Appl Phys B (2011) 105:415–420
DOI 10.1007/s00340-011-4626-9

Applied Physics B
Lasers and Optics

Spectral changes in stochastic light beams propagating in turbulent ocean

E. Shchepakina · N. Farwell · O. Korotkova

Received: 7 January 2011 / Published online: 9 July 2011

Abstract The behavior of the spectral composition of a typical stochastic beam in a turbulent ocean environ is revealed. The analysis is based on the extended Huygens–Fresnel principle and the coherence theory in the space-frequency domain. The optical source is assumed to be of the Gaussian Schell-model type with a single narrow Gaussian spectral line in the visible region. Optical turbulence in the ocean is assumed to be driven by temperature and salinity fluctuations. It is found that the well-known source correlation-induced spectral shift is compensated by turbulence at sufficiently large distances.

1 Introduction

The analysis of light colors in oceanic waters was a subject of extensive investigations for several decades [1–6]. However the majority of such studies were devoted to the interaction of sun light with water and only explored the spectral changes due to absorption and scattering by molecules and particles. Also, in a few papers, the propagation of laser light in the ocean was explored [7–9] where the accent was made on decoherence effects due to scattering/absorption. In these cases, i.e. either for an unbounded sun light wave, with a very wide spectrum, or for a bounded but monochromatic laser beam, the spectrum cannot change due to source correlations. In order to have the modification in the spectrum due to source-correlations the generated radiation must remain highly directional, have a narrow initial spectrum, and be partially coherent (stochastic). Then such beams have the ability to modify the spectral composition on propagation, even in a vacuum as a matter of fact, [10]. Moreover, as was shown recently, spectrum of partially coherent beams may exhibit even more complex behavior in some random media, such as atmospheric turbulence or human tissues [11, 12].

Spectral composition of beam-like light fields is sometimes employed as a carrier of a signal or an image, whether for communication or remote sensing [13, 14]. It is well known that for information transfer in random media, e.g. turbulent atmosphere or ocean, it is often preferable to use stochastic beams rather than deterministic [15, 16]. In the case of stochastic beams the signal-to-noise ratio of the detected signal after transmission through a random medium can be controlled to some extent [17]. Recently the interest in active optical underwater communications, imaging and sensing appeared [18–22] and it has become important to investigate how oceanic turbulence affects spectra of optical stochastic beams. This is the main purpose of the present work.

In this publication we only consider the case of the clean-water oceanic turbulence, i.e. we assume that the light wave is not affected by suspended particles, air bubbles, etc. The optical turbulence, i.e. temporal and spatial random variations in the index of refraction will be the only mechanism affecting the beam on propagation. As is well known, the fluctuations in the index of refraction of the ocean waters are induced primarily by temperature and salinity fluctuations [23]. However, a tractable analytical model for a spatial power spectrum accounting for both factors, which is critical for light propagation analysis, has not appeared until recently [24]. With this model in hand it becomes pos-

E. Shchepakina
Department of Technical Cybernetics, Samara State Aerospace University, Molodogvardeiskaya 151, Samara 443001, Russia

N. Farwell · O. Korotkova (✉)
Department of Physics, University of Miami, 1320 Campo Sano Drive, Coral Gables, FL 33146, USA
e-mail: korotkova@physics.miami.edu

sible to predict analytically how beams with arbitrary spectral and intensity distributions as well as with any coherence and polarization properties pass through the turbulent ocean [20, 25]. All the parameters entering the model [24] can be directly measured [26].

2 Stochastic beam propagation through oceanic turbulence

We begin by reviewing basic formulas for calculation of spectral changes in stochastic beams first in any linear medium and then applied specifically for oceanic propagation. Assume that the beam is generated in the plane $z = 0$ by a stochastic, statistically stationary source whose fluctuations are characterized by means of the cross-spectral density function [27]

$$W^{(0)}(\mathbf{r}_1^0, \mathbf{r}_2^0; \omega) = \langle U^{(0)*}(\mathbf{r}_1^0; \omega) U^{(0)}(\mathbf{r}_2^0; \omega)\rangle. \tag{1}$$

Here $\mathbf{r}_1^0 = (x_1^0, y_1^0, 0)$ and $\mathbf{r}_2^0 = (x_2^0, y_2^0, 0)$ are two-dimensional position vectors of points in the source plane, $U^{(0)}(\mathbf{r}; \omega)$ is the monochromatic realization of the optical field at angular frequency ω, $*$ denotes complex conjugate, angular brackets stand for the statistical average in the sense of coherence theory in the space-frequency domain [27]. Using the relation $\omega = c\lambda/2\pi$, where c is the speed of light in vacuum, λ is its wavelength, we will rewrite the cross-spectral density function (1) as

$$\mathcal{W}^{(0)}(\mathbf{r}_1^0, \mathbf{r}_2^0; \lambda) = \langle U^{(0)*}(\mathbf{r}_1^0; \lambda) U^{(0)}(\mathbf{r}_2^0; \lambda)\rangle. \tag{2}$$

Then the initial spectral density

$$\mathcal{S}^{(0)}(\mathbf{r}^0; \lambda) = \mathcal{W}^{(0)}(\mathbf{r}^0, \mathbf{r}^0; \lambda)$$

analyzes the dependence on wavelength rather than on angular frequency. Suppose that a beam-like field is generated by source (2) and propagates into positive half-space $z > 0$ filled with turbulent water column.

It was shown in [27] that upon propagation in any linear medium, which may be random, the optical field $U(\mathbf{r}; \lambda)$ is a solution of a Helmholtz equation of the form

$$\nabla^2 U(\mathbf{r}; \lambda) + k^2 n^2(\mathbf{r}) U(\mathbf{r}; \lambda) = 0,$$

where $k = 2\pi/\lambda$ is the wave number of light, and, $n(\mathbf{r})$ is the distribution of the index of refraction.

Upon propagation from the source plane to any plane with $z > 0$ the cross-spectral density function takes the form

$$\mathcal{W}(\mathbf{r}_1, \mathbf{r}_2; \lambda) = \int\int\int\int \mathcal{W}^0(\mathbf{r}_1^0, \mathbf{r}_2^0; \lambda) \mathcal{K}(\mathbf{r}_1^0, \mathbf{r}_2^0, \mathbf{r}_1, \mathbf{r}_2; \lambda)\, d^2\mathbf{r}_1^0\, d^2\mathbf{r}_2^0,$$

where $\mathcal{K}$ is the propagator, being the correlation of the Green's functions $\mathcal{G}(\mathbf{r}^0, \mathbf{r}; \lambda)$ of the medium, i.e.

$$\mathcal{K}(\mathbf{r}_1^0, \mathbf{r}_2^0, \mathbf{r}_1, \mathbf{r}_2; \lambda) = \langle \mathcal{G}^*(\mathbf{r}_1^0, \mathbf{r}_1; \lambda) \mathcal{G}(\mathbf{r}_2^0, \mathbf{r}_2; \lambda)\rangle_m.$$

Here angular brackets with subscript m stand for the ensemble average of the realizations of the fluctuating medium. If the beam travels in random medium then the propagator generally takes the form [28]

$$\begin{aligned}\mathcal{K}(\mathbf{r}_1^0, \mathbf{r}_2^0, \mathbf{r}_1, \mathbf{r}_2; \lambda) &= \left(\frac{1}{z\lambda}\right)^2 \exp\left[-i\pi \frac{(\mathbf{r}_1 - \mathbf{r}_1^0)^2 - (\mathbf{r}_2 - \mathbf{r}_2^0)^2}{z\lambda}\right] \\ &\quad \times \langle \exp[\psi^*(\mathbf{r}_1^0, \mathbf{r}_1; \lambda) + \psi(\mathbf{r}_2^0, \mathbf{r}_2; \lambda)]\rangle_m. \end{aligned} \tag{3}$$

In expression (3) the terms in the first line describe the effect of the free-space diffraction on the beam, and those in the second line include the perturbation of the complex phase $\psi(\mathbf{r}^0, \mathbf{r}, \lambda)$ caused by the refractive-index fluctuations of random medium between points $\mathbf{r}^0$ and $\mathbf{r}$ at wavelength λ. It was shown that if the source fluctuations are much stronger than those of random medium, which we assume to be the case in our study, then the phase term of (3) takes the form [29, 30]

$$\begin{aligned}&\langle \exp[\psi^*(\mathbf{r}_1^0, \mathbf{r}_1; \lambda) + \psi(\mathbf{r}_2^0, \mathbf{r}_2; \lambda)]\rangle_m \\ &\quad = \exp\left\{-\frac{4\pi^4 z}{3\lambda^2}\left[(\mathbf{r}_1 - \mathbf{r}_2)^2 + (\mathbf{r}_1 - \mathbf{r}_2)(\mathbf{r}_1^0 - \mathbf{r}_2^0) \right.\right. \\ &\qquad \left.\left. + (\mathbf{r}_1^0 - \mathbf{r}_2^0)^2\right] \int_0^\infty \kappa^3 \Phi_n(\kappa)\, d\kappa\right\},\end{aligned}$$

where $\Phi_n(\kappa)$ is the spatial power spectrum of refractive-index fluctuations [28].

In our analysis we will employ the model developed in [24] for clear-water oceanic turbulence which combines effects of temperature and salinity fluctuations in the water column. A particular case is considered here, when the eddy thermal diffusivity and the diffusion of the salt are equal. Then

$$\begin{aligned}\Phi_n(\kappa) &= 0.388 \times 10^{-8} \varepsilon^{-1/3} \kappa^{-11/3} \\ &\quad \times \left[1 + 2.35(\kappa\eta)^{2/3}\right] f(\kappa, w, \chi_T),\end{aligned} \tag{4}$$

where ε is the rate of dissipation of turbulent kinetic energy per unit mass of fluid which may vary in range from 10^{-1} m^2/s^3 to 10^{-10} m^2/s^3 (cf. [6], p. 25), $\eta = 10^{-3}$ m being the Kolmogorov micro scale (inner scale), and

$$f(\kappa, w, \chi_T) = \frac{\chi_T}{w^2}\left(w^2 e^{-A_T\delta} + e^{-A_S\delta} - 2w e^{-A_{TS}\delta}\right)$$

with χ_T being the rate of dissipation of mean-square temperature taking values in the range from 10^{-4} K^2/s to

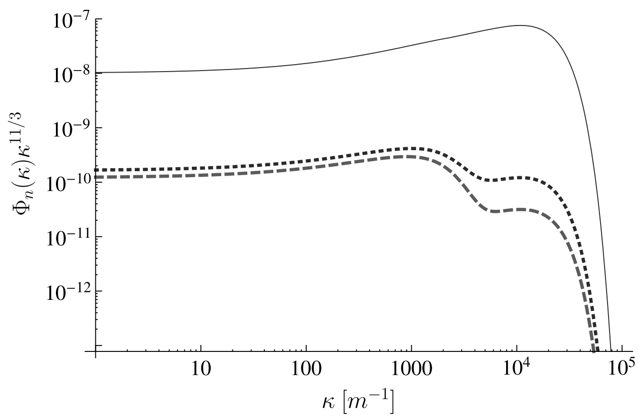

Fig. 1 Log–log plot of the oceanic power spectrum $\Phi_n(\kappa)$, calculated from (4) and normalized by the Kolmogorov power-law $\kappa^{-11/3}$, for $w=-0.1$ (*solid curve*), $w=-2.5$ (*dotted curve*), $w=-4.9$ (*dashed curve*)

10^{-10} K^2/s (cf. [6], p. 26), $A_T = 1.863\times10^{-2}$, $A_S = 1.9\times10^{-4}$, $A_{TS} = 9.41\times10^{-3}$, and $\delta = 8.284(\kappa\eta)^{4/3} + 12.978(\kappa\eta)^2$, w (non-dimensional) being the relative strength of temperature and salinity fluctuations, which in the ocean waters can vary in the interval $[-5; 0]$, attaining the lower bound for the maximum temperature-induced optical turbulence.

While parameters χ_T and ε primarily influence the height of the spectrum, the balance parameter w affects its shape. To illustrate typical dependence on w we show in Fig. 1 the spectrum (4) as a function of wave number κ, for several values of w.

3 Spectral changes in Gaussian Schell-model beams in turbulent ocean

In order to illustrate the dependence of the spectral changes on the parameters numerically, we will employ the isotropic Gaussian Schell-model beams [27]. The cross-spectral density matrix of such a beam in the source plane $z=0$ has the form

$$\mathcal{W}^{(0)}\big(\mathbf{r}_1^0,\mathbf{r}_2^0;\lambda\big) = \mathcal{I}_0(\lambda)\exp\left[-\frac{(\mathbf{r}_1^0)^2+(\mathbf{r}_2^0)^2}{4\sigma^2}\right] \times\exp\left[-\frac{(\mathbf{r}_1^0-\mathbf{r}_2^0)^2}{2\delta^2}\right]. \tag{5}$$

The parameters σ and δ characterize the rms source radius and the rms width of the spectral degree of coherence of the source, respectively. $\mathcal{I}_0$ is the initial spectral composition consisting of a single Gaussian spectral line, i.e.

$$\mathcal{I}_0(\lambda) = \exp\big[-(\lambda-\lambda_0)^2/\big(2\Lambda^2\big)\big],$$

with a peak value of one, being centered at wavelength λ_0 and having r.m.s. width Λ. In order for a field to possess a beam-like structure the following inequality must be satisfied [27]:

$$\frac{1}{4\sigma^2}+\frac{1}{\delta^2} \ll \frac{2\pi^2}{\lambda^2}.$$

On substituting the expression for the cross-spectral density source (5) and for the propagator the expression for the cross-spectral density function $\mathcal{W}(\mathbf{r}_1,\mathbf{r}_2;\lambda)$ can be derived (see [30], formula (B.12)):

$$\begin{aligned}\mathcal{W}(\mathbf{r}_1,\mathbf{r}_2;\lambda) &= \frac{\mathcal{I}_0}{\Delta^2(z)}\exp\left(-\frac{(\mathbf{r}_1+\mathbf{r}_2)^2}{8\sigma^2\Delta^2(z)}\right)\\ &\times\exp\left(-\left[\frac{1}{2\Delta^2(z)}\left(\frac{1}{4\sigma^2}+\frac{1}{\delta^2}\right)+M\big(1+\sigma^2\big)\right.\right.\\ &\left.\left.-\frac{M^2z^2\lambda^2}{8\pi^2\sigma^2\Delta^2(z)}\right](\mathbf{r}_1+\mathbf{r}_2)^2\right)\\ &\times\exp\left(\frac{i\pi(\mathbf{r}_2^2-\mathbf{r}_1^2)}{\lambda R(z)}\right)\end{aligned}$$

with

$$\Delta^2(z) = 1+\left(\frac{z\lambda}{2\pi\sigma}\right)^2\left(\frac{1}{4\sigma^2}+\frac{1}{\delta^2}\right)+\frac{Mz^2\lambda^2}{2\pi^2\sigma^2},$$

$$M = \frac{4\pi^4 z}{3\lambda^2}\int_0^\infty \kappa^3\Phi_n(\kappa)\,d\kappa,$$

$$R(z) = \frac{4\pi^2\sigma^2\Delta^2(z)z}{4\pi^2\sigma^2\Delta^2(z)+Mz^2\lambda^2-4\pi^2\sigma^2}.$$

Then

$$\mathcal{S}(\mathbf{r},\lambda) = \frac{\mathcal{I}_0(\lambda)}{\Delta^2(z)}\exp\left[-\frac{\mathbf{r}^2}{2\sigma^2\Delta^2(z)}\right].$$

In what follows we will be interested in evaluation of the normalized spectral density of the beam at distance $z\ge0$ from the source plane and at any transverse location (x,y), given by the expression [27]

$$\mathcal{S}_N(\mathbf{r};\lambda) = \mathcal{S}(\mathbf{r};\lambda)\bigg/\int_0^\infty \mathcal{S}(\mathbf{r};\lambda)\,d\lambda,$$

where $\mathcal{S}(\mathbf{r};\lambda) = \mathcal{W}(\mathbf{r},\mathbf{r};\lambda)$ is the spectral density of the field at position $\mathbf{r}=(x,y,z)$. Further, the shifted central frequently of the beam can be found from the expression [31]

$$\lambda_1(\mathbf{r}) = \int_0^\infty \lambda\mathcal{S}(\mathbf{r};\lambda)\,d\lambda\bigg/\int_0^\infty \mathcal{S}(\mathbf{r};\lambda)\,d\lambda. \tag{6}$$

The normalized spectral shift at position $\mathbf{r}$ may be quantified by $\varrho(\mathbf{r}) = \frac{\lambda_1(\mathbf{r})-\lambda_0}{\lambda_0}$ being *blue* if its value is positive and *red*

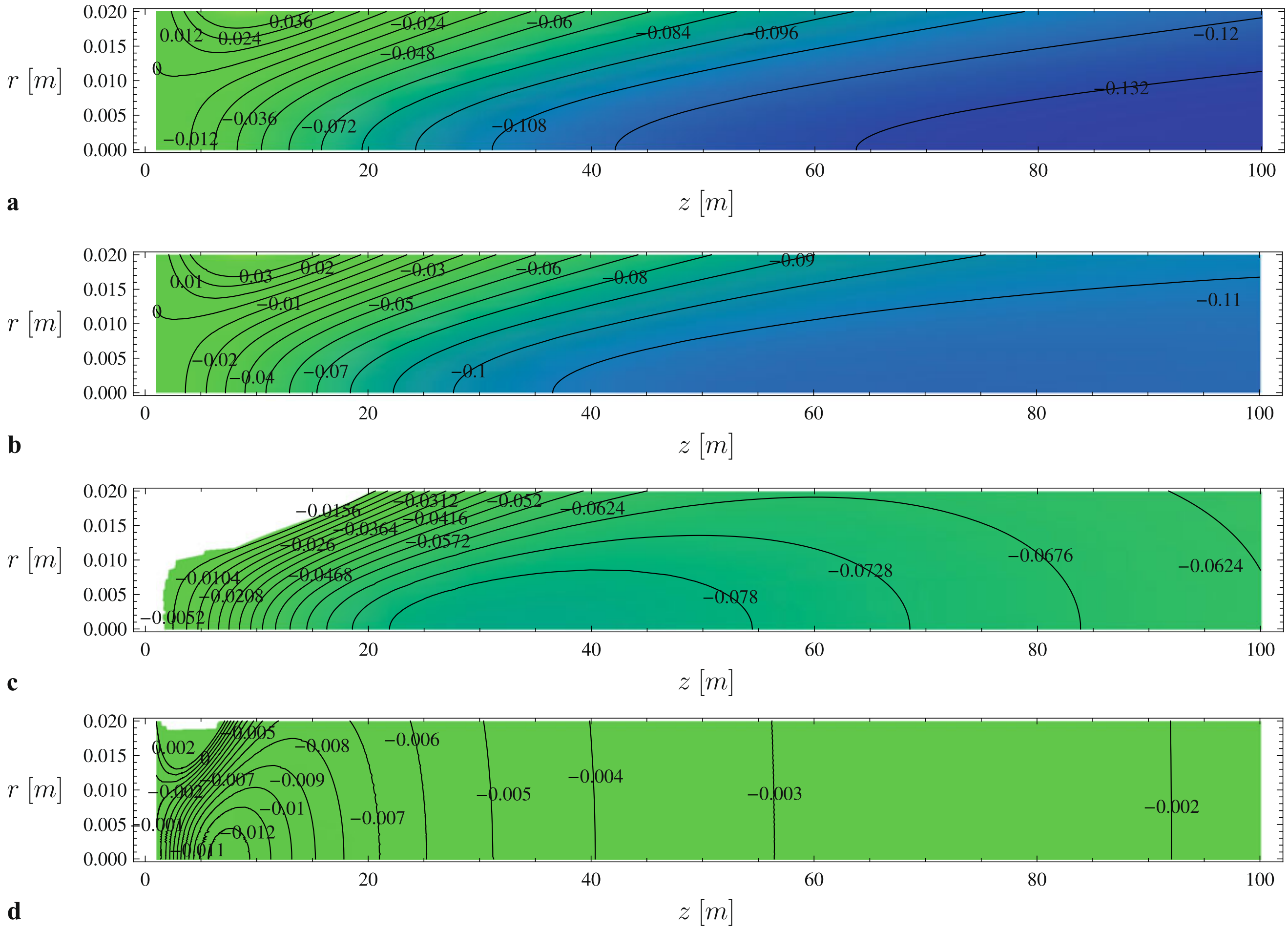

Fig. 2 Density plots of actual spectral shift λ_1 overlapped with contour plots of normalized spectral shift $\varrho = \frac{\lambda_1 - \lambda_0}{\lambda_0}$ as a function of z (*horizontal axis*, in meters) and r (*vertical axis*, in meters) for (**a**) $\chi_T = 10^{-10}$ K^2/s; (**b**) $\chi_T = 10^{-5}$ K^2/s, (**c**) $\chi_T = 10^{-4}$ K^2/s; (**d**) $\chi_T = 10^{-2}$ K^2/s; $\varepsilon = 10^{-4}$ $\mathrm{m}^2/\mathrm{s}^3$, $w = -4.5$

if it is negative. In (6) λ_0 is the central wavelength of the source, which we assume to be position-independent.

We will use the following parameters for the source, unless other parameters are specified in the figure captions: $\lambda_0 = 0.5435 \times 10^{-6}$ m, $\Lambda = \lambda_0/6$; $\sigma = 10^{-2}$ m; $\delta = 10^{-4}$ m.

4 Results

We will now investigate by means of colored density plots, the influence of several parameters of oceanic waters on spectral composition of propagating Gaussian Schell-model beams. In Fig. 2 the dependence of central wavelength and relative spectral shifts on the position of a point in the propagating beam is shown. In particular, the colored density plots provide the optical color corresponding to the central wavelength λ_1 and the contours correspond to level curves of the relative shift σ. Here four cases, (a)–(d), are considered in which different values of the dissipation rate of the mean-square temperature χ_T are assumed, from $\chi_T = 10^{-10}$ K^2/s, corresponding to a very weak turbulence (virtually vacuum) to $\chi_T = 10^{-2}$ K^2/s, which is associated with a fairly strong fluctuations. We included the numerical examples for the cases when χ is in the range 10^{-4} K^2/s to 10^{-2} K^2/s because the experimental data can easily be obtained with the error of two orders of magnitude ([6], p. 179). Parameter χ_T has the strongest influence on the beam, having similar meaning to the refractive-index structure parameter C_n^2 used in the atmospheric turbulence. We find from Fig. 2 that unlike for free-space propagation [see Fig. 2(a)] where the spectrum of the beam undergoes a blue shift (more so on the optical axis) for larger values of χ_T such shift is being suppressed by turbulence, leading to the spectral switch.

From Figs. 2(b)–2(d) we see that for larger values of χ_T (high local fluctuations of the refractive index) the switch moves toward the source and can occur within the first ten meters of propagation. In Fig. 2(d) the turbulence is so

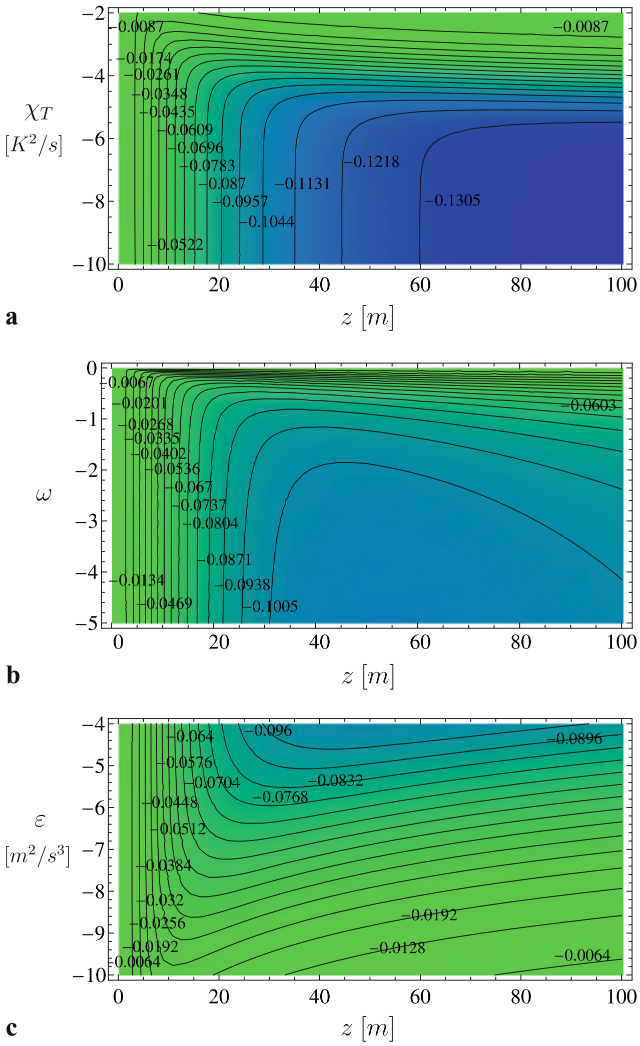

Fig. 3 Density plots of actual spectral shift λ_1 overlapped with contour plots of normalized spectral shift $\varrho = \frac{\lambda_1-\lambda_0}{\lambda_0}$ as a function of z (*horizontal axis*, in meters) and (**a**) χ_T, on a log scale for $\varepsilon = 10^{-4}$ m^2/s^3, $w = -2.5$, (**b**) w for $\varepsilon = 10^{-4}$ m^2/s^3, $\chi_T = 10^{-4.5}$ K^2/s, (**c**) ε, on a log scale (*vertical axis*) for $\chi_T = 10^{-4.5}$ K^2/s, $w = -2.5$; $r = 0$

strong that practically no change in central wavelength occurs, since the source-induced spectral shift is effectively mitigated very close to the source.

Figure 3 shows evolution of the central wavelength λ_1 (density plot) and relative wavelength shift σ (contours) on the optical axis ($\mathbf{r} = 0$) as a function propagation distance z from the source, and three major parameters of oceanic turbulence: (a) temperature mean-square dissipation rate χ_T; (b) temperature–salinity balance parameter w and (c) energy dissipation rate ε. Figure 3(a) suggests that the transition values for χ_T at which the turbulence is capable of suppressing the source correlation-induced spectral change is on the order of 10^{-5} K^2/s. On the other hand, we see from Fig. 3(b) that temperature–salinity parameter w must be fairly large (close to zero) in order to mitigate initially induced spectral change. This implies that salinity-induced optical turbulence must be minimal for fast reconstruction of the initial spectrum. Finally, Fig. 3(c) shows that the spectral changes are also sensitive to the kinetic energy dissipation rate ε, perhaps in a lesser degree: for given choice of χ_T and w the spectral shifts can only be reduced regardless of the value of ε.

5 Summary

In conclusion we have investigated the evolution of spectra of Gaussian Schell-model beams with initial Gaussian spectral profile on propagation in turbulent ocean. Under the assumptions of a strongly fluctuating source [29] and the temperature–salinity ocean spectrum model [24] we found that with propagation distance the spectral composition, which originally modifies due to the source correlations, can self-reconstruct after relatively short distances, on the order of tens of meters. Similar effect was recently predicted for the same class of beams on propagation on atmospheric turbulence [11], however for much longer range, on the order of tens of kilometers. We also note that the reconstruction effect for the beam spectrum in the ocean is similar to that for polarization properties [25]. The origin of such a recovery is in the competition of source and turbulence correlations: while the former modifies the spectrum at some short propagation range, the atter affects it more at larger propagation distances.

Our results are of importance for optical underwater imaging and communications, in situations where spectral encoding and spectral diversity techniques are employed.

Acknowledgements O. Korotkova's research is funded by the US AFOSR (grant FA 95500810102) and US ONR (grant N0016111RCZ0604). E. Shchepakina is supported by the Russian Foundation for Basic Research (grant 10-08-00154a).

References

1. N.G. Jerlov, *Marine Optics* (Elsevier, Amsterdam, 1976)
2. C. Mobley, *Light and Water* (Academic Press, New York, 1994)
3. K.S. Shifrin, *Physical Optics of Ocean Water* (American Institute of Physics, New York, 1988)
4. R.W. Spinrad, K.L. Carder, M.J. Perry (eds.), *Ocean Optics* (Oxford University Press, Oxford, 1994)
5. R.E. Walker, *Marine Light Field Statistics* (Wiley, New York, 1984)
6. S.A. Thorpe, *The Turbulent Ocean* (Cambridge University Press, Cambridge, 2005)
7. N. Swanson, Proc. SPIE **1750**, 397 (1992)
8. K. Arora, E.O. Sheybani, Microw. Opt. Techol. Lett. **6**, 151 (1993)
9. A. Perennou, J. Cariou, J. Lotrian, Pure Appl. Opt. **4**, 617 (1995)
10. E. Wolf, Phys. Rev. Lett. **56**, 1370 (1986)
11. O. Korotkova, E. Shchepakina, Opt. Lett. **35**, 3772 (2010)
12. W. Gao, Opt. Lett. **35**, 862 (2010)

13. J.D. Gibson (ed.), *The Communications Handbook* (CRC Press, Boca Raton, 2002)
14. J.B. Campbell, *Introduction to Remote Sensing*, 4th edn. (Taylor and Francis, London, 2007)
15. G. Gbur, E. Wolf, J. Opt. Soc. Am. A **19**, 1592 (2002)
16. Z. Tong, S. Sahin, O. Korotkova, Opt. Commun. **283**, 4512 (2010)
17. O. Korotkova, L.C. Andrews, R.L. Phillips, Opt. Eng. **43**, 330 (2004)
18. J.B. Snow, J.P. Flatley, D.E. Freeman, M.A. Landry, C.E. Lindstrom, J.R. Longacre, J.A. Schwartz, Proc. SPIE **1750**, 419 (1992)
19. S. Arnon, D. Kedar, J. Opt. Soc. Am. A **26**, 530 (2009)
20. W. Lu, L. Liu, J. Sun, J. Opt. A, Pure Appl. Opt. **8**, 1052 (2006)
21. W. Hou, Opt. Lett. **34**, 2688 (2009)
22. F. Hanson, M. Lasher, Appl. Opt. **49**, 3224 (2010)
23. R.J. Hill, J. Opt. Soc. Am. **68**, 1067 (1978)
24. V.V. Nikishov, V.I. Nikishov, Int. J. Fluid Mech. Res. **27**, 82 (2000)
25. O. Korotkova, N. Farwell, Opt. Commun. **284**, 1740 (2011)
26. M. Alford, D. Gerdt, C. Adkins, J. Atmos. Ocean. Technol. **23**, 121 (2006)
27. L. Mandel, E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995)
28. L.C. Andrews, R.L. Phillips, *Laser Beam Propagation in Random Media*, 2nd edn. (SPIE, Bellingham, 2005)
29. T. Shirai, A. Dogariu, E. Wolf, J. Opt. Soc. Am. A **20**, 1094 (2003)
30. W. Lu, L. Liu, J. Sun, Q. Yang, Y. Zhu, Opt. Commun. **271**, 1 (2007)
31. O. Korotkova, J. Pu, E. Wolf, J. Mod. Opt. **55**, 1199 (2008)