Applied Physics B (2018) 124:229
https://doi.org/10.1007/s00340-018-7101-z

Applied Physics B
Lasers and Optics

Apodization for improving the two-point resolution of coherent optical systems with defect of focus

Andra Naresh Kumar Reddy[1,2] · **Svetlana Nikolaevna Khonina**[2,3]

Received: 12 June 2018 / Accepted: 12 November 2018 / Published online: 17 November 2018
© Springer-Verlag GmbH Germany, part of Springer Nature 2018

Abstract

An amplitude apodization influenced defocused optical system used to form a coherent light illuminated two-point sources with an improved resolution is analyzed. The composite coherent image intensity distributions in the defocused and the Gaussian ($Y_{\mathrm{d}} = 0$) planes have been calculated and compared. It has shown the resolution of the apodized optical system is significantly increased for an excessive amount of defocusing effect. As is clearly evidenced by the presence of irradiance dip in the resulting intensity profiles can describe the super-two-point resolution phenomena in a maximum defocused plane ($Y_{\mathrm{d}} = 2\pi$).

1 Introduction

Usually, studies on the apertures have paid attention on improving the resolution of the image. Earlier and current approaches were proposed to improve this essential property. Aberrated apertures, results in images with degraded resolution, which represent a fundamental problem in many applications [1–3]. The optical system and the implemented techniques focus on reduction of the aberration and to increase the resolving power [4–8]. However, it differs from one optical system to another, controlled by the degree of coherence and the type of apodization to be employed. In the recent past, the coherent optical systems have become vital in a variety of applications. It is well-known the coherent image-forming system continued to be the worst case since a fully coherent beam produces more interference effects. However, even in this case, one can increase the resolution of the system by employing a suitable apodization technique in the pupil plane. Apodization represent the process of redistribution of the diffracted light field in the focal region of the optical system [9–15]. These modification results gained more attention in the presence of optical aberrations [4–8]. For the aberrated optical systems operating in partially coherent or incoherent light, the application of apodization was theoretically established by the past studies [7, 8, 15–17]. However, there were a few studies on enhancing the resolution of coherent optical systems [18–21]. In the past studies, both Rayleigh and Sparrow criteria were used in understanding the performance of the optical systems [22, 23]. Since they were incompetent to reveal complete characteristics of the diffracted image. So it is necessary to compute the intensity distributions produced by the two-point sources under different considerations. The investigation reported here may increase the range of applications. In this paper, we investigated the resolving power of a circular aperture illuminated by fully coherent light in the presence of defocusing aberration. This can be seen by calculating the resulting composite image intensity distributions from two-points and coherent sources with respect to each other. The circular aperture with an amplitude apodization property can alter the transmittance of the optical system which focuses the light flux enclosed in the principal part of the resulting diffraction pattern. Earlier, this type of pupil function was successful in modifying the two-point resolution produced by the aberration-free asymmetrically apodized optical systems under illumination of fully incoherent and partially coherent light [24]. This method is also applied to investigate the two-point resolution in the presence of the spherical aberration [25]. Technically, the current investigations on the apodized circular aperture are quite useful to

✉ Andra Naresh Kumar Reddy
naarereddy@gmail.com

1 Department of Physics of Complex Systems, Weizmann Institute of Science, 7610001 Rehovot, Israel

2 Samara National Research University, Moskovskoye Shosse, 34, 443086 Samara, Russia

3 Image Processing Systems Institute-Branch of the Federal Scientific Research Centre “Crystallography and Photonics” of Russian Academy of Sciences, 443001 Samara, Russia

interpret the situations in a real-time imaging experiments. This work might be productive for describing the interference of the two coherent images embedded into the longitudinal displacement of their principal maxima, results a linear decrease of the principal maxima intensity and the energy spreads out to maximize the ringing effect. Ultimately, the present study is valuable in dealing with the defocus error results from the composite coherent image intensity distribution produced by a telescope objective being observed not at the Gaussian point, but at a point displaced longitudinally from its Gaussian focus.

2 Formulation: far-field diffraction

Following the Fraunhofer-diffraction theory, the expression for the composite image intensity distributions from two-point sources formed by the circular pupil function in the image plane of the coherent optical systems are given by [8, 24, 26]:

$$I(Z) = |P(Z-Q)|^2 + \alpha|P(Z+Q)|^2 + 2\sqrt{\alpha}\mu(Z_0)|P(Z-Q)||P(Z+Q)|. \quad (1)$$

Over here the two-point sources are separated by the distance $2Q = Z_0$, α is the intensity difference between the two-coherent sources, $\mu(Z_0)$ remain the substantial part of the complex degree of the coherence of the illumination, Z is the dimensionless diffraction variable, $P(Z+Q)$ and $P(Z-Q)$ are the amplitude impulse responses of the coherent optical imaging system corresponding to the two-points, positioned at the distance of $Z_0/2$ on either side of the optical axis.

The amplitude impulse response functions of two-coherent sources are given by:

$$P(Z \pm Q) = 2\int_0^1 J_0[(Z \pm Q)r]\, r\mathrm{d}r. \quad (2)$$

In the above expression, J_0 is the Bessel function of first kind and zero order, r is the radial-coordinate in the pupil plane of the coherent optical system and it is the normalized distance of a general point on the exit pupil varying from zero to one. In the present investigation, by employing the amplitude apodization filter across the exit of the pupil function, then Eq. (2) Rewritten as:

$$P(Z \pm Q) = 2\int_0^1 f(r) J_0[(Z \pm Q)r] r\mathrm{d}r. \quad (3)$$

$f(r) = r^2$ represents the analytic expression for the amplitude apodizer, employed across the pupil function of the optical system. The amplitude transmittance of the proposed amplitude type super-resolving filter varies with the radial coordinate r. The amplitude transmittance increases from the midpoint to the frontier of the pupil function. The resulting amplitude impulse response of the apodized pupil function is given as Eq. (4):

$$P(Z \pm Q) = 2\int_0^1 r^2 J_0[(Z \pm Q)r] r\mathrm{d}r. \quad (4)$$

In this work, the defocusing wave front aberration is represented by $\psi(r,\theta)$, given by:

$$\psi(r,\theta) = \exp\left\{-iY_\mathrm{d}\frac{r^2}{2}\right\}. \quad (5)$$

From Eqs. (4) and (5), the amplitude response of the apodized pupil function with the defect-of-focus can be written as:

$$P(Z \pm Q) = 2\int_0^1 r^2 \exp\left\{-iY_\mathrm{d}\frac{r^2}{2}\right\} J_0[(Z \pm Q)r] r\mathrm{d}r, \quad (6)$$

where Y_d represents the limiting factor which controls the amount of defect-of-focus in the coherent optical system subjected to the amplitude apodization. Note that, $Y_\mathrm{d} = 0$ represents the Gaussian focal plane and non-zero values of Y_d are used to specify the out-of-focus planes. The field distribution in the different image planes of the optical system functions of amplitude apodization and the amount of defocusing effect in the pupil plane. However, $Y_\mathrm{d} = 0$, $\pi/2$, π, $3\pi/2$ and 2π are the different image planes. The solution of the problem has been developed by designing the apodized elements that operate in the paraxial domain of diffractive optics, which can generate small-angle far-field intensity distributions. In the present study, $\alpha = 0.20$, 0.40,0.60, 0.80 and 1.00 can be termed as the intensity ratio of the two-points under the coherent illumination of light and μ is being equal to ± 1.00.

3 Composite image intensity distributions

A resolution criterion like the Rayleigh and Sparrow are applicable in the assessment of optical systems. However, they are not used to reveal the salient characteristics of the resulting field distribution in the focal region of the optical systems. This problem can be solved by computing the composite image intensity distribution from two-points and fully coherent sources with respect to each other. From Eq. (6) and Eq. (1) the intensity distribution from the two equal-intensity point sources has been evaluated for the dimensionless variable Z_0 equal to 3.0, 3.5, 4.0, 4.5, and 5.0; the value

μ being taken as ± 1. The effect of amplitude apodization and the defect-of-focus (Y_d) on the structure of the composite intensity distribution have been investigated, are shown in Figs. 1 and 2.

In Fig. 1a, for the different distance separations (Z_0), the resulting intensity distributions from two-point sources produced by the coherent system. In the presence of maximum defocusing effect ($Y_d = 2\pi$), the peak value of the principal maxima is decreased, consequently, the energy in the side-lobes region has increased. It is observed the points are well resolved for the values of Z_0 varying from 3.5 to 5.0 [$Z_0 = 3.5$, which is a separation just less than that of the incoherent Rayleigh limit ($3.832 = 1.22\lambda/D_{\text{aperture diameter}}$)], as is clearly evidenced by the dips in the maximum defocused plane. The effect of a maximum defocusing is found to be significant in resolving the two-coherent point objects. In Fig. 1a, for $Z_0 = 3.0$ the points have a tendency to resolve, the curve is essential with the indication of a distinct dip in intensity. In Fig. 1b, the apodizer has smoothed the field distributions in the maximum defocused plane. In each

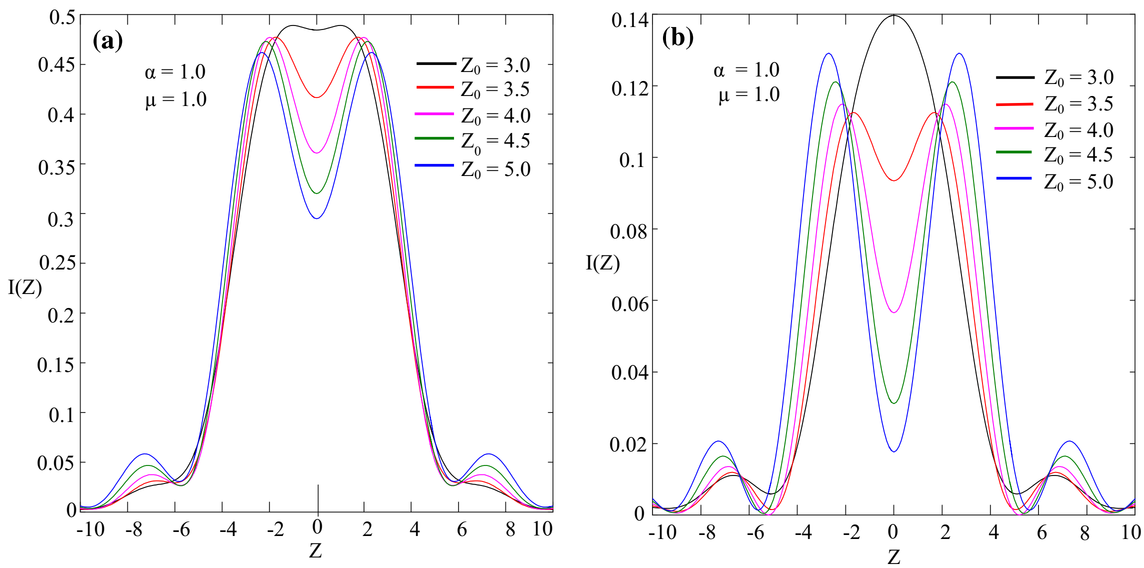

Fig. 1 Resultant coherent image intensity distribution as a function of various distance separations (Z_0)when the two-point sources are in the defocused plane $Y_d = 2\pi$: **a** for unapodized pupil and **b** for apodized pupil

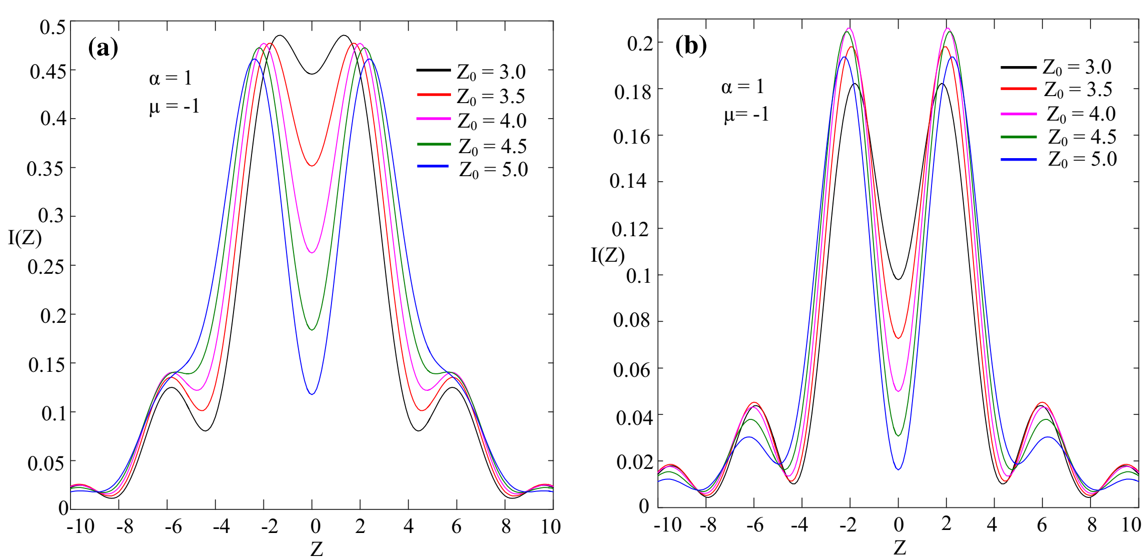

Fig. 2 Coherent intensity distribution of the two anti-phase points for different values of Z_0 in the presence of maximum defocusing effect ($Y_d = 2\pi$): **a** for unapodized pupil and b for apodized pupil

case, the two-point resolution is significantly increased as the apodizer is applied. But when the points are in the distance $Z_0 = 3.0$ the resolving power deteriorates and the curve is essentially flat with no indication of a dip in intensity. Therefore, for $Z_0 = 3.0$, the apodizer proved to a failure in producing the points to get separated. While the distance separations $Z_0 = 4.0$, 4.5 and 5.0, the resolution found to be excellent and demonstrates a super-two-point resolution phenomenon. In these cases, the maxima in the resultant distribution move away from each other producing a large-scale measured separation is shown in Fig. 1b.

Consider the two-coherent sources with a phase difference of π, situated at a distance $Z_0/2$ from the optical axis. The two-coherent sources are said to be in anti-phase ($\mu \to -1$), are imaged through a two-dimensional apodized optical system with defocus aberration. In Fig. 2a, as the value of Z_0 increases from 3.0 to 5.0, the points are said to be highly resolved in the maximum defocused plane ($Y_d = 2\pi$), as follows reaching an infinite value of resolving power (say in the specific cases when $Z_0 = 4.5$ and 5.0). Resulting intensity profile curves in Fig. 2a concludes that in the presence of maximum defect-of-focus the two coherent sources in phase opposition found with the superresolution, is evidenced by the presence of distinct dips. Therefore, in the presence of defocusing aberration, the two-coherent sources under antiphase illumination will be resolved regardless the energy which has succeeded into the side-lobes region. Figure 2b shows the composite image intensity distribution of two coherent points with equal intensity ratio, separated by various dimensionless diffraction units Z_0, when the image plane is fully defocused ($Y_d = 2\pi$). In this case, the apodization becomes significantly effective in improving the response of the optical system, and the resolution is found to be excellent in an apodized system for a maximum defocused plane, as shown in Fig. 2b. As Z_0 increases from 3.0 to 5.0 in steps of 0.5, the resolution increases, thus reaching an infinite value and, therefore, the dip, midway between the two coherent images has gained the lower intensity maximum values. In addition, the side-lobes region in the coherent image intensity distribution has modified by the apodizer such that the non-zero minima regions are shifting vertically towards their zero-amplitude positions.

Figure 3a presents the profile of the intensity distribution of two-equal intensities coherent sources, separated by $Z_0 = 4.0$ dimensionless diffraction units for a clear aperture under various degrees of defect-of-focus. It is observed that the points under coherent illumination are said to be resolved only when the defect-of-focus is more than $Y_d = \pi/2$. In this case, the resolution is found to increase with increasing value of defocusing parameter Y_d as shown in Fig. 3a. However, Fig. 3b illustrates the influence of amplitude apodization is pronounced in resolving the two-coherent points for all defocused planes. For instance, the apodization in a defocused plane with $Y_d = \pi/2$ has proved to be successful in resolving the two-coherent sources of equal intensity at $Z_0 = 4.0$. It is also observed that for all defocused planes, the quadratic amplitude apodization is found to be very effective in improving the resolution of two-points under coherent illumination, whereas the high resolving power is noticed at $Y_d = 3\pi/2$. As is proved by the maxima in the resultant distribution shift away from each other producing a large-scale separation as shown in Fig. 3b.

Figure 4a–c shows the intensity profile curves for various values of intensity ratio $\alpha = 0.2$–0.8 in steps of 0.2 for

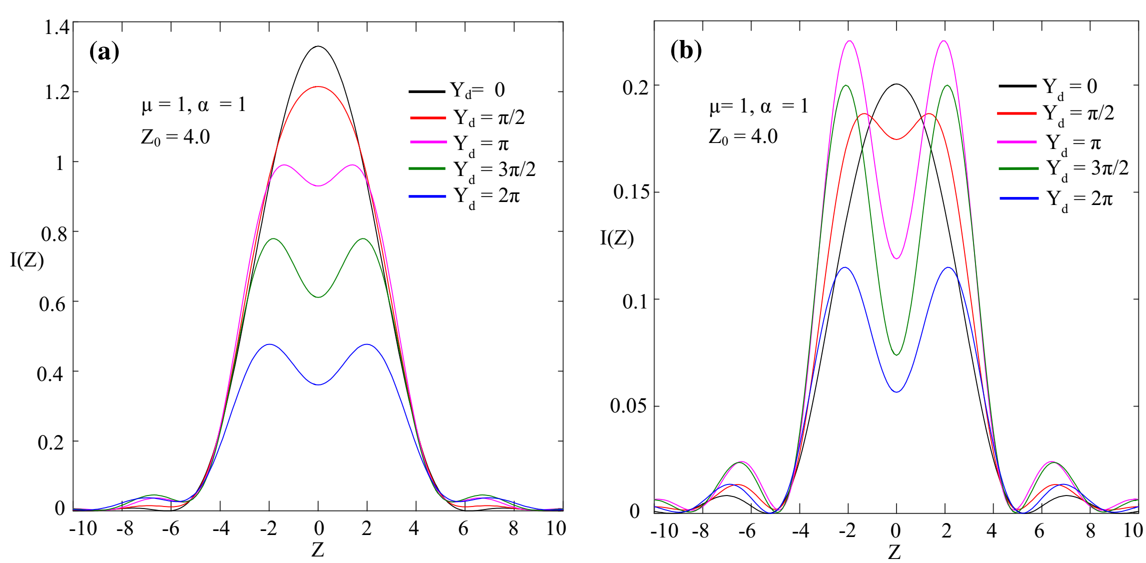

Fig. 3 Influence of defocus in resolving the two-coherent sources separated in the distance $Z_0 = 4.0$: **a** for unapodized pupil and **b** for apodized pupil

$Z_0 = 3.5$ (just less than that of the Rayleigh limit). In each case, other parameters are kept constant. Figure 4a illustrates the performance of the Airy pupil system in the Gaussian focal plane ($Y_{\mathrm{d}} = 0$). In this case, the two-point sources under coherent illumination are said to be unresolved for any value of α.The curve is essentially flat with no indication of a dip in the profile. In Fig. 4b, for a defocused optical system ($Y_{\mathrm{d}} = 2\pi$), the two maxima in the intensity distribution move outward producing a clear measured separation. In this case, the resolution of the optical system is increased for a maximum defocused plane. However, in the presence of amplitude apodization the resolution of two-coherent sources increases than that of the unapodized case, as shown in Fig. 4c. By employing the apodization across the pupil function, the position of the maxima in the resulting composite intensity distribution moves away from each other, results, the separation dips appear substantially larger than that of the case as shown in Fig. 4b. Figure 5 illustrates the effect of apodization on the performance of aberration-free coherent optical systems. In this case study, the resolution of the two points of equal intensity under coherent illumination functions of Z_0 values. It is seen from Fig. 5 that the points are unresolved for the values of Z_0 varying from 3.0 to 4.2 in steps of 0.4, the curves are found with no indication of distinct dip. As Z_0 increases from 4.2 to 4.6, the two-points are just resolved, as is clearly evidenced by the dip in the intensity profile. For $Z_0 = 5.0$, the apodization becomes significantly effective in resolving the points, the

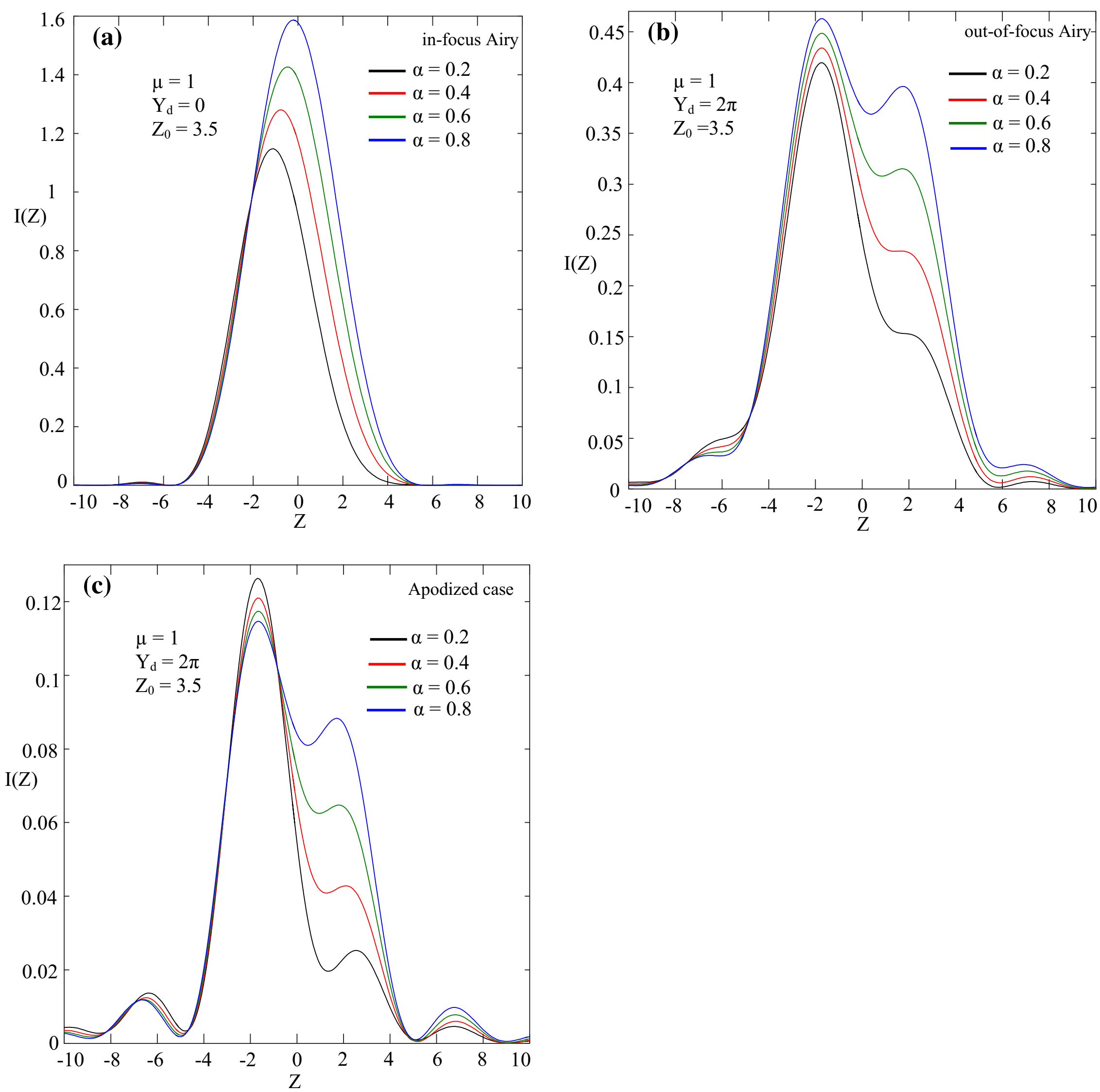

Fig. 4 Distribution of composite image intensity for two mutually coherent point sources with different values of intensity ratio α, separated by the distance $Z_0 = 3.5$ (beyond the Rayleigh limit): **a** for airy pupil without defocusing, **b** in the maximum defocused plane ($Y_{\mathrm{d}} = 2\pi$) and **c** increasing nature of the resolution for the apodization

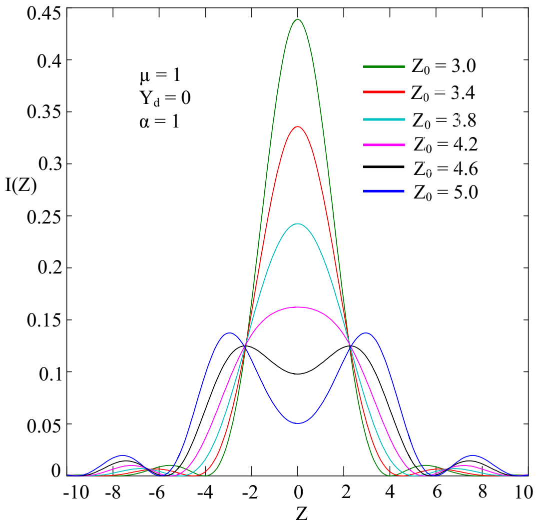

Fig. 5 The resolution enhancing with Z_0 in the amplitude apodized system for a Gaussian focal plane ($Y_d = 0$)

two maxima in the resultant profile move outward producing a more large-scale separation, and, therefore, the two-points are well resolved under coherent illumination ($\mu = 1$).

In Fig. 6a, the value $Z_0 = 3.5$, which is a separation just less than that for the incoherent Rayleigh limit ($Z_0 = 3.832$). The two anti-phase points are well resolved in a unapodized coherent system for a maximum out-of-focus plane ($Y_d = 2\pi$). Regardless the energy spreads out in the side-lobes region, the two principal maxima shift outward producing the clear dips for different values of α in a maximum defocused plane. Note that, the resolution increases with α, as shown in Fig. 6a. In the application of apodization, the performance of the defocused optical system gradually increases and as follows reaching an enormous value of resolving power under the anti-phase coherent illumination. It is seen from Fig. 6b that for all values of α, the points in phase opposition are said to be well resolved and no matter how large ringing in the coherent image. Traditionally, the enhancement of the side-lobes region degrades the axial resolution. In this case, the super-two-point resolution can be measured with an absolute accuracy, as is justified by the presence of steep dips in the intensity distribution.

4 Significance of the study

In this section, we demonstrated the importance of present investigation. For comparison, the results with different pupil engineering techniques are also presented. Figure 7a shows the resultant intensity distribution of two-point sources with widely varying in their intensities ($\alpha = 0.6$) as a function of μ values for highly apodized optical system. Here, the value $Z_0 = 3.832$ is the distance separation equal to the Rayleigh limit. From the family of intensity curves in Fig. 7a, we found that the two-point sources has a low probability to get resolve for the maximum out-of-focus plane ($Y_d = 2\pi$). However, the two-point sources are said to be not resolved while the curves are essentially flat with no indication of dip in intensity. Note that, in the absence of

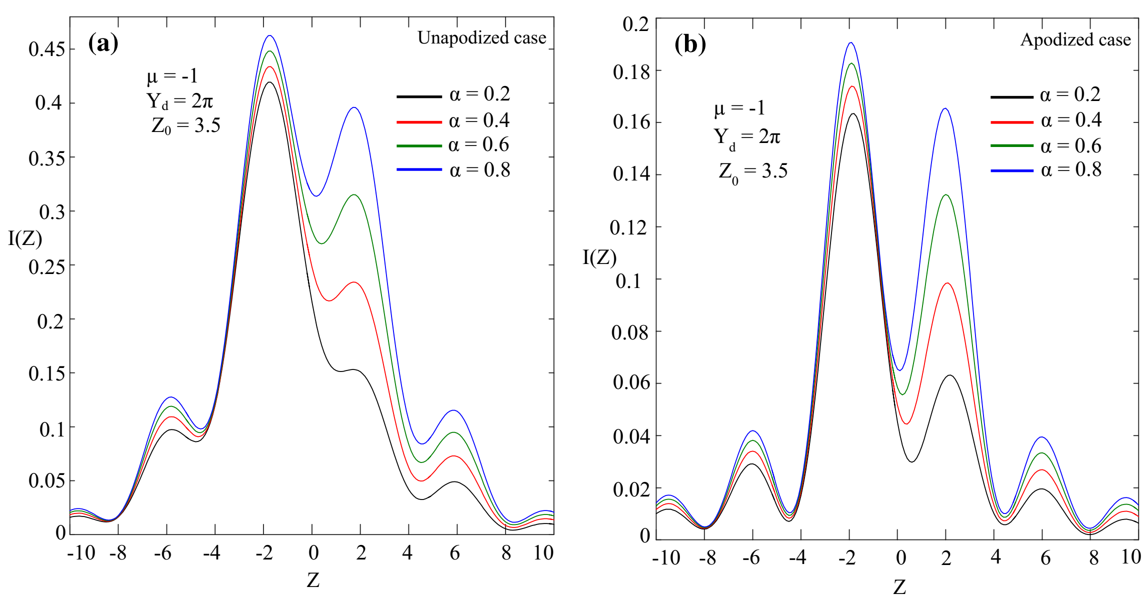

Fig. 6 Intensity distribution in the resultant coherent image of two anti-phase points separated by $Z_0 = 3.5$ for different values of intensity ratio α: **a** in the presence of maximum defocusing effect and **b** resolution increasing by the high amount in an apodized system for a maximum defocused plane

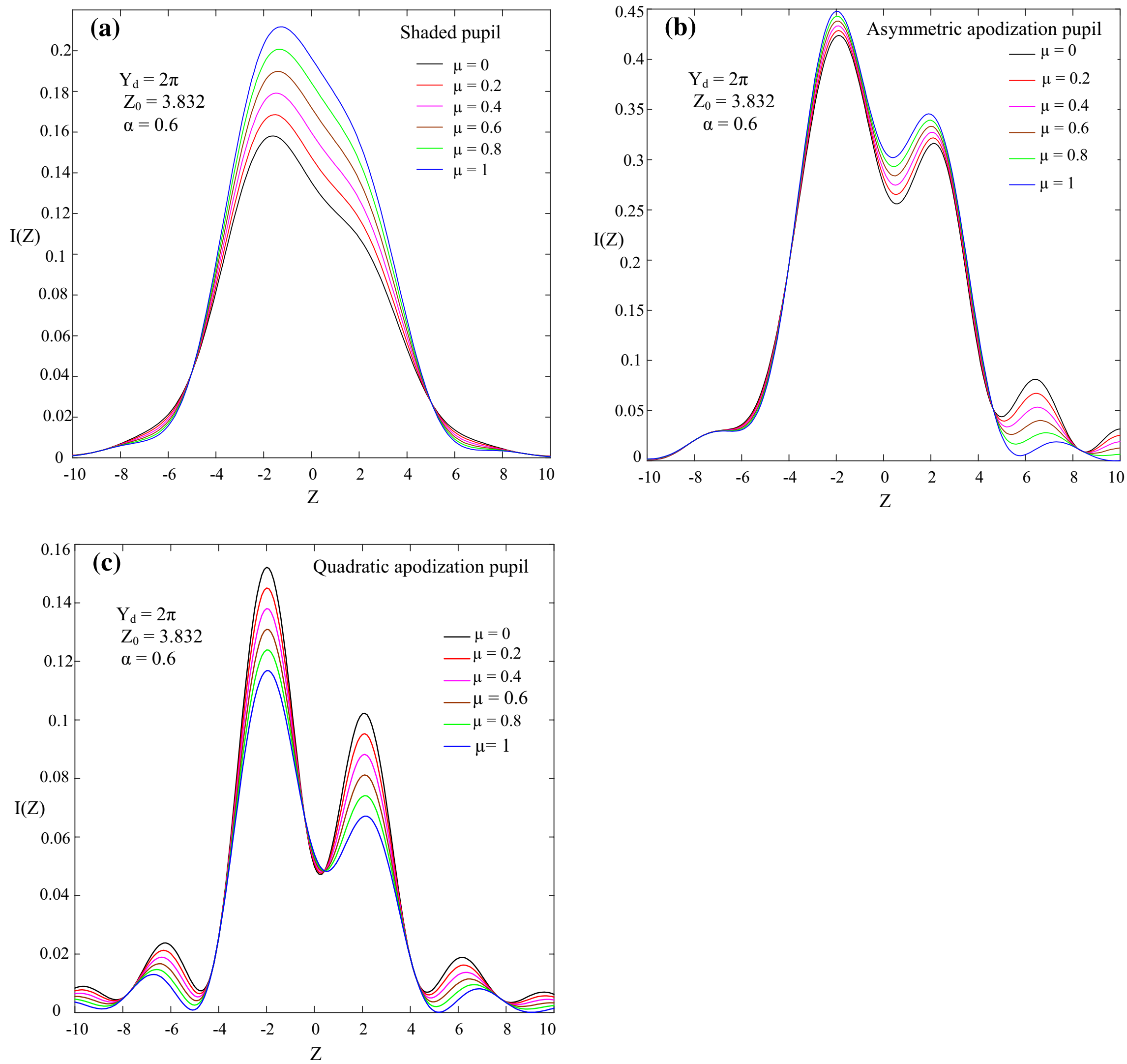

Fig. 7 Resolution of two overlapping point-sources under the different conditions of coherence, when they are separated by the distance (Z_0) equal to the Rayleigh limit (3.832): for **a** shaded pupil [26], **b** asymmetric pupil [8] and **c** quadratic pupil function

apodization function the two-points are said to be resolved with the defocusing effect [26]. As observed from the Fig. 7a that the increased coherence lowers the resolving power. Finally, we are concluding that this apodization function is absolutely not effective in resolving the two-points under coherent light illumination. Moreover, by employing the shaded pupil function, the resolution became worst even in the presence of defocusing effect [26]. In such cases, the use of quadratic apodization function improve the two-point resolution of aberrated coherent optical systems (as observed from Figs. 1b, 4c).

Figure 7b shows the resultant intensity distribution of two-point sources formed by asymmetrically apodized optical system for various degree of the coherence (μ). As observed from the intensity curves, the asymmetric pupil function is effective than the earlier case (Fig. 7a) in resolving the two-point sources separated by the Rayleigh limit distance. Here, a dip of intensity is noticed in the resulting intensity distributions and the two-point sources are said to be just resolved for all cases of coherence with the maximum defocusing effect ($Y_d = 2\pi$). It is also observed that the increase in coherence lowers the resolution of two-point sources. Moreover, the left half of the composite image intensity distribution is obtained at the cost of worsening the right half of the distribution [8, 24]. This apodization function is radially asymmetric which cannot decrease the spot size in all directions [24]. In earlier study [8], the authors omitted to include the defocusing effect in the annular regions of the pupil function and as it appears the defect-of-focus confined to the middle region of the pupil and, therefore, apparently is attributable to the mask itself. That is not helpful assumption. Surely the defocusing effect

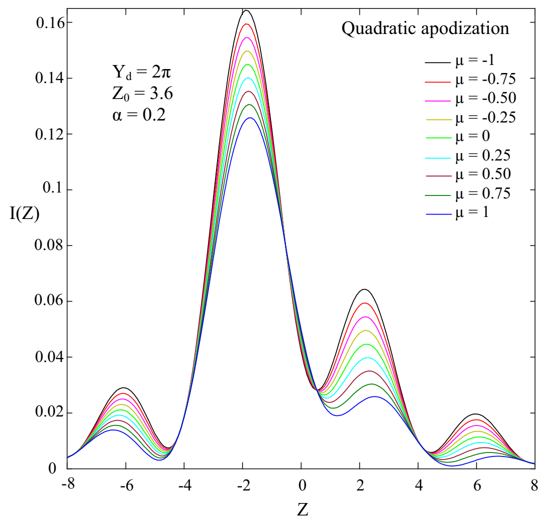

Fig. 8 Resolution of two nearest bright and faint sources for various values of μ (the point separation, $Z_0 = 3.6$ is just below the Rayleigh limit)

would originate in the upstream of telescope optics, and thus would apply to the entire pupil function. In such situations, our method has improved the resolution to a greater extent. This effect is clearly depicted in Fig. 7c that for all coherent conditions of μ, a clear steep dips are found in the resultant intensity distributions and, therefore, the two-points are said to be well resolved. Particularly, our pupil engineering technique is more efficient and suitable for improving the resolution under the coherent illumination. Unlike other pupil functions, the design of the quadratic apodization pupil is relatively simple, and the reason for selecting it for two-point resolution studies is that it has a radial symmetry, which ensures a decrease in the spot size in all directions. The latter property is important, since it ensures the separation of two closely located point-sources, regardless of their mutual arrangement. Such a behavior plays a vital role in improving the performance of coherent imaging systems.

Figure 8 shows that by employing the quadratic apodization function, for all values of μ, the two nearest neighborhood point-sources with unequal intensities are said to be well resolved below the Rayleigh limit. In all presented results, the shift in position of the two maxima in the resultant distribution producing a measured separation much larger than the real separation, therefore, the two-points are well-resolved. This evidence demonstrates the super-resolving nature of the pupil function. This study is quite useful in the astronomical and spectroscopy observations i.e., resolving the faint-object/weak-line in the vicinity of a bright-object/strong-line under different coherent conditions of light illumination.

Since, the method is efficient in resolving the two-point sources of different intensity ratios under different coherence conditions of light illumination, the high spatial coherence of light generates speckles. However, in the presence of excessive defocusing effect, the spatial coherence of the output light could be controlled when the point of observation is changed from the Gaussian focal plane to a defocused plane along the direction of light propagation. In the absence of defocusing, amplitude and phase modulation pupils could be more effective in reducing the speckles [27]. We plan to address these in future papers.

5 Conclusions

In this paper, the two-point resolution of coherent optical systems has been determined in different situations. From the results, we conclude that the defect-of-focus is found to be important in resolving the two-point sources in the case of quadratic amplitude apodization across the circular pupil function. For a maximum defocused plane ($Y_d = 2\pi$), the apodizer very much improves the resolution of a coherent imaging system that the maxima in the resultant intensity profile move outward producing an enormous value of measured separation. Even the combined effect of defocusing and apodization is successful in resolving the two-coherent sources separated by the distance less than that of the Rayleigh diffraction limit ($Z_0 = 3.832$). Note that, in the presence of defocusing aberration the apodized composite intensity distribution can provide the super-two-point resolution phenomena due to the growth of the sidelobes. For the apodized system in a defocused plane ($Y_d = 2\pi$), as the degree of coherence takes the value from $+1$ to -1, the corresponding resolving power is reaching an infinite value. Accordingly, the resolution of the two-coherent sources in phase opposition is said to be excellent. This method has significant impact on the performance of coherent and incoherent optical imaging systems employed in various situations such as diffraction limited planet imaging by combining coronagraph and pupil amplitude apodization technique helpful to detect the direct image of extra solar planets. This technique is very useful for improving the imaging of visible and near-infrared faint astrophysical sources close to bright ones. The quadratic apodization principle and its flexible design could improve real time two-point resolution experiments, e.g. inserting the quadratic amplitude aperture at the Fourier plane of optical microscope to improve real time imaging of closely moving biological structures.

Acknowledgements The Corresponding author is indebted to Asher A. Friesem and Nir Davidson, Department of Physics of Complex systems, Weizmann Institute of Science, Israel for useful discussions, which indeed improved the paper. This work was supported by the Israel Science Foundation (ISF) in part of optimizing the calculations and by the Ministry of Science and Education, Russia within the state assignment FSRC "Crystallography and Photonics" RAS in part of simulation results.

References

1. F. Aieta, P. Genevet, M. Kats, F. Capasso, Aberrations of flat lenses and aplanatic metasurfaces. Opt. Exp. **21**, 31530–31539 (2013)
2. P.J. Sands, Inhomogeneous lenses, III. Paraxial optics. J. Opt. Soc. Am. **61**, 1436–1443 (1971)
3. P.J. Sands, Third order aberrations of inhomogeneous lenses. J. Opt. Soc. Am. **60**, 1436–1443 (1970)
4. A. Naresh Kumar Reddy, M. Hashemi, S.N. Khonina, Apodzation of two-dimensional pupils with aberrations. Pramana J. Phys. **90**(6), 77 (2018)
5. A. Naresh Kumar Reddy, D. Karuna Sagar, Spherical aberration of the point spread function with asymmetric pupil mask. Adv. Opt. Technol. **2016**, 1608342 (2016)
6. L.N. Hazra, P.K. Purkait, M. De, Apodization of aberrated pupils. Can. J. Phys. **57**, 1340–1346 (1979)
7. D. Karuna Sagar, G. Bikshamaiah, M. Keshavulu Goud, S. Lacha, Goud, Defect of focus in two-line resolution with Hanning amplitude filters. J. Mod. Opt. **53**(14), 2011–2019 (2006)
8. A. Naresh Kumar Reddy, D. Karuna Sagar, S.N. Khonina, Complex pupil masks for aberrated imaging of closely spaced objects. Opt. Spectrosc. **123**(6), 940–949 (2017)
9. J.P. Mills, B.J. Thompson, *Selected Papers on Apodization: Coherent Optical Systems, SPIE Milestone Series*, vol. MS 119 (SPIE Optical Engineering Press, Ellingham, 1996)
10. M. Martinez-Corral, M.T. Caballero, E.H.K. Stelzer, J. Swoger, Tailoring the axial shape of the point spread function using Toraldo concept. Opt. Exp. **10**(1), 98–103 (2002)
11. N. Reza, L.N. Hazra, Toraldo filters with concentric unequal annuli of fixed phase by evolutionary programming. J. Opt. Soc. Am. A .**30**(2), 189–195 (2012)
12. L. Cheng, G.G. Siu, Asymmetric apodization. Meas. Sci. Technol. **2**(3), 198–202 (1991)
13. G.G. Siu, L. Cheng, D.S. Chiu, Improved side-lobe suppression in asymmetric apodization. J. Phys. D Appl. Phys. **27**(3), 459–463 (1994)
14. M. Kowalczyk, C.J. Zapata-Rodriguez, M. Martinez-Corral, Asymmetric apodization in confocal scanning systems. Appl. Opt. **37**(35), 8206–8214 (1998)
15. V.P. Nayar, N.K. Sharma, Two-point resolution of Gaussian aperture operating in partially coherent light using various resolution criteria. Appl. Opt. **17**(14), 2176–2180 (1978)
16. K. Yamamoto, Y. Ichioka, T. Suzuki, Performance of an apodized aperture in a defocused optical system under partially coherent illumination. Opt. Acta **23**(12), 965–986 (1976)
17. J.P. McGuire Jr., R.A. Chipman, Diffraction image formation in optical systems with polarization aberrations. II. Amplitude response matrices for rotationally symmetric systems. J. Opt. Soc. Am. A **8**(6), 833–841 (1991)
18. J.P. Mills, B.J. Thompson, Effect of aberrations and apodization on the performance of coherent optical systems I. The amplitude response. J. Opt. Soc. Am. A **3**(5), 694–703 (1986)
19. P. Richter, S. Tisza, The effect of lens aberrations on coherent optical filter. Opt. Acta **23**(12), 965–986 (1976)
20. G. Cesini, G. Guattari, P. De Santis, C. Palma, Two-point resolution with anti-phase coherent Illumination. I. One-dimensional systems. J. Opt. (Paris) **10**(2), 79–87 (1979)
21. S.M. Watson, J.P. Mills, S.K. Rogers, Two-point resolution criterion for multiaperture optical telescopes. J. Opt. Soc. Am. A **5**(6), 893–903 (1988)
22. L. Rayleigh, *Collected Papers*, vol. 3 (Cambridge University Press, London, 1902), p. 84
23. C.M. Sparrow, On spectroscopic resolving power. Astrophys. J. **44**, 76–86 (1916)
24. A. Naresh Kumar Reddy, D. Karuna, Sagar, Two-point resolution of asymmetrically apodized optical systems. Opt. Pura Apl. **46**(3), 215–222 (2013)
25. A. Naresh Kumar Reddy, S.N. Khonina, Two-point resolution with spherical aberration quadratic amplitude filters. Opt. Appl. **48**(4), 550–565 (2018)
26. R. Keshavulu, D. Sayanna, Karuna Sagar, S.L. Goud, Effect of defocusing on the sparrow limits for apodized optical systems. Opt. Commun. **217**, 59–67 (2003)
27. L. Wang, T. Tschudi, T. Haldorsson, P.R. Petursson, Speckle reduction in laser projection systems by diffractive optical elements. Appl. Opt. **37**(10), 1770–1775 (1998)