



# Bell nonlocality, entanglement, and entropic uncertainty in a Heisenberg model under intrinsic decoherence: DM and KSEA interplay effects

Mostafa Hashem<sup>1,2</sup> · Abdel-Baset A. Mohamed<sup>2</sup> · Saeed Haddadi<sup>3,4</sup> · Youssef Khedif<sup>5</sup> ·  
Mohammad Reza Pourkarimi<sup>6</sup> · Mohammed Daoud<sup>7,8</sup>

Received: 19 December 2021 / Accepted: 8 March 2022 / Published online: 29 March 2022  
© The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2022

## Abstract

Bell's inequalities are described by the sums of correlations including non-commuting observables in each of two systems. Bell's inequalities violation is possible since the accuracy of any joint measurement of mentioned observables would be limited by quantum uncertainty relations. In this work, we investigate the generating and robustness of two-qubit information resources including two-qubit Bell nonlocality, quantum entanglement, and entropic measurement uncertainty in a two neighboring spin-1/2 particles coupled via the Heisenberg XYZ interaction subjected to a transverse uniform magnetic field by applying Dzyaloshinskii–Moriya (DM) and Kaplan–Shekhtman–Entin–Wohlman–Aharony (KSEA) interactions under intrinsic decoherence. The influence of DM–KSEA interactions, external magnetic field, and intrinsic decoherence on the dynamics of quantum correlations in our mentioned model is analyzed. Interestingly, new dynamical features of Bell nonlocality, entanglement, and entropic uncertainty are obtained by regulating the initial state, system parameters, and decoherence. Therefore, our results provide a helpful understanding of such dynamics and might offer an insight into measurement estimating in open quantum systems.

## 1 Introduction

Numerous processes to generate the two-qubit information resources, namely, quantum correlations [1–3] and quantum coherence [4] have been exploited and used in several real physical systems. These significant concepts have tremendous advantages in different aspects of modern physics from information science to condensed matter theory [5, 6]. The quantum entanglement [2, 7–11], a direct result of the superposition principle, is considered at the forefront of resources which stimulates the advance of quantum technology. The occurrence of this purely quantum phenomenon manifests in an instantaneous disturbance of any part of a pair of spatially separated particles immediately after the other part undergoes a local measurement [7, 8]. This is done as if a certain channel is configured to secure a nonlocal correlation between the entangled system components. Such nonlocality makes the involved system enjoys by its distinguishing feature of being able to be described only as a whole rather than in its parts. Therefore, the nonlocality concept is a quintessential property of quantum entanglement. It is regarded as a fundamental resource for many quantum technology applications, such as quantum teleportation [12, 13], quantum

✉ Saeed Haddadi  
saeed@ssqig.com

Abdel-Baset A. Mohamed  
abdelbastm@aun.edu.eg

- <sup>1</sup> Faculty of Computers and Information Technology, The Egyptian E-Learning University, Giza 12611, Egypt
- <sup>2</sup> Department of Mathematics, Faculty of Science, Assiut University, Assiut, Egypt
- <sup>3</sup> Faculty of Physics, Semnan University, P.O.Box 35195-363, Semnan, Iran
- <sup>4</sup> Saeed's Quantum Information Group, P.O.Box 19395-0560, Tehran, Iran
- <sup>5</sup> Laboratory of High Energy and Condensed Matter Physics, Department of Physics, Faculty of Sciences Aïn Chock, University Hassan II, P.O. Box 5366, 20100 Maarif Casablanca, Morocco
- <sup>6</sup> Department of Physics, Salman Farsi University of Kazerun, Kazerun, Iran
- <sup>7</sup> Department of Physics, Faculty of Sciences, University Ibn Tofail, Kénitra, Morocco
- <sup>8</sup> Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34151 Trieste, Italy

dense coding [14], quantum cryptography [15], quantum metrology [16] and so on. Besides its excellent technological applications, nonlocality provides a new understanding of many conceptual issues in quantum physics. In terms of the Bell nonlocality concept [1, 8], the nonlocal character of a quantum state lies in the violation of the Bell inequalities. Strictly speaking, the realization of violation of the Bell inequalities for a quantum system led to the conclusion that there is an entanglement phenomenon [17]. In this respect, Buscemi has been establish a closer relationship between entanglement and nonlocality and shown that all entangled states are necessarily nonlocal [18].

Whenever the multipartite system is correlated, any local measurement made on one of its constituents is necessarily accompanied by a degree of uncertainty. This uncertainty is regarded as a primary tool to characterize the nonlocal correlation between the multipartite system parts. The uncertainty principle was recognized as one of the most striking features of quantum mechanics that genuinely differentiates the classical world from the quantum one [19–22]. Specifically, the famous Heisenberg uncertainty principle states that the momentum  $\hat{p}$  and position  $\hat{x}$  of a physical system cannot be simultaneously and precisely assigned [19, 20]. Aside from momentum and position, Robertson [21] and Schrödinger [23] showed that such limitation can be generalized to any arbitrary two incompatible observables. To make a connection between the uncertainty relation and quantum information theory, the entropic uncertainty relation was introduced [24, 25]. It is shown that this new version of uncertainty has attracted special attention in the literature (see for instance Refs. [26–42] and references therein). From an applicative point of view, the entropic uncertainty relation provides valuable physical resources for many quantum information tasks achievement; such as entanglement witnessing [43], quantum metrology [44], cryptographic security [45], quantum randomness [46], probing quantum correlations [47, 48] and quantum key distribution [49].

Quantum correlations in finite-dimensional systems are broadly recognized as crucial resources to improve the performance of quantum protocols in comparison to their classical analogue [50]. But the proper use of such quantum correlations is often obstructed by the decoherence phenomenon. Among the decoherence process, we distinguish intrinsic decoherence which was initially addressed by Milburn [51] and after intensively studied in many seminal works [52–56]. The Milburn intrinsic decoherence was mainly introduced to modify the usual Schrödinger equation that governs the quantum system time evolution. In fact, Milburn approach is based on the assumption wherein over sufficiently short time steps, the system under consideration does not evolve

continuously under the effect of a unitary evolution, but rather according to a stochastic sequence of identical unitary phase transformations. It is worth mentioning that the influence of intrinsic decoherence in quantum systems was intensively addressed in many works (see, for instance, [57–72]).

In the present work, we shall explore the intrinsic decoherence approach for engendering pairwise information resources, such as the two-qubit Bell nonlocality, entropic uncertainty, and entanglement. In this context, we deal with a two neighboring spin-1/2 particles coupled via the Heisenberg XYZ interaction subjected to a transverse magnetic field and under the interplay  $z$ -components of DM ( $z$ -DM) along with KSEA ( $z$ -KSEA) interactions controlled by the intrinsic decoherence effect. The characterization and quantification of quantum correlations as well as entropic uncertainty in this investigating model were recently studied [73, 74]. Usually, the choice of Heisenberg spin systems is motivated by the fact that such quantum systems are the potential candidate for implementing quantum communication and many other quantum information tasks.

This paper is outlined as follows. The Sect. 2 is devoted to reviewing some preliminaries about the quantum correlations that can be employed in this work. In Sect. 3, we introduce the model describing the quantum system consisting of a two-spin Heisenberg XYZ model under an external transverse homogeneous magnetic field by considering also the effects of the spin-orbit coupling both antisymmetric as well as symmetric contributions directed along the  $z$ -axis and studying its intrinsic evolution in the ambit of the Milburn master equation. The dynamical behaviors of the investigated quantum correlations in terms of the different parameters characterizing the considered system are examined and discussed through Sect. 4. Finally, our main results are summarized in Sect. 5.

## 2 Two-qubit information resource quantifiers

- **Bell nonlocality (BL)** Here, let us use the maximal amounts of the two-qubit Bell function  $B_{\max}(t)$  as an indicator of two-qubit nonlocality [75, 76]. This two-qubit nonlocality satisfies  $B_{\max}(t) > 2$ , namely Bell-CHSH inequality is violated [77]. The analytical representation of the two-qubit Bell function  $B_{\max}(t)$  is given by [78]

$$B_{\max}(t) = 2\sqrt{\xi_1 + \xi_2}, \quad (1)$$

in which  $\xi_1$  and  $\xi_2$  are the two largest eigenvalues of the matrix  $T^\dagger T$  with  $T = [t_{ij}]$  which is the correlation matrix

[76] of a two-qubit state  $\hat{M}_{AB}$ , and its elements are given by

$$t_{ij} = \text{tr}\{\hat{M}_{AB}(\sigma^i \otimes \sigma^j)\}, \tag{2}$$

where  $\sigma^{ij}$  ( $i, j = x, y, z$ ) are the Pauli spin matrices. In our investigation, the BL is quantified by

$$BL(\hat{M}_{AB}) = B_{\max}(t) - 1. \tag{3}$$

This means that two-qubit state has BL when this function satisfies  $BL(\hat{M}_{AB}) > 1$ .

- **Concurrence (CE)** The CE is one of the most common entanglement measures for a bipartite quantum system  $\hat{M}_{AB}$ . The CE is defined as [79, 80]

$$CE(\hat{M}_{AB}) = \max\{0, \sqrt{\mu_1} - \sqrt{\mu_2} - \sqrt{\mu_3} - \sqrt{\mu_4}\}, \tag{4}$$

where  $\mu_i$ 's are the positive eigenvalues of the matrix  $R = \hat{M}_{AB}(\sigma^x \otimes \sigma^y)\hat{M}_{AB}^*(\sigma^y \otimes \sigma^x)$  in decreasing order with  $\hat{M}_{AB}^*$  which is the complex conjugate of density matrix  $\hat{M}_{AB}$ . In general,  $0 \leq CE(\hat{M}_{AB}) \leq 1$ .

- **Entropic uncertainty (EU)** The concept of uncertainty was first proposed by Heisenberg [19] in 1927, and then it was formulated by Kennard [20] and Robertson [21] in terms of a standard deviation. After that, some researchers introduced new relations based on the Shannon and von Neumann entropies [81, 82]. Amongst them, let us use the relation provided by Berta et al. [83] for a two-particle system with a quantum memory ( $B$ ), which may be correlated with the measured particle ( $A$ ). This scenario can be run between two legitimate characters called Alice and Bob. The uncertainty of Bob about measurement outcome of Alice is bounded by [83]

$$S(P|B) + S(Q|B) \geq S(A|B) + \log_2 \frac{1}{c}, \tag{5}$$

where  $S(A|B) = S(\hat{M}_{AB}) - S(\hat{M}_B)$  is the conditional von Neumann entropy of density operator  $\hat{M}_{AB}$  with  $S(\hat{M}) = -\text{tr}(\hat{M} \log_2 \hat{M})$  (for a general density matrix  $\hat{M}$ ). And  $c = \max_{i,j}\{|\langle p_i|q_j \rangle|^2\}$  where  $|p_i\rangle$  and  $|q_j\rangle$  are eigenstates of the incompatible observables  $P$  and  $Q$ . In the left-hand side,  $S(X|B) = S(\hat{M}_{XB}) - S(\hat{M}_B)$  with  $X \in \{P, Q\}$  being the conditional von Neumann entropy of post measurement states  $\hat{M}_{PB} = \sum_i (|p_i\rangle_A \langle p_i| \otimes \mathbf{I}_B)\hat{M}_{AB}(|p_i\rangle_A \langle p_i| \otimes \mathbf{I}_B)$ ,  $\hat{M}_{QB} = \sum_j (|q_j\rangle_A \langle q_j| \otimes \mathbf{I}_B)\hat{M}_{AB}(|q_j\rangle_A \langle q_j| \otimes \mathbf{I}_B)$ . Herein,  $\hat{M}_B = \text{tr}_A(\hat{M}_{AB})$  and  $\mathbf{I}_B$  is the identity operator. According to Eq. (5), by resorting the incompatibility  $P$  and  $Q$  with the Pauli spin-1/2 operators  $\sigma^x$  and  $\sigma^z$ , respectively, the EU, left-hand side of Eq. (5), can be expressed as

$$EU(\hat{M}_{AB}) \equiv S(\hat{M}_{\sigma^x B}) + S(\hat{M}_{\sigma^z B}) - 2S(\hat{M}_B). \tag{6}$$

### 3 Physical model and its dynamics

This section considers a Heisenberg XYZ model consisting of two qubits, say  $A$  and  $B$ , under the external transverse uniform magnetic field with DM and KSEA (DM-KSEA) interactions governed along the  $z$ -axis as [73, 74]

$$\hat{H} = \sum_{i=x,y,z} J_i \sigma_A^i \sigma_B^i + \sum_{j=1,2} \left\{ D_j^z (\sigma_A^x \sigma_B^y + (-1)^j \sigma_A^y \sigma_B^x) \right\} + \sum_{k=A,B} B_k^z \sigma_k^z, \tag{7}$$

where  $\sigma_{A,B}^{x,y,z}$ 's denote the standard Pauli matrices which are spanned by the lower  $|0\rangle_k$  and upper  $|1\rangle_k$  ( $k = A, B$ ) states.  $B_k^z$  is the transverse homogeneous magnetic field in the  $z$ -direction at  $k$ -site.  $D_1^z$  is the strength of  $z$ -DM interaction resulting to orbit-spin coupling antisymmetric contribution,  $D_2^z$  represents the symmetric orbit-spin coupling contribution of the  $z$ -KSEA interaction, and  $J_i$  is the symmetrical spin-spin exchange coupling constants. Note that the special case  $D_1^z = D_2^z$  corresponds to a uniform spin-orbit interaction.

In the two-qubit standard computational basis states  $\{|\varpi_1\rangle = |11\rangle, |\varpi_2\rangle = |10\rangle, |\varpi_3\rangle = |01\rangle, |\varpi_4\rangle = |00\rangle\}$ , the Hamiltonian which represents that case is given by Eq. (7), and its the eigenstates  $|E_l\rangle$  ( $l = 1, 2, 3, 4$ ) and the eigenvalues  $E_l$  are

$$\begin{aligned} |E_1\rangle &= \Lambda_\alpha^+ (W_\alpha^+ |\varpi_1\rangle + |\varpi_4\rangle), & E_1 &= J_z + \alpha, \\ |E_2\rangle &= \Lambda_\beta^+ (W_\beta^+ |\varpi_2\rangle + |\varpi_3\rangle), & E_2 &= -J_z + \beta, \\ |E_3\rangle &= \Lambda_\beta^- (W_\beta^- |\varpi_2\rangle + |\varpi_3\rangle), & E_3 &= -J_z - \beta, \\ |E_4\rangle &= \Lambda_\alpha^- (W_\alpha^- |\varpi_1\rangle + |\varpi_4\rangle), & E_4 &= J_z - \alpha, \end{aligned} \tag{8}$$

with

$$\begin{aligned} \Lambda_\alpha^\pm &= \sqrt{\frac{1}{2} \pm \frac{1}{2\alpha}(B_A^z + B_B^z)}, & \Lambda_\beta^\pm &= \sqrt{\frac{1}{2} \pm \frac{1}{2\beta}(B_A^z - B_B^z)}, \\ W_\alpha^\pm &= \pm \frac{J_x - J_y + 2iD_2^z}{\alpha \pm (B_A^z + B_B^z)}, & W_\beta^\pm &= \pm \frac{J_x + J_y - 2iD_1^z}{\beta \pm (B_A^z - B_B^z)}, \\ \alpha &= \sqrt{(B_A^z + B_B^z)^2 + (J_x - J_y)^2 + 4(D_2^z)^2}, \\ \beta &= \sqrt{(B_A^z - B_B^z)^2 + (J_x + J_y)^2 + 4(D_1^z)^2}. \end{aligned} \tag{9}$$

Here, Milburn model [51] is used to explore the dynamics of the two-qubit information resources. The Milburn equation is given by

$$\frac{d}{dt} \hat{M}_{AB} = -i[\hat{H}, \hat{M}_{AB}] - \frac{\gamma}{2} [\hat{H}, [\hat{H}, \hat{M}_{AB}]], \tag{10}$$

where  $\hat{M}_{AB}$  is a general density matrix of the two-qubit and  $\gamma$  is the decoherence parameter.

Using the eigenvalues  $E_l$  and the eigenstates  $|E_l\rangle$ , the general solution for Eq. (10) is described by the following density matrix  $\hat{M}_{AB}(t)$  [57]

$$\hat{M}_{AB}(t) = \sum_{m,n=1}^4 \Lambda_{mn}(t) I_{mn}(t) \langle E_m | \hat{M}(0) | E_n \rangle |E_m\rangle \langle E_n|, \quad (11)$$

with the Schrödinger unitary interaction  $\Lambda_{mn}(t)$  and the intrinsic decoherence  $I_{mn}(t)$  terms which are described by

$$\Lambda_{mn}(t) = e^{-i(E_m - E_n)t}, \quad I_{mn}(t) = e^{-\frac{\gamma}{2}(E_m - E_n)^2 t}. \quad (12)$$

To obtain a particular analytical solution for Eq. (11), we consider that the two qubits are initially started with two different initial states; the uncorrelated state  $\hat{M}(0) = |\varpi_1\rangle \langle \varpi_4|$  and the maximally correlated Bell state

$$\hat{M}(0) = \frac{1}{2} [ (|\varpi_1\rangle + |\varpi_4\rangle) (\langle \varpi_1| + \langle \varpi_4|) ]. \quad (13)$$

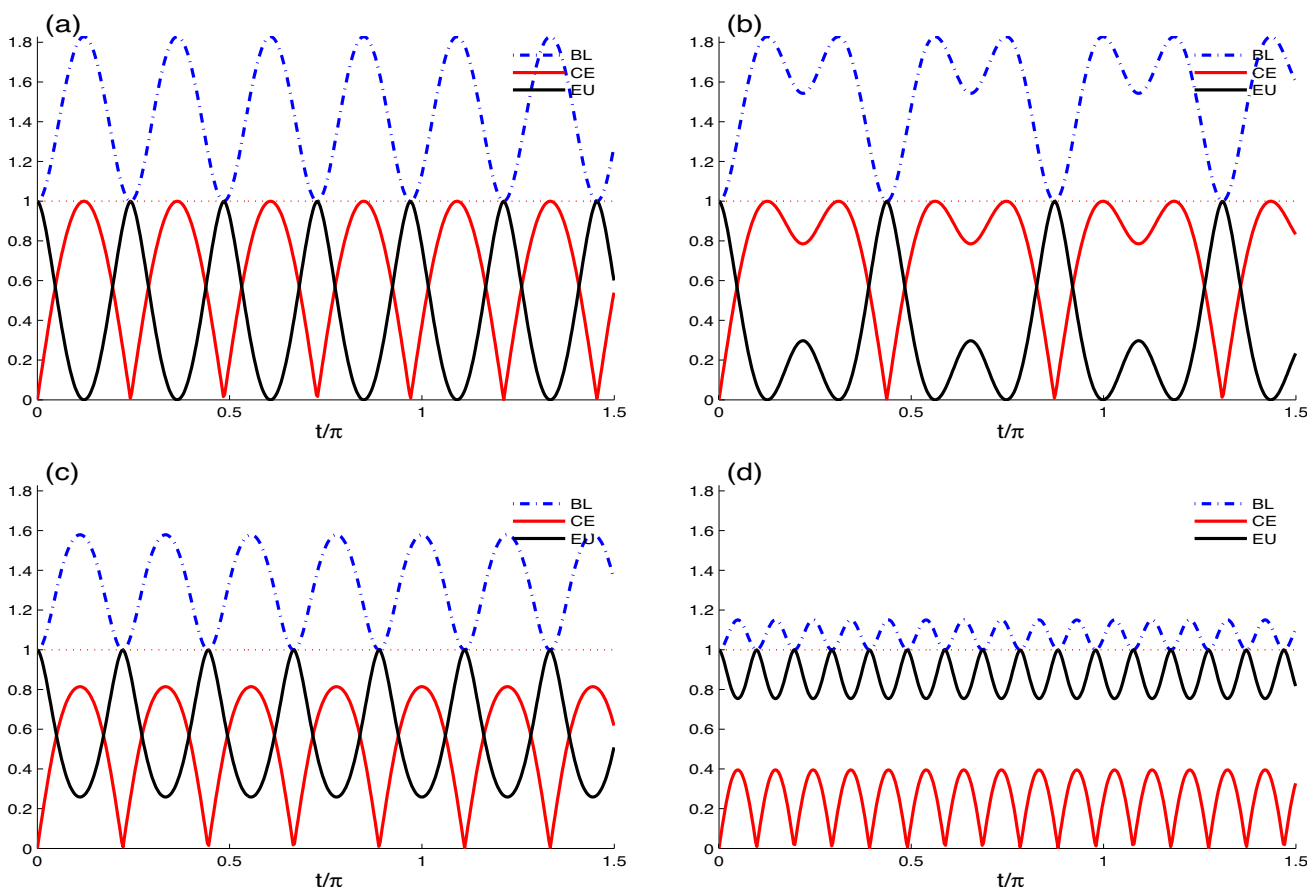
In the following, let us use the case of the uncorrelated state to explore the generation of the the two-qubit information

resources via BL, CE, and EU. While the case of the maximally correlated Bell state will be used to investigate the robustness of the two-qubit information resources against the Schrödinger unitary interaction and the intrinsic decoherence.

### 4 Dynamics of two-qubit information resources

#### 4.1 For initial separable state

The dynamics of the BL, CE, and EU are depicted in Fig. 1 for specific values of  $J_x = -1$ ,  $J_y = -0.5$ ,  $J_z = 2$ , and  $D_1^z = D_2^z = 1$  for different values of transverse uniform magnetic field  $B = B_A^z = B_B^z$  and in the absence of the intrinsic decoherence  $\gamma = 0$  when initial state is separable  $\hat{M}(0) = |\varpi_1\rangle \langle \varpi_4|$ . As shown in this figure, all criteria oscillate during the time and their frequencies increase with growing magnitude of  $B$  when  $B > 1/2$ . As mentioned before, the initial state is uncorrelated and hence, BL and



**Fig. 1** The dynamics of the BL, CE, and EU are shown for  $J_x = -1$ ,  $J_y = -0.5$ ,  $J_z = 2$ , and  $D_1^z = D_2^z = 1$  when the two-qubit system is initially in the uncorrected state  $\hat{M}(0) = |\varpi_1\rangle \langle \varpi_4|$  in the absence of

the intrinsic decoherence with different transverse uniform magnetic fields.  $B = 0$  in **a**,  $B = 0.5$  in **b**,  $B = 2$  in **c**, and  $B = 5$  in **d**

CE started from one and zero, respectively, and then they evolve between their minimal and maximal values within the time. Conversely, EU begins from its maximal value and it fluctuates in the opposite way of BL and CE.

In Fig. 2, all parameters are the same as in Fig. 1 but the system is affected by the intrinsic decoherence, that is  $\gamma = 0.02$ . It is obvious that BL, CE, and EU fluctuate but their amplitudes become less and finally, they reach steady values. According to this figure, they get the fixed values more quickly with increasing transverse homogeneous magnetic field  $B$ . One may conclude that the external magnetic field and intrinsic decoherence have a destructive effect on the quantum correlations when they affect the system simultaneously. Hence, Bob’s information about the results of Alice’s measurements in a decoherence environment decreases with increasing  $B$  during the time.

Figure 3 is devoted to study of the behavior of BL, CE, and EU in the absence and presence of decoherence for different values of the  $z$ -DM and  $z$ -KSEA interactions  $D = D_1^z = D_2^z$  and the other parameters are the same in Fig. 1b. As displayed in this figure, all functions oscillate throughout the time and when DM and KSEA interactions

are applied, their frequencies and sudden change points increase in the absence of decoherence [compare plots Fig. 3a and b]. In the presence of decoherence as well as DM–KSEA interactions, all quantities become fixed after some fluctuations (see Fig. 3d). However, there is no considerable difference between absence and presence of decoherence by comparing plots Fig. 3a and c, in the case that the DM–KSEA interactions are turned off.

### 4.2 For initial maximally correlated state

The time evolution of the robustness quantum information resources is shown in Fig. 4 for specific values of exchange couplings and  $D$ , and different values of transverse uniform magnetic field  $B$  with and without intrinsic decoherence when the two-qubit state is maximally entangled  $\hat{M}(0) = [(|\varpi_1\rangle + |\varpi_4\rangle)(\langle\varpi_1| + \langle\varpi_4|)]/2$  at the beginning time. It is obvious that BL and CE begin from their maximal values ( $BL(t=0)=2\sqrt{2} - 1 \approx 1.8284$  and  $CE(t=0)=1$ ) as expected from the assumed initial state, and conversely, EU starts from zero. Therefore, Bob can correctly guess Alice’s results at the initial moment the system interacts with the

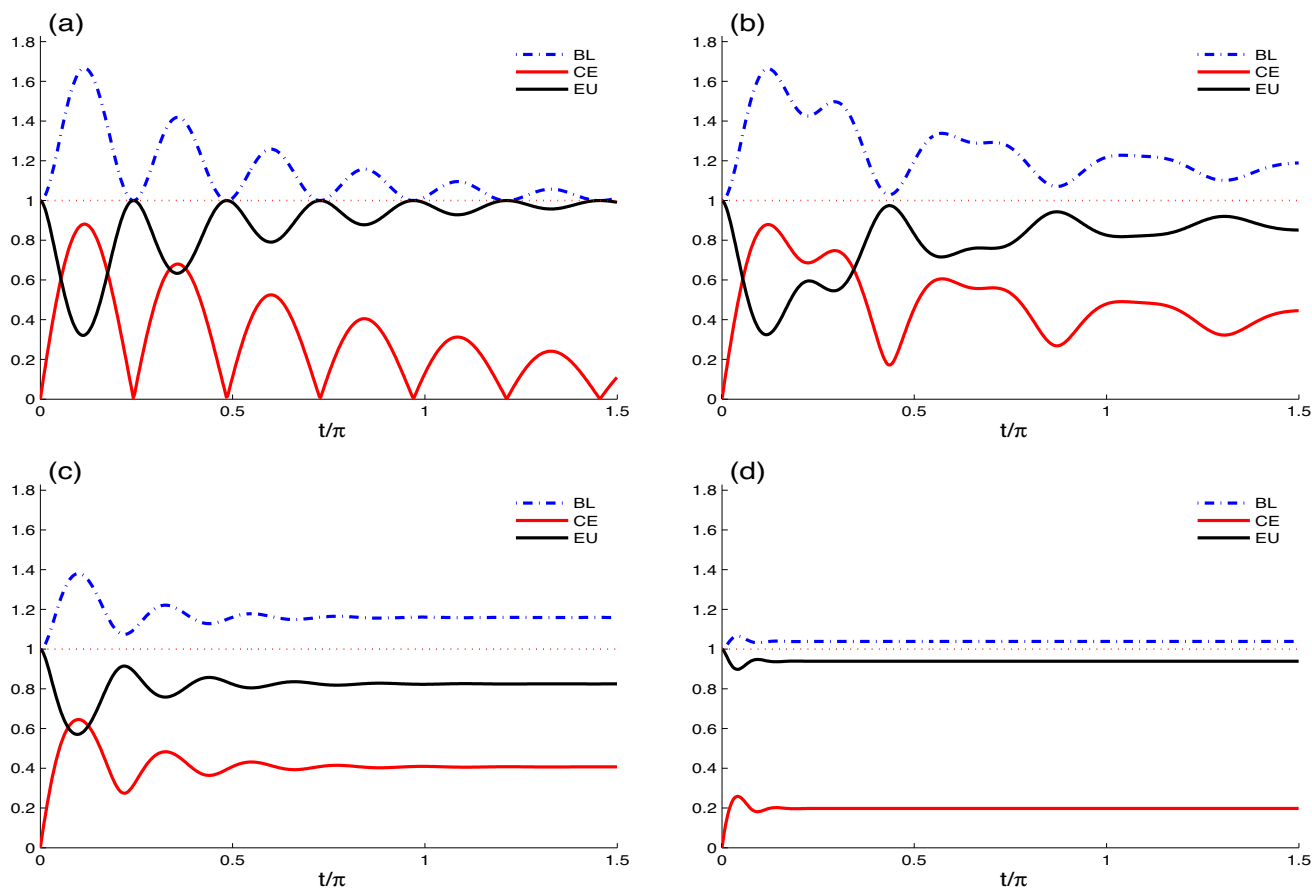
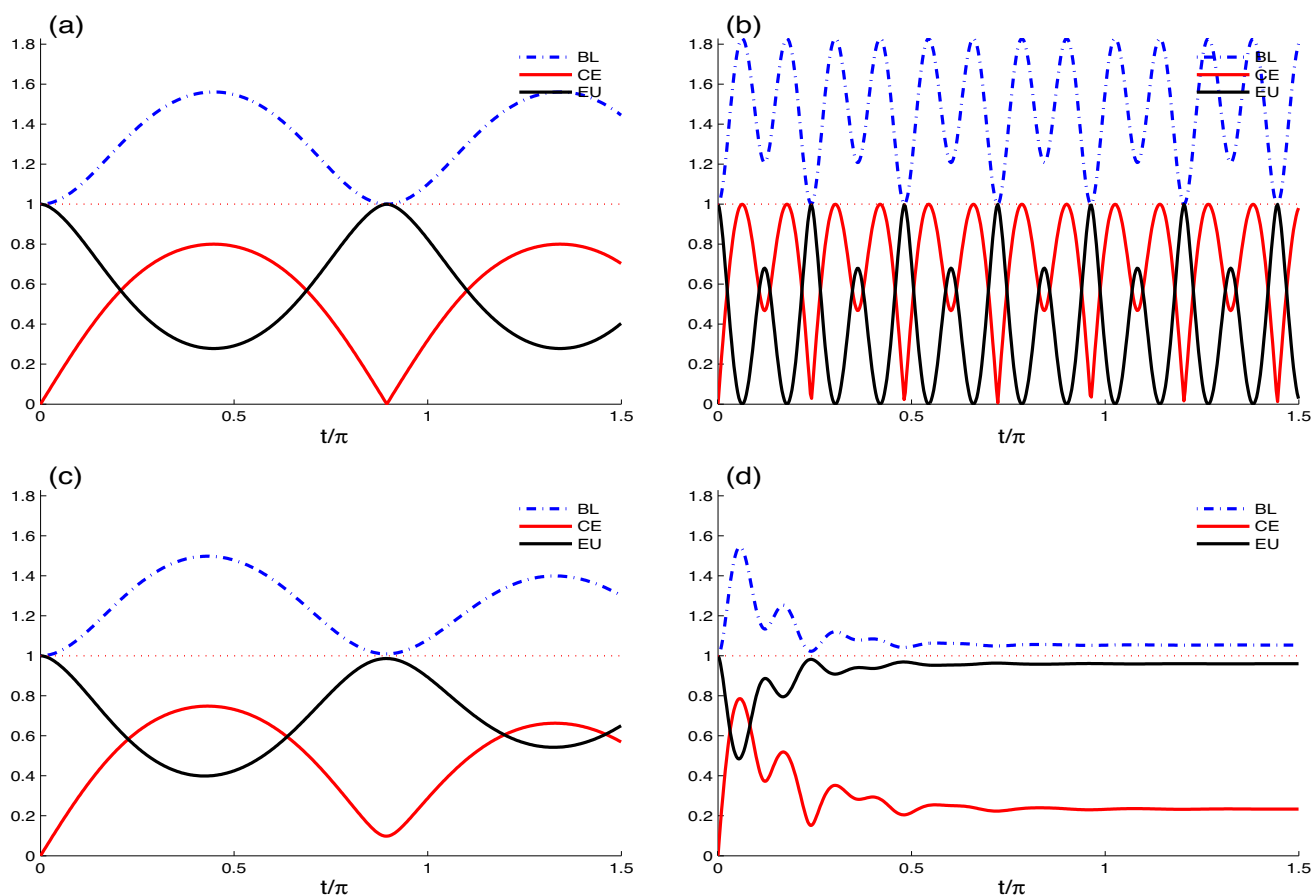


Fig. 2 The dynamics of the BL, CE, and EU are shown as in Fig. 1 but under the intrinsic decoherence effect,  $\gamma = 0.02$



**Fig. 3** The dynamics of BL, CE, and EU are shown as in Fig. 1b but for different strengths of  $z$ -DM and  $z$ -KSEA interactions.  $D = 0$  in **a** and  $D = 2$  in **b**. Graphs **c** and **d** as in **a** and **b** but under the intrinsic decoherence effect,  $\gamma = 0.02$

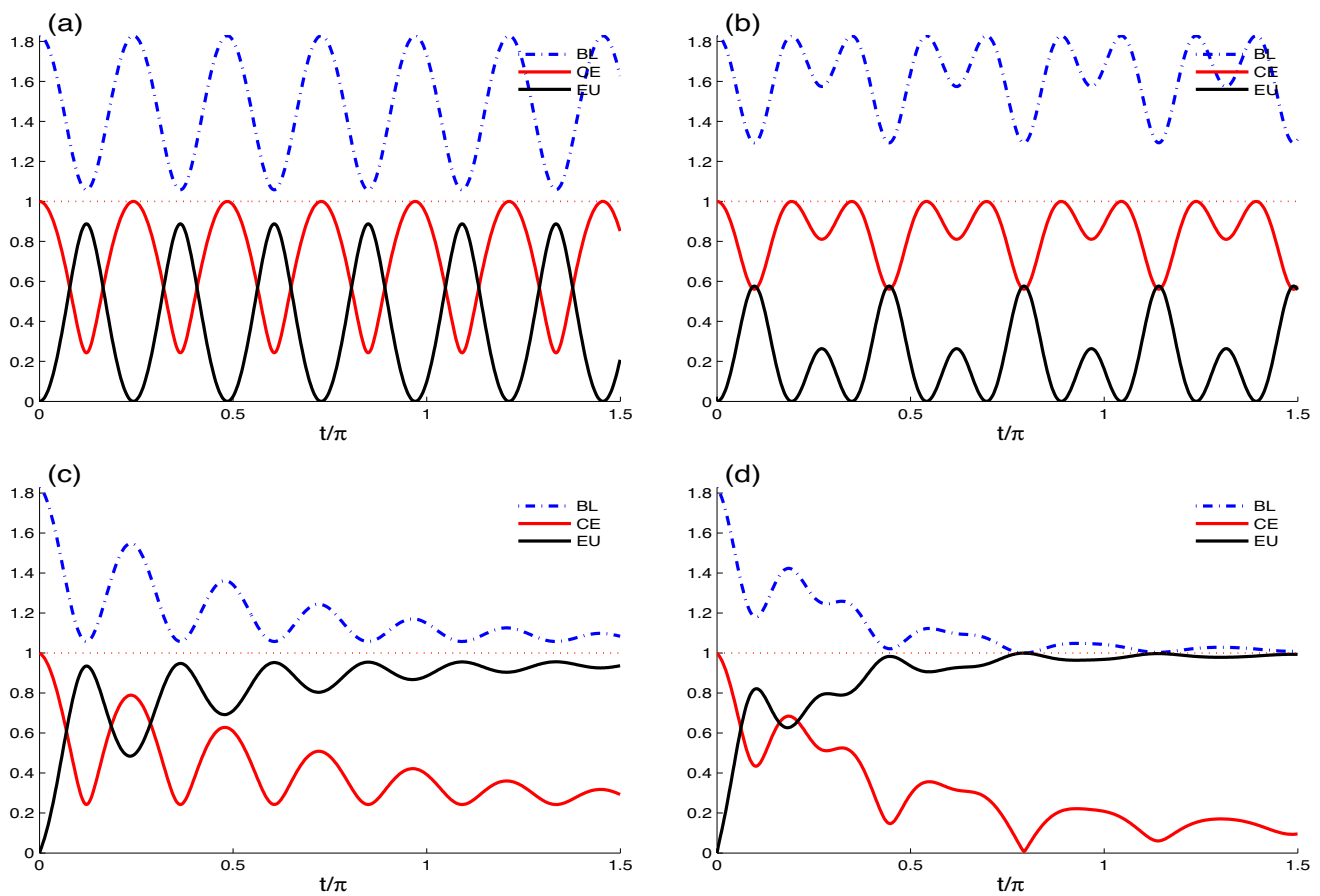
environment. By comparing plots Fig. 4a, b with c, d respectively, one can find that they oscillate over time but in the case of the existence of decoherence, their vibrations and sudden change points become less and they can be vanish faster for larger amount of  $B$ .

Finally, in Fig. 5 the dynamics of BL, CE, and EU have been drawn for those parameters in Fig. 1b and different values of DM-KSEA interactions  $D = D_1^z = D_2^z$ . This figure is similar to Fig. 3 except that the beginning state is a maximally entangled state and hence, there is no any uncertainty to measure incompatible observables at  $t = 0$ . Thus, we can see the destructive effects of  $D$  and intrinsic decoherence when they are used simultaneously. It is worth nothing that in all cases,

the behavior of EU is in opposite way of BL and CE during the time. In the other words, EU lessens with growing BL and CE.

### 5 Concluding remarks and outlook

In this work, we have studied the dynamics of Bell non-locality, entanglement, and entropic uncertainty for a two-qubit Heisenberg XYZ model with DM-KSEA interactions under intrinsic decoherence and different initial states. Specifically, we examined the time evolution of the BL, CE, and EU under the effect of DM-KSEA interactions, transverse uniform magnetic field, and intrinsic

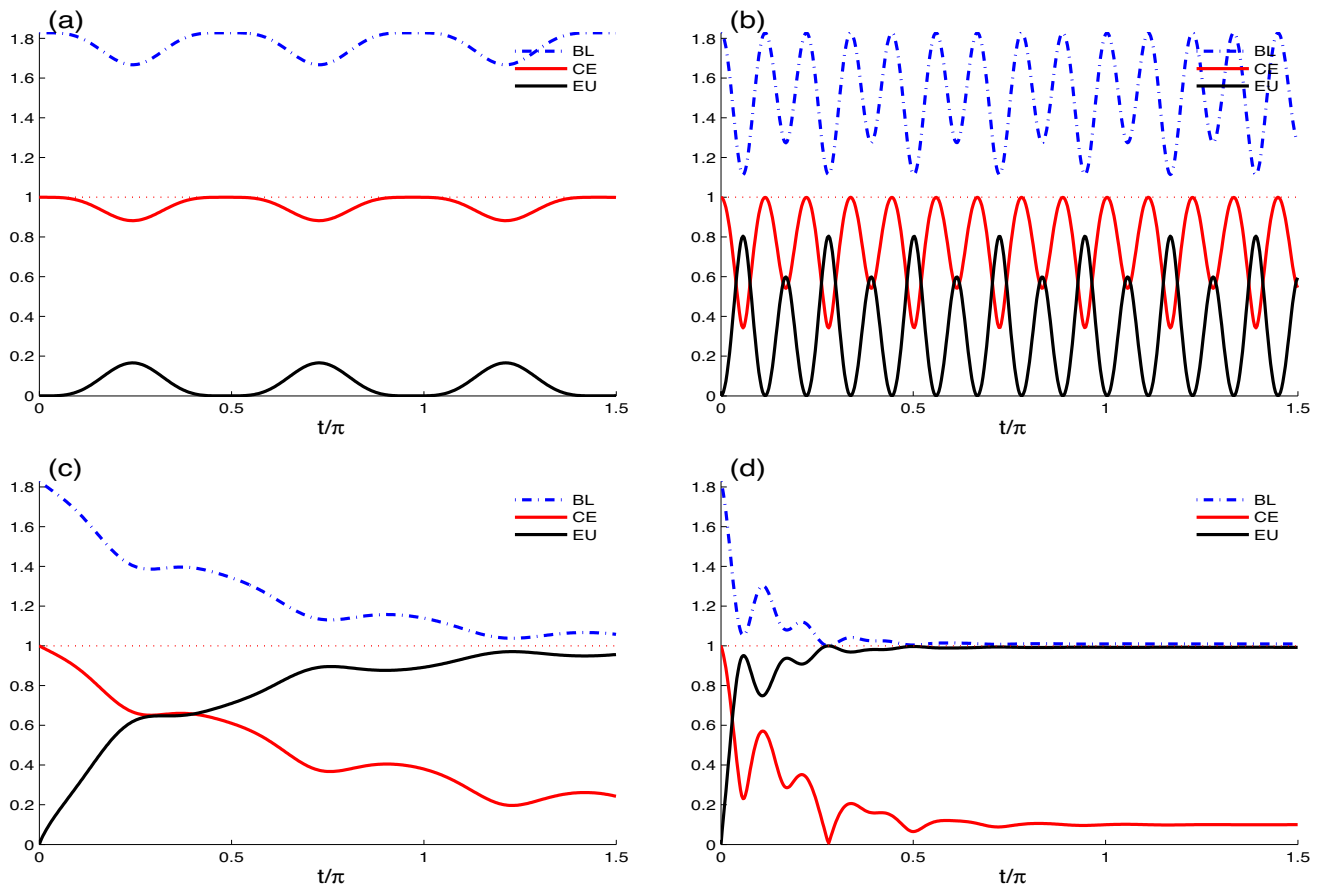


**Fig. 4** The dynamics of the robustness quantum information resources are shown for  $J_x = -1, J_y = -0.5, J_z = 2$ , and  $D_1^z = D_2^z = 1$  when the two-qubit system is initially in the maximally correlated state  $\hat{M}(0) = \frac{1}{2}[(|\varpi_1\rangle + |\varpi_4\rangle)(\langle\varpi_1| + \langle\varpi_4|)]$  in the absence of the

intrinsic decoherence with different transverse uniform magnetic fields.  $B = 0$  in (a) and  $B = 1$  in (b). Graphs c and d as in a and b but under the intrinsic decoherence effect,  $\gamma = 0.02$

decoherence. The results showed that the symmetric and antisymmetric orbit–spin couplings greatly affect the oscillations behaviors and sudden change points of quantum correlations. It was also found that external magnetic field and intrinsic decoherence have a significant effect on the dynamical behavior of quantum quantifiers. More precisely, we observed that by tuning DM–KSEA interactions as well as magnetic field strength to appropriate values, one can reach maximal Bell nonlocality and entanglement,

resulting in smaller measurement uncertainty even under intrinsic decoherence effects. Thereby, Bob would be able to guess Alice’s results with high precision, which is extremely required in realistic quantum information processing. Thus, we think our investigations may be beneficial to assist us to understand the behaviors of the quantum correlations and entropic measurement uncertainty in open quantum systems and their controlling.



**Fig. 5** The dynamics of the robustness quantum information resources are shown as in Fig. 1b but for different strength of DM and KSEA interactions.  $D = 0$  in **a** and  $D = 2$  in **b**. Graphs **c** and **d** as in **a** and **b** but under the intrinsic decoherence effect,  $\gamma = 0.02$

**Author Contributions** MH and ABAM have proposed the main idea and performed the calculations. SH, YK, and MRP all contributed to the development of the idea, writing and discussions of the manuscript, and analyzing the results. Thorough checking of the paper was done by all authors.

**Data availability** This work is theoretical research and has no the associated data.

## Declarations

**Conflict of interest** All authors declare that they have no recognized competing financial interests that could have emerged to influence the work reported in this research.

## References

1. M. Genovese, Research on hidden variable theories: A review of recent progresses. *Phys. Rep.* **413**, 319 (2005)
2. R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki, Quantum entanglement. *Rev. Mod. Phys.* **81**, 865 (2009)
3. D. Cavalcanti, P. Skrzypczyk, Quantum steering: a review with focus on semidefinite programming. *Rep. Prog. Phys.* **80**, 024001 (2017)
4. A. Streltsov, G. Adesso, M.B. Plenio, Colloquium: quantum coherence as a resource. *Rev. Mod. Phys.* **89**, 041003(2017)
5. J. Eisert, M. Cramer, M.B. Plenio, Colloquium: area laws for the entanglement entropy. *Rev. Mod. Phys.* **82**, 277 (2010)
6. G.D. Chiara, A. Sanpera, Genuine quantum correlations in quantum many-body systems: a review of recent progress. *Rep. Prog. Phys.* **81**, 074002 (2018)
7. E. Schrödinger, Discussion of probability relations between separated systems. *Proc. Camb. Philos. Soc.* **31**, 555 (1935)
8. A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777 (1935)
9. M.A. Nielsen, I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000)
10. V. Vedral, The role of relative entropy in quantum information theory. *Rev. Mod. Phys.* **74**, 197 (2002)
11. O. Gühne, G. Tóth, Entanglement detection. *Phys. Rep.* **474**, 1 (2009)
12. C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W.K. Wootters, Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.* **70**, 1895 (1993)
13. A. Kumar, S. Haddadi, M.R. Pourkarimi, B.K. Behera, P.K. Panigrahi, Experimental realization of controlled quantum teleportation of arbitrary qubit states via cluster states. *Sci. Rep.* **10**, 13608 (2020)



14. C.H. Bennett, S.J. Wiesner, Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states. *Phys. Rev. Lett.* **69**, 2881 (1992)
15. A.K. Ekert, Quantum cryptography based on Bell's theorem. *Phys. Rev. Lett.* **67**, 661 (1991)
16. V. Giovannetti, S. Lloyd, L. Maccone, Quantum-enhanced measurements: beating the standard quantum limit. *Science* **306**, 1330 (2004)
17. J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Proposed experiment to test local hidden-variable theories. *Phys. Rev. Lett.* **23**, 880 (1969)
18. F. Buscemi, All entangled quantum states are nonlocal. *Phys. Rev. Lett.* **108**, 200401 (2012)
19. W. Heisenberg, Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. *Z. Phys.* **43**, 172 (1927)
20. E.H. Kennard, Zur Quantenmechanik einfacher Bewegungstypen. *Z. Phys.* **44**, 326 (1927)
21. H.P. Robertson, The uncertainty principle. *Phys. Rev.* **34**, 163 (1929)
22. K. Kraus, Complementary observables and uncertainty relations. *Phys. Rev. D* **35**, 3070 (1987)
23. E. Schrödinger, Zum Heisenbergschen Unschärfepinzipp. Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch-mathematische Klasse **14**, 296 (1930)
24. H. Everett, "Relative State" formulation of quantum mechanics. *Rev. Mod. Phys.* **29**, 454 (1957)
25. I.I. Hirschman, A note on entropy. *Am. J. Math.* **79**, 152 (1957)
26. D. Wang, F. Ming, M.L. Hu, L. Ye, Quantum-memory-assisted entropic uncertainty relations. *Ann. Phys. (Berlin)* **531**, 1900124 (2019)
27. M.L. Hu, H. Fan, Quantum-memory-assisted entropic uncertainty principle, teleportation, and entanglement witness in structured reservoirs. *Phys. Rev. A* **86**, 032338 (2012)
28. M.L. Hu, H. Fan, Upper bound and shareability of quantum discord based on entropic uncertainty relations. *Phys. Rev. A* **88**, 014105 (2013)
29. M.L. Hu, H. Fan, Competition between quantum correlations in the quantum-memory-assisted entropic uncertainty relation. *Phys. Rev. A* **87**, 022314 (2013)
30. M.R. Pourkarimi, S. Haddadi, S. Haseli, Exploration of entropic uncertainty bound in a symmetric multi-qubit system under noisy channels. *Phys. Scr.* **96**, 015101 (2021)
31. S. Haddadi, S. Haseli, M.R. Pourkarimi, F. Mirmasoudi, Analyzing entropic uncertainty bound in two qubits coupled to a spin environment. *Phys. Scr.* **96**, 075104 (2021)
32. S. Haddadi, M. Ghominejad, A. Akhound, M.R. Pourkarimi, Exploring entropic uncertainty relation and dense coding capacity in a two-qubit X-state. *Laser Phys. Lett.* **17**, 095205 (2020)
33. S. Haseli, S. Haddadi, M.R. Pourkarimi, Entropic uncertainty lower bound for a two-qubit system coupled to a spin chain with Dzyaloshinskii-Moriya interaction. *Opt. Quant. Electron.* **52**, 465 (2020)
34. S. Haddadi, M. Ghominejad, A. Akhound, M.R. Pourkarimi, Entropic uncertainty relation and quantum coherence under Ising model with Dzyaloshinskii-Moriya interaction. *Laser Phys. Lett.* **18**, 085204 (2021)
35. S. Haddadi, M.R. Pourkarimi, S. Haseli, Relationship between quantum coherence and uncertainty bound in an arbitrary two-qubit X-state. *Opt. Quant. Electron.* **53**, 529 (2021)
36. S.J. Xiong, Z. Sun, J.M. Liu, Entropic uncertainty relation and quantum phase transition in spin-1/2 Heisenberg chain. *Laser Phys. Lett.* **17**, 095203 (2020)
37. S. Haddadi, M.R. Pourkarimi, D. Wang, Tripartite entropic uncertainty in an open system under classical environmental noise. *J. Opt. Soc. Am. B* **38**, 2620 (2021)
38. R.A. Abdelghany, A.B.A. Mohamed, M. Tammam, W. Kuo, H. Eleuch, Tripartite entropic uncertainty relation under phase decoherence. *Sci. Rep.* **11**, 11830 (2021)
39. H. Dolatkah, S. Haseli, S. Salimi, A.S. Khorashad, Tightening the tripartite quantum-memory-assisted entropic uncertainty relation. *Phys. Rev. A* **102**, 052227 (2020)
40. S. Haddadi, M.R. Pourkarimi, S. Haseli, Multipartite uncertainty relation with quantum memory. *Sci. Rep.* **11**, 13752 (2021)
41. S. Haddadi, M. Ghominejad, A. Akhound, M.R. Pourkarimi, Suppressing measurement uncertainty in an inhomogeneous spin star system. *Sci. Rep.* **11**, 22691 (2021)
42. S. Haddadi, M.L. Hu, Y. Khedif, H. Dolatkah, M.R. Pourkarimi, M. Daoud, Measurement uncertainty and dense coding in a two-qubit system: combined effects of bosonic reservoir and dipole-dipole interaction. *Results Phys.* **32**, 105041(2022)
43. F. Li, J.S. Xu, X.Y. Xu, K. Li, G.C. Guo, Experimental investigation of the entanglement-assisted entropic uncertainty principle. *Nature Phys.* **7**, 752 (2011)
44. A. Luis, A. Rodil, Alternative measures of uncertainty in quantum metrology: contradictions and limits. *Phys. Rev. A* **87**, 034101 (2013)
45. M. Tomamichel, E. Hänggi, The link between entropic uncertainty and nonlocality. *J. Phys. A: Math. Theor.* **46**, 055301 (2013)
46. G. Vallone, D.G. Marangon, M. Tomasin, P. Villoresi, Quantum randomness certified by the uncertainty principle. *Phys. Rev. A* **90**, 052327 (2014)
47. S. Haddadi, M.R. Pourkarimi, A. Akhound, M. Ghominejad, Quantum correlations and quantum-memory-assisted entropic uncertainty relation in two kinds of spin squeezing models. *Laser Phys. Lett.* **16**, 095202 (2019)
48. S. Haseli, S. Haddadi, M.R. Pourkarimi, Probing the entropic uncertainty bound and quantum correlations in a quantum dot system. *Laser Phys.* **31**, 055203 (2021)
49. Z. Chen, Y. Zhang, X. Wang, S. Yu, H. Guo, Improving parameter estimation of entropic uncertainty relation in continuous-variable quantum key distribution. *Entropy* **21**, 652 (2019)
50. K. Modi, A. Brodutch, H. Cable, T. Paterek, V. Vedral, The classical-quantum boundary for correlations: discord and related measures. *Rev. Mod. Phys.* **84**, 1655 (2012)
51. G.J. Milburn, Intrinsic decoherence in quantum mechanics. *Phys. Rev. A* **44**, 5401 (1991)
52. G.C. Ghirardi, A. Rimini, T. Weber, Unified dynamics for microscopic and macroscopic systems. *Phys. Rev. D* **34**, 470 (1986)
53. G.C. Ghirardi, P. Pearle, A. Rimini, Markov processes in Hilbert space and continuous spontaneous localization of systems of identical particles. *Phys. Rev. A* **42**, 78 (1990)
54. L. Diosi, Models for universal reduction of macroscopic quantum fluctuations. *Phys. Rev. A* **40**, 1165 (1989)
55. J. Ellis, S. Mohanty, D.V. Nanopoulos, Quantum gravity and the collapse of the wavefunction. *Phys. Lett. B* **221**, 113 (1989)
56. C.M. Caves, G.J. Milburn, Quantum-mechanical model for continuous position measurements. *Phys. Rev. D* **36**, 5543 (1987)
57. Z.H. He, Z. Xiong, Y. Zhang, Influence of intrinsic decoherence on quantum teleportation via two-qubit Heisenberg XYZ chain. *Phys. Lett. A* **354**, 79 (2006)
58. M. Abdel-Aty, New features of total correlations in coupled Josephson charge qubits with intrinsic decoherence. *Phys. Lett. A* **372**, 3719 (2008)
59. Q. Liang, W. An-Min, M. Xiao-San, Effect of intrinsic decoherence of Milburn's model on entanglement of two-qutrit states. *Commun. Theor. Phys.* **49**, 516 (2008)
60. M.B. Plenio, P.L. Knight, Decoherence limits to quantum computation using trapped ions. *Proc. R. Soc. Lond. A* **453**, 453 (1997)

61. L.M. Kuang, X. Chen, M.L. Ge, Influence of intrinsic decoherence on nonclassical effects in the multiphoton Jaynes-Cummings model. *Phys. Rev. A* **52**, 1857 (1995)
62. V. Bužek, M. Konôpka, Dynamics of open systems governed by the Milburn equation. *Phys. Rev. A* **58**, 1735 (1998)
63. M.L. Hu, H.L. Lian, State transfer in intrinsic decoherence spin channels. *Eur. Phys. J. D* **55**, 711 (2009)
64. M.L. Hu, Disentanglement dynamics of interacting two qubits and two qutrits in an XY spin-chain environment with the Dzyaloshinsky-Moriya interaction. *Phys. Lett. A* **374**, 3520 (2010)
65. M.L. Hu, H. Fan, Robustness of quantum correlations against decoherence. *Ann. Phys.* **327**, 851 (2012)
66. M. Essakhi, Y. Khedif, M. Mansour, M. Daoud, Intrinsic decoherence effects on quantum correlations dynamics. *Opt. Quant. Electron.* **54**, 103 (2022)
67. M. Mansour, S. Haddadi, Bipartite entanglement of decohered mixed states generated from maximally entangled cluster states. *Mod. Phys. Lett. A* **36**, 2150010 (2021)
68. W.Y. Sun, D. Wang, B.L. Fang, Z.Y. Ding, H. Yang, F. Ming, L. Ye, Intrinsic relations of bipartite quantum resources in tripartite systems. *Ann. Phys. (Berlin)* **531**, 1800358 (2019)
69. W.Y. Sun, D. Wang, L. Ye, Dynamics and recovery of genuine multipartite Einstein-Podolsky-Rosen steering and genuine multipartite nonlocality for a dissipative Dirac system via the Unruh effect. *Ann. Phys. (Berlin)* **530**, 1700442 (2018)
70. W.Y. Sun, D. Wang, J.D. Shi, L. Ye, Exploration quantum steering, nonlocality and entanglement of two-qubit X-state in structured reservoirs. *Sci. Rep.* **7**, 39651 (2017)
71. W.Y. Sun, D. Wang, J. Yang, L. Ye, Enhancement of multipartite entanglement in an open system under non-inertial frames. *Quantum Inf. Process.* **16**, 90 (2017)
72. W.Y. Sun, J. Chen, A. Ding, D. Wang, L. Ye, Exploration of quantum correlations in an open system with Unruh effect under a Schwarzschild space-time. *Laser Phys. Lett.* **16**, 115201 (2019)
73. M.A. Yurischev, On the quantum correlations in two-qubit XYZ spin chains with Dzyaloshinsky-Moriya and Kaplan-Shekhtman-Entin-Wohlman-Aharony interactions. *Quantum Inf. Process.* **19**, 336 (2020)
74. Y. Khedif, S. Haddadi, M.R. Pourkarimi, M. Daoud, Thermal correlations and entropic uncertainty in a two-spin system under DM and KSEA interactions. *Mod. Phys. Lett. A* **36**, 2150209 (2021)
75. N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Bell nonlocality. *Rev. Mod. Phys.* **86**, 419 (2014)
76. R. Horodecki, P. Horodecki, M. Horodecki, Violating Bell inequality by mixed spin-1/2 states: necessary and sufficient condition. *Phys. Lett. A* **200**, 340 (1995)
77. A.F.G. Solis-Labastida, M. Gastelum, J.G. Hirsch, The Violation of Bell-CHSH Inequalities Leads to Different Conclusions Depending on the Description Used. *Entropy* **23**, 872 (2021)
78. A.S.F. Obada, A.B.A. Mohamed, M. Hashem, M.M. Elkhateeb, Trace distance discord and Bell-function correlations beyond entanglement in two SC-qubits interacting with a dissipative SC-cavity. *Laser Phys.* **30**, 055203 (2020)
79. S. Hill, W.K. Wootters, Entanglement of a pair of quantum bits. *Phys. Rev. Lett.* **78**, 5022 (1997)
80. W.K. Wootters, Entanglement of formation of an arbitrary state of two qubits. *Phys. Rev. Lett.* **80**, 2245 (1998)
81. D. Deutsch, Uncertainty in quantum measurements. *Phys. Rev. Lett.* **50**, 631 (1983)
82. H. Maassen, J.B.M. Uffink, Generalized entropic uncertainty relations. *Phys. Rev. Lett.* **60**, 1103 (1988)
83. M. Berta, M. Christandl, R. Colbeck, J.M. Renes, R. Renner, The uncertainty principle in the presence of quantum memory. *Nature Phys.* **6**, 659 (2010)