Applied Physics B Lasers and Optics

Measurements and analysis of diode laser modulation wavelength at high accuracy and response rate

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Received: 10 April 2019 / Accepted: 28 December 2019 / Published online: 1 February 2020 © Springer-Verlag GmbH Germany, part of Springer Nature 2020

Abstract

It is a key procedure of measuring the diode laser wavelength in the wavelength modulation spectroscopy (WMS) technique since it determines the selection of specifc modulation amplitude and frequency and thus the overall accuracy of the WMS technique. However, the wavelength modulation frequency of lasers is usually from tens of kHz to hundreds of kHz, which makes the traditional methods difficult to measure the wavelength with the sufficient accuracy and time response rate. Therefore, in this paper, we developed a method to measure the modulated wavelength with improved accuracy and time response rate by using a customized long fber ring etalon. In the method, the free spectral range (FSR) of the etalon was determined by using two adjacent absorption lines of water. And the amplitude of the laser wavelength and its phase relative to the driving voltage were determined by means of interference peak identifcation and sinusoidal ftting. Finally, we used the developed method to measure dynamic wavelengths as well as phases of a distribute feedback (DFB) diode laser with the modulation frequency from 1 to 500 kHz and the modulation voltage from 0.2 to 1 V. Based on the measurements, the response characteristics of both the linear and nonlinear wavelengths as well as phases with modulation frequency and amplitude were obtained, which provide necessary data for the application of quantitative and high-repetition WMS technique in combustion diagnostics.

This work was supported by the National Natural Science Foundation of China under Grant Nos. 91541203, 91641112 and 91441201, and the Foundation Project of State Key Laboratory of Laser Interaction with Matter under Grant No. SKLLIM1609.

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1 Introduction

Tunable diode laser (TDL) absorption-based sensors have been extensively studied as methods for non-intrusive measurements of gas properties in combustion due to their robust, compact, low-cost nature $[1–5]$ $[1–5]$ $[1–5]$. Wavelength modulation spectroscopy (WMS) is a derivative form of the absorption spectroscopy that has been increasingly applied for measurements in harsh environments in virtue of its improved sensitivity and noise-rejection capabilities [[6–](#page-5-2)[8](#page-5-3)]. In WMS technique, the output wavelength of the TDL is modulated at a specifc frequency (i.e., from kHz to MHz), so that the corresponding absorption information is shifted to high frequencies, which facilitates the identifcation and detection of absorption signals by get rid of various common noise sources [[6–](#page-5-2)[8\]](#page-5-3). Therefore, the response of output wavelength of the TDL to modulation voltage becomes a key parameter, which determines the selection of specifc modulation voltage amplitude and frequency. Furthermore, quantitative measurements of the WMS technique require accurate knowledge of laser wavelength modulation parameters (i.e., the amplitude of the laser wavelength and its phase relative to the driving voltage) $[9-12]$ $[9-12]$ $[9-12]$.

Extensive theoretical analysis of the modulation properties of TDLs haves reported over many years [\[13](#page-5-6)[–17](#page-6-0)]. And a relatively accurate theoretical model for the magnitude and phase of the wavelength as a function of the modulation frequency has been established. However, the theoretical model is based on approximate conditions, and the various parameters in the model depend on the specifc TDLs [\[18](#page-6-1)]. Thus, it is necessary to measure the modulation properties of TDLs. This is due to the diferent TDLs, their response characteristics are diferent [[18\]](#page-6-1). For WMS applications, the wavelength modulation amplitude of diode laser depends on the specific absorption line width, usually around 0.1 cm^{-1} in an atmospheric pressure environment [\[9](#page-5-4), [12](#page-5-5)]. And the wavelength modulation frequency depends on the specifc measurement repetition, which is usually hundreds kHz for kilohertz-repetition-rate combustion measurements [\[7](#page-5-7)[–9,](#page-5-4) [11](#page-5-8)]. Therefore, there is a need for the accuracy of wavelength measurement system to reach at least 0.01 cm−1 and the response rate to reach at least megahertz to yield a highaccuracy measurement of the dynamic wavelength. To date, the commercial wavelength measuring devices cannot meet the above requirements in terms of measurement accuracy and time response rate. For example, the time response rate of the latest wavelength meters is usually around several kHz [[19,](#page-6-2) [20\]](#page-6-3).

To address the above issues, several types of customized wavelength measurement methods, mostly based on interferometry principles, have been developed to measure and analyze the dynamic wavelength characteristics of diode lasers. For example, Liu et al. [\[21,](#page-6-4) [22\]](#page-6-5) measured the wavelength scanning characteristics of DFB (Distribute Feedback) diode lasers using an all-fber Mach–Zehnder interferometer. Lytkine et al. [[23](#page-6-6)] used Fabry–Perot etalon to measure the dynamic wavelength of a vertical cavity surface emitting diode laser (VCSEL) under 100 Hz sawtooth scanning. However, those methods only realized the dynamic wavelength measurement under the condition of scanning wavelength of the TDLs, not the modulation wavelength. Because measuring the modulation wavelength of TDLs is more difficult than the scanning wavelength due to the small wavelength modulation amplitude $({\sim}0.1 \text{ cm}^{-1})$ and the high wavelength modulation frequency $($ ~ 100 kHz).

To measure and analyze the modulation characteristics of TDLs, the previous research works mainly used the change in the spectrum of the harmonic signal in WMS to infer the frequency response characteristics of laser wavelength with the modulation frequency $[24-27]$ $[24-27]$ $[24-27]$. This method was applied to measure the magnitude and phase lag of the wavelength modulation as a function of the modulation frequency for several specifc lasers including a 2 μm DFB laser [\[24\]](#page-6-7), 0.763–2.3 μm VCSELs [[25,](#page-6-9) [26\]](#page-6-10) and a 3 μm interband cascade laser [\[27\]](#page-6-8). However, this method is based on the assumption that the laser wavelength changes linearly with the injection current and thus only linear wavelength modulation parameters can be measured. In practice, however, the wavelength of the diode laser varies nonlinearly with the injection current, especially at large current modulation amplitude, which is demonstrated in this paper.

In this paper, we developed a method to measure the modulated wavelength with improved accuracy and time response rate using a customized long fber ring etalon and a high-speed data acquisition card. In the proposed method, the free spectral range (FSR) of the etalon was determined using two adjacent absorption lines of water. And the amplitude of the laser wavelength and its phase relative to the driving voltage was determined by means of interference peak identification and sinusoidal fitting. Based on the developed method, we measured the dynamic modulation wavelength of DFB diode lasers at the modulation frequency from 1 to 500 kHz and the modulation voltage from 0.2 to 1 V which corresponds to a modulation current from 4 to 20 mA. The wavelength measuring accuracy can be better than 0.0005 cm−1 and the time response rate can be better than 25 MHz.

2 Description of the method

The wavelength modulation mechanism of DFB laser is due to the change of equivalent refractive index in the resonant cavity when the injection current of the laser changes [\[18](#page-6-1)]. In practice, the injection current of the DFB laser can be adjusted by the driving voltage of the laser controller. The driving voltage (*U*) of the diode laser can be written as:

$$
U(t) = u \cdot \sin(2\pi ft) \tag{1}
$$

Then the varying wavelength $(cm⁻¹)$ of the laser can be expressed as:

$$
\Delta v = a_1 \cdot \sin(2\pi ft + \varphi_1) + a_2 \cdot \sin(4\pi ft + \varphi_2)
$$
 (2)

where u is the modulation amplitude (V) , f is the modulation frequency (Hz); a_1 and a_2 are linear modulation depth and nonlinear modulation depth (cm⁻¹) of wavelength, respectively; φ_1 and φ_2 are the corresponding phase shifts (rad) with respect to the driving voltage. Thus, for a specifc drive voltage, the dynamic parameters of wavelength to be measured are a_1 , φ_1 , a_2 , φ_2 .

Figure [1](#page-2-0) shows the schematic setup of the dynamic wavelength and phase measurement system. The DFB laser used here is a fiber-coupled laser (NTT Electronics, NLK1E5EAAA) emitting wavelength near to 1.4 µm. An ILX Lightwave LDC-3724 is used to control the laser temperature and current. A function generator (Tektronix, AFG310) is used to modulate the injection current and thus the wavelength of laser. The current–voltage ratio is

Fig. 1 Schematic setup of the dynamic wavelength and phase measurement system

20 mA/V for LDC-3724 laser controller. A 1×2 fiber coupler is used to split the laser into two beams. One beam passes through the free air in order to monitor the change of laser intensity. The other beam passes through a ring fber etalon, which is customized using a 2×2 optical fiber coupler and has a cavity structure with a length of 120 cm. Two high speed detectors (Thorlabs, DET 10D/M) are utilized to detect the transmitted laser signals. The detected signals are recorded by a data acquisition system with a sampling rate of 1.25 GHz (Spectrum, M4i-2212). The data acquisition system also synchronously collects the driving voltage signal so as to obtain the phase information of the driving voltage.

The free spectral range (FSR) of the ring cavity etalon can be calculated by FSR=*c*/*nl*, where *c* is the speed of light in vacuum, *n* is the refractive index of the medium in the optical cavity, and *l* is the length of the optical cavity. However, the length of cavity *l* and the refractive index *n* vary with diferent materials and temperatures, and it is not easy to get accurate values. Instead, this work uses two adjacent absorption lines of water (i.e., 7139.61009 cm^{-1} and 7139.08913 cm−1) in free air to determine the FSR of the ring fiber etalon with a uncertainty of 0.00001 cm^{-1} . More specifcally, the laser wavelength is tuned to the above selected absorption lines. The scanned laser transmitted through the free air thus exhibits two absorption depressions, whose center positions corresponds to the center wavelength of the two selected absorption lines. In contrast, the laser passing through the ring fber etalon exhibits various interference peaks during the wavelength scanning. Figure [2](#page-2-1) presents the typical laser intensity in the measurement of free spectral range of the ring fber etalon. The wavelength interval between the two adjacent interference peaks is the FSR of the etalon, which is calculated to be 0.0057 cm⁻¹ by counting the number of interference peaks between the two absorption lines.

The information of modulation wavelength parameters (i.e., a_1 , φ_1 , a_2 , φ_2) are hidden in the interference peaks,

Fig. 2 Laser intensity in the measurement of free spectral range of the ring fber etalon

so there is a need to accurately identify the interference peaks. Figure [3](#page-2-2) shows the original and fltered signals of interference peaks under sinusoidal wavelength modulation. The driving voltage of the laser is 0.5 V and the

Fig. 3 Typical interference peaks and its data processing process under sinusoidal wavelength modulation. **a** The measured interference peaks in two modulation periods. **b** The enlarged view of the blue dashed border in (**a**)

Fig. 4 Processed interference signal and peaks identifcation

frequency is 50 kHz in Fig. [3.](#page-2-2) As seen, the original interference signals are fltered twice. The frst flter uses a low-order low-pass flter, the main purpose of which is to remove the burrs from the original signal and retain the interference peaks position information, as indicated by the green dotted line in Fig. [3](#page-2-2). The second flter uses a high-order low-pass flter, which is mainly to obtain the baseline of the original interference peaks signal, as indicated by the red dashed line in Fig. [3.](#page-2-2) Based on the two fltered signals, the high signal-to-noise ratio interference peaks signal can be obtained by performing natural logarithm operation of the two fltered laser signals. The results are shown in Fig. [4](#page-3-0). It can be seen that the processed interference signal is clean and free of burrs, and then the position of each peaks can be accurately found through the peak-to-valley identifcation function, as indicated in the red triangle points in Fig. [4.](#page-3-0)

The wavelength interval between the adjacent interference peaks in Fig. [4](#page-3-0) is the free spectral range of the ring fiber etalon, which is measured to be 0.0057 cm⁻¹. Based on the positions of interference peaks shown in Fig. [4](#page-3-0), the wavelength between adjacent interference peaks can be obtained. Figure [5b](#page-3-1) shows a time-dependent wavelength calculated using interference peaks position information, and then a_1 and φ_1 in Eq. [\(2](#page-1-0)) can be obtained by fitting the wavelength data with a Sine function. A further sinusoidal fitting of the residual error in Fig. [5b](#page-3-1) results in a_2 and φ_2 in Eq. ([2](#page-1-0)), as shown in Fig. [5](#page-3-1)c. Although the free spectral range of the ring fiber etalon is 0.0057 cm⁻¹, a_1 and a_2 are obtained on the basis of sinusoidal ftting. It can be seen from Fig. [5c](#page-3-1) that the fnal wavelength measurement accuracy is better than 0.0005 cm−1. As for the time response rate of the wavelength data, since the sampling rate of the acquisition card in the measurement system is 1.25 GS, even if a complete interference peak requires 50 data points to be recorded, the wavelength measurement rate of the developed method can reach 25 MHz.

Fig. 5 The synchronous measured driving voltage and laser wavelength as a function of time. **a** The measured driving voltage at 350 mV, 500 mV, 650 mV and their sinusoidal ftting. **b** The measured laser wavelengths and their sinusoidal ftting. **c** The residual error of (**b**) and their sinusoidal fitting at doubling frequency

3 Measurement results and analysis

Based on the measurement system and data processing procedure described in Sect. 2, the dynamic wavelength parameters $(a_1, \varphi_1, a_2, \varphi_2)$ at different modulation amplitudes and frequencies can be acquired. Figure [6](#page-4-0) shows the relationship of linear wavelength modulation depth a_1 and phase φ_1 of diode laser wavelength to driving voltage at diferent modulation frequencies. It can be seen that a_1 has a linear relationship with the driving voltage amplitude, and the slope of line varies for diferent modulation frequencies. For example, it decreases with the increase of modulation frequency, as shown in Fig. [6](#page-4-0)a. As for the φ_1 , Fig. 6b shows that the φ_1 is almost constant at diferent driving voltages, but it decreases with the increase of modulation frequency.

Defining the linear wavelength modulation rate $k_1 = a_1/u$, Fig. [7](#page-4-1) shows the measured k_1 and φ_1 as a function of modulation frequency. It can be seen that k_1 decays rapidly with the increase of modulation frequency. According to the nonlinear fitting of the measured data, it shows that k_1 has the second order exponential decay, rather than the simple frst order exponential decay, as shown in Fig. [7](#page-4-1)a. In contrast, φ_1 decays almost linearly with the increase of modulation frequency, as shown in Fig. [7b](#page-4-1). The basic characteristic of DFB diode laser is that the output wavelength (expressed by wavenumber) decreases with the increasing modulation voltage, so $\varphi_1 = \pi$ under ideal conditions. However, according to the measurement results shown in Fig. [7](#page-4-1)b, φ_1 only approaches to π at very low modulation frequency and gradually deviates

Fig. 6 Linear wavelength modulation depth a_1 (**a**) and phase φ_1 (**b**) varying with driving voltage at diferent modulation frequencies

to π as the modulation frequency increases. The sub-figure in Fig. [7b](#page-4-1) shows that the phase of the response wave delays with a smaller φ_1 , and the delay increases with the increase of the modulation frequency.

Figure [8](#page-4-2) shows the relationship between measured nonlinear wavelength modulation depth a_2 and phase φ_2 and the modulation frequency with diferent driving voltages. According to Fig. [8](#page-4-2), the following conclusions can be drawn: (1) a_2 increases with the increase of modulation voltage, and a_2 decays nonlinearly with the increase of modulation frequency; (2) similar to the φ_1 , φ_2 is almost the same at diferent modulation voltages, but it decays linearly with the increase of modulation frequency.

In WMS applications, the ratio of a_2 to a_1 is important because this value determines the distortion of WMS spectrum caused by the nonlinear response of diode laser wave-length. Figure [9](#page-5-9) shows a_2/a_1 as a function of modulation frequency and voltage. The following conclusions can be drawn from Fig. [9](#page-5-9): (1) there is no obvious variation tendency of a_2/a_1 with modulation frequency, and the fluctuations shown in Fig. [9](#page-5-9)a are mainly caused by the measurement errors; (2) a_2/a_1 increases with the increase of modulation voltage, and the relationship between a_2/a_1 and modulation voltage is linear, as shown by the linear ftting in Fig. [9](#page-5-9)b.

Fig. 7 Measured linear wavelength modulation rate $k_1 = a_1/u$ (a) and phase φ_1 (**b**) as a function of modulation frequency

Fig. 8 Nonlinear wavelength modulation depth a_2 (a) and phase φ_2 (**b**) varying with driving voltage at diferent modulation frequencies

Fig. 9 The ratio of nonlinear wavelength modulation depth a_2 to linear wavelength modulation depth a_1 varying with modulation frequency (**a**) and driving voltage (**b**)

As is known, if we want to increase the measurement repetition of WMS technique, we need to increase the wavelength scanning frequency and correspondingly the wavelength modulation frequency $[9-12]$ $[9-12]$. According to the measurement results in this paper, the measurement repetition of WMS technique is limited by two points: (1) the highest modulation current of diode laser, which is due to a large modulation current required for the high modulation frequency; (2) WMS spectrum distortion caused by nonlinear wavelength modulation, because the higher the modulation frequency, the greater the corresponding modulation current, resulting in the greater a_2 .

4 Conclusions

This work developed a method for the measurement of modulation characteristics of DFB diode laser. The wavelength measuring accuracy can be better than 0.0005 cm⁻¹ and the time response rate can be better than 25 MHz. Based on the developed method, the response characteristics of both the linear and nonlinear wavelengths as well as phases with modulation frequency and amplitude were obtained. The main conclusions are as follows: (1) the linear wavelength modulation depth a_1 varies linearly with the modulation voltage, and the slope of line decays with the increase of modulation frequency in a second-order exponential function; (2) the linear wavelength phase φ_1 remains constant with diferent modulation voltages, but it decreases with the increase of modulation frequency; (3) the ratio of the nonlinear wavelength modulation depth a_2 to linear wavelength modulation depth a_1 (a_2/a_1) increases linearly with modulation voltage, but a_2/a_1 remains almost constant with different modulation frequency; (4) the variation of nonlinear wavelength phase φ_2 with modulation frequency and modulation voltage is similar to φ_1 .

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