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Changes in the state of polarization of partially coherent flat-topped beam in turbulent atmosphere for different source conditions

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Abstract The polarization states, i.e. the size, the shape and the orientation of the polarization ellipse of partially coherent flat-topped (PCFT) beams passing through atmospheric turbulence are studied in detail. The effects are studied of different source conditions on the polarization states of a PCFT beam propagating through atmospheric turbulence. Based on the unified theory of the polarization states for random electromagnetic beams, we have established the detailed formula for calculating the change of the polarization states of such beams.

The polarization states behavior of PCFT beams passing through atmospheric turbulence for any arbitrary order of a flat-topped beam "N" under different source conditions were investigated.

1 Introduction

Studies of light beams propagating through the turbulent atmosphere are very important for many applications, such as tracking, remote sensing and free-space optical communications and many investigations have been done on propagation properties of various laser beams in turbulent atmosphere [1–28].

In the past three decades, the statistical properties of scalar partially coherent (stochastic) beams propagating in the atmospheric turbulence have been investigated extensively [14–21]. It has been known that the spectral density, the degree of coherence and the degree of polarization of a random electromagnetic beam may change on propagation [29–39]. The unified theory of coherence and polarization (Wolf et al.) [35, 36], made it possible to determine the changes in statistical properties of such beams, including the spectral density, the spectral degree of coherence, and the state of polarization, i.e. the size, the shape and the orientation of the polarization ellipse.

In this paper, we derive a general analytic formula for the elements of the cross-spectral density matrix of an isotropic, semi isotropic and anisotropic PCFT beam propagating in turbulent atmosphere. Furthermore, the changes in the state of polarization, i.e. the size, the shape and the orientation of the polarization ellipse of such beams were investigated.

2 Cross-spectral density matrix of anisotropic PCFT beam in atmospheric turbulence and its polarization properties

We consider field propagation from the source plane $z = 0$ to $z > 0$ where the turbulence atmosphere exists. (ρ, z) is the position vector of a point in the receiver plane, where ρ denotes a two dimensional transverse vector perpendicular to the direction of the beam propagation (see Fig. 1).

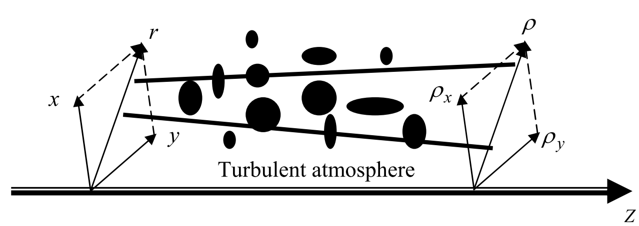

Fig. 1 Illustrating the notation related to propagation of a laser beam

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In order to describe the second-order coherence properties of a polychromatic and partially coherent beam, a 2×2 cross-spectral density correlation matrix was introduced [33–36, 40–46]:

$$\overleftrightarrow{W}(\vec{\rho}_1,\vec{\rho}_2,z)\equiv W_{ij}(\vec{\rho}_1,\vec{\rho}_2,z)=\langle E_i(\vec{\rho}_1,z)E_j^*(\vec{\rho}_2,z)\rangle$$
$$(i=x,y;\ j=x,y), \tag{1}$$

where angular brackets represent the ensemble average over the medium statistic, the asterisk denotes the complex conjugate, the subscripts (ij) label Cartesian components of a typical realization and $E(\vec{\rho},z)$ of the electric field in two mutually orthogonal directions perpendicular to the direction of propagation of the beam (the z-direction). The propagation of each of the two transverse components of the electromagnetic beam in the turbulent atmosphere can be studied with the following extended Huygens–Fresnel integral [18–25]:

$$E(\vec{\rho},z)=-\frac{ik\exp(ikz)}{2\pi z}\iint E(\vec{r},z=0)\exp\left[ik\frac{(\vec{\rho}-\vec{r})^2}{2z}\right]$$
$$\times\exp\left[\psi(\vec{\rho},\vec{r},z)\right]d^2r, \tag{2}$$

where $E(\vec{r},z=0)$ and $E(\vec{\rho},z)$ are the field distribution in source and receiver plane, respectively, and ψ represent the phase function that depends on the properties of the medium.

The cross-spectral density matrix of PCFT beams propagating in turbulent atmosphere reads [33–36, 40–46]

$$W_{ij}(\vec{\rho}_1,\vec{\rho}_2,z)$$
$$=\left(\frac{k}{2\pi z}\right)^2\iint d^2r_1\iint d^2r_2\,W_{ij}^{(0)}$$
$$\times(\vec{r}_1,\vec{r}_2,z=0)$$
$$\times\exp\left[-ik\frac{(\vec{\rho}_1-\vec{r}_1)^2-(\vec{\rho}_2-\vec{r}_2)^2}{2z}\right]$$
$$\times\left\langle\exp\left[\psi(\vec{\rho}_1,\vec{r}_1,z)+\psi^*(\vec{\rho}_2,\vec{r}_2,z)\right]\right\rangle_m, \tag{3}$$

where k is the wave number and the sharp brackets with subscript "m" denotes the average over an ensemble of statistical realizations of the turbulent atmosphere. The last term in the integrand of (3) can be expressed as [44]

$$\left\langle\exp\left[\psi(\vec{\rho}_1,\vec{r}_1,z)+\psi^*(\vec{\rho}_2,\vec{r}_2,z)\right]\right\rangle$$
$$=\exp\left[-0.5D_\psi(\vec{r}_1-\vec{r}_2)\right]$$
$$\cong\exp\left[-\left(\frac{1}{\rho_0^2}\right)\left((\vec{r}_1-\vec{r}_2)^2+(\vec{r}_1-\vec{r}_2)(\vec{\rho}_1-\vec{\rho}_2)\right.\right.$$
$$\left.\left.+(\vec{\rho}_1-\vec{\rho}_2)^2\right)\right], \tag{4}$$

where $D_\psi(\vec{r}_1-\vec{r}_2)$ is the structure function of the refractive index fluctuation, $\rho_0=(0.545C_n^2k^2z)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent medium with C_n^2 being the structure constant.

Let us consider a light beam with a PCFT profile where its cross-spectral density at the source plane ($z=0$) is characterized by [22–25]:

$$W_{ij}^{(0)}(\vec{r}_1,\vec{r}_2,z=0)$$
$$=\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}A_iA_jB_{ij}\frac{(-1)^{m+n-2}}{N^3}\binom{N}{n}\binom{N}{m}$$
$$\times\exp\left\{-\left[\frac{n\vec{r}_1^2}{4\sigma_i^2}+\frac{m\vec{r}_2^2}{4\sigma_j^2}+\frac{l(\vec{r}_1-\vec{r}_2)^2}{2\delta_{ij}^2}\right]\right\}. \tag{5}$$

Here $\binom{N}{m}$, $\binom{N}{n}$ denotes the binomial coefficient, N is the order of the flat-topped beams, A_i, A_j, B_{ij} are the coefficients and σ_i^2, σ_j^2 are the waist sizes of an elliptical Gaussian beam in x- and y-directions, respectively. δ_{ij}^2 is the effective width of the spectral degree of coherence of source.

To evaluate (3), it is convenient to introduce the new variable of integration,

$$u=\frac{\vec{r}_1+\vec{r}_2}{2},\qquad v=\vec{r}_1-\vec{r}_2. \tag{6}$$

Substituting (6), (5) and (4) into (3) and calculating the related integral we obtain

$$W_{ij}(\rho_1,\rho_2,z)$$
$$=A_iA_jB_{ij}\frac{k^2}{4\pi^2z^2}\exp\left[-\frac{ik}{2z}(\vec{\rho}_1^2-\vec{\rho}_2^2)\right]$$
$$\times\exp\left[-\frac{(\vec{\rho}_1-\vec{\rho}_2)^2}{\rho_0^2}\right]$$
$$\times\sum_m^N\sum_n^N\sum_l^N\frac{(-1)^{m+n}}{N^3}\binom{N}{m}\binom{N}{n}$$
$$\times\exp\left[\frac{-k^2}{4\alpha_1z^2}\left[(\rho_{2x}-\rho_{1x})^2+(\rho_{2y}-\rho_{1y})^2\right]\right]$$
$$\times\frac{\pi^2}{\alpha_1\alpha_2}\exp\left[\frac{\beta_2^2+\beta_3^2}{4\alpha_2}\right], \tag{7}$$

where

$$\alpha_1 = \frac{n}{4\sigma_i^2} + \frac{m}{4\sigma_j^2}, \qquad \beta_1 = \frac{n}{4\sigma_i^2} - \frac{m}{4\sigma_j^2} + \frac{ik}{z},$$
$$\alpha_2 = \frac{n}{16\sigma_i^2} + \frac{m}{16\sigma_j^2} + \frac{l}{2\delta_{ij}^2} + \frac{1}{\rho_0^2} - \frac{\beta_1^2}{4\alpha_1}$$
$$\beta_2 = \frac{-ik}{2z}(\rho_{2x} + \rho_{1x}) + \left(\frac{1}{\rho_0^2} + \frac{ik}{2\alpha_1 z}\right)(\rho_{1x} - \rho_{2x}),$$
$$\beta_3 = \frac{-ik}{2z}(\rho_{2y} + \rho_{1y}) + \left(\frac{1}{\rho_0^2} + \frac{ik}{2\alpha_1 z}\right)(\rho_{1y} - \rho_{2y}). \tag{8}$$

All the information about the polarization properties of an anisotropic PCFT beam may be deduced from a restricted form of the correlation matrix (7) [36].

The spectral degree of polarization at the point (ρ, z) was given by the formula

$$P(\vec{\rho}, z) = \sqrt{1 - \frac{4\mathrm{Det}\ \overleftrightarrow{W}(\vec{\rho}, z, \omega)}{[\mathrm{Tr}\ \overleftrightarrow{W}(\vec{\rho}, z, \omega)]^2}}, \tag{9}$$

where Det $\overleftrightarrow{W}$ is the determinant of $\overleftrightarrow{W}$ and Tr stands for the trace.

It was shown that the elements of the matrix (7) defined the spectral polarization state of the polarized portion of the beam at any point (ρ, z), i.e. the shape and the orientation of the polarization ellipse [35].

The orientation angle, say, θ_0, is for the smallest angle which the major axis of polarization ellipse makes with the x-direction, Therefore rotating the x-, y-coordinate system in the counter-lock wise sense with respect to the positive z-direction we are presented with the following formula [35]:

$$\theta_0(\vec{\rho}, z) = \frac{1}{2}\arctan\left(\frac{2\mathrm{Re}[W_{xy}(\vec{\rho}, z, \omega)]}{W_{xx}(\vec{\rho}, z, \omega) - W_{xy}(\vec{\rho}, z, \omega)}\right). \tag{10}$$

Also it was shown that the squares of the magnitudes of the major and the minor semi-axes of the polarization ellipse are given by the expressions:

$$A_{\mathrm{major}}^2(\vec{\rho}, z) = \frac{\sqrt{(W_{xx}-W_{yy})^2-4|W_{xy}|^2} + \sqrt{(W_{xx}-W_{yy})^2-4[\mathrm{Re}W_{xy}]^2}}{2},$$
$$A_{\mathrm{minor}}^2(\vec{\rho}, z) = \frac{\sqrt{(W_{xx}-W_{yy})^2-4|W_{xy}|^2} - \sqrt{(W_{xx}-W_{yy})^2-4[\mathrm{Re}W_{xy}]^2}}{2}. \tag{11}$$

The shape of the ellipse was characterized by the ratio, sometimes called the "degree of ellipticity":

$$\varepsilon = \frac{A_{\mathrm{minor}}}{A_{\mathrm{major}}} \tag{12}$$

The above equations serve as the basic formula for studying the polarization properties of anisotropic PCFT beams propagating through atmospheric turbulence.

3 Discussion

The spectral degree of polarization for the PCFT beam was determined in (9), and the change in the spectral degree of polarization on propagation distance in turbulence is studied by choosing different conditions source parameters.

To represent the partial coherence, the spectral degree of correlation δ_{ij} should be smaller than the σ_i and σ_j, i.e. δ_{ij} is not in the order of the beam size [15].

Figure 2 demonstrates the changes in the spectral degree of polarization along the z-axis ($\rho = 0$) of PCFT beams passing through a turbulent atmosphere with different source parameters and for different order of the flat-topped beam N. In Fig. 2(a) it is assumed that $\sigma_x = \sigma_y$ and $\delta_{xx} = \delta_{yy}$, and it shows that the spectral degree of polarization of the beam reaches a maximum value in propagation distance and then returns to its value in the source plane after propagation over a long distance. This is in agreement with the previous results for $N = 1$, which are related to Gaussian shell model (GSM) beam [28–30, 34, 35]. In addition Fig. 2(a) clearly demonstrates that the degree of polarization for PCFT beam increases with the flat-topped order N increasing.

Figure 2(b) shows the spectral degree of polarization for a beam generated by a semi isotropic source, i.e. the source with $\sigma_x \neq \sigma_y$ and $\delta_{xx} = \delta_{yy}$. From Fig. 2(b), it is seen that the degree of polarization decreases as the propagation distance increases.

Also it is seen that increasing of the flat-topped order N will decrease the amount of the spectral degree of polarization for a PCFT beam propagating through atmospheric turbulence up to a special propagation distance. For propagation distances larger than this, the spectral degree of polarization does not follow this regularity. The reason for this behavior is that the effect of atmospheric turbulence on $W_{ij}(\vec{\rho}_1, \vec{\rho}_2, z)$ with different σ_x, σ_y and N values is changed [27].

Also it is seen that increasing of the flat-topped order N will result in a decrease of the special propagation distance.

Figure 2(c) shows the spectral degree of polarization of a beam generated by an anisotropic source, i.e. a source with $\sigma_x \neq \sigma_y$ and $\delta_{xx} \neq \delta_{yy}$. It is evident from this figure that the same general behavior for the spectral degree of polarization in Fig. 2(b) exists, but for this case, the spectral degree of polarization of the beam reaches a minimum and maximum value in propagating distance. Since in Fig. 2(c), δ_{xx} is not equal to δ_{yy}, therefore the effect of atmosphere on the $W_{xx}(\vec{\rho}_1, \vec{\rho}_2, z)$ is different from the effect of atmosphere on the $W_{yy}(\vec{\rho}_1, \vec{\rho}_2, z)$. This different atmospheric effect justifies the different behavior of Figs. 2(b) and 2(c).

Further investigation shows that the same special propagation distance in Fig. 2(c) exists for Fig. 2(b) and the spectral degree of polarization has a minimum value in this special propagation distance.

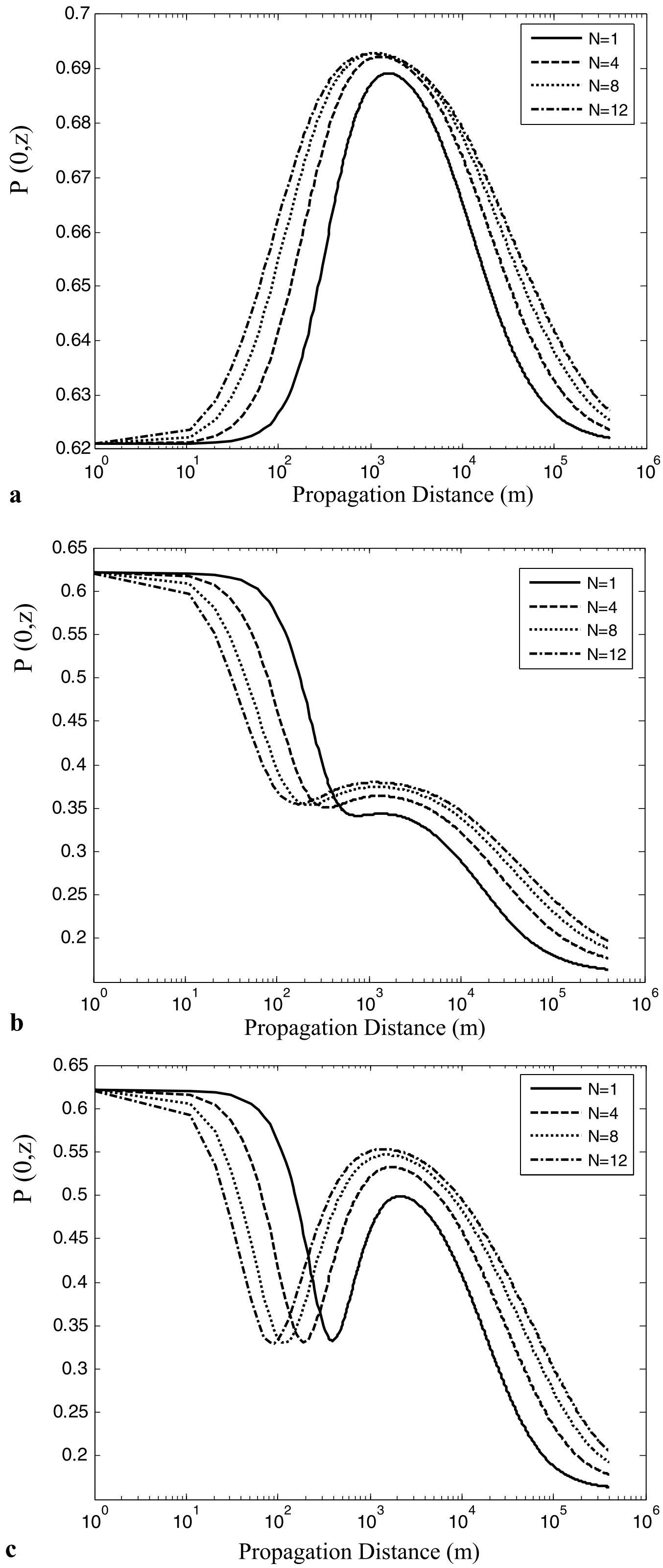
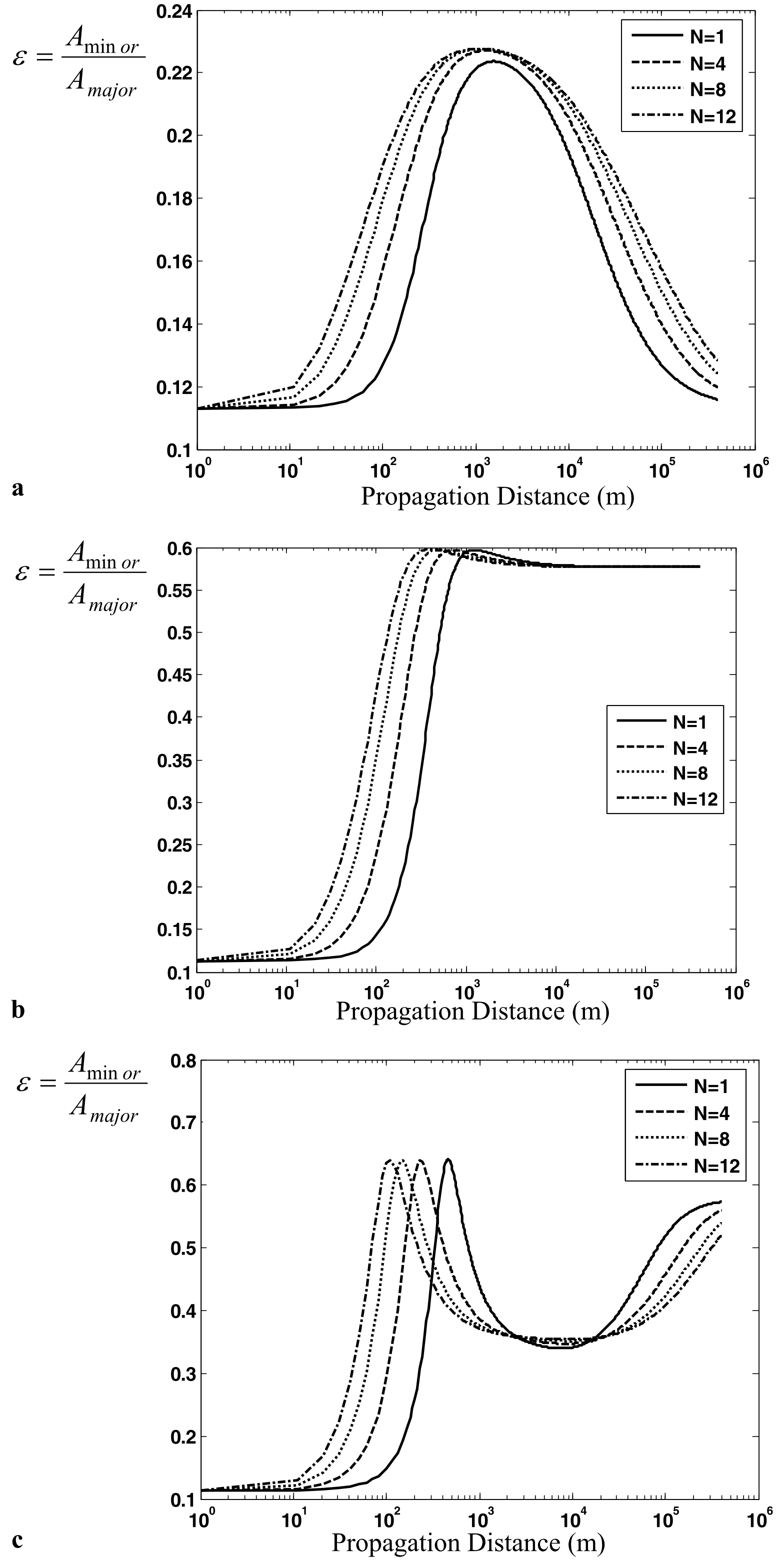


Fig. 2 Changes in the spectral degree of polarization along the z-axis of PCFT beam propagating through the turbulent atmosphere with different N with the parameter $\lambda = 632.8$ μm, $A_x = 2$, $A_y = 1$, $B_{xy} = 0.2\exp(\frac{i\pi}{3})$, $\delta_{xy} = 4$ mm, $C_n^2 = 10^{-14}$ m$^{-2/3}$. (**a**) The isotropic source: $\sigma_x = \sigma_y = 2$ cm, $\delta_{xx} = \delta_{yy} = 2$ mm; (**b**) the semi isotropic source: $\sigma_x = 1$ cm, $\sigma_y = 2$ cm, $\delta_{xx} = \delta_{yy} = 2$ mm; (**c**) the anisotropic source: $\sigma_x = 1$ cm, $\sigma_y = 2$ cm, $\delta_{xx} = 2$ mm, $\delta_{yy} = 3$ mm

We were interested in determining the behavior of the degree of ellipticity ε (corresponding to (12)) with respect to the propagation distance for several orders of the circular flat-topped beams N. In Fig. 3(a), it is assumed that the source is an isotropic one. The degree of ellipticity of the laser beam "ε" reaches a maximum value in propagation

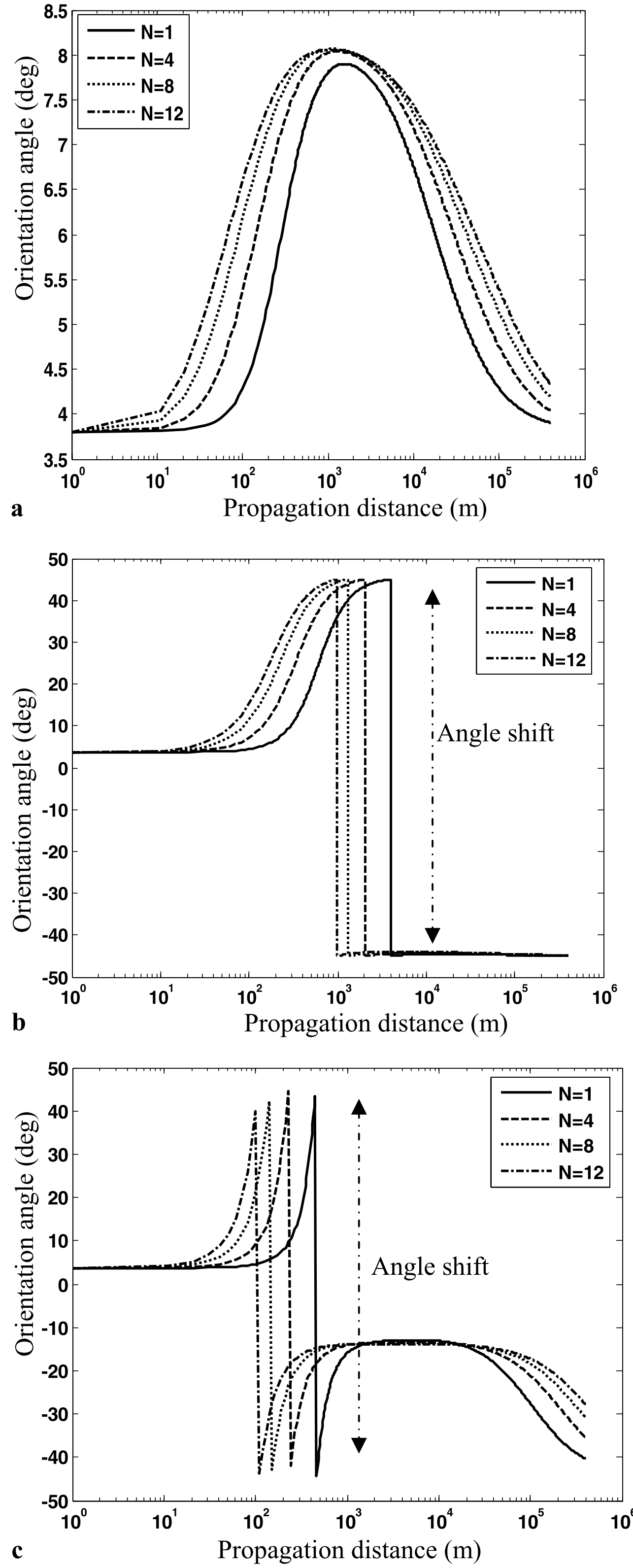
Fig. 3 The changes in the degree of ellipticity $\varepsilon = \frac{A_{\mathrm{minor}}}{A_{\mathrm{major}}}$ along the z-axis of a PCFT beam propagating through the turbulent atmosphere with different N. The source parameters are the same as in Fig. 2



distance and then returns to its value in the source plane after propagation in long distance. Also, Fig. 3(a) clearly demonstrates that increasing of the flat-topped order N, will increase the amount "ε" for a PCFT.

Figure 3(b) shows change of "ε" versus the propagation distance for the case that the source is semi isotropic. It can be seen that increasing of the propagation distance will increase the amount "ε", and in the far field "ε" of various

Fig. 4 The change in the orientation angle θ_0 along the z-axis of PCFT beam propagating through the turbulent atmosphere with different N. The source parameters are the same as in Fig. 2



flat-topped beams, with any arbitrary N value, it approaches a definite number.

In Fig. 3(c), we can see the effect of anisotropic source on the degree of ellipticity for the laser beam "ε". It is evident from this figure that on increasing of the propagation distance the value of "ε" reaches a minimum and maximum in propagating distance, and also increasing the various flat-topped beams "N" will decrease the amount "ε" for a PCFT beam propagating through atmospheric turbulence up to a special propagation distance and for propagation distances larger than it, the amount "ε" does not imitate this regularity and the amount "ε" has for this special propagation distance a maximum value.

In the next step, we are interested in determining the change in the orientation angle θ_0 by using (10).

Figure 4(a) shows the change of θ_0 for isotropic PCFT beam versus propagation distance and for several orders of the circular flat-topped beams N. It is evident from this figure that the behavior of θ_0 is nearly the same as the behavior of "ε" and the spectral degree of polarization. The θ_0 reaches a maximum value in propagating distance and then returns to its value in the source plane after propagation over a long distance. Increasing the flat-topped order N will increase the amount θ_0.

Figure 4(b) shows the change of θ_0 for a semi isotropic PCFT beam versus propagation distance. It is seen that increasing of the propagation distance will increase θ_0 for a PCFT beam propagating through atmospheric turbulence up to a special propagation distance and for propagation distances larger than it, the laser beam is rotated, and θ_0 become changed into $-\theta_0$; this behavior is called an angle shift. Also it is obvious that θ_0 for the PCFT beam increases as the flat-topped order N increases and an angle shift exists for any arbitrary N value but the special propagation distance the related angle shift increases as the flat-topped order N increases.

Figure 4(c) shows the change of θ_0 for anisotropic PCFT beam versus propagation distance. Further investigation shows that the same shift angle at special propagation distance in Fig. 4(b) exists for Fig. 4(c) but θ_0 has a maximum value after this special propagation distance.

The reason of th shift angle in Fig. 4(b) and Fig. 4(c) is the same for the spectral degree of polarization behavior in Fig. 2(b) and Fig. 2(c).

4 Conclusion

In summary, general analytic expressions for the state of polarization, i.e. the size, the shape and the orientation of the polarization ellipse, of an isotropic, semi isotropic and anisotropic partially coherent flat-topped beam propagating in the turbulent atmosphere have been derived.

With the help of calculations we have then studied the changes in the state of polarization for the beams mentioned. In particular, we found that the behavior of the orientation (θ_0) is nearly the same as for the behavior of the shape (ε) and the spectral degree of polarization. The θ_0, ε and the spectral degree of polarization reach a maximum value in propagating distance and then return to the value in the source plane after propagation over a long distance. And increasing of the flat-topped order N will increase the amount θ_0, ε and the spectral degree of polarization.

It is also found that there is a special propagation distance for a semi isotropic and anisotropic PCFT beam propagating in turbulent atmosphere; for propagation distance smaller and larger than this, the behavior of the shape (ε) and the spectral degree of polarization relation to order flat-topped N are varied.

Finally, we found a specific distance for the orientation of semi isotropic and anisotropic PCFT beam propagating in a turbulent atmosphere, which for a propagation distance larger than it the orientation (θ_0) becomes changed into $-\theta_0$; this behavior is called an angle shift.

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