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Optical feedback-induced dynamics and nonclassical photon statistics of semiconductor microcavity laser

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Abstract

We study a coherently driven semiconductor laser cavity containing a single quantum dot(QD) (as gain medium) with optical feedback under Markovian approximation. We have obtained coupled operator equations for the model Hamiltonian using standard input-output formalism of cavity-QED and have found that these equations do not have any finite steady state solutions. We have also used an exact numerical framework based on Matlab platform qotoolbox, to compute the temporal dynamics of the mean excitation number of laser cavity mode under high feedback coupling regime. We have further studied the photon correlations of both the cavity mode as well as external feedback mode to feedback identify the laser parameters and coupling strength that give the nonclassical sub-Poissonian photon statistics. This work is useful for coherent control of photon statistics and photon correlations in the semiconductor laser with optical feedback.

1 Introduction

The simplest way to increase the interaction between photons and atoms is to confine them inside a optical resonator. This system is described by well known Jaynes-Cumming model for cavity-Quantum Electrodynamics (cavity-QED) [1]. Inside the resonator, two main loss processes that affect the coherent dynamics of the system are: spontaneous decay from the excited atomic-level to ground level and the leakage of photons outside the cavity [2]. When the coupling strength of the atom with cavity mode dominates over the decoherence processes, strong- coupling regime of cavity-QED is achieved [3]. Such a regime has been obtained in various microscopic systems like optical cavities with trapped ions [4] as well as in semiconductor microcavities, where excitons in quantum dot act as two-level system (TLS) [5]. One of the most important applications of strong coupling regime is photon blockade effect, where cavity resonance frequency is modified in such a way that a second photon can not enter the cavity before the first leaks out [6]. Many other interesting and useful quantum optical phenomena have also been observed in cavities, such as photon antibunching and squeezed light [7], stationary occupation inversion [8] and sub-natural linewidths [9]. Optical cavities have been also used for slow light propagation using electromagnetic induced transparency (EIT) [10]. Furthermore, two mode entanglement has been generated in between two spatially separated cavities [11], where entanglement dynamics can be controlled via cavity parameters. Due to all these significant features, cavity structures are most promising candidate for technical implementation of quantum information algorithms as well as construction of a quantum network.

Rapidly evolving experimental progress in cavity-QED has led to the coherent feedback scheme, where quantum coherent output of a given system is directly used as feedback into an input channel. Coherent feedback for coupled cavities system was first given by Wiseman and Milburn [12] and due to its coherence preserving nature, it become more useful as compared to its measurement based counterpart [13–16]. Since then, seminal experiments using coherent feedback are used in various ways, to enhance the efficiency of intrinsic quantum processes [17–20], tune the coupling between different system components [21], alter the stability landscape of the whole quantum system [22–24] and implementing quantum computation tasks [25]. Similarly, in the semiconductor nanostructure domain, it is well known that a semiconductor laser with external optical feedback can demonstrate very complex non linear dynamics depending upon the feedback phase and strength [26–29]. Furthermore, in more generalized scenario, a semiconductor laser with finite

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number of emitters and photons has been used with output power in high-gain (mW) regime. In all these works, a semi-classical treatment of light shows a very diverse dynamics which have also been observed experimentally [30]. On the other hand, for a low-intensity (low-gain) regime, where a very small number of emitters are involved, it requires a full quantum treatment, as the range of validity of the semi-classical description is not clear. However, a fully analytical quantum optical treatment for the output field statistics of a microcavity laser with optical feedback, and above threshold in the few emitter regime (low gain regime) has been given in [31]. Therefore, recent advances in semiconductor nanotechnology have also lead to realisation of photonic devices in the quantum limit. One of the promising nanostructures are Quantum Dots (QDs) which act as gain medium in state-of-the art microcavity lasers. In the semiconductor lasers, cavity quantum electrodynamics makes possible that a large fraction, β, of the spontaneous emission is coupled into the lasing mode. This leads into the designing of semiconductor microcavity lasers with ultra-low thresholds and a few QDs or even a single QD as gain medium [34–38].

In our present work, we have studied a fully quantized theory of optical feedback in a semiconductor microcavity laser coupled with a single two level emitter, i.e., Quantum Dot (QD) under Markovian approximation [41–43]. In this scenario, time delay corresponding to the feedback loop is considered to be very small as compared to cavity mode decay lifetime, since a longer feedback loop enforces non-Markovian memory kernel [44–46].

This paper is organized as follows. In Sect. 2, we have considered the model Hamiltonian and its operator equations using the well known input-output formalism for cavity-QED. In the same section, we have also given an exact numerical solution for the temporal dynamics of the mean cavity field excitation number using qotoolbox platform in matlab. Section 3 discuses the results with the photon statistics of the lasing cavity as well as the feedback mode including the cross-correlation between them. We have concluded our results in Sect. 4.

2 The model Hamiltonian

The basic scheme of our study is shown in Fig. 1. We have considered a single quantum dot (QD) inside a semiconductor laser containing predominantly only a single lasing mode and it further includes light-matter interaction inside the laser cavity [31]. The coherent exchange phenomena between QD and microcavity mode leads to Vacuum Rabi oscillations. Although, the coexistence of vacuum Rabi oscillation and laser oscillation seems to be contradictory to each other, but it has recently been studied theoretically as well as experimentally that the strong-coupling effect could

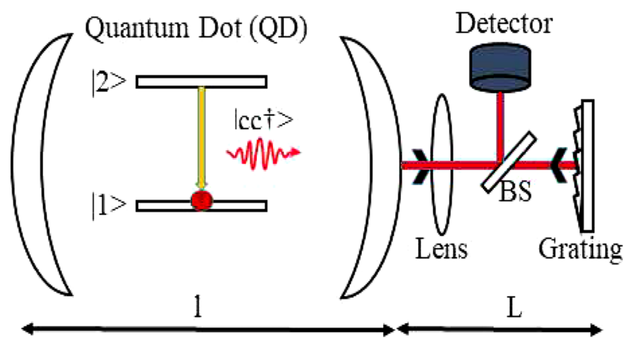

Fig. 1 Single mode semiconductor laser cavity containing a single quantum dot (QD) as well as with an external optical feedback through a grating placed at a distance L, as shown in Fig.

be sustained in laser oscillations [32, 33]. The electronic transition of QD is assumed to be in resonant with this lasing mode. This leads to a lower laser threshold as compared to off resonant QD. Furthermore, the laser cavity is also getting a coherent optical feedback through a grating fixed at a distance L as investigated experimentally also in [47, 48]. In these experimental works, output power fluctuations in a grating external cavity diode laser shows peculiar chaotic behaviour. We have also considered here that length l of the semiconductor cavity is very much greater than L, such that the feedback delay time can be neglected. This enables us to study the dynamics of the optical system under Markovian approximation.

Our system Hamiltonian under rotating wave approximation is given by,

$$\hat{H} = \hat{H}_0 + \hat{V} \tag{1}$$

$$\hat{H}_0 = \hbar\left[\omega_{QD}\hat{A}_{22} + \omega_c\hat{c}^\dagger\hat{c} + \omega_d\hat{d}^\dagger\hat{d}\right] \tag{2}$$

$$\hat{V} = \hbar\hat{c}\left(g\hat{A}_{21} + G\hat{d}^\dagger\right) + H.C. \tag{3}$$

where ω_c is the cavity photon frequency of mode $\hat{c}$, ω_d is the frequency of the optical feedback mode $\hat{d}$ and ω_{QD} is the frequency of the QD, whereas $A_{ij} = |i\rangle\langle j|$ represents the two level QD system with A_{21} is the excitation operator from level 1 to level 2. In Equation (3), g and G are respective coupling strengths of cavity mode $\hat{c}$ with QD and feedback mode $\hat{d}$, whereas $H.C.$ stands for Hermitian conjugate.

2.1 Analytical approach

The dynamics of the system is given using Quantum Langevin equations [50, 51],

$$\dot{\hat{c}} = -i\omega_c\hat{c} - i\left(g^*\hat{A}_{12} + G^*\hat{d}\right) - \frac{k_c}{2}\hat{c} + \tau \quad (4)$$

$$\dot{\hat{d}} = -i\omega_d\hat{d} - iG\hat{c} \quad (5)$$

$$\dot{\hat{A}}_{12} = -i\omega_{QD}\hat{A}_{12} - ig\hat{c} - \frac{\Gamma}{2}\hat{A}_{12} \quad (6)$$

where k_c represents decay of the cavity mode, Γ represents spontaneous decay of two level quantum dot (QD) and τ is the noise operator of cavity mode. The inhomogeneity τ for a cavity must be ascribed to the incoming part of the cavity field $\hat{c}_{in}$, i.e., $\tau = \alpha\hat{c}_{in}$, where α is an unknown coefficient. Therefore, Eq. (4) can be written as,

$$\frac{d\hat{c}}{dt} = -i\omega_c\hat{c} - i\left(g^*\hat{A}_{12} + G^*\hat{d}\right) - \frac{k_c}{2}\hat{c} + \alpha\hat{c}_{in} \quad (7)$$

now the time reversal of (7) must be equivalent to a change in sign and, we need to replace the incoming field with outgoing field as,

$$\frac{d\hat{c}}{d(-t)} = i\omega_c\hat{c} + i\left(g^*\hat{A}_{12} + G^*\hat{d}\right) - \frac{k_c}{2}\hat{c} + \alpha\hat{c}_{out} \quad (8)$$

the boundary condition at one side of the mirror inside cavity is given by [50],

$$\hat{c} = k\left(\hat{c}_{in} + \hat{c}_{out}\right) \quad (9)$$

where k denotes the the fluctuations of the cavity mode.

For consistency of equations (7) − (9), it requires $\alpha = k_c k$. Hence, the relationship between k and k_c is the manifestation of the quantum fluctuation-dissipation theorem [51] and is given as $k^2 k_c = 1$ by C. W. Gardiner et al for cavity field operators in [50, 51]. So, the relation $k^2 k_c = 1$ gives us $k = \frac{1}{\sqrt{k_c}}$ and hence finally we get, $\alpha = k_c k = \sqrt{k_c}$

$$\dot{\hat{c}} = -i\omega_c\hat{c} - i\left(g^*\hat{A}_{12} + G^*\hat{d}\right) - \frac{k_c}{2}\hat{c} + \sqrt{k_c}\hat{c}_{in} \quad (10)$$

$$\dot{\hat{d}} = -i\omega_d\hat{d} - iG\hat{c} \quad (11)$$

$$\dot{\hat{A}}_{12} = -i\omega_{QD}\hat{A}_{12} - ig\hat{c} - \frac{\Gamma}{2}\hat{A}_{12} \quad (12)$$

For the complete study of a given quantum system, we must need to find the expectation value of quantum mechanical observable, so the above set of equations (10)-(12) will reduce to

$$\left\langle\dot{\hat{c}}\right\rangle = -i\omega_c\langle\hat{c}\rangle - i\left(g^*\left\langle\hat{A}_{12}\right\rangle + G^*\left\langle\hat{d}\right\rangle\right) - \frac{k_c}{2} \quad (13)$$

$$\left\langle\dot{\hat{d}}\right\rangle = -i\omega_d\left\langle\hat{d}\right\rangle - iG\langle\hat{c}\rangle \quad (14)$$

$$\left\langle\dot{\hat{d}}\right\rangle = -i\omega_d\left\langle\hat{d}\right\rangle - iG\langle\hat{c}\rangle \quad (15)$$

where expectation value of the noise operator is always zero i.e. $\langle\hat{c}_{in}\rangle = 0$ as shown also in standard text [51–54]. These coupled set of equations (13)-(15) do not have any finite steady state solutions. However, for this Hamiltonian we can calculate the time evolution of the various second-order quantum correlations like our earlier work of cavity based quantum system [55].

2.2 Numerical analysis

The above Hamiltonian in the interaction picture in terms of the various detunings read as,

$$\hat{V}^{int} = \hbar\hat{c}\left(g\hat{A}_{21}e^{-i\Delta_c t} + G\hat{d}^\dagger e^{-i\Delta_d t}\right) \quad (16)$$

where $\Delta_c = (\omega_{QD} - \omega_c)$ and $\Delta_d = (\omega_d - \omega_c)$. Furthermore, we focus on the case of resonant coupling between QD and the lasing cavity mode as well as in between the cavity and the feedback mode i.e. $\Delta_c = \Delta_d = 0$. Here, we would like to mention that a single QD on resonance with the cavity mode results in an increase of the laser efficiency and a lower laser threshold as compared to a QD in off-resonance as also shown in the seminal experiment [38].

Therefore, for a small distance in between the laser cavity and the feedback grating (Markovian limit) [49], the corresponding density operator $\hat{\rho}$ of the system obeys the von Neumann equation with Lindblad terms for the different decay channels.

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}[H, \hat{\rho}] + \frac{\Gamma}{2}L[\hat{\rho}] + \frac{k_c}{2}L[\hat{\rho}] \quad (17)$$

For the cavity field decay, Langevin L is given by

$$L[\hat{\rho}] = 2\hat{c}\hat{\rho}\hat{c}^\dagger - \hat{c}^\dagger\hat{c}\hat{\rho} - \hat{\rho}\hat{c}^\dagger\hat{c} \quad (18)$$

whereas for the spontaneous decay process of QD, Langevin L is given by

$$L[\hat{\rho}] = 2\hat{A}_{12}\hat{\rho}\hat{A}_{21} - \hat{A}_{22}\hat{\rho} - \hat{\rho}\hat{A}_{22} \quad (19)$$

Here Γ is the spontaneous decay rate of the QD and k_c is the decay rate for the cavity mode $\hat{c}$. The average number of photons n_{SQD} emitted spontaneously by a single resonant QD with the cavity mode is given by [38, 39].

$$n_{SQD} = \frac{\beta \tau_{ph} f_{QD}}{\tau_{sp}} \tag{20}$$

where β denotes the spontaneous emission (SE) coupling factor, which represents the fraction of the spontaneous emission coupled into the cavity mode and f_{QD} is the occupation probability of the QD by a single exciton. Here, τ_{ph} and τ_{sp} represents the photon lifetime and the spontaneous emission lifetime of QD, respectively. When the average number of photons inside the microcavity laser is larger than unity, then only we have lasing phenomena from our proposed system, i.e. the emission of a coherent beam of photons [40]. As β and f_{QD} can not exceed beyond 1, Equation (20) essentially requires the photon lifetime to be larger than the spontaneous emission lifetime of the QD, to realise a single QD laser. So, in throughout our numerical simulations we have taken the decay rate for cavity mode k_c smaller than the spontaneous decay rate Γ of the QD. From experimental point of view, it is well known that both time scales can be tailored efficiently using the high-quality resonator structures. It has been also found that QD based laser has very low threshold power as compared to the conventional gaseous laser [38].

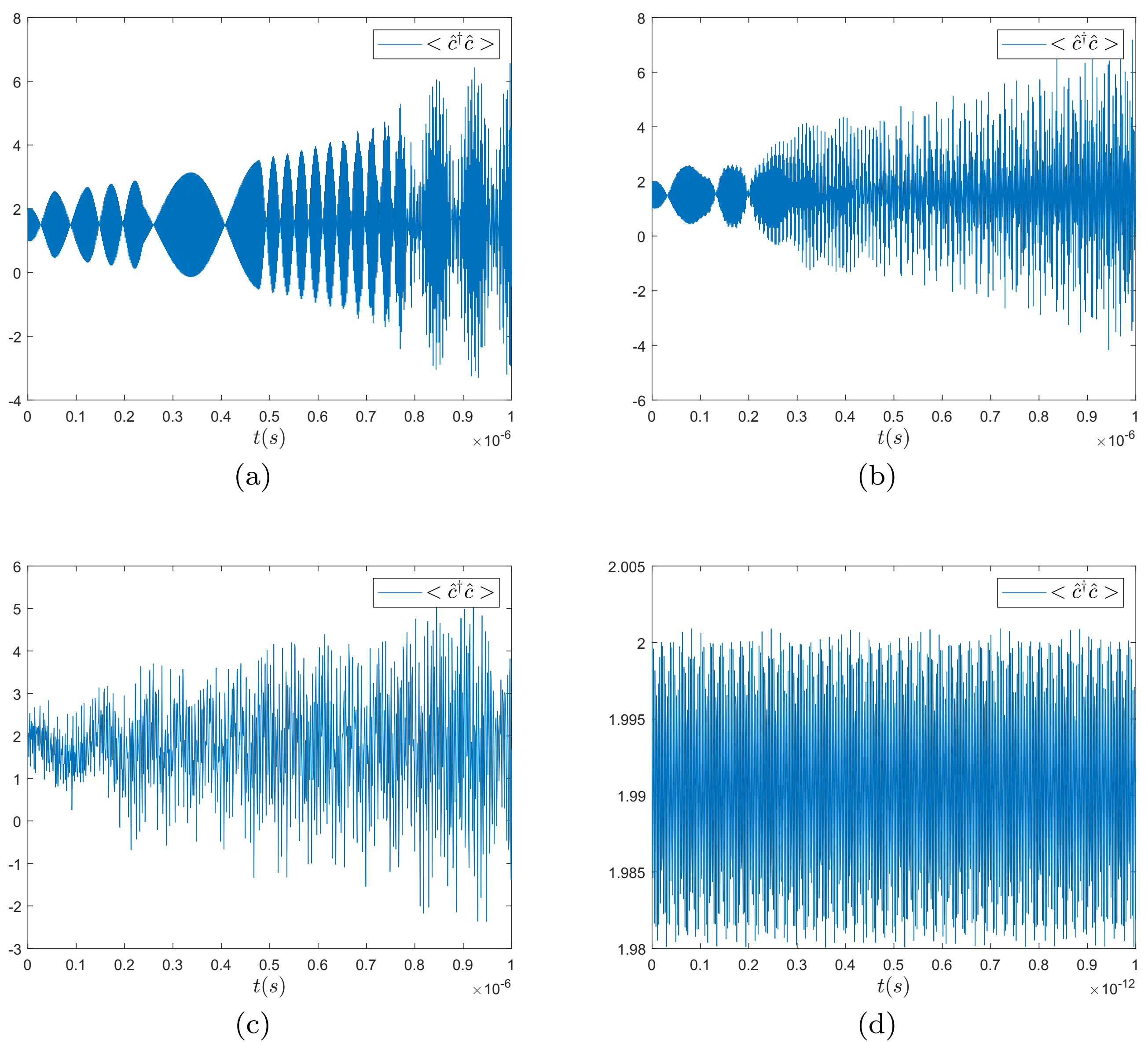

Fig. 2 Time evolution of mean number of cavity field excitation $\langle \hat{c}^\dagger \hat{c} \rangle$ for cavity decay rate $\kappa_c = 0.5 \times 10^{-12}\ s^{-1}$ and spontaneous decay rate of QD, $\Gamma = 0.6 \times 10^{-9}\ s^{-1}$. **a** For the ratio of $\frac{g}{G} = 1000$. **b** For the ratio of $\frac{g}{G} = 100$. **c** For the ratio of $\frac{g}{G} = 10$. **d** For the ratio of $\frac{g}{G} = 0.1$

We have used here an object-oriented open-source framework based on matlab for solving the dynamics of this feedback based quantum system [56]. In this framework named qotoolbox, quantum mechanical Hamiltonians including time-dependent systems, are usually build up from operators and states defined by a quantum object class, and then subsequently passed on to a choice of master equation or Monte Carlo solvers. Furthermore, we have also used this qotoolbox for studying coherently driven Raman transition in bimodal cavity [57]. So, based on our previously used numerical method, we have studied temporal dynamics and nonclassical photon statistics of a semiconductor laser with optical feedback. Here, we have used odesolve routine for density matrix evolution at first given in [56]. We would also like to mention here that a fully open-source framework designed for simulating open quantum dynamics using programming language python named as Quantum Toolbox in Python (QuTiP) has been given in [58].

We have studied here the time evolution of the density operator ρ of the system given in eq. (17), with the help of the numerical integration solver odesolve routine developed in [56]. Initially, we have considered that the QD is in ground state and both the cavity mode and the feedback mode are in their respective arbitrary fock states. In other words, we have defined initial density matrix in numerical simulation for this quantum system like earlier works on density matrix simulations of cavity-QED [57, 58]. It can be seen that, for a very small feedback coupling strength G i.e. $\left(\frac{g}{G} = 1000\right)$, the mean cavity field excitation $\langle \hat{c}^\dagger \hat{c} \rangle$ shows collapse and revival phenomena (like our earlier work on a two level system inside the bimodal cavity [57]) over the microsecond time scale as shown in the Fig. 2a. Although, it can also be observed that due to continuous optical feedback, $\langle \hat{c}^\dagger \hat{c} \rangle$ shows irregular behaviour with time as the feedback coupling through the external mode begin to dominate over the Rabi coupling strength of the QD with the laser cavity mode. As the feedback coupling strength G further increases gradually, $\langle \hat{c}^\dagger \hat{c} \rangle$ follows irregular behaviour as shown in the Fig. 2b, 2c. Finally, a higher value of G, leads to completely irregular behaviour of $\langle \hat{c}^\dagger \hat{c} \rangle$ over the picosecond time scale as shown in the Fig. 2d. So, depending upon the feedback coupling strength G, we can have different kinds of behaviour for the mean excitation number of the cavity mode c i.e. from collapse and revival phenomena to the irregular type behaviour. Our numerical results of quantum dynamics can be further analysed through the Fast Fourier Transform (FFT) to check the possibility for deterministic as well as chaotic properties. However, in the high gain regime, non linear dynamics of a single- mode semiconductor laser with optical feedback has been studied through the numerical simulation of classical Lang Kobayashi model [26]. The numerical simulations of classical Lang Kobayashi model always leads to the deterministic chaos.

3 Photon statistics

In many seminal experiments, semiconductor laser has been used to generate photon bunching of the lasing mode, although photon bunching can also be observed without the application of the feedback if other disturbances act on the light fluctuations. Photon bunching in semiconductor lasers has been already observed for a nearly degenerate weak light mode coexisting with the strong lasing mode [63]. The weak mode exhibits super-Poissonian statistics without feedback after getting disturbed by the strong mode [64].

Here, we have studied the feedback induced effects on the photon statistics of a semiconductor laser. In single mode regime the observation of bunched photon statistics is possible due to the fact that coherent feedback in semiconductor lasers induces chaotic emission. For bunched photons the photon-photon correlation $g^{(2)}(0) > 1$; whereas $g^{(2)}(0) = 1$ gives the pure lasing limit.

The second-order self-correlation functions for the cavity mode c, feedback mode d and the cross correlation between modes c and d, for zero time delay are, respectively, defined as

$$g_c^{(2)}(0) = \frac{\langle \hat{c}^{\dagger 2} \hat{c}^2 \rangle}{\langle \hat{c}^\dagger \hat{c} \rangle^2} \tag{21}$$

$$g_d^{(2)}(0) = \frac{\langle \hat{d}^{\dagger 2} \hat{d}^2 \rangle}{\langle \hat{d}^\dagger \hat{d} \rangle^2} \tag{22}$$

$$g_{cd}^{(2)}(0) = \frac{\langle \hat{c}^\dagger \hat{d}^\dagger \hat{d} \hat{c} \rangle}{\langle \hat{d}^\dagger \hat{d} \rangle \langle \hat{c}^\dagger \hat{c} \rangle} \tag{23}$$

Since we know that the cross correlation between the two bosonic modes represents nonclassical behaviour for $g_{cd}^{(2)}(0) < 1$, or in other words follows sub-Poissonian photon statistics [65]. The regions where $g_{cd}^{(2)}(0) = 1$ and $g_{cd}^{(2)}(0) > 1$ are similarly referred to as Poissonian and super-Poissonian, respectively. The second-order cross-correlation between two different bosonic modes are further used to study the entanglement dynamics between them [65].

Here, we have studied the effects of the coupling ratio $\left(\frac{G}{g}\right)$ on the time evolution of $g_c^{(2)}(0)$, $g_d^{(2)}(0)$ and $g_{cd}^{(2)}(0)$ using qotoolbox framework in Matlab. We have again used *odesolver* routine for density matrix evolution and then calculated the expectation value of higher order quantum correlations which are related to photon statistics [56–58]. It can be seen that the second-order autocorrelation function for the cavity mode $g_c^{(2)}(0)$, without feedback coupling ($G = 0$), strictly follows sub-Poissonian photon statistics

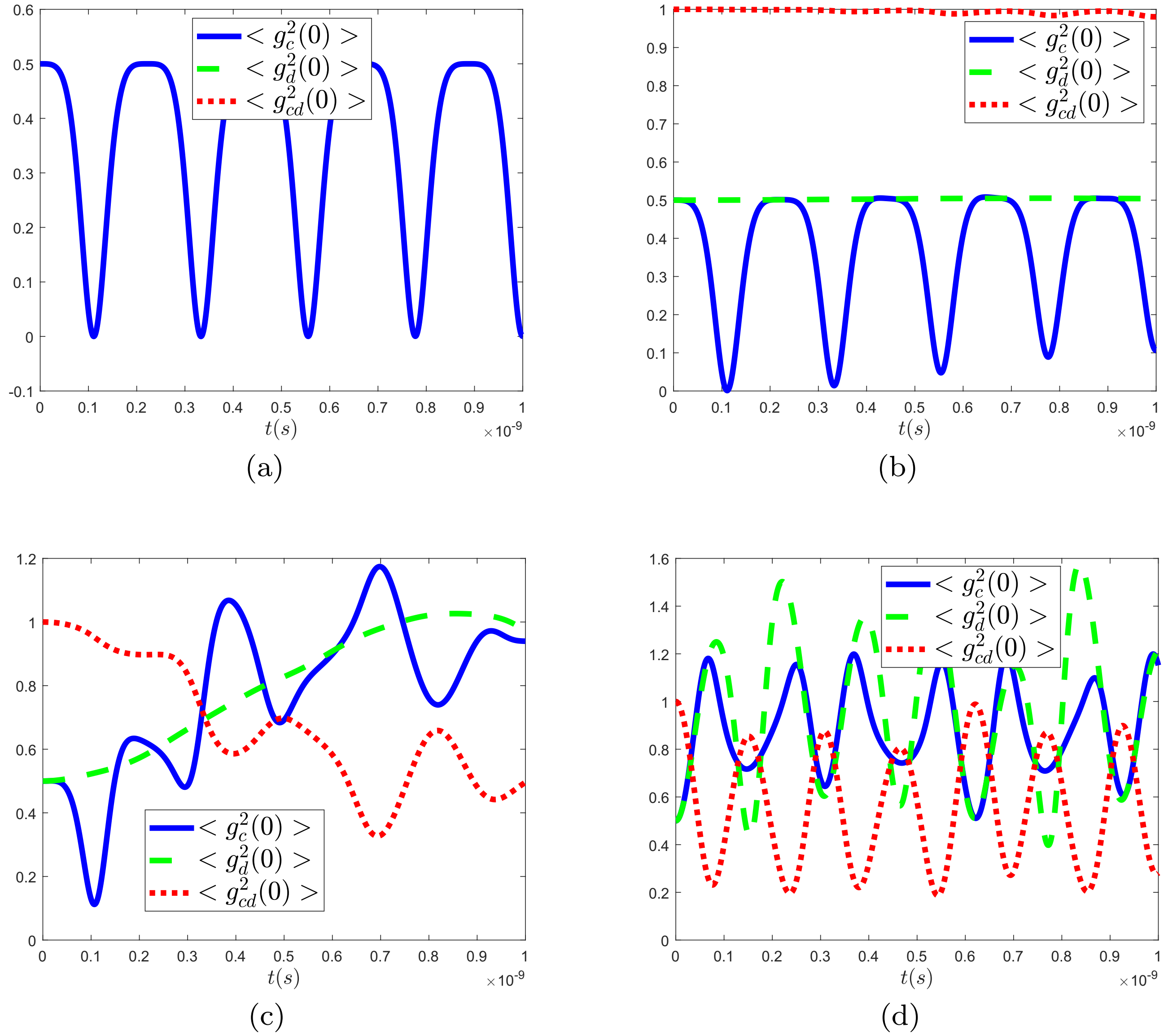

Fig. 3 Second-order autocorrelations $g_c^{(2)}(0)$ (blue solid line), $g_d^{(2)}(0)$ (green dash line) and $g_{cd}^{(2)}(0)$(red dot line) versus coupling ratio g/G for cavity decay rate $\kappa_c = 0.5 \times 10^{-12}\ s^{-1}$ and spontaneous decay rate of QD, $\Gamma = 0.6 \times 10^{-9}\ s^{-1}$. **a** $\frac{G}{g} = 0$; **b** $\frac{G}{g} = 10^{-2}$; **c** $\frac{G}{g} = 10^{-1}$; **d** $\frac{G}{g} = 1$.

as shown in Fig. 3a; as well as even for a weak feedback coupling strength G, also shown in Fig. 3b. So, the laser operation get disturbed by the emission of single photons and leads to antibunching phenomena in the photon statistics of the emitted light [61]. In all these cases, the coherent interaction between single QD and cavity mode dominates over the optical feedback coupling. So, the cavity mode always shows strong nonclassical behaviour as it is strongly coupled with a single QD and generates single photons on demand, also demonstrated in the seminal experiments [59–61]. In this regime, second-order autocorrelation function for the feedback mode $g_d^{(2)}(0)$, also follows sub-Poissonian photon statistics, whereas the cross correlation between them $g_{cd}^{(2)}(0)$ always obeys Poissonian photon statistics. Furthermore, as the feedback coupling strength G increases gradually, $g_c^{(2)}(0)$ starts to vary from super-Poissonian to sub-Poissonian over time, whereas $g_d^{(2)}(0)$ becomes Poissonian and $g_{cd}^{(2)}(0)$ strictly follows sub-Poissonian statistics as shown in Fig. 3c. So, the feedback can induce bunched photon statistics for the cavity mode, as shown experimentally also [63, 64]. This is due to the irregular oscillatory dynamics of the mean light-field intensity arising from feedback which, leads to bunching phenomena even above the lasing threshold. Furthermore, when G becomes comparable to g, cross correlation function $g^{(2)}_{cd}(0)$ still follows sub-Poissonian photon statistics as shown in Fig. 3d. This may further leads to the possible entanglement generation in between the cavity mode and investigation of the feedback mode in our model Hamiltonian.

4 Conclusion

We have studied a single mode semiconductor laser cavity coupled with a single quantum dot (QD) as well as having an external optical feedback through a grating under Markovian approximation. We have obtained the operator equations through the quantum langevin equations of input-output formalism. Furthermore, we have also given an exact numerical solutions based on Matlab using qotoolbox for the temporal dynamics of the system. We have found that mean cavity field excitation shows quantum collapse and revival phenomena like cavity-QED to purely irregular behaviour depending upon the various coupling strengths. We have also studied the photon statistics of cavity as well as feedback mode including the cross correlation between them for various coupling ratio strength $\left(\frac{G}{g}\right)$. We have identified the regimes, where all the three correlations display strong sub-Poissonian photon statistics. For a comparable value of (g *and* G), cross correlation between them shows a nonclassical behaviour which may be further explored to investigate the possible entanglement dynamics in our proposed quantum system. Our study is also useful for optical feedback based control on nonclassical generation of light from semiconductor nanostructures.

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Declarations

Conflict of interest The author has no conflicts of interest to declare that are relevant to the content of this article.

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