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Propagation properties of apertured laser beams with amplitude modulations and phase fluctuations through atmospheric turbulence

X. Ji · X. Li

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Abstract The propagation properties of apertured laser beams with amplitude modulations (AMs) and phase fluctuations (PFs) through atmospheric turbulence are studied in detail both analytically and numerically. The analytical expressions for the average intensity, power in the bucket (*PIB*) and Strehl ratio (S_R) of apertured laser beams with AMs and PFs propagating through atmospheric turbulence are derived. It is found that the worse the phase fluctuation and the higher the amplitude modulation are, the less laser beams are affected by turbulence. Furthermore, apertured Gaussian beams are more sensitive to turbulence than apertured laser beams with AMs and PFs. The average intensity of apertured laser beams with AMs and PFs may be even larger than that of apertured Gaussian beams due to turbulence. In particular, the influence of turbulence on the average maximum intensity of apertured laser beams with PFs and AMs may become serious if an unsuitable truncated parameter is chosen, which should be avoided in practice.

1 Introduction

In many practical cases, for instance, of high-power lasers propagating through large Nd glass laser facility, the gain non-uniformity, gain saturation, aperture diffraction and nonlinear optical effects inevitably result in beam distortions on both amplitude and phase. As pointed out in Ref. [1], such high-power laser beams can be analyzed by using statistical optics methods. Comparisons with experimental results obtained from Novette (Novette is a large solid-state laser system at the Lawrence Livermore National Laboratory) and with large nonlinear propagation codes such as MALAPROP [2] (MALAPROP is the comprehensive, ab initio computer simulation code) simulations reveal that statistical optics methods can provide reliable estimates [1]. The propagation and focusing of laser beams with amplitude modulations (AMs) and phase fluctuations (PFs) have been studied in Ref. [3]. An approximate analytical study of laser beams with AMs and PFs through a multi-apertured ABCD system has been performed in Ref. [4]. On the other hand, it is very important to study the propagation of laser beams through atmospheric turbulence for many practical applications such as the remote sensing and atmospheric optical communication, etc. [5–7]. Recently, much work has been carried out concerning the influence of turbulence on the intensity distribution, the beam width, the angular spread, the polarization, the coherence, the spectrum, the M^2-factor, radius of curvature and the scintillation index of different laser beams [8–20]. To the best of our knowledge, propagation properties of apertured laser beams with AMs and PFs through atmospheric turbulence have not been examined until now. The aim of this paper is to study the influence of turbulence on propagation properties of apertured laser beams with AMs and PFs, where the average intensity, power in the bucket (*PIB*) and Strehl ratio (S_R) are taken as the characteristic parameters.

2 Average intensity

According to Ref. [1], in the space-time domain the laser beam with AMs and PFs is characterized by the mutual intensity $J\ (x_1', x_2', z=0)$ in the two-dimensional case of the form

X. Ji (✉) · X. Li
Department of Physics, Sichuan Normal University, Chengdu 610068, China
e-mail: jiXL100@163.com

$$J(x_1', x_2', z=0) = I \exp\left[-\frac{(x_1'-x_2')^2\sigma_P^2}{L_P^2}\right] + \sigma_A^2 \exp\left[-\left(\frac{1}{L_A^2}+\frac{\sigma_P^2}{L_P^2}\right)(x_1'-x_2')^2\right], \quad (1)$$

where L_A and L_P are the scale of the AMs and PFs, σ_A^2 and σ_P^2 measure the strengths of the intensity modulations and the amplitude of the phase error, respectively. Usually I is much larger than the noise intensity σ_A^2, and has a Gaussian profile, i.e.,

$$I = \exp\left(-\frac{x_1'^2 + x_2'^2}{w_0^2}\right), \quad (2)$$

with w_0 being the waist width.

It is noted that (1) is valid under two key assumptions, i.e., the first is that the mutual intensity be isoplanatic, and the second is that the laser field be treated as a Gaussian random phasor [1].

For $x_1' = x_2' = x'$, (1) reduces to the intensity in the plane $z = 0$, i.e., $I(x', z=0) = \exp(-2x'^2/w_0^2) + \sigma_A^2$. It is clear that the intensity in the plane $z = 0$ turns into Gaussian exponential raised by an amount σ_A^2, and is independent of PFs. Figure 1 gives intensity distributions $I(x', z=0)$ of laser beams with AMs and PFs in the plane $z = 0$, which indicates that $I(x', z=0)$ is not a Gaussian profile if $\sigma_A^2 \neq 0$.

Assume that the laser beam with AMs and PFs is incident upon a slit with full width $2a$ oriented along the x axis in

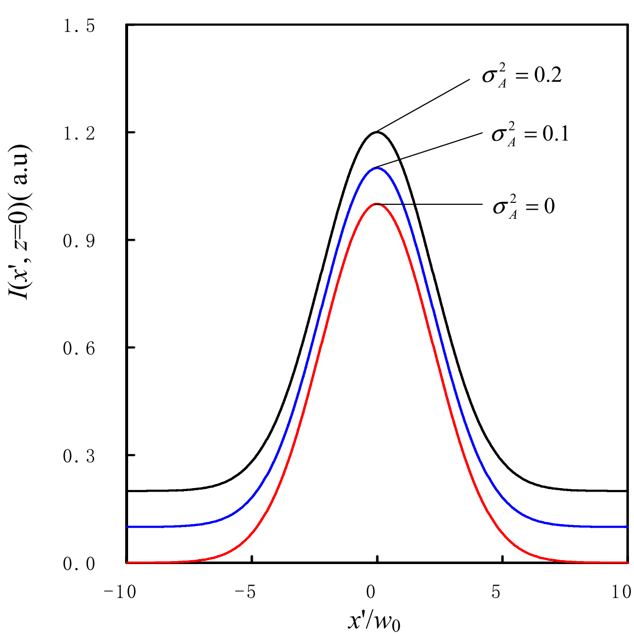

Fig. 1 Intensity distributions $I(x', z=0)$ of laser beams with AMs and PFs. $w_0 = 0.05$ m

the source plane $z = 0$. Based on the extended Huygens-Fresnel principle, the average intensity of the apertured laser beam with AMs and PFs propagating through atmospheric turbulence reads [6]

$$\langle I(x,z)\rangle = \frac{k}{2\pi z}\int_{-a}^{a}\int_{-a}^{a} dx_1'\, dx_2'\, J(x_1', x_2', z=0) \times \exp\left\{\left(\frac{ik}{2z}\right)\left[(x_1'^2 - x_2'^2) - 2(x_1'-x_2')x\right]\right\} \times \left\langle \exp\left[\psi(x_1', x) + \psi^*(x_2', x)\right]\right\rangle_m, \quad (3)$$

where k is the wave number related to the wave length λ by $k = 2\pi/\lambda$. $\psi(x', x)$ is the complex phase function that depends on the properties of the turbulence medium. $\langle\rangle$ denotes average over the field ensemble, while $\langle\rangle_m$ denotes average over the ensemble of the turbulent medium, and [21]

$$\left\langle \exp\left[\psi(x_1', x) + \psi^*(x_2', x)\right]\right\rangle_m \cong \exp\left[-\frac{(x_1'-x_2')^2}{\rho_0^2}\right], \quad (4)$$

where $\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ is the spatial coherence radius of a spherical wave propagating in turbulence, and C_n^2 is the refraction index structure constant which describes how strong the turbulence is.

The window function of the slit is described by the rectangular function of the form

$$T(x) = \begin{cases} 1 & |x| \le a \\ 0 & |x| > a \end{cases}. \quad (5)$$

$T(x)$ can be expanded into a finite sum of complex-valued Gaussian functions [22]

$$T(x) = \sum_{i=1}^{M} F_i \exp\left(-\frac{G_i x^2}{a^2}\right), \quad (6)$$

where the coefficients F_i, G_i and the number M are evaluated by a computation fitting given by Wen and Breazeale in Tab. I of Ref. [22] for $M = 10$, and are omitted here. According to Wen and Breazeale, the method of the finite expansion of the aperture function is applicable to the Fraunhofer and Fresnel regions except for the extreme near-field (<0.12 times the Fresnel distance) [22].

Bearing this point in mind, (3) can be rewritten as

$$\langle I(x,z)\rangle = \frac{k}{2\pi z}\sum_{i=1}^{M}\sum_{j=1}^{M} F_i F_j^* \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx_1'\, dx_2'\, J(x_1', x_2', z=0) \times \exp\left(-\frac{G_i x_1'^2 + G_j^* x_2'^2}{a^2}\right) \times \exp\left\{\left(\frac{ik}{2z}\right)\left[(x_1'^2 - x_2'^2) - 2(x_1'-x_2')x\right]\right\}$$

$$\times \left\langle \exp\left[\psi\left(x_1',x\right)+\psi^*\left(x_2',x\right)\right]\right\rangle_m. \tag{7}$$

On substituting from (1) and (4) into (7), and recalling the integral formula

$$\int_{-\infty}^{\infty}\exp\left(-C^2x^2+Dx\right)\mathrm{d}x=\frac{\sqrt{\pi}}{C}\exp\left(\frac{D^2}{4C^2}\right), \tag{8}$$

after very tedious but straightforward integral calculations we have

$$\begin{aligned}\langle I(x,z)\rangle &= \frac{k}{2z}\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{F_iF_j^*}{D_1}\exp\left(-\frac{k^2B_1}{4D_1^2z^2}x^2\right)\\ &\quad+\frac{\sigma_A^2k}{2z}\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{F_iF_j^*}{D_2}\exp\left(-\frac{k^2B_2}{4D_2^2z^2}x^2\right),\end{aligned} \tag{9}$$

where

$$\alpha=\frac{L_\mathrm{P}/w_0}{\sigma_\mathrm{P}},\qquad \beta=\frac{L_\mathrm{A}}{w_0},\qquad \delta=\frac{a}{w_0}, \tag{10}$$

$$B_1=\frac{G_i+G_j^*}{w_0^2\delta^2}+\frac{2}{w_0^2},\qquad B_2=\frac{G_i+G_j^*}{w_0^2\delta^2}, \tag{11}$$

$$\begin{aligned}D_1 &= \left[\frac{G_iG_j^*}{w_0^4\delta^4}+B_2\left(\frac{1}{w_0^2}+\frac{1}{w_0^2\alpha^2}+\frac{1}{\rho_0^2}\right)\right.\\ &\quad+\frac{\mathrm{i}k}{2z}\left(\frac{G_i-G_j^*}{w_0^2\delta^2}\right)+\frac{1}{w_0^4}+\frac{2}{w_0^2}\left(\frac{1}{w_0^2\alpha^2}+\frac{1}{\rho_0^2}\right)\\ &\quad\left.+\frac{k^2}{4z^2}\right]^{1/2},\end{aligned} \tag{12}$$

$$\begin{aligned}D_2 &= \left\{\frac{G_iG_j^*}{w_0^4\delta^4}+B_2\left[\frac{1}{w_0^2}\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)+\frac{1}{\rho_0^2}\right]\right.\\ &\quad\left.+\frac{\mathrm{i}k}{2z}\left(\frac{G_i-G_j^*}{w_0^2\delta^2}\right)+\frac{k^2}{4z^2}\right\}^{1/2}.\end{aligned} \tag{13}$$

Equation (9) together with (10)–(13) is the closed-form average intensity of the apertured laser beam with AMs and PFs in turbulence.

3 Power in the bucket (*PIB*) and Strehl ratio (S_R)

The *PIB* is a useful method for characterizing different laser beams and is essentially a measure of laser power focusability in the far field. The *PIB* clearly indicates how much fraction of the total beam power is within a given bucket, which is defined as [23]

$$PIB=\frac{\int_{-h}^{h}\langle I(x,z)\rangle\,\mathrm{d}x}{\int_{-\infty}^{\infty}\langle I(x,z)\rangle\,\mathrm{d}x}, \tag{14}$$

where h is the bucket half width chosen. The larger the *PIB* is, the higher the laser power focusability is.

The substitution from (9) into (14), the *PIB* of the apertured laser beam with AMs and PFs in turbulence is given by

$$PIB=\frac{\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*B_1^{-1/2}Erf\left(\frac{kB_1^{1/2}}{2D_1z}h\right)+\sigma_A^2\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*B_2^{-1/2}Erf\left(\frac{kB_2^{1/2}}{2D_2z}h\right)}{\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*B_1^{-1/2}+\sigma_A^2\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*B_2^{-1/2}}, \tag{15}$$

where *Erf* is the error function.

The Strehl ratio is defined as [24]

$$S_R=\frac{I_{\max}}{I_{0\max}}, \tag{16}$$

where $I_{\max}$, $I_{0\max}$ is the maximum intensity of a real beam and maximum intensity of a ideal beam, respectively. In this paper, the apertured laser beam with AMs and PFs in turbulence is taken as the real beam, and that in free space is taken as the ideal beam. The smaller value of S_R is, the more the maximum intensity is affected by turbulence.

From (9), it is clear that the maximum intensity is at the point $x=0$ both for atmospheric turbulence and free space cases. Letting $x=0$, and substituting from (9) into (16) yields the Strehl ratio of the apertured laser beam with AMs and PFs in turbulence, i.e.,

$$S_R=\frac{\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{F_iF_j^*}{D_1}+\sigma_A^2\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{F_iF_j^*}{D_2}}{\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{F_iF_j^*}{D_3}+\sigma_A^2\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{F_iF_j^*}{D_4}}, \tag{17}$$

where

$$\begin{aligned}D_3 &= \left[\frac{G_iG_j^*}{w_0^4\delta^4}+\frac{B_2}{w_0^2}\left(1+\frac{1}{\alpha^2}\right)+\frac{\mathrm{i}k}{2z}\left(\frac{G_i-G_j^*}{w_0^2\delta^2}\right)+\frac{1}{w_0^4}\right.\\ &\quad\left.+\frac{2}{w_0^4\alpha^2}+\frac{k^2}{4z^2}\right]^{1/2},\end{aligned} \tag{18}$$

$$D_4 = \left[\frac{G_i G_j^*}{w_0^4 \delta^4} + \frac{B_2}{w_0^2} \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} \right) + \frac{\mathrm{i}k}{2z} \left(\frac{G_i - G_j^*}{w_0^2 \delta^2} \right) + \frac{k^2}{4z^2} \right]^{1/2}. \tag{19}$$

Equations (9), (15) and (17) are expressions for the average intensity, *PIB* and S_R of the apertured laser beam with AMs and PFs propagating through atmospheric turbulence, which are main analytical results obtained in this paper. Some interesting results can be treated as special cases of (9), (15) and (17), e.g., (9), (15) and (17) reduce to the corresponding formulae of the apertured laser beam with only PFs when $\sigma_A = 0$, (9), (15) and (17) reduce to the corresponding formulae of the apertured laser beam with only AMs when $\alpha \to \infty$ (i.e., $\sigma_P = 0$), and (9), (15) and (17) reduce to the corresponding formulae of the unapertured laser beam with AMs and PFs when $\delta \to \infty$ (i.e., $a \to \infty$), which are all omitted here in order to save space

4 Numerical examples and analysis

Numerical calculations were performed to illustrate the influence of turbulence on the average axial intensity distributions $I(0, z)$, average transversal intensity distributions $I(x, z)$, *PIB* and S_R of apertured laser beams with AMs and PFs. In the following numerical examples (i.e., Figs. 2–11) $\lambda = 1.06$ µm and $w_0 = 0.05$ m are taken, where dashed curves and solid curves are for laser beams in free space and in turbulence, respectively.

The smaller α (i.e., the larger σ_P) means that the worse the phase fluctuation is. Figures 2, 3, 4 and 5 are for the case of apertured laser beams with only PFs. In free space (i.e., $C_n^2 = 0$), average intensity distributions are different for different values of α, and the average maximum intensity (see Figs. 2 and 3) and the *PIB* (see Fig. 4) increase with increasingα. However, in turbulence (i.e., $C_n^2 \neq 0$) the curves of the average intensity distribution for different val-

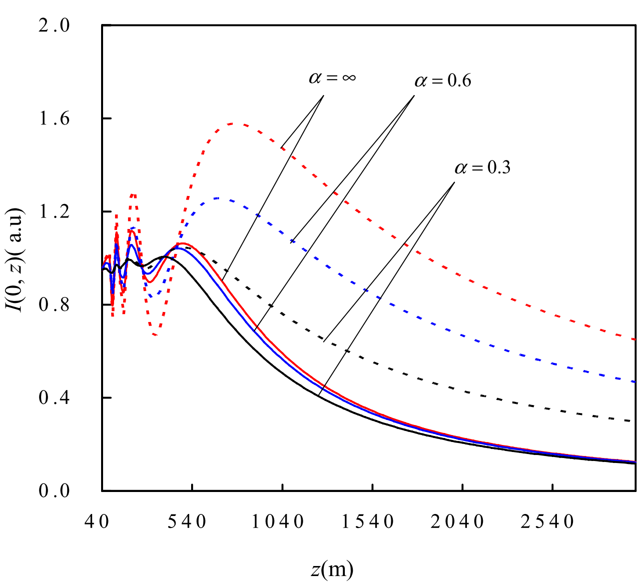

Fig. 2 Average axial intensity distributions $I(0, z)$ of apertured laser beams with only PFs. $\sigma_A = 0$, $\delta = 0.5$, "—" $C_n^2 = 10^{-13}$ m$^{-2/3}$, "- - -" $C_n^2 = 0$

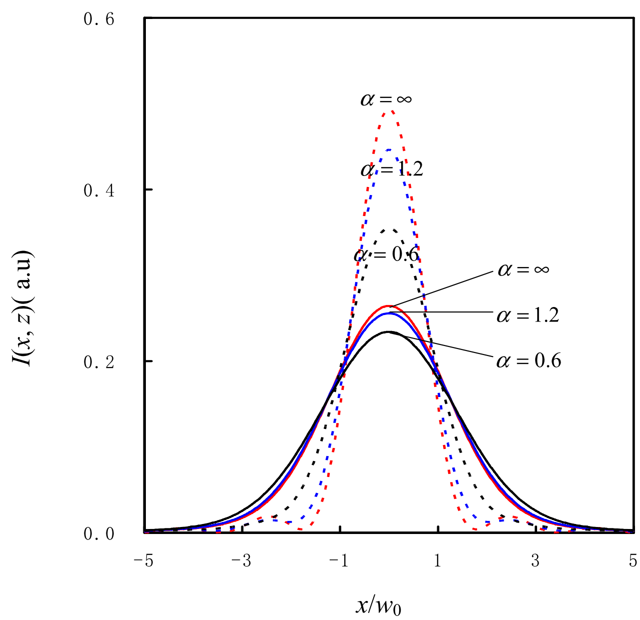

Fig. 3 Average transversal intensity distributions $I(x, z)$ of apertured laser beams with only PFs. $\sigma_A = 0$, $\delta = 0.5$, $z = 4$ km, "—" $C_n^2 = 10^{-14}$ m$^{-2/3}$, "- - -" $C_n^2 = 0$

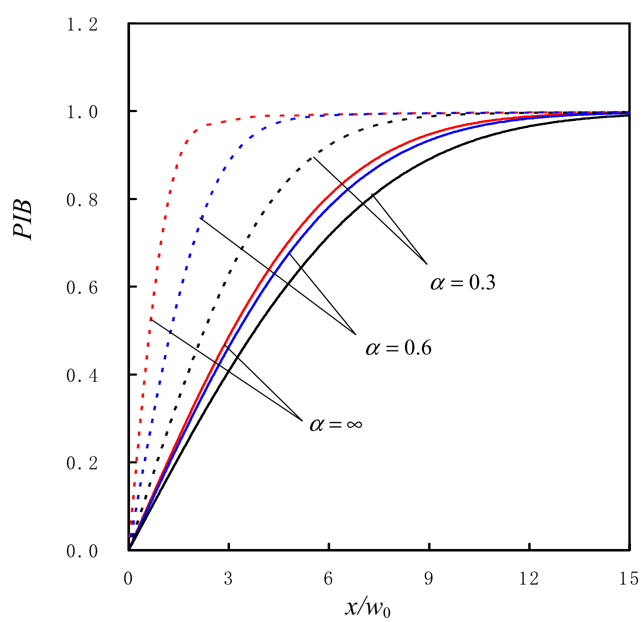

Fig. 4 *PIB* curves of apertured laser beams with only PFs. $\sigma_A = 0$, $z = 10$ km, $\delta = 1$, "—" $C_n^2 = 10^{-14}$ m$^{-2/3}$, "- - -" $C_n^2 = 0$

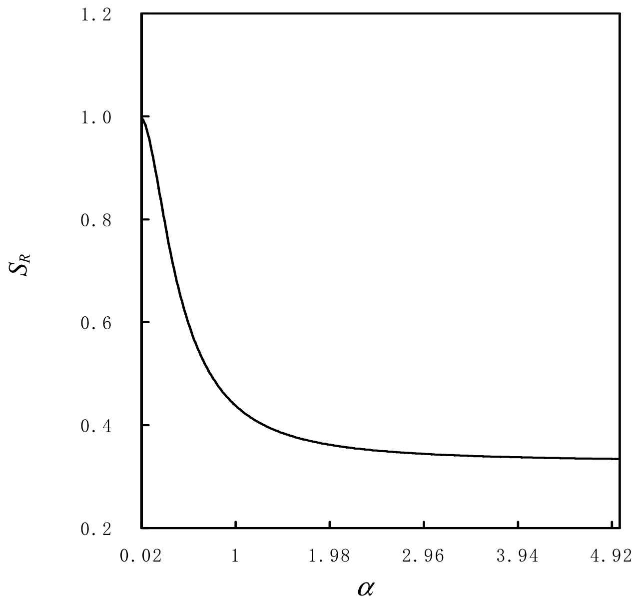

Fig. 5 Strehl ratio S_R of apertured laser beams with only PFs. $\sigma_A = 0$, $z = 10$ km, $\delta = 1$, $C_n^2 = 10^{-14}$ m$^{-2/3}$

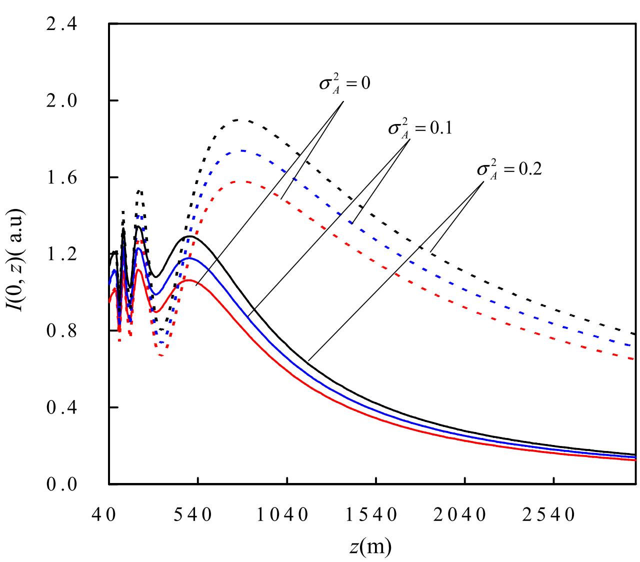

Fig. 6 Average axial intensity distributions $I(0, z)$ of apertured laser beams with only AMs. $\alpha \to \infty$ ($\sigma_P = 0$), $\delta = 0.5$, $\beta = 1$, "—" $C_n^2 = 10^{-13}$ m$^{-2/3}$, "- - -" $C_n^2 = 0$

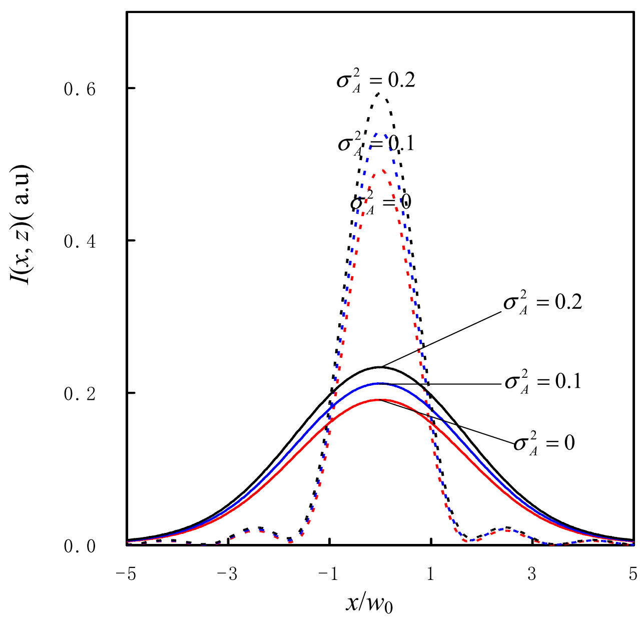

Fig. 7 Average transversal intensity distributions $I(x, z)$ of apertured laser beams with only AMs. $\alpha \to \infty$ ($\sigma_P = 0$), $\delta = 0.5$, $\beta = 1$, $z = 4$ km, "—" $C_n^2 = 2 \times 10^{-14}$ m$^{-2/3}$, "- - -" $C_n^2 = 0$

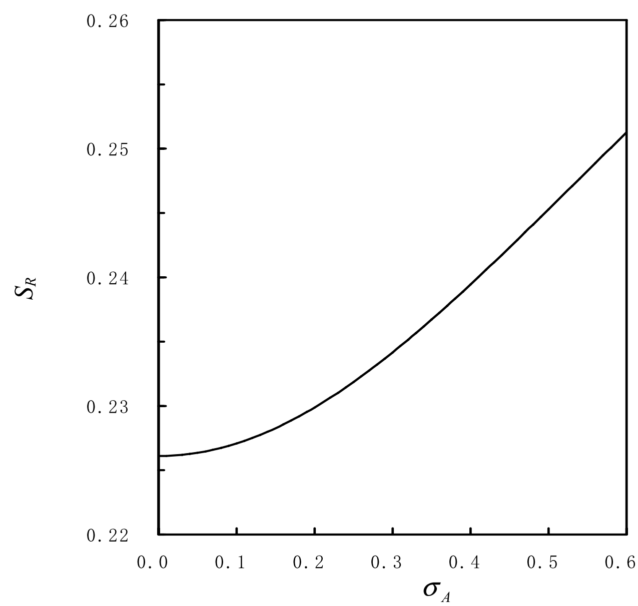

Fig. 8 Strehl ratio S_R of apertured laser beams with only AMs. $\alpha \to \infty$ ($\sigma_P = 0$), $z = 5$ km, $\delta = 1$, $\beta = 1$, $C_n^2 = 2 \times 10^{-14}$ m$^{-2/3}$

ues of α are close to each other (see Figs. 2 and 3), and the same behavior is for *PIB* curves (see Fig. 4). Furthermore, Fig. 3 shows that there are small lobes due to aperture in the average transversal intensity distribution in free space, but it becomes a Gaussian-like profile due to turbulence. In addition, Fig. 5 indicates that S_R decreases with increasing α. Therefore, the worse the phase fluctuation is, the less laser beams are affected by turbulence.

The larger σ_A implies that the higher the amplitude modulation is. Figures 6, 7 and 8 are for the case of apertured laser beams with only AMs. From Figs. 6, 7 and 8 it can be seen that the effects of turbulence on propagation properties of apertured laser beams with only AMs are similar to those of apertured laser beams with only PFs. The higher the amplitude modulation is, the less laser beams are affected by turbulence. On comparing the behaviors of the propagation property between apertured laser beams with only AMs and those with only PFs, we can see that apertured laser beams

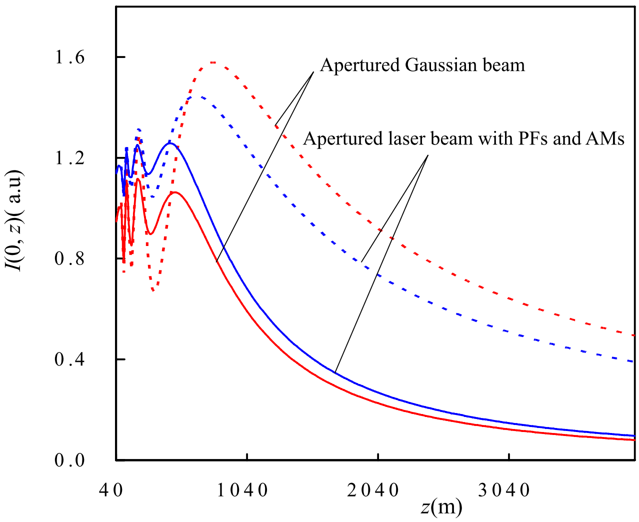

Fig. 9 Average axial intensity distributions $I(0,z)$ of apertured laser beams with both AMs and PFs. $\alpha=0.5$, $\delta=0.5$, $\beta=1$, $\sigma_A^2=0.2$, "—" $C_n^2=10^{-13}\ \mathrm{m}^{-2/3}$, "- - -" $C_n^2=0$

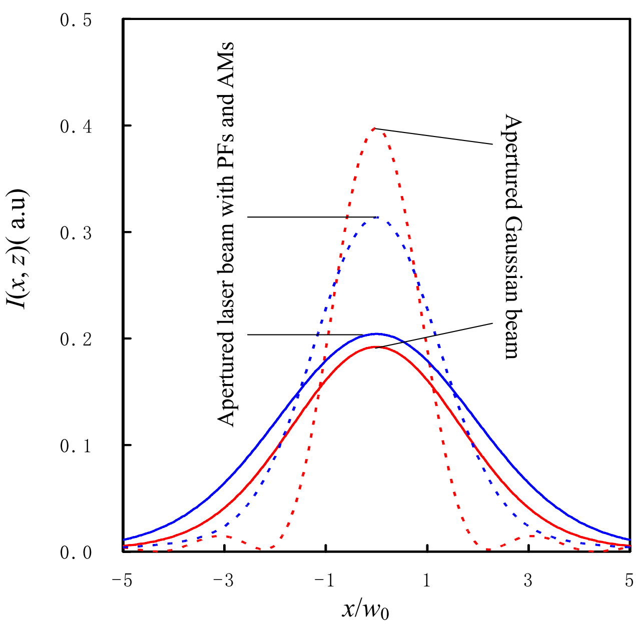

Fig. 10 Average transversal intensity distributions $I(x,z)$ of apertured laser beams with both AMs and PFs. $\alpha=0.5$, $\delta=0.5$, $\beta=1$, $\sigma_A^2=0.2$, $z=5$ km, "—" $C_n^2=10^{-14}\ \mathrm{m}^{-2/3}$, "- - -" $C_n^2=0$

with only PFs are more sensitive to turbulence than apertured laser beams with only AMs.

A comparative study of propagation properties of apertured laser beams with and without PFs and AMs is given in Figs. 9, 10 and 11. Figures 9 and 10 indicate that in free space the average intensity of apertured Gaussian beams is larger than that of apertured laser beams with PFs and AMs, but the average intensity of apertured Gaussian beams may be even smaller than that of apertured laser beams with PFs and AMs due to turbulence. From Fig. 11 it can be seen

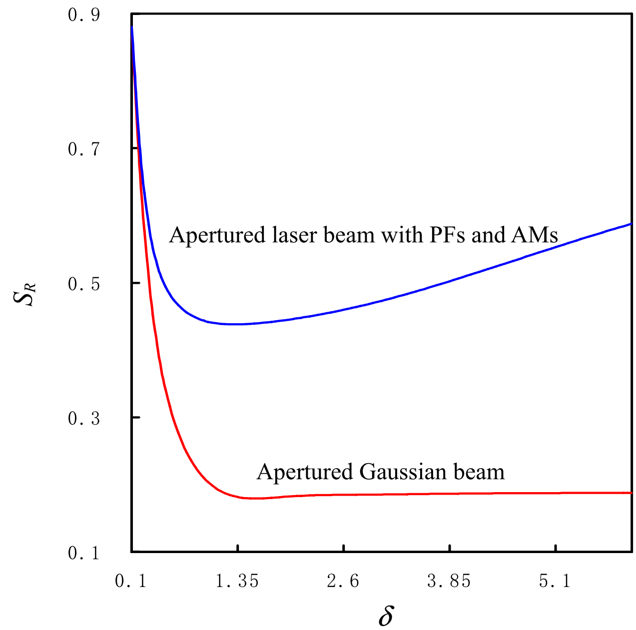

Fig. 11 Strehl ratio S_R of apertured laser beams with both AMs and PFs. $\alpha=0.5$, $\beta=1$, $\sigma_A^2=0.2$, $z=10$ km, $C_n^2=10^{-14}\ \mathrm{m}^{-2/3}$

that for apertured laser beams with PFs and AMs S_R is larger than that for apertured Gaussian beams. For apertured Gaussian beams S_R decreases with increasing δ, namely, the average maximum intensity is more and more sensitive to turbulence as δ increases. However, for apertured laser beams with PFs and AMs S_R is not monotonic versus δ, and there exists a minimum of S_R. For example, a minimum of S_R appears when $\delta=1.3$ (see Fig. 11). It means for apertured laser beams with PFs and AMs the average maximum intensity may be more seriously affected by turbulence within a certain range of δ.

5 Conclusion

The propagation properties of apertured laser beams with AMs and PFs through atmospheric turbulence have been studied in detail both analytically and numerically. The analytical expressions for the average intensity, *PIB* and S_R of apertured laser beams with AMs and PFs propagating through atmospheric turbulence have been derived, which may reduce to some interesting cases (e.g., apertured laser beams with only AMs or only PFs in turbulence, and apertured laser beams with AMs and PFs in free space). It has been found that the turbulence results in a beam spreading and a degradation of the average intensity. However, the worse the phase fluctuation and the higher the amplitude modulation are, the less laser beams are affected by turbulence. The average intensity of apertured laser beams with PFs and AMs may be even larger than that of apertured

Gaussian beams due to turbulence. Furthermore, apertured laser beams with only PFs are more sensitive to turbulence than those with only AMs. In particular, the influence of turbulence on the average maximum intensity of apertured laser beams with PFs and AMs may become serious if the unsuitable truncated parameter δ is chosen, which is different from the behavior of apertured Gaussian beams and should be avoided in practice.

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