Appl Phys B (2011) 103:695–700
DOI 10.1007/s00340-010-4346-6

Applied Physics B
Lasers and Optics

Autocorrelation of self-mixing speckle in an EDFR laser and velocity measurement

D. Han · S. Chen · L. Ma

Received: 9 August 2010 / Revised version: 7 November 2010 / Published online: 17 December 2010

Abstract Autocorrelation analysis for a velocity measurement based self-mixing speckle in an erbium-doped fiber ring (EDFR) laser is presented. The expression for the velocity measurement is deduced. Theoretical analysis reveals that the velocity of a random surface is inversely proportional to the autocorrelation time of the self-mixing signal produced in the EDFR laser. In experiments, a linear fit between the velocity and the reciprocal of autocorrelation time (autocorrelation frequency) is obtained for velocity measurement. In a range of velocities from 100 mm/s to 350 mm/s, the measurement errors are evaluated to be less than 5%. The signal length, the measurement distance, and the pump power are experimentally studied for gaining knowledge of the influence on the velocity measurement.

1 Introduction

A beam of coherent light illuminates a moving diffuse object, part of the light back-scattered from the object reenters the laser cavity, causing variations of the lasing frequency and the output power; this phenomena was known as self-mixing speckle [1–7]. Self-mixing speckle produced in a fiber laser has been studied for applications of velocity sensing [8–10] in recent years, the experimental setup based on an EDFR laser was presented and the model analysis was made to this system. In theoretical analysis, the generation of a self-mixing speckle signal was illustrated and the expression of variations of the output power was deduced; in experiments, speckle signals were obtained and analyzed by the method of fractal, and an empirical relationship between the signal and the velocity was explored to realize velocity sensing. However, the previous studies have not theoretically deduced the expression of velocity that relates speckle signals; for velocity measurement, it is inconvenient.

Aiming at the problem above and for a further study of velocity measurement, a correlation-based analysis is applied in this paper, which is a common method of speckle signal processing and has been applied to self-mixing speckle in laser diodes for velocity and displacement measurement [11–13]. In this analysis, the autocorrelation function of the detected speckle signal in a self-mixing fiber laser is theoretically deduced, and then the measurement velocity is retrieved from the speckle signal. The speckle signals are experimentally obtained and processed by autocorrelation. The expression that relates the velocity and autocorrelation frequency is obtained. To evaluate the validity of the velocity measurement, the measurement result is evaluated in dependence on the time of the speckle signal length, the external cavity length and the pump power.

2 Experimental system and model analysis

The experimental system for velocity measurement using self-mixing speckle in an EDFR laser is shown in Fig. 1. The fiber laser is built by the length of the erbium-doped fiber, a wavelength-division multiplexer (WDM), a 980-nm pump; each connected in order, by aid of a circulator and a coupler, forms an active measurement system based on fiber ring cavity. The circulator ensures a unidirectional laser operation-the light is guided to the moving object and backs to the laser cavity. A fiber Bragg grating (FBG) functions as a reflector as well as 1550 nm wavelength selector. The

D. Han (✉) · S. Chen · L. Ma
Physics Experiment Center of Nanchang University, Nanchang 330031, China
e-mail: dfhan@ncu.edu.cn
Fax: +86-791-3969493

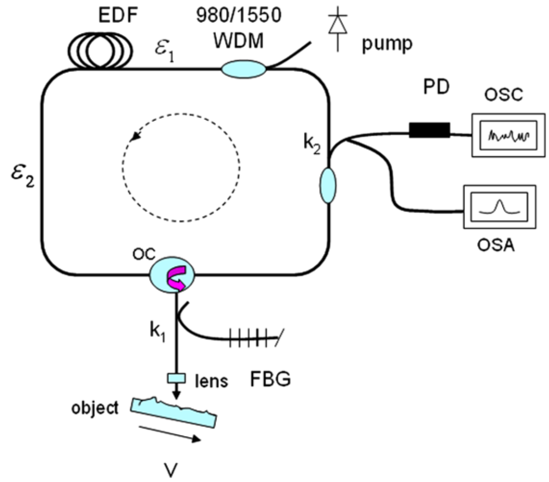

Fig. 1 Experimental system for velocity measurement using self-mixing speckle in an EDFR laser

laser beam projects onto the moving object through a focusing lens, part of the light back-scattered from the moving object is picked up and coupled into the ring cavity, producing modulation of the frequency and the output power. To detect variations of the output power, a photodiode (PD) detector is applied to transform the light intensity into electrical signal and then received by an oscilloscope (OSC).

Model analysis is made to the experimental system by reference to [8]. Using the theory of light amplification, the equation of power in the EDFR system is written as

$$P_s^s\left[\alpha_s L-\frac{(A-1)P_s^{\text{out}}}{P_s^s}-\ln A\right]=P_p^{\text{in}}\left[1-\exp\left(-\alpha_p L+\frac{P_s^s}{P_p^s}(\alpha_s L-\ln A)\right)\right] \tag{1}$$

with

$$A=k_2\varepsilon_1\varepsilon_2\left[k_1^2\mu+(1-k_1)^2 r_f\right], \tag{2}$$

where P_p^{in} and P_s^{out} are the pump power at the input and the laser power at the output of the EDFR laser. In (1), the subscripts p and s refer to the pump and laser wavelengths, respectively. $P_p^s(P_s^s)$ is the saturation power. $\alpha_p(\alpha_s)$ is the small signal absorption coefficient, and L is length of the erbium-doped fiber. In (2), $\varepsilon_1(\varepsilon_2)$ is the cavity loss in the erbium-doped fiber, respectively. $k_1(k_2)$ is the ratio of the power-split in two couplers. r_f is the reflection ratio of the FBG, and μ is the coefficient defined as the ratio of the power fed back into the laser to that exiting the fiber at the lens (near the moving object), then we have

$$\mu\propto\left|U(x_0)\right|^2\Big/\left|E(x_0)\right|^2, \tag{3}$$

where x_0 is the coordinate of the point at facet of the fiber where the light output and input, $E(x_0)$ is the output optical field that passes to the surface of the moving object, $U(x_0)$ is the complex amplitude of the speckle that feeds back into the laser.

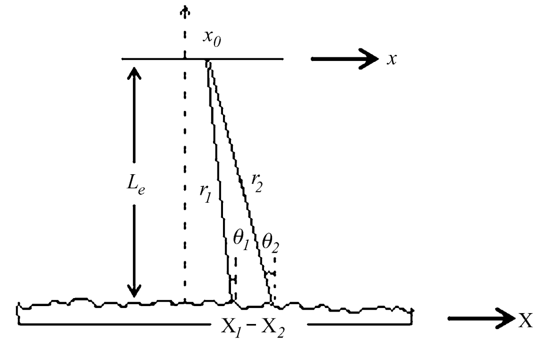

Fig. 2 Schematic diagram for light scattering in the diffraction region

Figure 2 shows the one-dimensional geometry of speckle formation in the Fresnel diffraction region, and $U(x_0)$ can be calculated by

$$U(x_0)=\int t(x_0)\cdot E(x_0)\cdot\exp(ikr)\cdot\exp\left[ik\cdot 2h(X)\right]\times\frac{\exp(ikr)}{r}\cdot\cos\theta\,dX \tag{4}$$

in the approximation of Fresnel diffraction, where $k=2\pi/\lambda$ is the laser wave vector, $t(x_0)$ is for the transmission functions corresponding to the lens and the fiber, X are the coordinates of the point on the diffracting region, $h(X)$ is the altitude function of the random surface, L_e is the external cavity length from point x_0 to the random surface, and r is the distance from point x_0 to point X. $\exp(ikr)$ is the phase change of that light from point x_0 to the diffracting region, $\exp[ik\cdot 2h(X)]$ is the phase modulation of the random surface, and $\frac{\exp(ikr)}{r}\cdot\cos\theta$ is the transfer function that the light scattering from point X to point x_0.

Equation (1) is solved numerically for P_s^{out} and finally the laser output power is obtained from

$$P_{\text{detector}}=(1-k_2)\varepsilon_2\left[k_1^2\mu+(1-k_1)^2 r_f\right]\cdot P_s^{\text{out}} \tag{5}$$

In the above analysis, excited state absorption (ESA) becomes weak when the laser is pumped into the 980-nm band and amplified spontaneous emission (ASE) diminishes when the laser is well above threshold. Therefore both ASE and ESA are neglected.

3 Autocorrelation analysis of self-mixing speckle signal

The variation of the output power is detected by the PD, which represents the speckle signal. We set $\Delta I(t)$ as the detected intensity of the signal at time t, and after a moment of time τ, this becomes $\Delta I(t+\tau)$, and then the normalized autocorrelation function of the dynamic speckle signal can be written as

$$\gamma_I(\tau) = \frac{\int \Delta I(t)\Delta I(t+\tau)\,dt}{\int \Delta I(t)\Delta I(t)\,dt} \tag{6}$$

In (5), we can know that the variable μ dominates the variation of the output power; therefore the autocorrelation of the speckle signal can be deduced by looking into that of the variable μ, and we have

$$\gamma_I(\tau) \simeq \gamma_\mu(x_0;\tau) \tag{7}$$

where $\gamma_\mu(x_0,\tau)$ is the normalized space–time autocorrelation function of the intensity that locates on the point x_0. In order to get $\gamma_\mu(x_0,\tau)$, we need to calculate the space–time autocorrelation function of the variable μ, which is defined as follows:

$$\begin{aligned}\Gamma_\mu(x_0;t_1,t_2) &= \langle \mu(x_0;t_1)\mu^*(x_0;t_2)\rangle \\ &= \iint \langle \exp\{i2k[h(X_1;t_1)-h(X_2;t_2)]\}\rangle \\ &\quad \times \exp[i2k(r_1-r_2)] \\ &\quad \times \cos(\theta_1)\cos(\theta_2)/(r_1r_2)\,dX_1\,dX_2 \end{aligned} \tag{8}$$

where (4) has been used, $\langle\cdots\rangle$ stands for an ensemble average, and r_1 and r_2 are the distances of the point x_0 from the point X_1, X_2, respectively.

Assuming that the altitude distributions of the random surface obey a Gaussian probability density, we have the following relation:

$$\begin{aligned}&\langle \exp\{i2k[h(X_1;t_1)-h(X_2;t_2)]\}\rangle \\ &\quad = \exp\{-(2k)^2[\omega^2 - \Gamma_h(X_1-X_2)]\}\end{aligned} \tag{9}$$

where ω is the root-mean-square deviation roughness, $\Gamma_h(X_1-X_2) = \langle h(X_1)h(X_2)\rangle$ is the autocorrelation function of the surface altitude. Usually $\Gamma_h(X_1-X_2)$ is a narrow function with respect to the correlation separation X_1–X_2, indicating that the range of X_1–X_2 is very small in the order of correlation length of the random surface. Then we only need to consider the case with X_1 close to X_2, and we have $\theta_1 \approx \theta_2$.

With the relation $\sin\theta_1 \approx \sin\theta_2 = (X_1-x_0)/r_1$, we can get

$$r_2 - r_1 \approx \Delta X \sin\theta_1 \tag{10}$$

With the variable substitutions of $X_2 = X_1 + \Delta X$ and $X_1 = r_1\sin\theta_1 + x_0$ in the integral, (8) can be rewritten as

$$\begin{aligned}\Gamma_\mu(x_0;\Delta X) &= (1/L_e)\iint \exp\{-(2k)^2[\omega^2-\Gamma_h(-\Delta X)]\} \\ &\quad \times \exp[-i2k\Delta X\cdot\sin\theta_1] \\ &\quad \times \cos^3\theta_1\,d\sin\theta_1\,d\Delta X\end{aligned} \tag{11}$$

Here we have used $\cos\theta_2 \approx \cos\theta_1$, $r_2 \approx r_1 = L_e/\cos\theta_1$, $dX_1 = r_1 d\sin\theta_1$, and $dX_2 = d\Delta X$.

The inclination factor $\cos\theta_1$ does not vary considerably within the significant part of the scattered intensity, the approximation $\cos\theta_1 \approx 1$ is made, and then (11) can be simplified as

$$\begin{aligned}\Gamma_\mu(x_0;\Delta X) &= (1/L_e)\iint \exp\{-(2k)^2[\omega^2-\Gamma_h(-\Delta X)]\} \\ &\quad \times \exp[-i2k\Delta X\cdot\sin\theta_1]d\sin\theta_1 d\Delta X\end{aligned} \tag{12}$$

The right side of (12) can be considered as the pair of Fourier transform and inversion of $\exp\{-(2k)^2[\omega^2-\Gamma_h(-\Delta X)]\}$, and the autocorrelation function turns out to be

$$\Gamma_\mu(x_0;\Delta X) = (1/L_e)\exp\{-(2k)^2[\omega^2-\Gamma_h(\Delta X)]\} \tag{13}$$

The random surface is described by a variety of practical surfaces ranging from growth fronts to ground interfaces, and the height autocorrelation function of the kind of surface is given by

$$\Gamma_h(\Delta X) = \omega^2\exp[-(\Delta X/l_0)^2] \tag{14}$$

where ΔX is the correlation separation, and l_0 is the horizontal correlation length, representing the average grain size of the random surface.

Substituting (14) to (13), the normalized autocorrelation function is obtained by

$$\begin{aligned}\gamma_\mu(x_0;\Delta X) &= |\Gamma(x_0;\Delta X)|^2 / |\Gamma(x_0;0)|^2 \\ &= \exp\{-2(2k)^2\omega^2[1-\exp(-\Delta X/l_0)^2]\}\end{aligned} \tag{15}$$

In the velocity measurement, if the light is located at X_1 at time t, after a moment of time τ, it will be located at $X_2 = X_1 + v\tau$, where v is the velocity of the surface. Using this relationship and (7), thus, the space autocorrelation of the speckle intensity located at the point x_0 will be transfer to a temporal autocorrelation of the speckle signal.

$$\gamma_I(\tau) = \exp\{-2(2k)^2\omega^2[1-\exp(-v\tau/l_0)^2]\} \tag{16}$$

Equation (16) is the expression of velocity measurement, and it reveals that the velocity is inversely proportional to the autocorrelation time. In the velocity measurement, $\gamma_I(\tau)$ can be calculated by using speckle signals obtained from experiment.

4 Experiment results and discussion

The experiment is implemented with the experimental system. A rotating aluminum plate is used as the object to be measured. The laser beam is focused to illuminate the rough surface of the plate. The velocity of the surface is

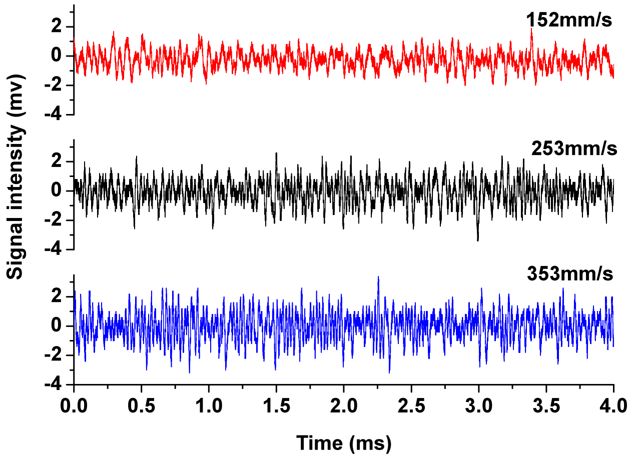

Fig. 3 Self-mixing speckle signals obtained at different velocities

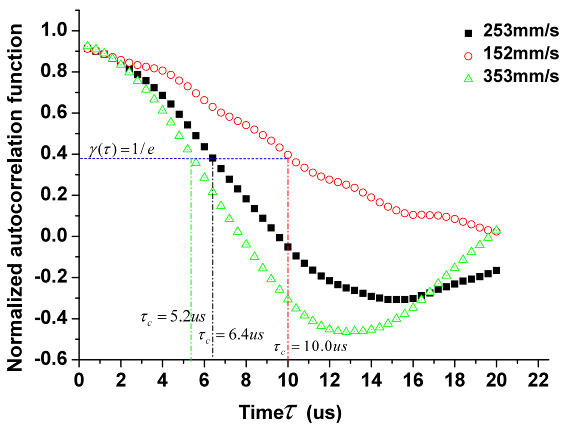

Fig. 4 Normalized autocorrelation function of speckle signals at different velocities

tuned by a servo-motor. The laser power is 1.0 mW, the ratios $k_1 = 0.5$, $k_2 = 0.9$, respectively, and the external cavity length is 3.0 mm. Self-mixing speckle signals in different velocities are acquired as shown in Fig. 3. It is seen that the amplitude of each signal fluctuates at random, and the faster the velocity is, the faster the amplitude varies. Using (6), the normalized function of the signal is illustrated in Fig. 4. For a velocity, the autocorrelation curve begins to decline with time τ increase, and after it reaches a minimum, starts to increase. In the first decline segment, the autocorrelation time τ_c is determined when the autocorrelation value reaches $1/e$. It is seen that as the velocity increases, the autocorrelation time decreases.

A group of speckle signals with velocities of (63.6 mm/s, 114.4 mm/s, 157.6 mm/s, 204 mm/s, 234.4 mm/s, 275.6 mm/s, 302 mm/s, 334.4 mm/s, 374 mm/s) are acquired, and each signal is calculated to obtain the autocorrelation frequency. The velocities corresponding to the autocorrelation frequencies are plotted in Fig. 5, a linear re-

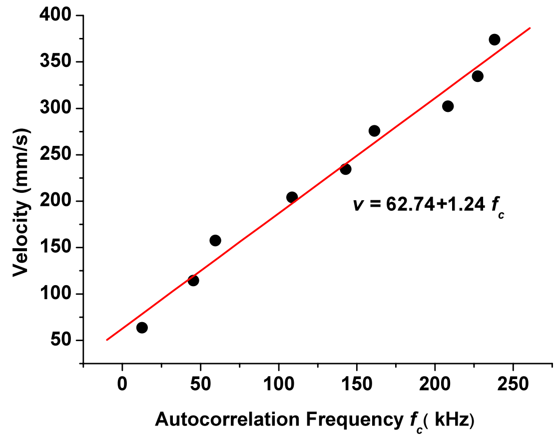

Fig. 5 Linear relationship between the velocity and the autocorrelation frequency

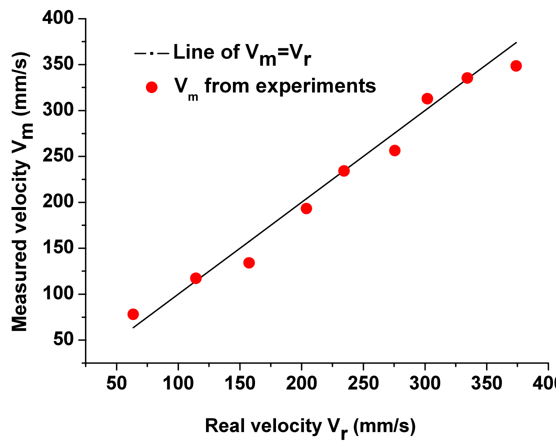

Fig. 6 Comparison between the measured velocities and the real velocities

lationship between the velocity and the autocorrelation frequency is obtained. The linear fit is expressed by

$$v = 62.74 + 1.24 \times f_c \text{ (mm/s)} \tag{17}$$

Equation (17) is the expression for the velocity measurement obtained from experiments. The validity of the expression can be evaluated by the deviation of the measurement. We calculate the autocorrelation frequency of speckle signal from the experiment, and substitute it into (7), and then the measurement velocity will be obtained. Repeating the operation with a group of speckle signals, we get a group of measurement velocities. The measurement velocities are compared with the real velocities that we are setting in experiments. Figure 6 shows the result of comparison for a range from 60 mm/s to 400 mm/s. The circle dots represent the value from experiment, and the line represents that the measurement velocity is equal to the real velocity. It is shown that the measurement velocities deviate from the real velocities.

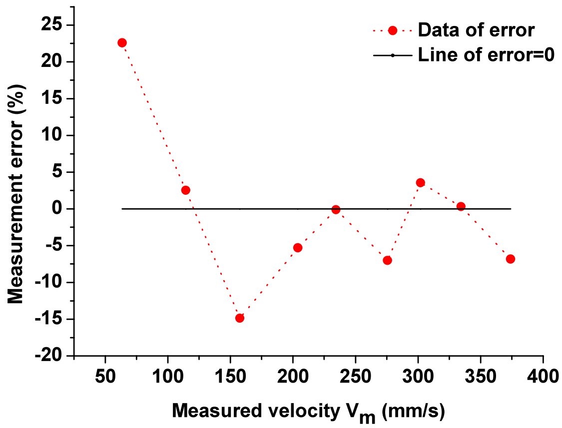

Fig. 7 Measurement errors

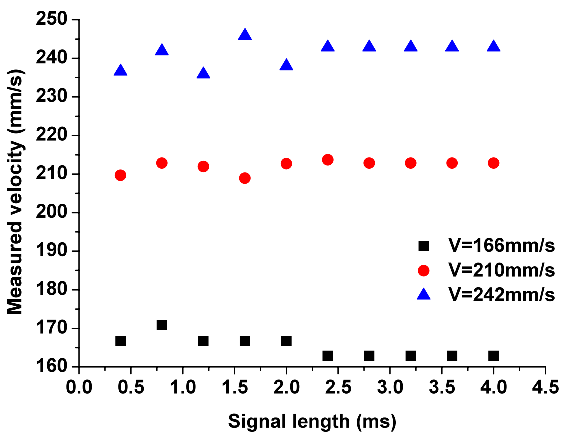

Fig. 8 Measured velocities at different signal lengths of time

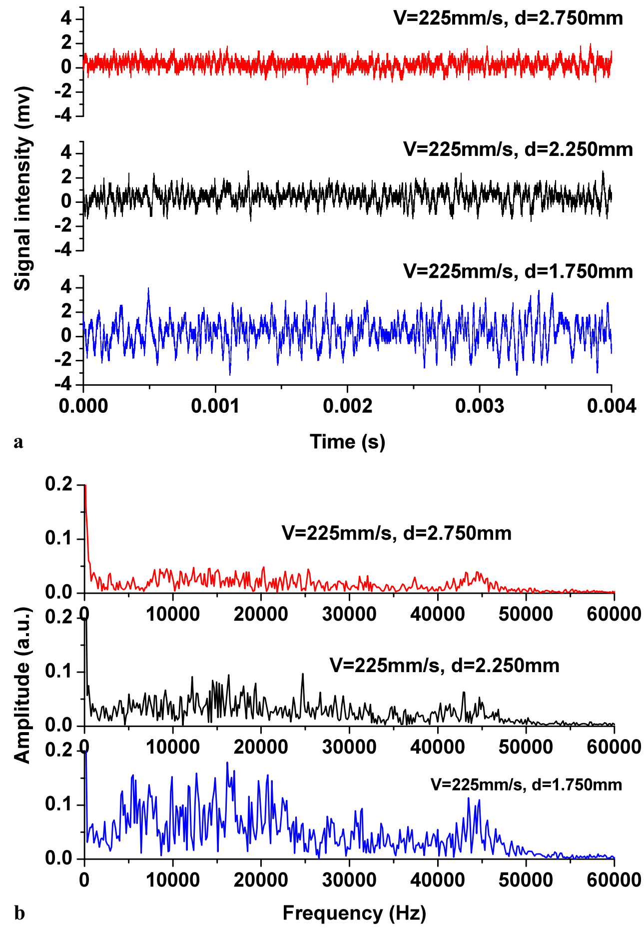

Fig. 9 (**a**) The speckle signals to different lengths of external cavity in the velocity 225 mm/s; (**b**) their spectrum

Based on the result in Fig. 6, the measurement error is calculated and shown in Fig. 7; we see that in the range of 100 mm/s to 350 mm/s, most of the error points are less than 5%, beyond this range, the percent error become larger. This phenomenon can be explained from the ratio of signal to noise (SNR). In experiments, the signal and the noise are simultaneously sampled. For a given sampling rate, there is a range of signal frequency that makes the measurement with high SNR. If the velocity to frequency is out of the range, the errors from the measurement become large. In this study, the sampling rate of signal is 4.0e-7s, which is applicable to the signals to velocities in the range of 100 mm/s to 350 mm/s.

In autocorrelation calculation, the signal length of time is important for the measurement result. To find the influence on velocity measurement for different signal lengths, a group of signals with different lengths are acquired at velocity 166 mm/s in experiments. These signals are processed by autocorrelation and the measurement results are obtained as shown in Fig. 8. It is seen that when the signal length is greater than 2.5 ms, no significant variations in the measured value occur with the changes of signal length. Repeating the experiments to the velocities of 210 mm/s and 242 mm/s, similar results appeared, though some measured values are obviously fluctuant before 2.5 ms. We define the time of 2.5 ms as the autocorrelation length, which is commonly determined by sampling rate of the experimental system and the feature of the object under test.

In the experiments, the self-mixing signal is affected by the external cavity length between the laser and the target surface. Three signals to the velocity of 225 mm/s are observed. They are corresponding to the external cavity lengths of 2.75 mm, 2.25 mm, and 1.75 mm, respectively. Figure 9 shows the signals and their spectrum, and it is seen that with increase of the external cavity length, the signal amplitude decreases, but there is no change in frequency range. At the same frequency range of signals, the measured velocities are same.

Along the line from the laser to the target surface, the range of the distance that can measure the velocity correctly is studied. The external cavity length is up to 5.0 mm, the signals that can be detected are very weak. At this time, due to low SNR, the measured value is unstable and holds a large error. The external cavity length is below 0.8 mm, the signals that can be detected are very strong, and then the outflow of amplification in the EDF laser occurs. Therefore, the distance from 0.8 mm to 5.0 mm is the range that can measure the velocity correctly. The range of measurement distance is based on a stabilized pump. When the power of the pump is changed, the range will change in the same direction. As the self-mixing signal is dependent on the length of external cavity and the pump strength, a stabilized pump and a suitable measurement distance are important in experiments.

5 Conclusion

In this paper, the self-mixing speckle in an EDFR laser is studied by autocorrelation analysis. The expression for velocity measurement is deduced. The validity of the measurement is demonstrated in experiments; the result shows that in the range of velocity from 100 mm/s to 350 mm/s, the measurement errors are evaluated to be less than 5%. The result is equivalent to that of the self-mixing speckle in the laser diode. Due to the advantages of the fiber laser, correlation-based self-mixing speckle in an EDFR laser is suitable to build a fiber network with more stable signal and quick response in velocity sensing.

Acknowledgements This work was supported by the National Natural Science Foundation of China (60808019) and the Natural Science Foundation of Jiangxi Province (2008GQW0002).

References

1. T. Shibata, S. Shinohara, H. Ikeda, H. Yoshida, T. Sawaki, M. Sumi, IEEE Trans. Instrum. Meas. **45**, 499 (1996)
2. S.K. Ozdemir, T. Takasu, S. Shinohara, H. Yoshida, M. Sumi, Appl. Opt. **38**, 1968 (1999)
3. S.K. Ozdemir, S. Takamiya, S. Ito, S. Shinohara, H. Yoshida, IEEE Trans. Instrum. Meas. **49**, 1029 (2000)
4. M. Lu, M. Wang, H. Hao, H. Huan, Guangxue Xuebao/Acta Opt. Sinica **24**, 1229 (2004)
5. M. Lu, M. Wang, H. Hao, Guangxue Xuebao/Acta Opt. Sinica **25**, 190 (2005)
6. M. Wang, M. Lu, H. Hao, J. Zhou, Opt. Commun. **260**, 242 (2006)
7. D. Han, M. Wang, J. Zhou, Opt. Express **14**, 3312 (2006)
8. D. Han, M. Wang, J. Zhou, IEEE Photonics Technol. Lett. **19**, 1398 (2007)
9. D. Han, M. Wang, J. Zhou, Opt. Express **16**, 3204 (2008)
10. D. Han, X. Zuo, L. Ma, Proc. SPIE **7514**, 75140R (2009)
11. S.K. Ozdemir, S. Ito, S. Shinohara, H. Yoshida, M. Sumi, Appl. Opt. **38**, 6859 (1999)
12. J. Hast, M. Okkonen, H. Heikkinen, L. Krehut, R. Myllyla, Opto-Electron. Rev. **14**, 125 (2006)
13. S.K. Ozdemir, S. Takamiya, S. Shinohara, H. Yoshida, Meas. Sci. Technol. **11**, 1447 (2000)