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Moment and kurtosis parameter of partially coherent cosh-Gaussian beam in turbulent atmosphere

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Abstract An analytical expression for arbitrary moments of the cosh-Gaussian–Shell beam in turbulent atmosphere is derived. As a special case, kurtosis parameters of collimated and focused cosh-Gaussian beam with and without turbulent atmosphere are studied in detail. It can be seen from the study that the kurtosis parameters of Gaussian beam do not remain constant in turbulent atmosphere. Similar to the kurtosis parameters of a cosh-Gaussian beam at source plane, the kurtosis parameters at focal plane without turbulent atmosphere are independent of the propagation distance and vary with the parameters of the beam. But the variation of the kurtosis parameters at focal plane is different from that at source plane. However, the kurtosis parameters of collimated and focused cosh-Gaussian beam both vary with the propagation distance and gradually converge to 3 along the z-axis in turbulent atmosphere. Compared with a collimated beam, the kurtosis parameters of a focused beam converge more quickly.

1 Introduction

It is well known that a cosh-Gaussian beam can be regarded as the superposition of decentered Gaussian beams [1–3]. Various intensity profiles which can be used in some important applications can be obtained by altering the parameters of a cosh-Gaussian beam [3–5]. The characterization of the propagation and transform for a cosh-Gaussian beam have been studied extensively [2–10]. Because the moments of laser beams are considered to be the important parameters for describing their propagation, the properties concerning the second and fourth moment of a cosh-Gaussian beam have been investigated [11, 12].

In recent years, the propagation of many types of beam in turbulent atmosphere has attracted more and more attention due to its wide application [13–22]. As an important role, the properties of cosh-Gaussian beam in turbulent atmosphere have also been studied [2–7]. In the present paper, attention is paid to the moment of cosh-Gaussian beam in turbulent atmosphere. The paper is organized as follows. First, the analytical formula for the moment of arbitrary order is derived. As we know, the intensity profile of cosh-Gaussian beam is not of circular symmetry, i.e., the intensity varies with respect to the angle, and cannot be expressed as a function of distance from a central point only. Therefore, the variation of the moment with respect to the angle is then analyzed. Third, the variation of any moment in turbulent atmosphere is studied. As an example, the kurtosis parameter of a cosh-Gaussian beam in turbulent atmosphere is investigated in detail.

2 Moment of cosh-Gaussian–Schell beam at source plane

In general, a cosh-Gaussian–Schell beam is expressed in the form that the variables x and y can be separated. Namely, its cross-spectral density function can be denoted by [3–5, 23]

$$W_0(x_{01}, x_{02}, y_{01}, y_{02}) = \mathcal{W}_0(x_{01}, x_{02})\mathcal{W}_0(y_{01}, y_{02}) \tag{1}$$

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where (x_{01}, y_{01}) and (x_{02}, y_{02}) are the coordinates of two different points at source plane, and

$$\mathcal{W}_0(\chi_{01}, \chi_{02}) = \cosh(\Omega_0\chi_{01})\cosh(\Omega_0\chi_{02}) \times \exp\left(-\frac{\chi_{01}^2+\chi_{02}^2}{w_0^2}\right) \times \exp\left[-\frac{(\chi_{01}-\chi_{02})^2}{2\sigma_0^2}\right] \times \exp\left[-\frac{ik(\chi_{01}^2-\chi_{02}^2)}{2R_0}\right] \quad (2)$$

can be interpreted as one-dimensional cross-spectral density function of a cosh-Gaussian–Schell beam. In (2), Ω_0, which has the units of m^{-1}, is the parameter associated with the cosh part, $k = 2\pi/\lambda$ and λ is wavelength, w_0, σ_0 and R_0 denote the waist width, spatial correlation length and phase front radius of curvature of the beam, respectively.

Therefore, the two-dimensional Wigner transform of a cosh-Gaussian–Schell beam can be taken as the product of two independent one-dimensional Wigner transforms, i.e.

$$h_0(p_{0x}, p_{0y}, \theta_{0x}, \theta_{0y}) = \hbar_0(p_{0x}, \theta_{0x})\hbar_0(p_{0y}, \theta_{0y}). \quad (3)$$

where $p_{0x} = (x_{01}+x_{02})/2$, $p_{0y} = (y_{01}+y_{02})/2$, $q_{0x} = x_{01}-x_{02}$ and $q_{0y} = y_{01}-y_{02}$; $k\theta_{0x}$ and $k\theta_{0y}$ are the wave vector component along x-axis and y-axis. One-dimensional Wigner transforms of cosh-Gaussian–Schell beam can be written as

$$\hbar_0(p_{0\chi}, \theta_{0\chi}) = \frac{k}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathcal{W}_0\left(p_{0\chi}+\frac{q_{0\chi}}{2}, p_{0\chi}-\frac{q_{0\chi}}{2}\right) \times \exp(-ikq_{0\chi}\theta_{0\chi})\,dq_{0\chi}. \quad (4)$$

By using the equation $\cosh(x) = (e^x + e^{-x})/2$ and substituting (2) into (4), we obtain

$$\hbar_0(p_{0\chi}, \theta_{0\chi}) = \frac{k\omega_0}{4\sqrt{2\pi}}\sum_{i_1=0}^{1}\sum_{i_2=0}^{1}\exp\left\{-\frac{2}{w_0^2}p_{0\chi}^2 + \Lambda^+\Omega_0 p_{0\chi} - \frac{\omega_0^2}{8}\left[2k\left(\frac{p_{0\chi}}{R_0}+\theta_{0\chi}\right)+i\Lambda^-\Omega_0\right]^2\right\}. \quad (5)$$

where $\Lambda^+ = (-1)^{i_1} + (-1)^{i_2}$, $\Lambda^- = (-1)^{i_1} - (-1)^{i_2}$ and $1/\omega_0^2 = 1/w_0^2 + 1/\sigma_0^2$. Because the moment for arbitrary beam can be expressed as

$$\langle p_{0x}^{n_1}p_{0y}^{n_2}\theta_{0x}^{m_1}\theta_{0y}^{m_2}\rangle = \frac{1}{P}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}p_{0x}^{n_1}p_{0y}^{n_2}\theta_{0x}^{m_1}\theta_{y}^{m_2} \times h_0(p_{0x}, p_{0y}, \theta_{0x}, \theta_{0y})\,dp_{0x}\,dp_{0y}\,d\theta_{0x}\,d\theta_{0y}, \quad (6)$$

where P is the total power. The moments for any beam whose cross-spectral density function can be expressed by (1), equal to the product of the moments along x- and y-axis, i.e.,

$$\langle p_{0x}^{n_1}p_{0y}^{n_2}\theta_{0x}^{m_1}\theta_{0y}^{m_2}\rangle = \langle p_{0x}^{n_1}\theta_{0x}^{m_1}\rangle\langle p_{0y}^{n_2}\theta_{0y}^{m_2}\rangle. \quad (7)$$

Therefore, the moments along x-axis or y-axis can be rewritten as

$$\langle p_{0\chi}^{n}\theta_{0\chi}^{m}\rangle = \frac{1}{P_\chi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}p_{0\chi}^{n}\theta_{0\chi}^{m}\hbar_0(p_{0\chi}, \theta_{0\chi})\,dp_{0\chi}\,d\theta_{0\chi}, \quad (8)$$

where $P_\chi = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hbar_0(p_{0\chi}, \theta_{0\chi})\,dp_{0\chi}\,d\theta_{0\chi}$ is one-dimensional power of cosh-Gaussian beam. Substituting (5) into (8) and performing the integration, we get

$$\langle p_{0\chi}^{n}\theta_{0\chi}^{m}\rangle = \frac{k\omega_0}{4w_0}\operatorname{sech}\left(\frac{\gamma^2}{4}\right) \times \sum_{i_1=0}^{1}\sum_{j_1=0}^{1}\sum_{v=0}^{n}\sum_{t=0}^{[v/2]}\frac{2^{-(3m+2n+v-2t)/2}i^{m+n+2(v-t)}n!v!}{v!(n-v)!(v-2t)!t!} \times \left(\frac{w_0^2}{R_0}\right)^{v-2t}\left(\frac{w_0}{k\omega_0}\right)^{m-v+2t+1} \times A^{m-n}\exp\left[\frac{i^{2(i_1+j_1)}}{4}\gamma^2\right] \times H_{n-v}\left[-\frac{\gamma}{2Aw_0}\left(2i\Lambda^+ + \frac{\Lambda^- k\omega_0^2}{R_0}\right)\right] \times H_{m+v-2t}\left[\frac{\gamma}{2\sqrt{2}AR_0w_0}\left(i\Lambda^+ kw_0^2 - 2\Lambda^- R_0\right)\right], \quad (9)$$

where $A = (\frac{4}{w_0^2} + \frac{k^2\omega_0^2}{R_0^2})^{1/2}$, $\gamma = w_0\Omega_0$, $[v/2]$ denote the integer part of $v/2$ and $H_n(x)$ is Hermite polynomial.

3 Variation of moment in different direction

There are many types of beams whose optical fields are circular symmetry, namely, all points on each circle take the same value, such as fundamental Gaussian, super-Gaussian,

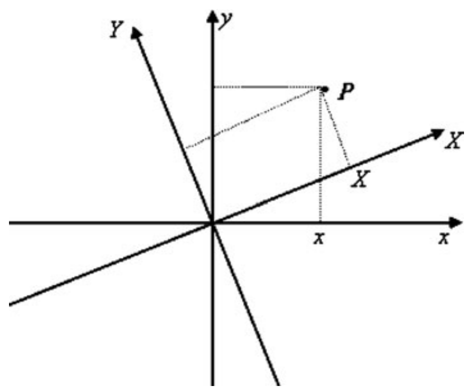

Fig. 1 Rotation of coordinates in two dimensions

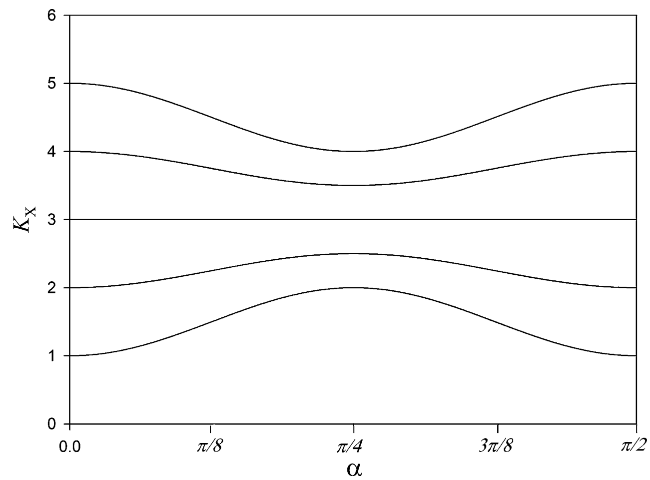

Fig. 2 The variation of K_X with angle α for different K_x

Bessel, annular and dark hollow beams. There also are many types of beam whose optical fields can be expressed as $u(x, y) = u(x)u(y)$, namely, the optical fields can be expressed as the product of x and y parts, and the x part is the same as the y part, such as fundamental Gaussian, cosh-Gaussian, cos-Gaussian sinh-Gaussian and sin-Gaussian beams. For the beams whose optical fields are not circular symmetry, their moments in different direction need to be investigated.

The variable in arbitrary direction is denoted by X (see Fig. 1). If the angle between x-axis and X-axis is α, X and θ_X can be presented as

$$\begin{cases} X = x\cos(\alpha) + y\sin(\alpha), \\ \theta_X = \theta_x\cos(\alpha) + \theta_y\sin(\alpha). \end{cases} \tag{10}$$

From (9) we can find that $\langle x^n\theta_x^m\rangle = 0$ if $n + m$ is odd and $\langle x^n\theta_x^m\rangle = \langle y^n\theta_y^m\rangle$. From (9) and (10), any moment along X can be obtained. For example,

$$\langle X^2\rangle = \langle x^2\rangle, \qquad \langle\theta_X^2\rangle = \langle\theta_x^2\rangle \tag{11}$$

and

$$\langle X^4\rangle = \frac{1}{4}\{\langle x^4\rangle[3 + \cos(4\alpha)] + 3\langle x^2\rangle^2[1 - \cos(4\alpha)]\}. \tag{12}$$

Namely, the second moments in any direction are equal to each other, and the fourth moment varies with the angle α.

The kurtosis parameter can be used to describe the sharpness of any beam and is defined as the ratio of the fourth-order moment to the square of the second-order moment [24]. From (11) and (12) we can see that the kurtosis parameter along X is

$$K_X = \frac{\langle X^4\rangle}{\langle X^2\rangle^2} = \frac{1}{4}\{K_x[3 + \cos(4\alpha)] + 3[1 - \cos(4\alpha)]\}. \tag{13}$$

where K_x is the kurtosis parameter along x. Equation (13) shows that the kurtosis parameter is different in different direction. If a beam whose optical field can be denoted by $u(x, y) = u(x)u(y)$ is circular symmetry, for example fundamental Gaussian beam, namely, $K_X = K_x$, from (13) we can see that its kurtosis parameter equals 3 in any direction. If we set $K_x = 3$ or $K_X = 3$, from (13) we can obtain $K_X = 3$ or $K_x = 3$. Namely, if the kurtosis parameter in any direction equal 3, its optical field must be circular symmetry. The variation of K_X with angle α for different K_x is plotted in Fig. 2.

One can see that the kurtosis parameter in any direction keeps invariant when $K_x = 3$. If $0 \le K_x < 3$, the kurtosis parameter along x or y is smallest. If α has values in the interval $[0, \pi/4]$, the kurtosis parameter increases with the increase of α. The largest kurtosis parameter is in the direction of $\alpha = \pi/4$ and equal to $(K_x + 3)/2$. If $K_x > 3$, the kurtosis parameter along x or y is the largest. If α has values in the interval $[0, \pi/4]$, the kurtosis parameter decreases with the increase of α. The smallest kurtosis parameter is in the direction of $\alpha = \pi/4$ and equal to $(K_x + 3)/2$.

4 Propagation of moment in turbulent atmosphere

The propagation of the cross-density of arbitrary beam at any plane can be calculated by using the Huygens–Fresnel diffraction integral relating to random inhomogeneous media as follow [25]:

$$\begin{aligned} &W(p_x, p_y, q_x, q_y, z) \\ &= \left(\frac{k}{2\pi z}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dp_{0x}\,dq_{0x}\,dp_{0y}\,dq_{0y} \\ &\quad\times W_0\left(p_{0x} + \frac{q_{0x}}{2}, p_{0x} - \frac{q_{0x}}{2}, p_{0y} + \frac{q_{0y}}{2}, p_{0y} - \frac{q_{0y}}{2}, 0\right) \\ &\quad\times \exp\left\{\frac{ik}{z}[(p_x - p_{0x})(q_x - q_{0x})\right. \end{aligned}$$

$$+(p_y-p_{0y})(q_y-q_{0y})\big]-H(q_x,q_y,q_{0x},q_{0y})\Big\}, \quad (14)$$

where $p_x=(x_1+x_2)/2$, $p_y=(y_1+y_2)/2$, $q_x=x_1-x_2$ and $q_y=y_1-y_2$, $H(q_x,q_y,q_{0x},q_{0y})$ represents the effects of turbulence and is given as [26]

$$H(q_x,q_y,q_{0x},q_{0y}) = 4\pi^2k^2z\int_0^1 d\xi\int_0^\infty\big\{1-J_0\big[\kappa\big|\xi\vec{q}_0+\kappa(1-\xi)\vec{q}\big|\big]\big\} \times \Phi(\kappa)\kappa\,d\kappa. \quad (15)$$

Here κ is the spatial wave number, (x_1,y_1) and (x_2,y_2) are the coordinates of two different points at z-plane; $\vec{q}_0=(q_{0x},q_{0y})$ and $\vec{q}=(q_x,q_y)$; J_0 and $\Phi(\kappa)$ are the Bessel function of zero order and the spatial power spectrum of the refractive index fluctuations, respectively. Because $H(q_x,q_y,q_{0x},q_{0y})$ cannot be expressed as $H(q_x,q_{0x})\times H(q_y,q_{0y})$, the moment of arbitrary beam through turbulence is expressed as

$$\langle p_x^{n_1}p_y^{n_2}\theta_x^{m_1}\theta_y^{m_2}\rangle = \frac{1}{P}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}p_x^{n_1}p_y^{n_2}\theta_x^{m_1}\theta_y^{m_2} \times h(p_x,p_y,\theta_x,\theta_y)\,dp_x\,dp_y\,d\theta_x\,d\theta_y \quad (16)$$

where $h(p_x,p_y,\theta_x,\theta_y)$ is WDF of $W(p_x,p_y,q_x,q_y,z)$. Substituting (14) into (16) we can derive

$$\langle p_x^{n_1}p_y^{n_2}\theta_x^{m_1}\theta_y^{m_2}\rangle = \langle G(p_{0x},p_{0y},\theta_{0x},\theta_{0y})\rangle \quad (17)$$

where [27, 28]

$$G(p_{0x},p_{0y},\theta_{0x},\theta_{0y}) = \frac{1}{z^2}\left(\frac{k}{2\pi}\right)^4 \times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}p_x^{n_1}p_y^{n_2} \times\theta_x^{m_1}\theta_y^{m_2} \times\exp\big[-ik(\theta_xq_x+\theta_yq_y-\theta_{0x}q_{0x}-\theta_{0y}q_{0y})\big] \times\exp\bigg\{\frac{ik}{z}\big[(p_x-p_{0x})(q_x-q_{0x}) +(p_y-p_{0y})(q_y-q_{0y})\big] -H(q_x,q_y,q_{0x},q_{0y})\bigg\} \times dp_x\,dp_y\,d\theta_x\,d\theta_y\,dq_x\,dq_y\,dq_{0x}\,dq_{0y}. \quad (18)$$

By using the properties of the Dirac delta function

$$\delta^{(n)}(s)=\frac{1}{2\pi}\int_{-\infty}^{\infty}(-ix)^n\exp(-isx)\,dx, \quad n=0,1,2,\ldots, \quad (19)$$

and

$$\int_{-\infty}^{\infty}f(x)\delta^{(n)}(x)\,dx=(-1)^nf^{(n)}(0), \quad n=0,1,2,\ldots, \quad (20)$$

(18) can be simplified as [27, 28]

$$G(p_{0x},p_{0y},\theta_{0x},\theta_{0y}) = i^{n_1+n_2+m_1+m_2}k^{-(n_1+n_2+m_1+m_2)}z^{n_1+n_2} \times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta^{(n_1)}(q_{0x}-q_x) \times\delta^{(n_2)}(q_{0y}-q_y)\delta^{(m_1)}(q_x)\delta^{(m_2)}(q_y) \times\exp\bigg[\frac{ik}{z}(-q_xp_{0x}+q_{0x}p_{0x}-q_yp_{0y}+q_{0y}p_{0y}) +ik(\theta_{0x}q_{0x}+\theta_{0y}q_{0y})-H(q_x,q_y,q_{0x},q_{0y})\bigg] \times dq_x\,dq_y\,dq_{0x}\,dq_{0y}. \quad (21)$$

With the help of (21), any moment of cosh-Gaussian–Shell beam in turbulent atmosphere can be obtained.

5 Numerical calculation

Many characteristics of beam can be expressed by its moment. Different moments have different meanings. As an example, the variation of the kurtosis parameter of a cosh-Gaussian beam is studied in the present paper. From (9) we can obtain the kurtosis parameter at source plane as

$$K_{0x}=\frac{\langle p_{0x}^4\rangle}{\langle p_{0x}^2\rangle^2} =\frac{[1+\exp(\gamma^2/2)][3+\exp(\gamma^2/2)(3+6\gamma^2+\gamma^4)]}{[1+(1+\gamma^2)\exp(\gamma^2/2)]^2} \quad (22)$$

It can be seen from (22) that K_{0x} is determined by the product of Ω_0 and w_0. The variation of K_{0x} with γ is shown in Fig. 3. It can be seen from Fig. 3 that K_{0x} equals 3 when $\gamma=0$ (fundamental Gaussian beam) and decreases with increasing γ. When γ is large enough K_{0x} trends to 1.

If the spatial power spectrum of the refractive index fluctuations is independent of the propagation distance, from (17) and (21) the moments at z-plane can be derived as

$$\langle p_x^2\rangle = \langle (p_{0x}+z\theta_{0x})^2\rangle + \frac{2}{3}\pi^2 z^3 \int_0^\infty \Phi(\kappa)\kappa^3\, d\kappa, \tag{23}$$

$$\langle p_x^2 p_y^2\rangle = \langle p_x^2\rangle\langle p_y^2\rangle + \frac{\pi^2 z^5}{10k^2}\int_0^\infty \Phi(\kappa)\kappa^5\, d\kappa \tag{24}$$

and

$$\begin{aligned}\langle p_x^4\rangle &= \langle (p_{0x}+z\theta_{0x})^4\rangle \\ &\quad + 4\pi^2 z^3 \langle (p_{0x}+z\theta_{0x})^2\rangle \int_0^\infty \Phi(\kappa)\kappa^3\, d\kappa \\ &\quad + \frac{4}{3}\pi^4 z^6 \left[\int_0^\infty \Phi(\kappa)\kappa^3\, d\kappa\right]^2 \\ &\quad + \frac{3\pi^2 z^5}{10k^2}\int_0^\infty \Phi(\kappa)\kappa^5\, d\kappa \end{aligned} \tag{25}$$

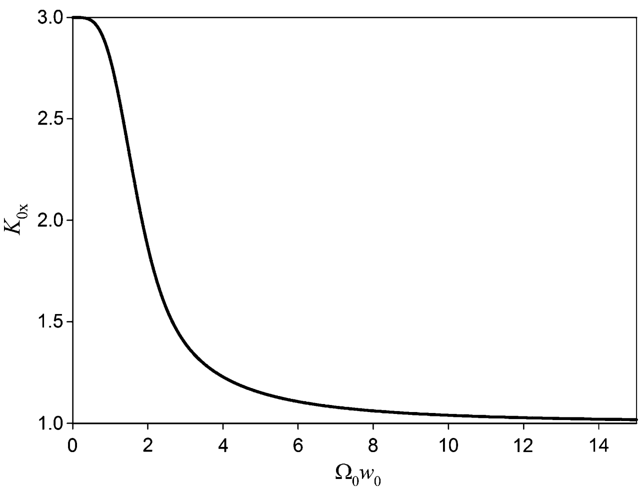

Fig. 3 Fig.3 Variation of the kurtosis parameter with the parameters of a cosh-Gaussian beam at source plane

Before proceeding with calculations of the kurtosis parameters in turbulent atmosphere, we examine the variation of the kurtosis parameters in vacuum. For collimated ($R_0 \to \infty$) and completely coherent ($\sigma_0 \to \infty$) beam, the kurtosis parameters in vacuum can be expressed as

$$\begin{aligned} K_x &= \frac{\langle (p_{0x}+z\theta_{0x})^4\rangle}{\langle (p_{0x}+z\theta_{0x})^2\rangle^2} \\ &= \frac{1+\exp(\frac{\gamma^2}{2})}{[k^2 w_0^4 + 4z^2 - \gamma^2 z^2 + \exp(\frac{\gamma^2}{2})(k^2 w_0^4 + k^2\gamma^2 w_0^4 + 4z^2)]^2} \\ &\quad \times \left\{ k^4 w_0^8 \left[3 + \exp\left(\frac{\gamma^2}{2}\right)(3+6\gamma^2+\gamma^4)\right] \right. \\ &\quad + 24k^2 w_0^4 z^2 \left[1-\gamma^2+\exp\left(\frac{\gamma^2}{2}\right)(1+\gamma^2)\right] \\ &\quad \left. + 16z^4\left[3-6\gamma^2+\gamma^4+3\exp\left(\frac{\gamma^2}{2}\right)\right]\right\} \end{aligned} \tag{26}$$

When z is large enough, from (25) we can derive

$$\lim_{z\to\infty} K_x = \frac{[1+\exp(\frac{\gamma^2}{2})][3-6\gamma^2+\gamma^4+3\exp(\frac{\gamma^2}{2})]}{[1-\gamma^2+\exp(\frac{\gamma^2}{2})]^2}. \tag{27}$$

We can find that the kurtosis parameters of a completely coherent ($\sigma_0 \to \infty$) beam at focal plane ($R_0 \to z$) in vacuum can also be expressed by (27). The variation of the kurtosis parameters of a collimated cosh-Gaussian beam along x-axis with the propagation distance is shown in Fig. 4a. It

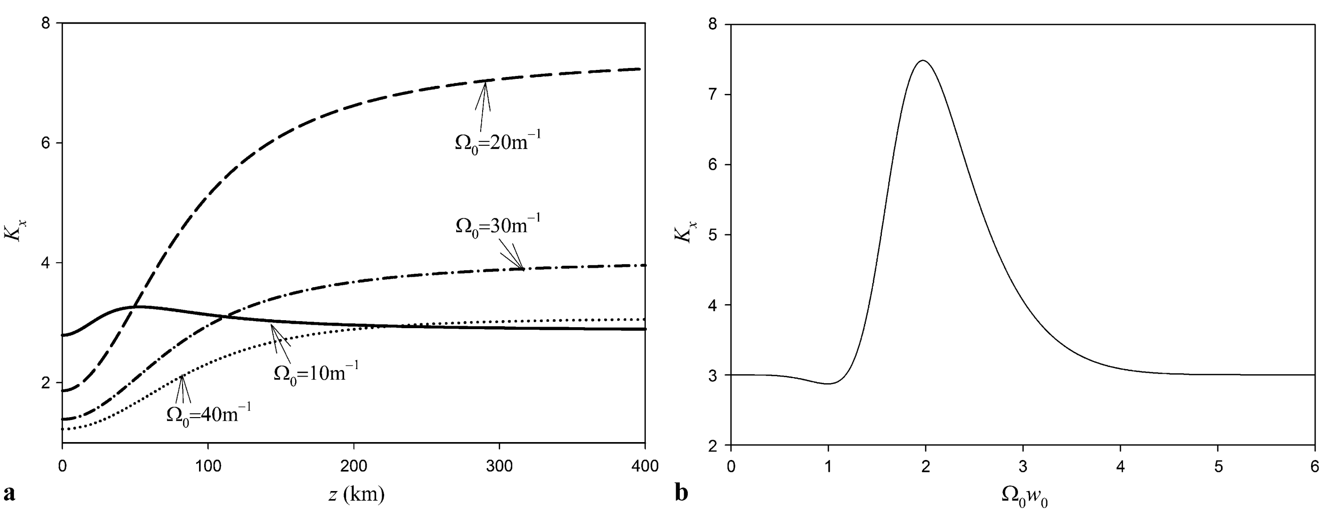

Fig. 4 Variation of the kurtosis parameter of a completely coherent cosh-Gaussian beam in vacuum; (**a**) collimated beam along z-axis; (**b**) focused beam at focal plane

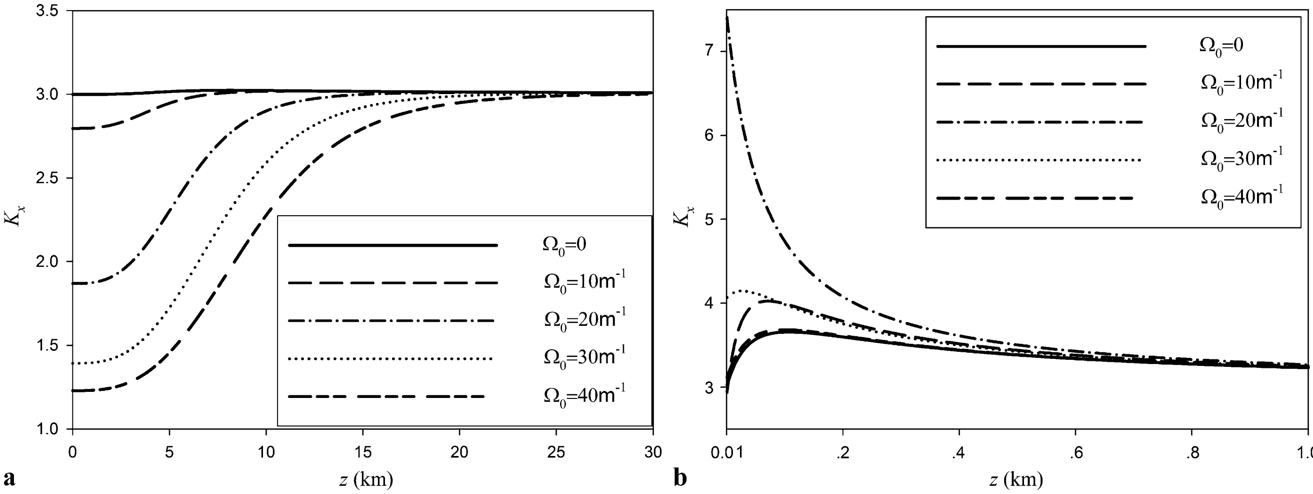

Fig. 5 The variation of the kurtosis parameter of a completely coherent cosh-Gaussian beam in turbulence where $\lambda = 1.06 \times 10^{-6}$ m, $C_n^2 = 10^{-14}$ m$^{-2/3}$, $w_0 = 0.1$ m and $l_0 = 0.1$ m; (**a**) collimated beam along z-axis; (**b**) focused beam at focal plane

can be seen from Fig. 4a that kurtosis parameters of a collimated cosh-Gaussian beam increase at the beginning for any γ. When γ is small, for example $\gamma = 1$, the kurtosis parameters then decrease. If z is long enough the kurtosis parameters trend to a constant. The variation of kurtosis parameters with γ at focal plane (or z is large enough) is shown in Fig. 4b. The comparison between Fig. 4b and Fig. 3 shows that the variation of the kurtosis parameters with γ at focal plane (or z is large enough) is different from that at source plane. When z is large enough or $z = R$ (at focal plane), the kurtosis parameter decreases slightly at the beginning and reaches a minimum value of 2.88 where $\gamma = 1.05$. When $\gamma > 1.05$, the kurtosis parameter increases until it reaches a maximum value of 7.49 where $\gamma = 1.97$. With further increase of γ, the kurtosis parameter decreases. From (27) we can see that when γ is large enough, the kurtosis parameter equals 3.

To study the variation of the kurtosis parameter in turbulence, in the present paper the Tatarskii spectrum is adopted, i.e.,

$$\Phi(\kappa) = 0.033 C_n^2 \kappa^{-11/3} \exp\left(-\frac{\kappa^2}{\kappa_m^2}\right) \tag{28}$$

where $\kappa_m = 5.92/l_0$ (l_0 being the inner scale of turbulence), C_n^2 is the structure constant and is assumed to be constant over the propagation path. From (9), (23) and (25) the variation of the kurtosis parameter of cosh-Gaussian beam in turbulence can be calculated. Figures 5a and 5b show the variation of the kurtosis parameter of collimated and focused beam in turbulent atmosphere, respectively. It can be seen that the kurtosis parameter of a collimated cosh-Gaussian beam increases with z when $\Omega_0 \neq 0$. If $\Omega_0 = 0$, the kurtosis parameter varies slightly. Namely, the kurtosis parameter of a Gaussian beam does not keep constant in turbulence. With the increase of z, the kurtosis parameter gradually converges to 3 for any Ω_0. The variation of the kurtosis parameter of a focused cosh-Gaussian beam is more complex than that of a collimated cosh-Gaussian beam. For some Ω_0, such as $\Omega_0 = 0, 10, 30, 40$ m^{-1} the kurtosis parameter increases at the beginning. After reaching the maximum value it decreases and gradually converges to 3. But for $\Omega_0 = 20$ m^{-1}, the kurtosis parameter decreases with z and converges to 3 when z is large enough.

From (10), (23)–(25) we can obtain the kurtosis parameter along X as

$$K_X = \frac{1}{4}\left\{K_x\left[3 + \cos(4\alpha)\right] + 3(1+T)\left[1 - \cos(4\alpha)\right]\right\}, \tag{29}$$

where

$$T = \frac{\pi^2 z^5}{10 k^2 \langle p_x^2 \rangle^2} \int_0^\infty \Phi(\kappa)\kappa^5 \, d\kappa. \tag{30}$$

If we set $K_X = K_x = K$ (foundational Gaussian beam), from (29) we can obtain

$$K = 3(1+T). \tag{31}$$

Namely, the kurtosis parameter of a Gaussian beam in turbulence can be expressed by (31). The variations of T with different propagation distance and Ω_0 are plotted in Fig. 6.

It can be seen that T increases at the beginning, and then decreases along the z-axis. When z is small, T is different with different Ω_0. With the increase of z, the values of T with different Ω_0 approximately equal each other. The value

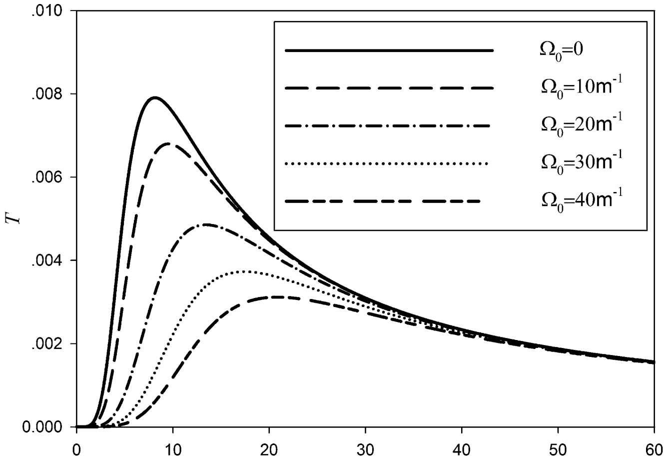

Fig. 6 The variations of T with different propagation distance and Ω_0 where $\lambda = 1.06 \times 10^{-6}$ m, $C_n^2 = 10^{-14}$ m$^{-2/3}$, $w_0 = 0.1$ m and $l_0 = 0.1$ m; (**a**) collimated beam; (**b**) focused beam at focal plane

of T for a focused cosh-Gaussian beam is much larger than that for a collimated beam.

6 Conclusion

An analytical expression for arbitrary moments of a cosh-Gaussian–Shell beam in turbulent atmosphere is derived. With the help of the formula, the kurtosis parameters of collimated and focused cosh-Gaussian beams in turbulent atmosphere are studied. The specific conclusions derived from the study can be listed as follows.

1. Kurtosis parameters equal 3 in vacuum and $3(1+T)$ [see (31)] in turbulent atmosphere for a Gaussian beam.
2. Kurtosis parameters of a cosh-Gaussian beam decrease with the increase of $\Omega_0 w_0$ at source plane. Its kurtosis parameters equals 3 (maximum) when $\Omega_0 w_0 = 0$ and trends to one (minimum) when $\Omega_0 w_0$ is large enough.
3. In vacuum, the variation of the kurtosis parameters of a collimated cosh-Gaussian beam along the z-axis is different with different $\Omega_0 w_0$. But the kurtosis parameters of a focused cosh-Gaussian beam at focal plane are independent on z. The variation of the kurtosis parameters at focal plane is different from that at source plane.
4. Kurtosis parameters of collimated and focused cosh-Gaussian beam gradually trend to 3 along the z-axis in turbulent atmosphere. The kurtosis parameters of a focused beam converge more quickly than those of a collimated beam.
5. The variation of the kurtosis parameters of a focused beam with angle is much bigger than that of a collimated beam in turbulent atmosphere.

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