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Discrimination and estimation for dephasing sources of trapped ion qubits

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Abstract
In this paper, we propose a handy method to discriminate and estimate the dephasing noise sources of trapped ion qubits, which are mainly magnetic field noise and laser frequency noise. In our method, the conventional Ramsey experiment is used to measure the dephasing time T_2^* and calculate the total linewidth of dephasing noise. Transitions with different magnetic field sensitivities and error propagation theory are employed to quantitatively estimate the linewidth of each noise. We experimentally demonstrated this method by taking advantage of five magnetically sensitive transitions of single trapped $^{40}Ca^+$, and the linewidth of the laser frequency noise and magnetic field noise is determined to be 493.3 ± 8.1 Hz and 181.4 ± 3.5 μG, respectively, in our system. This method also can be used to separate magnetic field noise and other dephasing noise in other atomic system such as neutral atoms.

1 Introduction

The development of quantum technology in the past decades makes quantum computers to be possible. The trapped ion system had been proven to be one of the most promising platforms to realize quantum computing [1]. For gate-based quantum computing with trapped ions [2], the quantum gates are realized by coherently manipulating the internal and external degrees of freedom of ions, and the long coherence time is required to perform a large amount of quantum gates [3–7] during the computing process. Experimentally, the coherence time of ion qubits is affected by imperfect parameters such as background gas collision, the spontaneous emission effect, the laser frequency and intensity fluctuation and the instable magnetic field [8]. Generally, the ions are stored in the ultra high vacuum chamber, laser intensity fluctuation can be easily reduced by servo system and the energy levels with extremely low decay rate (long depopulation time T_1) are chosen to encode qubit, hence the coherence time is limited by the dephasing time T_2, which is mainly affected by the instable magnetic field and the driving laser frequency noise [8, 9].

To extend the coherence time of trapped ions, it is important to investigate the decoherence contributions from each dephasing source. The fluctuation of magnetic field could be quantitatively estimated by measuring the noise spectrum of magnetic field coils [10, 11]; however, this method could not provide the information of laser frequency noise. Previously, the laser and magnetic phase noise were differentiated using a custom-designed entangled state [9], however, this method requires high controllability of two ion qubits and a complicated experimental setup. Normally, the Ramsey interferometry is a standard tool for estimating the dephasing time T_2^*, but the contributions of each dephasing source cannot be distinguished by this method [12]. Dynamic decoupling is also a good method for diagnosing the noise spectrum causing dephase [11, 13–15], but it still cannot separate different dephasing noise sources.

In this paper, we propose a handy method to quantitatively distinguish the dephasing contributions of each source, which only involves the conventional Ramsey experiments and the transitions with different magnetic sensitivities [12, 16, 17]. The dephasing time T_2^* of each transition can be measured using the conventional Ramsey experiment, and the total linewidth of the dephasing noise, which is assumed to obey Gaussian distribution in our system, could be obtained. By fitting the total linewidth of dephasing noise with random error propagation theory [18], the linewidth of independent dephasing noise sources

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can be quantitatively estimated. To improve the accuracy of measurement results, we make full use of all magnetically sensitive transitions rather than just two of them. Experimentally, we demonstrate this method using a single trapped $^{40}Ca^+$, which has five magnetically sensitive quadrupole transitions, and the laser frequency noise and magnetic field noise are determined to be 493.3 ± 8.1 Hz and 181.4 ± 3.5 μG in our system. We further notice that this method is not only limited to ions, but also suitable for separating magnetic field noise and other dephasing noise in atomic qubit systems such as neutral atoms.

2 Theoretical model

To demonstrate our method, here, we take the $^{40}Ca^+$ ion as an example. As shown in Fig. 1, there are ten quadrapole transitions between the sublevels of $S_{1/2}$ and $D_{5/2}$ states when a weak external magnetic field B is applied along the quantization axis, but only five of them show different sensitivities to the magnetic field fluctuations, and the magnetic potential gap can be calculated by

$$\Delta E_B = g_i \mu_B B, \tag{1}$$

where $g_i = |m_S g_{S_{1/2}} - m_D g_{D_{5/2}}|$, Landé g-factor $g_{S_{1/2}} \approx 2.0$ and $g_{D_{5/2}} \approx 1.2$. The coefficients of magnetic potential energy gaps for five transitions are summarized in Table 1.

In addition to the magnetic field fluctuations, other noise sources such as the frequency noise of the laser also limit the coherence time of the qubit. Experimentally, total noise contributions can be estimated by the Ramsey interferometric technique [19], which is a good tool to explore the phase coherence between an external field and atomic two-level system. A complete Ramsey sequence involves one $\pi/2$ pulse, a free precession process and the other second $\pi/2$ pulse. By scanning the free precession time, a fringe pattern

$$P_D = \frac{1}{2}(A\cos(\omega\tau + \phi_0) + 1) \tag{2}$$

can be obtained, where P_D denotes the population of D state, ω is the oscillation frequency of the fringe, τ is the free

Table 1 The coefficients of magnetic potential gaps for different optical transitions between states $S_{1/2}(m_J = -1/2)$ and $D_{5/2}(m_J = -5/2, -3/2, -1/2, 1/2, 3/2)$

	$-1/2 \leftrightarrow -5/2$	$-1/2 \leftrightarrow -3/2$	$-1/2 \leftrightarrow -1/2$	$-1/2 \leftrightarrow 1/2$	$-1/2 \leftrightarrow 3/2$
g_i	2	0.8	0.4	1.6	2.8

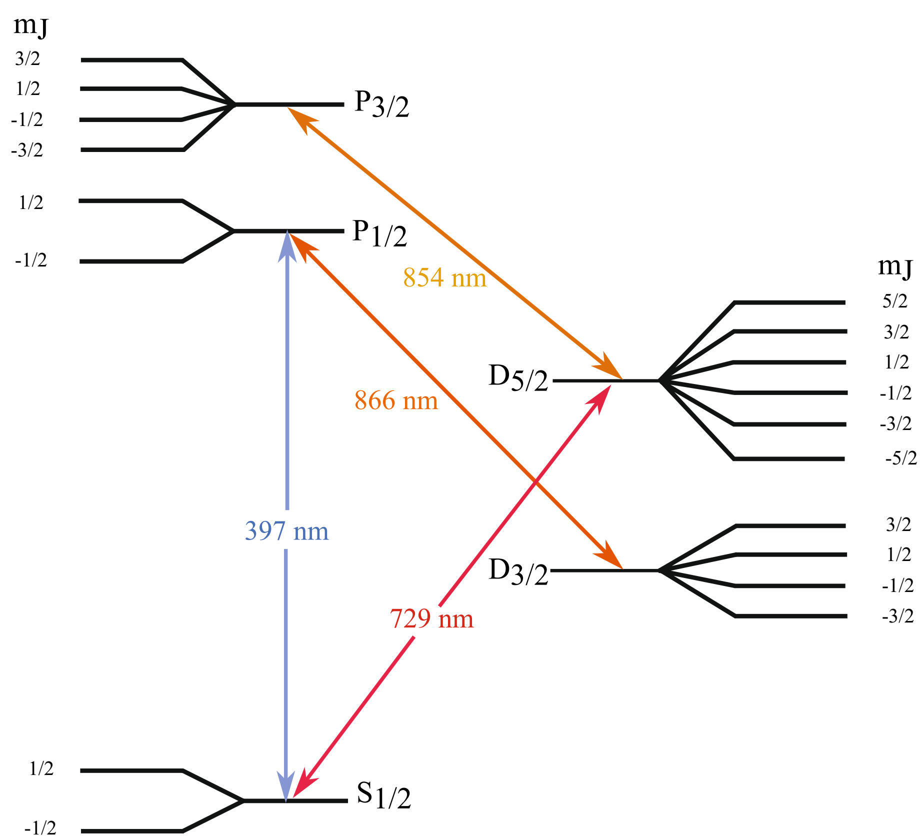

Fig. 1 The lowest three energy levels of $^{40}Ca^+$. The qubit states are $S_{1/2}$ and $D_{5/2}$ with lifetime about 1.17 s [20]. The narrow linewidth 729 nm laser beam is used to coherently control the qubit states. Laser light at 397 nm is used for Doppler cooling, optical pumping and quantum state detection. 854 nm and 866 nm laser beams are used for repumping the ion out of D states

precession time, A is the amplitude of fringe and ϕ_0 is the phase offset.

If no noise exists in the experimental process, perfect phase coherence results in $A = 1$. Any dephasing noise would cause the decay of fringe amplitude. In the experiment, it is reasonable to assume the fluctuation of detuning $\delta = \nu_L - \nu$ obeys Gaussian distribution

$$P(\delta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right), \tag{3}$$

where ν is the quadrupole transition frequency, ν_L is the laser frequency. The full width at half maximum $\delta\nu$ of $P(\delta)$ is linked to its standard deviation σ by

$$\sigma = \frac{\delta\nu}{2\sqrt{2\ln 2}}. \tag{4}$$

The fringe containing dephasing can be measured by averaging an ensemble of single experiment, and its amplitude A can be calculated by the average of the phase factor $\delta\tau$

$$\begin{aligned} A(\tau) &= |\langle \exp(i\delta\tau)\rangle| = |\int_{-\infty}^{+\infty} d\delta P(\delta)\cos(\delta\tau)| \\ &= \exp\left(-\frac{\sigma^2\tau^2}{2}\right). \end{aligned} \tag{5}$$

Generally, the dephasing time T_2^* is defined as the free precession time when the fringe amplitude decreases to 1 / e. Based on Eq. (4) and Eq. (5), T_2^* is connected to the noise model by

$$\delta\nu = \frac{4\sqrt{\ln 2}}{T_2^*}. \tag{6}$$

The linewidth of the frequency noise δ_B induced by magnetic field fluctuations can be calculated by

$$\delta_B = g_i \mu_B \Delta B, \tag{7}$$

where ΔB is the linewidth of the magnetic field noise. Based on the experimental measurement of T_2^* and transitions with different magnetic sensitivities, we can estimate the linewidth of laser frequency noise and magnetic noise by taking the assumption that the two kinds of noise are completely independent [18] and obey Gaussian distribution, and the total linewidth can be fitted by

$$\delta\nu = \sqrt{\delta_l^2 + \delta_B^2}, \tag{8}$$

where δ_l denotes the linewidth of laser frequency noise.

3 Experimental setup

A single $^{40}Ca^+$ ion is loaded into the blade-shaped linear Paul trap using the two-step photoionization method with laser light at 423 nm and 370 nm [21]. The radial confinement is formed by applying an RF signal (13.2 MHz) to one pair electrodes and connecting the other pair to the ground. A DC voltage of 900 V sent to the two end-caps provides the confinement along the axial direction and the corresponding axial secular frequency is measured to be $2\pi x1.41$ MHz. Laser light at 397 nm is used in experiment for Doppler cooling, optical pumping and state detection, and laser light at 866 nm and 854 nm are used for pumping the ion out of D state. The fluorescence of the ion is collected by a photomultiplier tube and an intensified CCD camera. The electronic state ($S_{1/2}$ and $D_{5/2}$) of the ion can be distinguished by using the electron shelving technique [22] with a detection time of 300 µs.

The quadrupole transition $S_{1/2} \leftrightarrow D_{5/2}$ is realized using a commercial diode laser at wavelength of 729 nm, whose frequency is stabilized to an ultra stable high finesse Fabry–Perot cavity and its linewidth is about 20 Hz. In the experiment, the 729 nm laser is used for sideband cooling of ion's motional mode, manipulation of quantum gates and frequency locking of 397 nm and 866 nm lasers. The optical and control setup of 729 nm laser is shown in Fig. 2. Geometrically, the 729 nm laser beam goes through the two end-cap electrodes and the wave vector almost overlaps with trap axis, resulting in the Lamb–Dicke parameter $\eta \approx 0.08$. The polarization of the 729 nm laser beam is properly adjusted to couple all the quadrupole transitions. AOM1 is used to stabilize the laser intensity so as to suppress the laser intensity

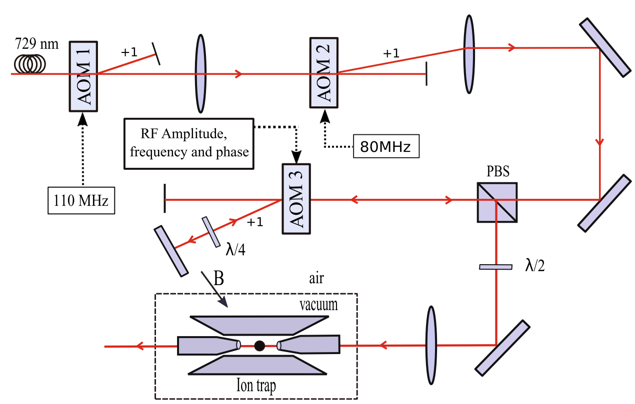

Fig. 2 The optical setup and control of 729 nm laser beam. AOM 1 working at 110 MHz is used for the laser power stabilization. AOM 2 supplied with an 80 MHz RF signal is used for the laser frequency shifting. AOM 3 working at 270 MHz with a double-pass configuration controls the overall frequency, phase and amplitude of the 729 nm laser beam. The half-wave plate is installed for adjusting the polarization of the laser beam so that all the transitions can be excited

fluctuation induced decoherence and distortion of Ramsey fringe patterns. AOM 2 is used to shift the laser frequency to transition resonance by 80 MHz. The amplitude, frequency and phase of the beam are controlled by AOM 3, which has a double pass optical configuration and is driven by an RF signal with frequency of 270 MHz. It is also used as a switch for laser pulses generation in experimental sequences. To reduce the magnetic fluctuations induced by the AC power line, we trigger the experimental sequence at the fixed phase of AC power line. Note that we have measured the resonance shift of each transition at different phase of the AC-power line and the trigger time is properly chosen to minimize the frequency shift during the Ramsey experiments.

4 Experimental results and analysis

Experimentally, the resonance for each transition should be identified before we implement Ramsey experiment. First, the ion is cooled down to the motional ground state by Doppler and sideband cooling. Then, we scan the frequency of AOM 3 with fixed pulse time and search the resonant frequencies for all the transitions, afterwards, we calibrate the laser power to set the same Rabi frequency for the five transitions at about 50 kHz, which guarantees that laser-induced decoherence impact is same for all transitions and the long-term drift of laser frequency during each measurement can be neglected since it is much less than the linewidth of each transition.

The dephasing time of the qubit is measured by scanning the free precession time of the Ramsey experiment up to 1200 μs and each data point is an average of 100 experiments. The data points are then fitted using the empirical function $p_\uparrow = \frac{1}{2}[\exp(-t^2/T_2^{*2})\cos(\omega t + \phi_0) + 1]$ to obtain the dephasing time. Figure 3a shows the example for dephasing time measurement of $S_{1/2}(m_J = -1/2) \leftrightarrow D_{5/2}(m_J = -1/2)$. It can be seen that the Ramsey fringe fits well with the empirical function. To obtain accurate measurement on coherence time, we repeat the Ramsey experiment for 50 times for each transition and take the average as the final measurement of dephasing time. Finally, the linewidth of laser frequency noise and the magnetic field noise can be obtained by fitting the data with Eq. (8). As shown in Fig. 3b, the linewidth of the laser frequency noise in our system is 493.3 ± 8.1 Hz, and the linewidth of the magnetic field noise is about 181.4 ± 3.5 μG.

Since the fluctuation of magnetic field distorts the Ramsey fringes, the error bars in Fig.3b increase with the magnetic sensitivity. For the transition $S_{1/2}(m_J = -1/2) \leftrightarrow D_{5/2}(m_J = -1/2)$, the magnetic field-induced fluctuation is about 203 Hz, which is about half of the laser frequency noise, therefore, the dominate noise for this transition is laser frequency noise. We further found that the laser frequency noise is caused by the limited response of the laser diode to current modulation as stated in Ref [12].

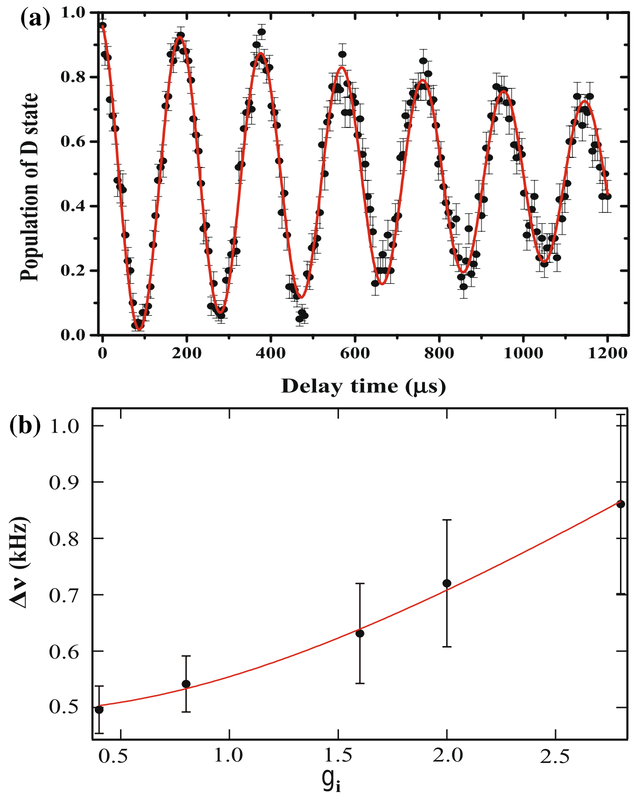

Fig. 3 **a** Ramsey fringe for $S_{1/2}(m_J = -1/2) \leftrightarrow D_{5/2}(m_J = -1/2)$. The solid line is the fitting using empirical function $p_\uparrow = \frac{1}{2}(\exp(-t^2/T_2^{*2})\cos(\omega t + \phi_0) + 1)$ and the coherence time T_2^* extracted is around 1.3 ms. **b** Fitting of total frequency noise with Eq. (8). The dots are the total linewidth of the dephasing noise, and the solid line is the fitting with Eq. 8. The laser and the magnetic field noise are 493.3 ± 8.1 Hz and 181.4 ± 3.5 μG, respectively

The problem could be overcome using the ultra-high finess cavity as a filter for laser beam and amplifying the transmitted laser beam with injected-locking method, or using another laser source with less frequency noise.

To further verify our method and experimental results above, we build another 729 nm laser system based on the Ti:sapphire laser that has much less frequency noise than the diode one. We repeat our measurement on transition $S_{1/2}(m_J = -1/2) \leftrightarrow D_{5/2}(m_J = -1/2)$, and the linewidth is around 156 μG for magnetic field noise if we ignore the laser frequency noise. This number is very close to the one we measured with diode laser and it proves the validity of our method and experimental results.

5 Conclusion

In this work, we proposed and demonstrated a convenient and useful method for estimating the magnetic- and laser-induced frequency noise in optical qubit, and we demonstrated this method with a single trapped $^{40}Ca^{+}$ ion. By measuring the dephasing time of the five magnetically sensitive transitions of a single $^{40}Ca^{+}$ ion, we discriminated and estimated the linewidth of laser frequency noise and magnetic field noise to be 493.3 ± 8.1 Hz and 181.4 ± 3.5 μG, respectively. In fact, there are other kinds of noise in the experiment such as electric field noise, patch charges on electrodes, but they only contribute to decoherence of motional mode and show negligible affection to qubit in the Lamb–Dicke regime. Since this method is independent of the ion species, it is a general method for all ion qubits, besides it can be used to separate the magnetic field noise and other kinds of dephasing noise in other atomic systems such as neutral atoms.

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References

1. J.I. Cirac, P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995)
2. M.A. Nielsen, I.L. Chuang, *Quantum computation and quantum information* (Cambridge University Press, Cambridge, 2010)
3. F. Schmidt-kaler, H. Häffner, M. Riebe, S. Gulde, G.P.T. Lancaster, T. Deuschle, C. Becher, C.F. Roos, J. Eschner, R. Blatt, Nature **422**, 408 (2003)
4. M. Riebe, H. Häffner, C.F. Roos, W. Hänsel, J. Benhelm, G.P.T. Lancaster, T.W. Körber, C. Becher, F. Schmidt-Kaler, D.F.V. James, Nature **429**, 734 (2004)
5. T. Monz, K. Kim, W. Hänsel, M. Riebe, A.S. Villar, P. Schindler, M. Chwalla, M. Hennrich, R. Blatt, Phys. Rev. Lett. **102**, 040501 (2009)
6. T. Monz, P. Schindler, J.T. Barreiro, M. Chwalla, D. Nigg, W.A. Coish, M. Harlander, W. Hänsel, M. Hennrich, R. Blatt, Phys. Rev. Lett. **106**, 130506 (2011)
7. H. Häffner, W. Hänsel, M. Riebe, C.F. Roos, J. Benhelm, R. Blatt, D. Chek-al kar, M. Chwalla, T. Körber, U.D. Rapol, M. Riebe, P.O. Schmidt, C. Becher, O. Gühne, W. Dür, Nature **438**, 643 (2005)
8. F. Schmidt-kaler, S. Gulde, M. Riebe, T. Deuschle, A. Kreuter, G. Lancaster, C. Becher, J. Eschner, H. Häffner, R. Blatt, J. Phys. B Atomic Mol. Opt. Phys. **36**, 623 (2003)
9. C.F. Roos, M. Chwalla, K. Kim, M. Riebe, R. Blatt, Nature **443**, 316 (2006)
10. B. Merkel, K. Thirumalai, J. Tarlton, V. Schäfer, C. Ballance, T. Harty, D. Lucas, Rev. Sci. Inst. **90**, 044702 (2019)
11. Y. Wang, M. Um, J. Zhang, S. An, M. Lyu, J.N. Zhang, L.M. Duan, D. Yum, K. Kim, Nat. Photon. **11**, 646 (2017)
12. N. Akerman, N. Navon, S. Kotler, Y. Glickman, R. Ozeri, N. J. Phys. **17**, 113060 (2015)
13. J. Bylander, S. Gustavsson, F. Yan, F. Yoshihara, K. Harrabi, G. Fitch, D.G. Cory, D. Yum, Y. Nakamura, J.S. Tsai, W.D. Oliver, Nat. Phys. **7**, 565 (2011)
14. G.A. Álvarez, D. Suter, Phys. Rev. Lett. **107**, 230501 (2011)
15. L.Z. He, M.C. Zhang, C.W. Wu, Y. Xie, W. Wu, P.X. Chen, Chin. Phys. B **27**, 120303 (2018)
16. X. Zhang, Y. Shen, J. Zhang, J. Casanova, L. Lamata, E. Solano, M.H. Yung, J.N. Zhang, K. Kim, Nat. Commun. **6**, 7917 (2015)
17. C.F. Roos, Controlling the quantum state of trapped ions. PhD dissertation, University of Innsbruck (2000)
18. H.H. Ku, J. Res. Natl. Bur. Stand. C Eng. Instrum. **70C**(4) (1966)
19. N.F. Ramsey, Phys. Rev. **76**, 996 (1949)
20. P.A. Barton, C.J.S. Donald, D.M. Lucas, D.A. Stevens, A.M. Steane, D.N. Stacey, Phys. Rev. A **62**, 032503 (2000)
21. D. Lucas, A. Ramos, J. Home, M. McDonnell, S. Nakayama, J.P. Stacey, S. Webster, D. Stacey, A. Steane, Phys. Rev. A **69**, 012711 (2004)
22. C. Roos, T. Zeiger, H. Rohde, H.C. Nägerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler, R. Blatt, Phys. Rev. Lett. **83**, 4713 (1999)