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Relativistic self-focusing and self-channeling of Gaussian laser beam in plasma

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Abstract In the present paper, we have studied the self-focusing of a laser beam in a relativistic plasma. We have set up the non-linear differential equation for the beam width parameter of the main beam by using the moment theory approach and solved it numerically by the Runge–Kutta method. The results obtained are in agreement with the findings of the simulation (3D PIC). A new stable form of self-channeling propagation has also been observed.

1 Introduction

The interaction of intense laser pulses with plasmas is important for a number of applications such as particle acceleration by plasma waves [1–4] for inertial confinement fusion. These applications require the pulse to propagate over several Rayleigh lengths without loss of energy. With the advent of compact short pulse tetra watt laser systems, one can attain focused intensities as high as 10^{18} W/cm^2 and, as a result, it has become possible to investigate the new regime of relativistic laser–matter interactions. One of the most promising topics of study is relativistic self-focusing and self-channeling of an intense laser beam in a plasma. In order to generate relativistic electron oscillation and pair production [5, 6] by laser pulses in plasmas, self-focusing and dielectric increase are desirable [7]. The relativistic self-focusing length has been derived by Hora [7] on the basis of geometrical optics.

When the power of the beam is very high, it causes an electron oscillatory velocity comparable to the velocity of light, which modifies the effective dielectric constant of plasma and hence effects the self-focusing of beam. Non-linearity in this case arises due to increase in mass of electron, which oscillates at relativistic velocities in intense laser field. Relativistic laser plasma interactions have been studied in detail by many workers both theoretically and experimentally [7–16] and reviewed thoroughly [17, 18]. Pukhov and Meyer-ter-vehn [19] reported three dimensional particle in cell (3D PIC) simulations for laser pulse propagation in a near critical underdense plasma far above the threshold for relativistic filamentation. The key feature predicted by these simulations is the formation of the narrow, single propagation channel, containing the significant part of laser energy. The simulation reveals the importance of relativistic electrons traveling with light pulse and generating multi-mega gauss magnetic field that strongly influences the light propagation. Most of the analysis of self-focusing are based on Paraxial Ray approximation [20, 21]. The problem with this approach is that it overemphasizes the field closest to the beam axis with subsequent loss of accuracy. The moment theory approach developed by Vlasov et al. [22] does not suffer from this defect. This theory was used to study equilibrium beam radius of a self-trapped Gaussian beam in case of ponderomotive non-linearity [23]. We see from the plasma-wave solution of the Maxwellian equations, that electromagnetic waves in vacuum are transversal, but for beams of finite diameter, the pure transversality is no longer valid, even in vacuum. Lax et al. [24] have shown that there is a longitudinal electric field component in an electromagnetic beam in a vacuum if the paraxial ray approximation is applied up to first order. For the sake of simplicity only the transversal components of laser field are evaluated and longitudinal components are not taken in to consideration in

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the present paper. However, longitudinal components should be taken for an exact formulation, while dealing with non-linear phenomenon [25].

In the present paper, the authors have followed the moment theory approach for studying the self-focusing of a laser beam in a plasma with relativistic non-linearity. In Sect. 2, we have set up and solved the wave equation for the laser beam. A detailed discussion of results is presented in Sect. 3.

2 Propagation of Gaussian laser beam

Consider the propagation of a laser beam of angular frequency ω_0 in a relativistic plasma along z-axis. The initial intensity distribution is assumed to be Gaussian. The slowly varying electric field E satisfies the following wave equation:

$$\nabla^2 E - \nabla(\nabla . E) + \frac{\omega_0^2}{c^2}\epsilon E = 0. \tag{1}$$

Even if E has a longitudinal components [21], the second term $\nabla(\nabla . E)$ of (1) can be neglected, provided $\frac{c^2}{\omega_0^2}|\frac{1}{\epsilon}\nabla^2 \ln\epsilon| \ll 1$,

$$\nabla^2 E + \frac{\omega_0^2}{c^2}\epsilon E = 0. \tag{2}$$

On substituting

$$E = E_0(r,z)\exp\left[\iota\{\omega_0 t - k_0 z\}\right] \tag{3}$$

into (2), one obtains in the Wentzel–Kramers–Brillouin (WKB) approximation

$$\iota\frac{dE_0}{dz} = \frac{1}{2k_0}\nabla_\perp^2 E_0 + \chi\left(E_0 E_0^\star\right)E_0 \tag{4}$$

where $\chi(E_0 E_0^\star) = \frac{k_0}{2\epsilon_0}(\epsilon - \epsilon_0)$ and $\epsilon = \epsilon_0 + \epsilon_1(|E_0|^2)$ where $\epsilon_0 = 1 - \frac{\omega_p^2}{\omega_0^2}$ and $\epsilon_1(|E_0|^2)$ are the linear and non-linear parts of the dielectric constant respectively. $k_0 = \frac{\omega_0}{c}\sqrt{\epsilon_0}$ and $\omega_p = \sqrt{\frac{4\Pi n_e e^2}{m_e}}$ are propagation constant and plasma frequency respectively.

Now from the definition of the second order moment, the mean square radius of the beam is given by

$$\langle a^2\rangle = \frac{\iint (x^2+y^2)E_0 E_0^\star\, dx\, dy}{I_0}. \tag{5}$$

From this one can obtain the following equation:

$$\frac{d^2\langle a^2\rangle}{dz^2} = \frac{4I_2}{I_0} - \frac{4}{I_0}\iint Q\left(E_0 E_0^\star\right) dx\, dy \tag{6}$$

where I_0 and I_2 are the invariants of (4) [23]:

$$I_0 = \iint |E_0|^2\, dx\, dy, \tag{7}$$

$$I_2 = \iint \frac{1}{2k_0^2}\left(|\nabla_\perp E_0|^2 - F\right) dx\, dy \tag{8}$$

where from Vlasov et al. [22]

$$F\left(E_0 E_0^\star\right) = \frac{1}{k_0}\int \chi\left(E_0 E_0^\star\right) d\left(E_0 E_0^\star\right) \tag{9}$$

and

$$Q\left(E_0 E_0^\star\right) = \left[\frac{E_0 E_0^\star \chi(E_0 E_0^\star)}{k_0} - 2F\left(E_0 E_0^\star\right)\right]. \tag{10}$$

For $z > 0$, we assume an energy conserving Gaussian ansatz for laser intensity [20, 21],

$$E_0 E_0^\star = \frac{E_{oo}^2}{f_0^2}\exp\left\{-\frac{r^2}{r_0^2 f_0^2}\right\}. \tag{11}$$

From (5), (7) and (11), it can be shown that

$$I_0 = \pi r_0^2 E_{oo}^2, \tag{12}$$

$$\langle a^2\rangle = r_0^2 f_0^2 \tag{13}$$

where f_0 is dimensionless beam width parameter and r_0 is the beam width at $z = 0$.

Now, from (6), (7), (8), (9), (10), (11), (12) and (13) one can get

$$\frac{d^2 f_0}{d\xi^2} + \frac{1}{f_0}\left(\frac{df_0}{d\xi}\right)^2 = \frac{2k_0^2}{\pi E_{00}^2 f_0}\left[I_2 - \iint Q\left(|E_0|^2\right) dx\, dy\right] \tag{14}$$

where $\xi = (z/k_0 r_0^2)$ is the dimensionless propagation distance. Equation (14) is the basic equation for studying the self-focusing of a Gaussian laser beam in a non-linear, non-absorptive medium. The relativistic general expression for the plasma frequency is given by

$$\omega_{pr}^2 = \frac{\omega_p^2}{\gamma_0} \tag{15}$$

where γ_0 is the relativistic factor given by

$$\gamma_0 = \left(1 + \alpha E_0 E_0^\star\right)^{\frac{-1}{2}} \tag{16}$$

where $\alpha = \frac{e^2}{m_0^2\omega_0^2 c^2}$.

For a relativistic plasma,we can express the non-linear part of the dielectric constant by the following equation:

$$\epsilon_1 = \frac{\omega_p^2}{\omega_0^2}\left[1 - \left(1 + \alpha E_0 E_0^\star\right)^{\frac{-1}{2}}\right]. \tag{17}$$

With the help of (8), (9), (10), (11), (14) and (17), we get

$$\frac{d^2 f_0}{d\xi^2} + \frac{1}{f_0}\left(\frac{df_0}{d\xi}\right)^2 = \frac{1}{f_0^3} + \left(\frac{\omega_p r_0}{c}\right)^2 \frac{1}{f_0}\left(\frac{2f_0^2}{\alpha E_{00}^2}\left[1-\sqrt{1+\frac{\alpha E_{00}^2}{f_0^2}} + \log\left(\frac{\sqrt{1+\alpha E_{00}^2/f_0^2}+1}{\sqrt{1+\alpha E_{00}^2/f_0^2}-1}\right)\right]\right). \quad (18)$$

The initial conditions of the plane wavefront are $\frac{df_0}{d\xi}=0$ and $f_0=1$ at $\xi=0$. Equation (18) describes the changes in the beam width parameter of a Gaussian beam on account of the competition between diffraction divergence and non-linear focusing terms as the beam propagates in the relativistic plasma.

3 Discussion

In the present investigation, we have considered only the relativistic non-linearity. We have not taken into account the ponderomotive non-linearity. In fact, relativistic self-focusing occurs almost instantaneously in the time of the order of a period of the optical oscillation, while the ponderomotive self-focusing arises later in time because of the motion of plasma from the axis of beam. Therefore, the ponderomotive self-focusing only adds to relativistic self-focusing and does not obstruct the relativistic self-focusing. Equation (18) governs the behavior of dimensionless beam width parameter f_0 of beam as a function of dimensionless distance of propagation ξ. We have solved this equation numerically for the following set of parameters: $\omega_0 = 1.778\times10^{15}$ rad s^{-1}, $(\frac{w_0 r_{em}}{c})^2 = 35$, $\alpha E_{00}^2 = 4.0, 4.5, 5.0$, $\frac{\omega_p}{\omega_0} = 0.5$ and also for different values of αE_{00}^2 and $\frac{\omega_p}{\omega_0}$. The results are depicted in the form of graphs in Figs. 1, 2 and 3.

The first term on the right hand side of (18) represents the diffraction phenomenon. The second term, which arises due to the relativistic non-linear effect resulting from relativistic mass correction, represents the non-linear refraction. The diffraction term leads to diffractional divergence of the beam, while the non-linear term is responsible for self-focusing of beam due to relativistic effect. The relative magnitude of these terms determines the focusing/defocussing behavior of the beam. If the first term on right hand side of (18) dominates over the second term, the beam diverges, on the other hand if second term over power the first term, focusing of beam takes place.

Figure 1 represents the variation of beam width parameter f_0 of beam with dimensionless distance of propagation ξ for three different values of intensities viz. $\alpha E_{00}^2 = 4.0, 4.5, 5.0$. From Fig. 1, it is observed that with the increase in value of intensity parameter, the diffraction of beam starts earlier. This is due to dominance of diffraction term over the non-linear self-focusing term with increase in intensity. It is further observed from Fig. 1 that

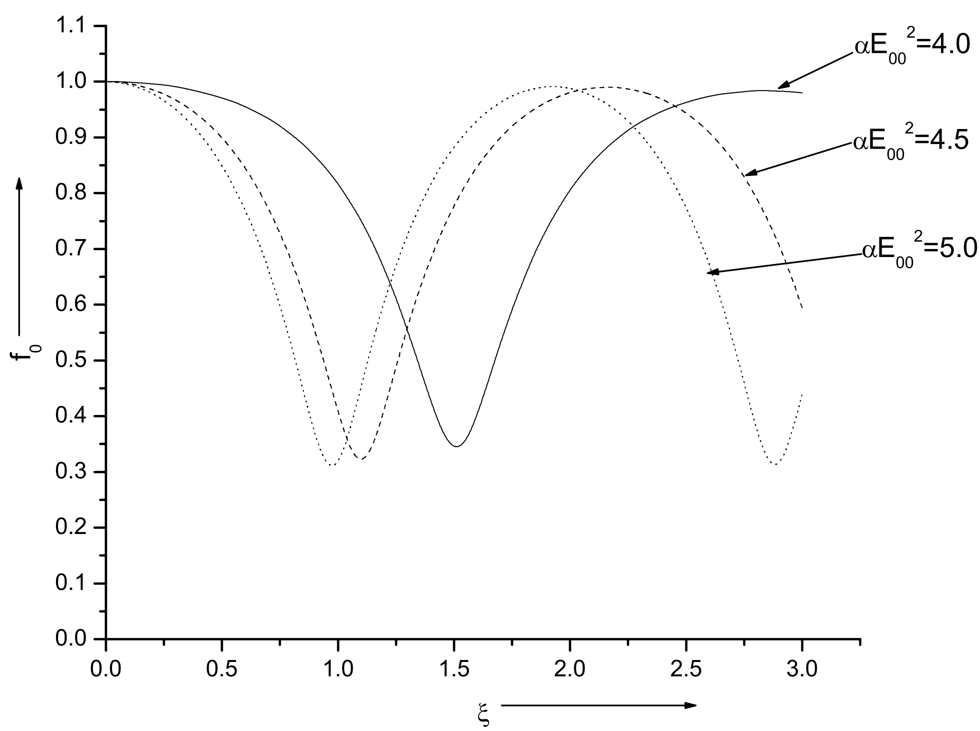

Fig. 1 Variation of beam width parameter f_0 against the normalized distance of propagation $\xi = \frac{Z}{R_d}$ for different values of intensities viz. $\alpha E_{00}^2 = 4.0, 4.5, 5.0$ and for the following set of parameters: $(\frac{\omega_0 r_{em}}{c})^2$=35.0, $\frac{\omega_p}{\omega_0}=0.5$, $\omega_0 = 1.778\times10^{15}$ rad s^{-1}

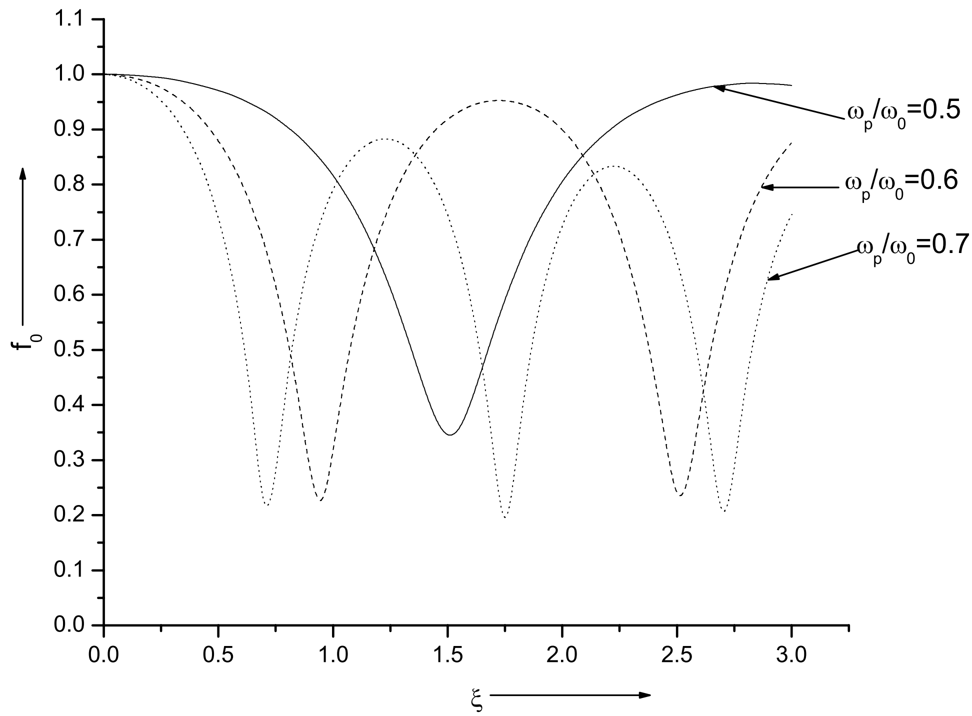

Fig. 2 Variation of beam width parameter f_0 against the normalized distance of propagation $\xi = \frac{Z}{R_d}$ for different values of $\frac{\omega_p}{\omega_0}$ viz. $\frac{\omega_p}{\omega_0} = 0.5, 0.6, 0.7$ and for intensity $\alpha E_{00}^2 = 4.0$ and for the parameters mentioned in Fig. 1

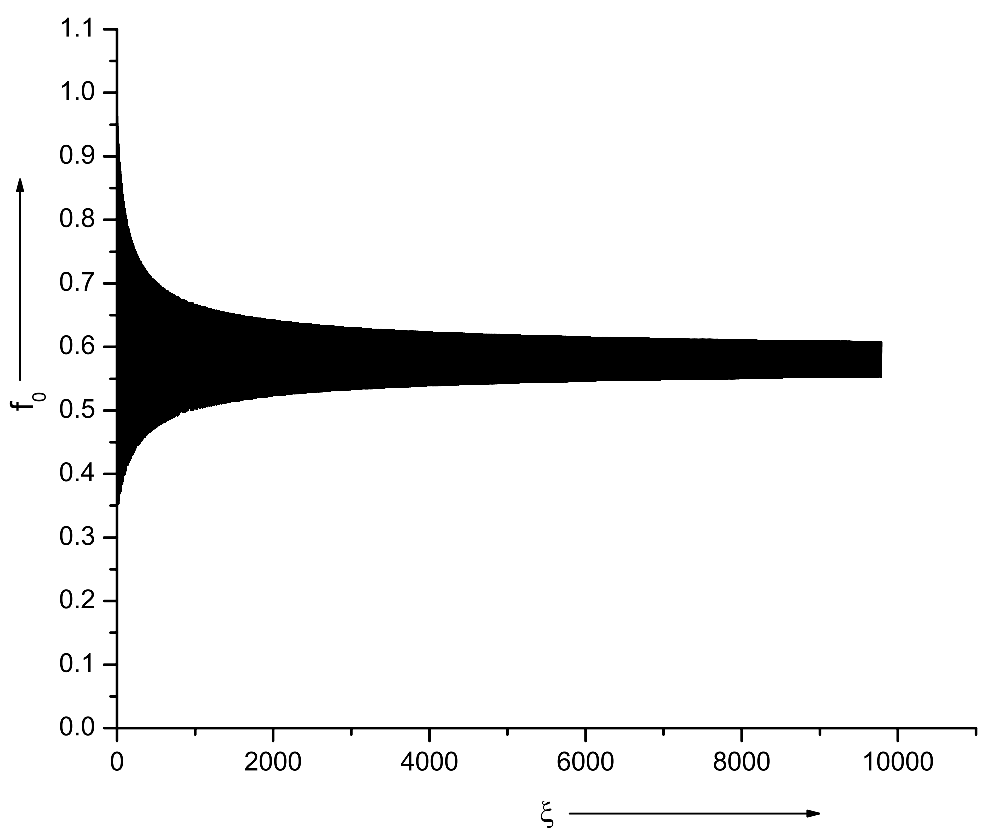

Fig. 3 Variation of beam width parameter f_0 against the normalized distance of propagation $\xi = \frac{Z}{R_d}$ for $\alpha E_{00}^2 = 4.0$, $\frac{\omega_p}{\omega_0} = 0.5$ and for the parameters mentioned in Fig. 1

the self-focusing length decreases with increase in intensity of the beam. This is due to the fact that at relativistic intensity, a quasi-stationary magnetic field is generated, the pinching effect of which adds to self-focusing [19]. Hauser et al. [14] investigated the relativistic self-focusing length of the intense laser beams in plasma by applying the technique of WKB approximation and observed that self-focusing length of the beam decreases with increase in intensity of the beam up to 2.44×10^{18} W/cm^2. We have taken $\alpha E_{00}^2 = 4.0, 4.5, 5.0$, which corresponds to intensities 3.68×10^{15}, 4.14×10^{15} and 4.6×10^{15} W/cm^2, respectively, and observed that as we increase the intensity of the beam, self-focusing length decreases and therefore supports the results of Hauser et al. [14].

Figure 2 depicts the variation of beam width parameter f_0 of the beam with the dimensionless distance of propagation ξ for three different values of the plasma density $\frac{\omega_p}{\omega_0} = 0.5, 0.6, 0.7$, respectively, and for $\alpha E_{00}^2 = 4.0$. From the figure it is observed that focusing of the beam takes place earlier with increase in plasma density. It is further observed that self-focusing becomes stronger with increase in plasma density. This is due to the fact that at relativistic intensities, if we increase the plasma density a beam with more relativistic electrons travels with the laser pulse, which generates a higher current and consequently a very high quasi-stationery magnetic field is generated. As a result the pinching effect of the magnetic field becomes stronger, which further adds to self-focusing. This prediction is in agreement with the simulation (3D PIC) results reported by Pukhov and Meyer-ter-vehn [19]. It is further observed from Fig. 2 that there is a periodic self-focusing due to the dynamic balance between two competing non-linear effects, i.e. diffraction and non-linear refraction. Diffraction due to the first term leads to a decrease of the beam intensity on the one hand and relativistic non-linearity due to the second term leads to an increase of beam intensity i.e. self-focusing on the other hand. Sadighi et al. [16] investigated the propagation of a Gaussian beam in an underdense plasma with upward increasing density ramp and reported that the relativistic effect is more pronounced in the region of increasing plasma density. As a result, the laser beam focuses more during propagation in a plasma density ramp. In the present analysis, we have not used the density ramp. In fact we have taken $\frac{\omega_p}{\omega_0} = 0.5, 0.6, 0.7$ and observed that as we increase the plasma density, self-focusing becomes stronger and thus supports the results of [16].

Figure 3 represents the variation of beam width parameter f_0 of the laser beam with the dimensionless distance $\xi = \frac{Z}{R_d}$ of propagation. Figure 3 represents a very interesting result as far as the fast ignitor scheme is concerned. We know that the fast ignitor concept due to Tabak et al. [4] presents an exciting new route to the achievement of ignition in inertial confinement fusion. In this scheme, an ultra intense laser pulse is guided through the channel to the edge of the core where it is absorbed producing a large flux of MeV electrons. These electrons then deposit their energy within the core, thus heating it to ignition temperatures before hydrodynamic disassembly can take place. In this process, a channeling pulse must propagate through plasma regions without being absorbed or without beam break-up due to filamentation. So, it is very important for the fast ignition scheme to understand if such propagation is possible. Now, in our investigations such self-channeling of a laser pulse is observed. The laser pulse propagates through the plasma region without any absorption up to several Rayleigh lengths as is evident from Fig. 3. Jones et al. [13] investigated the self-focusing effects of higher power laser beam with a dense laser produced plasma. In their analysis, they investigated the filament radius as a function of depth into the plasma at different times (cf. Fig. 3, Ref. [13]). It is obvious from the graph (Fig 3, Ref. [13]), that at any given, fixed time step, there is a sharp shrinking of the laser beam with the distance of propagation. A similar behavior is observed in the present investigation.

The results of the present analysis are useful in understanding the physics of high power laser driven fusion as well as to confirm the role of relativistic electrons traveling with a light pulse, reported by Pukhov and Meyer-ter-vehn [19] in their (3D PIC) simulation study.

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