha_{01})] = -2VH^*L_2$, $\delta_{c02}/\delta_{c01} = -\alpha_{01}/\alpha_{02} = -L_2/L_1$, and $D \equiv 1$. Different $\delta_{c0i}$ signs can be achieved by using different magnetic field directions (two magnetic systems face each other by identical poles or a system with a specially shaped field [25]). The larger the difference of the absorption coefficients, the closer $L_1$ to $L^*$ and $L_2$ to zero. The total MOE length in the case of the compensation scheme exceeds $L^*$ by $(\alpha_{02}+\alpha_{01})/(\alpha_{02}-\alpha_{01})$ times (for $\alpha_{02}/\alpha_{01} = 5$, $L_1 + L_2 = 1.5L^*$). In the order of smallness, it is first necessary to compensate or minimize $\mathrm{Im}(T_{21})'|_{\mathrm{p=0}}$ to reduce the $\gamma_P$. Here, the problem may be divided into two cases: (1) for materials with $\xi < 0$, for which $\mathrm{Im}(T_{21})'|_{\mathrm{p=0}} = 0$ can be obtained by choosing a crystallographic orientation without quartz rotator and (2) for materials with $\xi > 0$, for which this cannot be done. At first, consider the case $\xi > 0$ and, for definiteness and simplicity of analytical calculations, we assume that crystalline MOEs have the [111] orientation. In this case the coefficients $A_1 = A_2 = A$, $B_1 = B_2 = B$ and are defined as

$$A = h\sin(2\varphi)(1+2\xi)/3,$$
$$B = h\cos(2\varphi)(1+2\xi)/3.$$
$$h = \frac{1}{u}\int_0^u dz\int_0^z F_h(\zeta)d\zeta, \tag{7}$$

where $u = (r/r_h)^2$, $r$ and $\varphi$ are the polar radius and angle, and $r_h$ is the characteristic beam radius. By substituting (7) into (1)–(4) we obtain that for fixed $\delta_{c02}$, $\delta_{c01}$ ($\delta_{c02} \neq \delta_{c01}$) and $D = 1$ it is impossible to fulfill $\mathrm{Im}(T_{21})'|_{\mathrm{p=0}} = 0$ for any arbitrary $\varphi$, because is need to simultaneously meet $\sin(2\theta_r + \pi/4) = 0$ and $\cos(2\theta_r + \pi/4) = \delta_{c02}\sin(\delta_{c01}/2)/(\delta_{c01}\sin(\delta_{c02}/2))$. It is impossible to compensate $\gamma_V$ and $\gamma_P$ simultaneously. However, it is possible to find the value of $\theta_r$ at which $\gamma_P$ will be minimal. By differentiating with respect to $\theta_r$ and equating the derivative to zero we obtain that $\gamma$ will be minimal at $\theta_r = 3\pi/8 + \pi k$, and to an accuracy of terms of order $O(p^3)$, we obtain

$$\gamma = \left\{\frac{Q}{\lambda\kappa}\frac{\alpha_{01}\alpha_{02}}{\alpha_{02}-\alpha_{01}}\frac{\pi}{4VH^*}P_{laser}\right\}^2$$
$$a_1\frac{(2\xi+1)^2}{18}\left[\frac{\sin(\delta_{c01}/2)}{\delta_{c01}} - \frac{\sin(\delta_{c02}/2)}{\delta_{c02}}\right]^2,$$
$$\delta_{c01} = \frac{\pi}{2}\frac{\alpha_{02}}{\alpha_{02}-\alpha_{01}}, \quad \delta_{c02} = -\frac{\pi}{2}\frac{\alpha_{01}}{\alpha_{02}-\alpha_{01}},$$
$$a_1 = \int_0^\rho h^2F_h du. \tag{8}$$

In the case $\xi_i < 0$, according to [20], the condition $\mathrm{Im}(T_{21})'|_{\mathrm{p=0}} = 0$ is fulfilled for any values of $\xi_1$, $\xi_2$, $\delta_{c01}$, $\delta_{c02}$, $\theta_r$ and $D$, if MOEs are cut in the [C] orientation and are rotated by the angles $\Phi_1 = \delta_{c01}/4$ and $\Phi_2 = \theta_r + \delta_{c01}/2 + \delta_{c02}/4$.

For two different materials ($\xi_1 \neq \xi_2$), $\mathrm{Im}(T_{21})'|_{p=0}=0$ and $\mathrm{Re}(T_{21})''|_{p=0}=0$ cannot be met simultaneously, and for $\xi_1=\xi_2<0$, $\mathrm{Im}(T_{21})'|_{p=0}=0$ and $\mathrm{Re}(T_{21})''|_{p=0}=0$ will be fulfilled simultaneously if $\Phi_1=\Phi_2$ and system (9) is satisfied

$$\begin{cases} \frac{\sin(\delta_{c01}/2)}{\delta_{c01}}\sin\left(\frac{\delta_{c01}}{2}-2\Phi_1\right)+ \\ +\frac{\sin(\delta_{c02}/2)}{\delta_{c02}}\sin\left(2\theta_r+\delta_{c01}+\frac{\delta_{c02}}{2}-2\Phi_1\right)=0, \\ \frac{\delta_{c01}-\sin(\delta_{c01})}{2\delta_{c01}^2}+\frac{\delta_{c02}-\sin(\delta_{c02})}{2\delta_{c02}^2}+ \\ +2\frac{\sin(\delta_{c01}/2)}{\delta_{c01}}\frac{\sin(\delta_{c02}/2)}{\delta_{c02}}\sin\left(2\theta_r+\frac{\delta_{c01}}{2}+\frac{\delta_{c02}}{2}\right)=0, \end{cases} \tag{9}$$

It is clear from system (9) that the last equality is the equation for the magnitude of the angle of polarization rotation in the quartz rotator $\theta_r$, by substituting which into the first equality we obtain the angle of rotation of MOE crystals, $\Phi_1$. By substituting (6) into (9) we see that the solutions of system (9) do not depend on $\xi$. In the absence of a quartz rotator $\theta_r=0$, system (9) has no solutions and always has solutions for any $|\delta_{c01}/\delta_{c02}|=\alpha_{02}/\alpha_{01}>1.42$. When $\alpha_{02}/\alpha_{01}$ tends to the infinity, we have

$$\sin(2\theta_r+\pi/4)\xrightarrow[x\to\infty]{}-\sqrt{2}(\pi-2)/2\pi,$$
$$\tan(2\Phi_1)\xrightarrow[x\to\infty]{}\begin{cases}1-2[\pi-2]/\left[\sqrt{\pi^2+4\pi-4}+\pi+2\right],\\ 1+2[\pi-2]/\left[\sqrt{\pi^2+4\pi-4}-\pi-2\right].\end{cases} \tag{10}$$

Thus, there exist two solutions for $\theta_{r1}\approx-30^{\circ}$ and $\theta_{r2}\approx 75^{\circ}$, each of which corresponds to the value of the angle $\Phi_1$ ($18.6^{\circ}\pm 90^{\circ}\cdot m$ and $60.3^{\circ}\pm 90^{\circ}\cdot m$, respectively, *m is* an integer). It can also be noted that for $\alpha_{02}/\alpha_{01}>5$, the change in the $\theta_{ri}$ value does not exceed 1.4% and using of $\theta_{r1}\approx$-30$^{\circ}$ and $\theta_{r2}\approx 75^{\circ}$ do not critically change the compensation efficiency. The fulfillment of (9) for MOEs of identical material but with different absorption coefficients will allow simultaneous fulfillment of $\mathrm{Im}(T_{21})'|_{p=0}=0$, $\mathrm{Re}(T_{21})'|_{p=0}=0$ and $\mathrm{Re}(T_{21})''|_{p=0}=0$, and from (2) we will obtain $\Gamma=(\mathrm{Im}(T_{21})''|_{p=0})^2p^4/4+O(p^5)$. It is impossible to obtain an analytical expression for $\gamma$ in simple form.

## 3 Analysis of the effectiveness of the proposed Faraday isolator scheme

When creating a Faraday isolator, it is necessary to understand which of the contributions from thermally induced linear birefringence or from inhomogeneous Faraday rotation due to the temperature dependence of the Verdet constant to thermally induced depolarization will be decisive at a high laser power and what optical scheme from Fig. 2 to choose. To do this, lets define a coefficient $\varsigma$ equal to the ratio of these contributions to each other: $\varsigma=(\gamma_V/\gamma_P^{\min})^{1/2}$ as was done in Ref. [6] for TGG crystal ($\xi>1$) or $\varsigma=(\gamma_V/\gamma_{[C]}^{\min})^{1/2}$ provided that $\gamma_V+\gamma_{[C]}^{\min}=10^{-3}$ as in Ref. [20] for materials with $\xi<0$. By generalizing for all magnetooptical materials this coefficient will be written in the form:

$$\varsigma=\begin{cases}0.35\left|\frac{\lambda VH^*}{Q}(\alpha_T+\beta_T)\right|, & \xi\geq 1,\\ 0.35\left|\frac{1}{\xi}\frac{\lambda VH^*}{Q}(\alpha_T+\beta_T)\right|, & 0>\xi>1,\\ 30\left|\frac{\lambda VH^*}{Q}(\alpha_T+\beta_T)\frac{(\xi-1)}{(2\xi+1)(2\xi-3)}\right|, & \xi\leq 0.\end{cases} \tag{11}$$

The equality of this coefficient to unity corresponds to the equality of the contributions from thermally induced linear birefringence or from inhomogeneous Faraday rotation. The material parameters determining this coefficient depend on the wavelength and temperature, so it will vary both from material to material and depending on the conditions in which this material will be used. Hence, the proposed compensation scheme should be used for the materials and under the conditions when $\varsigma>1$. Contrariwise, when $\varsigma<1$, it is more efficient to use FI schemes with compensation of the contribution from thermally induced linear birefringence [12, 20]. The known parameters of a number of magneto-optical materials and the calculated values $\zeta$ are presented in Table 1. The emphasis in the table is on materials with $\xi<0$ and the parameters are given at room temperature measured at $\lambda=1$ µm.

It is clear from the table that the NTF crystal based on the data from [27] is most promising for use in the proposed scheme ($\zeta=3.56$). However, the material parameters of the NTF crystal were measured more accurately and refined in the Ref. [28] ($\xi$ became farther away from -0.5, $\alpha_0 Q$ increased by more than 60 times), which led to a change in the $\zeta$ value by 1.9 times. So, this material is less interesting in terms of its application in the proposed compensation scheme. During cryogenic cooling, due to an increase in the Verdet constant and a decrease in the $Q$ constant, the $\zeta$ coefficient increases significantly. As a consequence, in cryogenic Faraday isolators, the problem of compensating $\gamma_V$ becomes decisive. For example, for a widely used TGG crystal during cryogenic cooling, the $\zeta$ coefficient increases by a factor of 81, and at a temperature of 77 K, the thermally induced depolarization is completely determined by the $\gamma_V$ [6]. A similar situation will most likely be observed for all materials listed

**Table 1** Material parameters of the magneto-optical media

| | $\xi$ | $\|Q\|$, $\times 10^{-7}$ 1/K | $\alpha_0$, 1/m | $\kappa$, W/(m $\times$ K) | $V$, rad/(T $\times$ m) | $\beta_T$ $\times 10^{-3}$, 1/K | $L^*$, cm | $\varsigma$ | References |
|---|---|---|---|---|---|---|---|---|---|
| TSAG | – 101 | 0.173 | 0.3 | 3.6 | – 46.2 | – 3.5 | 0.68 | 1.86 | [26] |
| NTF | – 0.37 | 34.4 | 0.014 | 1 | – 32 | – 3.4 | 0.98 | 3.56 | [27, 28] |
| NTF | – 0.26 | 34.4 | 0.85 | 1 | – 32 | – 3.4 | 0.98 | 1.91 | [28] |
| KTF | – 4.9 | 18 | 0.015 | 1.67 | – 34 | – 3.4 | 0.92 | 0.27 | [29] |
| $EuF_2$ | – 0.95 | 17 | 4 | 1.6 | – 48.4 | – 3.4 | 0.65 | 3.43 | [30] |
| Tb:$CaF_2$ | – 0.61 | 33.1 | 0.001 | 2.2 | – 4 | – 3.4 | 7.85 | 0.57 | [31] |
| TCZ | – 0.29 | 370 | 0.3 | 2 | – 48.5 | – 3.4 | 0.65 | 0.31 | [32] |
| TGG 300 K | 2.25 | 17 | 0.13 | 4.5 | – 37 | – 3.4 | 0.9 | 0.07 | [33] |
| TGG 77 K | 1.78 | 2.98 | 0.26 | 4.8 | – 141 | – 13 | 0.22 | 5.7 | [6] |

in Table 1, and the problem under discussion will most acutely arise in cryogenic FIs. Consider the efficiency of the proposed compensation scheme on an example of two single crystals: $EuF_2$ ($\varsigma_{EuF2} = 3.43$) and TGG at 77 K ($\varsigma_{TGG77} = 5.7$) (Fig. 3). For comparison the integral thermally induced depolarization for an traditional FI (Fig. 2a) and the scheme proposed in Ref. [18] were plot. The plots for the scheme with compensation $\gamma_V$ and the scheme proposed in Ref. [18] numerically calculated for crystals with [111] orientation are shown in Fig. 3a, b and for [C] orientation in Fig. 3c. The numerical calculation was carried out without the weak birefringence approximation while maintaining all other approximation.

The analysis of the data in Fig. 3 shows that the efficiency of compensation increases with increasing the $x = \alpha_{02}/\alpha_{01}$ ratio and with increasing parameter $\varsigma$ of the material. For materials with $\xi < 0$ in the [C] orientation, the efficiency of the proposed compensation scheme depends on the value of

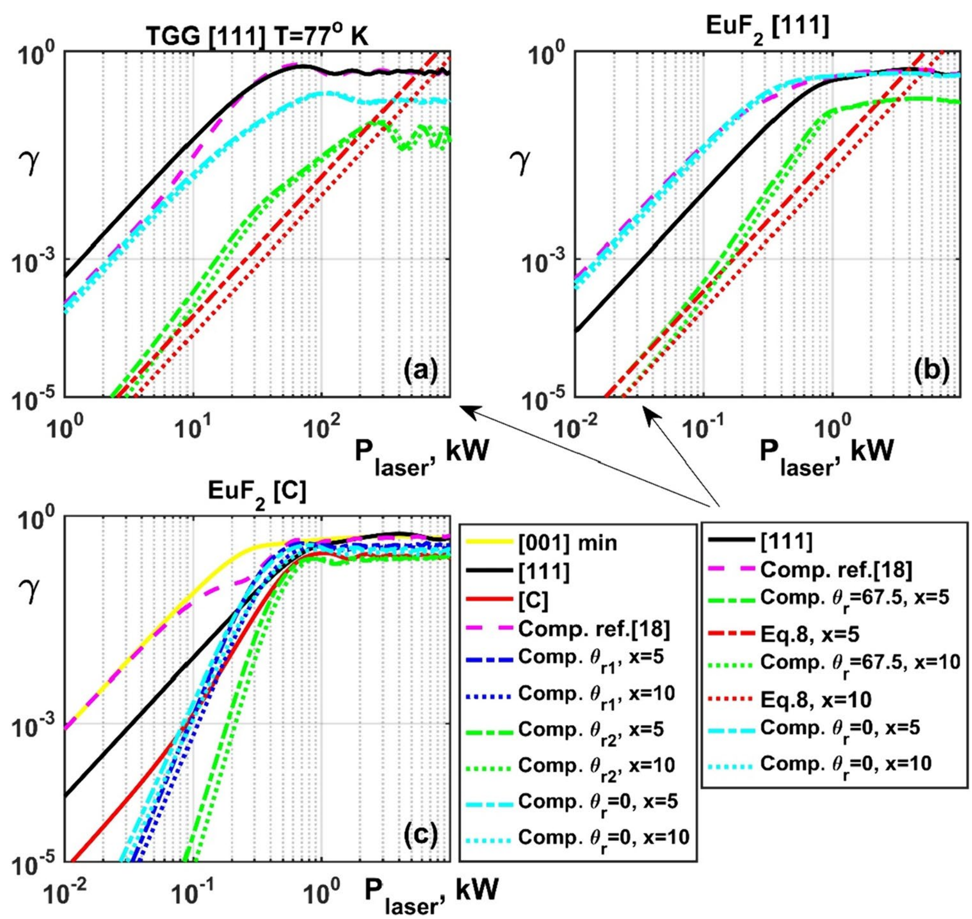


**Fig. 3** Integral thermally induced depolarization versus laser radiation power for conventional FI with [001], [111] and [C] orientations—solid curves; FI with compensation [18]—magenta dashed curve; for FI with compensation of temperature dependence of the Verdet constant for $x = \alpha_{02}/\alpha_{01} = 5$—dash-dotted curves and $x = \alpha_{02}/\alpha_{01} = 10$—dotted curves and for two different angles of rotation of polarization plane in quartz rotator: $\theta_{r1}$—blue curves, $\theta_{r2}$—green curves and without quartz rotator—cyan curves. **a** For cryogenically cooled TGG crystal with [111] orientation; **b** for $EuF_2$ crystal with [111] orientation; **c** for $EuF_2$ crystal with [C] orientation

the angle of polarization rotation in the quartz rotator and is higher for $\theta_{r2} \approx 75°$. The use of the proposed scheme with quartz rotator $\theta_{r1}$ for $EuF_2$ crystal as MOE material does not bring any significant benefits. The $P_{max}$ value of the FI with compensation of $\gamma_V$ based on cryogenically cooled TGG crystals with [111] orientation and $\alpha_{02}/\alpha_{01} = 10$ increases by 13.3 times compared to the traditional cryogenic FI with MOE in the [111] orientation. The $P_{max}$ value of the FI with compensation $\gamma_V$ based on $EuF_2$ crystals with [111] orientation and $\alpha_{02}/\alpha_{01} = 10$ increases by 4.7 times compared to the traditional FI on the same crystal with [111] orientation and by 1.8 times compared to the traditional FI on the same crystal with [C] orientation. If in the FI scheme with compensation of $\gamma_V$ the $EuF_2$ crystals with [C] orientation and $\alpha_{02}/\alpha_{01} = 10$ is used, the $P_{max}$ value increases by 1.22 and 2.4 for $\theta_{r1}$ and $\theta_{r2}$, respectively, compared to the traditional FI on the same crystal with [C] orientation. Direct pass losses in this case do not exceed 5% up to $P_{max}$ power. Numerical simulation has shown that in the proposed FI scheme with compensation of $\gamma_V$, using a quartz rotator with $\theta_r = 75^{\circ}$ (or $\theta_r = -30^{\circ}$) by varying the angle $\Phi_1 = \Phi_2$ the value of $\gamma$ (in the region of interest to us $\gamma \approx 10^{-3}$) can be returned close to the value obtained at $\theta_r$ and $\Phi_1$ calculated from system (9) for a specific $x$ for any $x > 3$. That is, the difference between the value of $\theta_r$ from the optimal one within ~ 3% is not critical and in practice can be compensated for by turning the MOEs (by changing the angle $\Phi_1 = \Phi_2$). The efficiency of using the proposed scheme of compensation does not depend on the absolute value of absorption coefficient only on the $\alpha_{02}/\alpha_{01}$ ratio. If the crystal growth technology allows reducing the absorption coefficient by several times, then the $P_{max}$ value will increase by the same factor in the traditional FI scheme and in the scheme with compensation proposed in this work. As predicted above, the use of the scheme proposed in [18] leads to a decrease in the isolation ratio and the value of $P_{max}$ (Fig. 3, magenta dashed curves). Similarly, excluding the quartz rotator from the proposed FI scheme with compensation, even though the $\gamma_V$ is compensated, an increase in the contribution from thermally induced linear birefringence leads to a significant decrease in the isolation ratio and the value of $P_{max}$ (Fig. 3, cyan curves).

The independence of the parameters of the proposed compensation scheme on $\xi$ and the weak dependence on $\alpha_{02}/\alpha_{01} >> 1$ significantly simplifies the practical use of the scheme. To implement the proposed scheme, two boules of material with different absorption are required. The absorption coefficient of a material can be easily increased by simply adding a small amount of impurity during its production [34]. The $\alpha_{02}/\alpha_{01}$ ratio are easily determined from measurements of thermally induced depolarization in each of the elements separately. Then the MOEs should be cut to the required length, at which the depolarization in them will be equal and they will be sufficient for the required Faraday rotation in the existing magnetic system. The required ratio $\delta_{c02}/\delta_{c01}$ and the total Faraday rotation angle $\delta_{c02} + \delta_{c01}$ can be achieved by moving the prepared MOEs along the axis of the magnetic system. The $\delta_{c02}/\delta_{c01}$ ratio does not depend on the MOEs temperature and will remain unchanged upon cooling. And finally, for MOE cut in [111] orientation should be used a 67.5° quartz rotator, and for samples cut in orientation [C]—a 75° quartz rotator. The angle $\Phi_1 = \Phi_2$ is set experimentally according to the minimum thermally induced depolarization at high average laser power. Proposed scheme can be used when the MOEs are in different conditions (for example, one of them is at cryo, the other is at room temperature). In this case, due to different values of thermo-optical constants under these conditions, even when using one material, the efficiency of the compensation scheme will be like that with the use of different materials.

## 4 Conclusion

The main conclusions of the study are the following:

(1) A new scheme of the Faraday isolator with compensation of thermally induced depolarization has been proposed. The direction of polarization rotation in the elements is opposite and they are made of the material with different parameters. This allows compensating for the contribution to thermally induced depolarization from inhomogeneous Faraday rotation due to the temperature dependence of the Verdet constant and a properly selected quartz rotator with new restrictions enables partial compensation for the contribution from the thermally induced linear birefringence. The angles of Faraday rotation (≡MOE lengths) are determined only by the ratio of the parameters of the used materials.

(2) The coefficient $\varsigma$ for determining which of the FI schemes with compensation for a particular material is preferable has been proposed. The new FI scheme is most efficient with the use of MOEs made of materials with the parameter $\varsigma >> 1$, with the efficiency increasing with the increase of $\varsigma$.

(3) The highest efficiency of compensation is observed when the MOEs are made of the same material but have different properties (for example, different absorption coefficients). The efficiency of the FI scheme when MOEs are made from identical material with differing absorption coefficients increases with increasing $\alpha_{02}/\alpha_{01}$ ratio.

(4) For materials with $\xi < 0$, for the absorption coefficients ratio $\alpha_{02}/\alpha_{01} > 1.42$, there always exist values of the angle of rotation of polarization plane in a quartz rotator ($\theta_r$) and angle of rotation of MOE with [C] orientation ($\Phi_1 = \Phi_2$) at which the first terms of the expansion of thermally induced depolarization in a small parameter will be compensated for up to the term proportional to the fourth

power of $P_{laser}$. In this case, as $\alpha_{02}/\alpha_{01}$ tends to the infinity, the values of optimal $\theta_r$ and $\Phi_1 = \Phi_2$ tend to finite values.

5) According to the calculations, the $P_{max}$ value of the FI with the proposed scheme of depolarization compensation on the basis of a cryogenically cooled TGG crystal with $\alpha_{02}/\alpha_{01} = 10$ increases by a factor of 13.3, and on the basis of $EuF_2$ crystals by a factor of 2.4 (instead of a factor of 1 and 1.3, respectively, for the compensation scheme from [12, 20]) compared to a traditional FI.

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**Author contributions** I.S. was responsible for problem formulation, theoretical and numerical analysis, drawing up conclusions, writing text and figures preparation

**Data availability** The data that support the findings of this study are available from the corresponding author upon reasonable request.

## Declarations

**Competing interests** The authors declare no competing interests.

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