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Laser diode feedback interferometry in flowing Brownian motion system: a novel theory

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Abstract A novel theory is developed for the laser-diode feedback interferometry in the flowing Brownian motion system irradiated by a focused Gaussian incident field. The back-emitted power from the laser cavity and the corresponding time autocorrelation function are derived with the aid of both the modified Lang–Kobayashi rate equations and dynamic light scattering theory. As a result, the time autocorrelation function of the back-emitted power from the laser diode includes a new character parameter dependent on the nature of laser diode and a new Gaussian term dependent on the transit time of the illuminated target through the Gaussian beam waist, which may also be applied to solid-state lasers.

Keywords Laser diode feedback interferometry · Flowing Brownian motion system · Nanoparticle · Time autocorrelation function · Dynamic light scattering · Lang–Kobayashi rate equations

1 Introduction

Laser feedback interferometry (LFI), which is also known as laser self-mixing interferometry (LSMI), is an interferometric sensing technique based on the optical mixing of the field in the laser cavity with the weak field back-reflected or back-scattered by a remote target. The pioneering works were reported by Rudd [1] and Churnside [2, 3] using gas lasers such as He–Ne laser and CO_2 laser. The basic idea of self-mixing modulation effect in class-B lasers was initiated by Otsuka [4, 5] using $LiNdP_4O_{12}$ (LNP) solid-state laser and later the self-mixing effect in the laser diode was reported by Lang and Kobayashi [6], Shinohara et al. [7], Beheim and Fritschusing [8] and others. The feedback interferometry in class-B lasers has attracted attentions mainly because class-B lasers are much more sensitive to perturbations from the outside world than the conventional gas lasers [9]. The advantages of this technique with class-B lasers are sensitive, compact, low-cost, and simple in experimental setup etc. Thanks to these advantages many applications using this technique have been demonstrated, including ranging [10, 11], velocitometry [7, 12–22], displacement [23], vibrometry [24–26] and so on.

A new application of the feedback interferometry in class-B lasers is the analysis of particle size and flow velocity [27–37]. Otsuka et al. [27–33] measure the particle size and flow velocity by a laser-diode-pumped thin-slice solid-state laser feedback interferometry. The average particle size is determined by a Lorentz fitting of the measured power spectrum. Liu et al. [34] measure the particle size by an erbium-doped fiber ring laser feedback interferometry technique. Zakian et al. [35, 36] measure the particle size and flow velocity by a laser-diode feedback interferometry (LDFI). Wang et al. [37] extract the particle size distribution from the power spectrum of LDFI with the aid of an improved inverse algorithm.

In the above experiments, the laser feedback interferometry can be theoretically described as follows. The light emitted from a laser cavity is focused onto the particles. Some of the light scattered by particles in the sample cell may be focused by the same lens into the laser cavity and interfere with the original field. The gain of the cavity is changed and therefore the steady-state condition of the compound cavity will experience a modulation. The modulation produces

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a Doppler shift of the original optical frequency, which is dependent on the motion of the particles. For instance, particles moving at a single velocity would produce a single frequency peak in the power spectrum of the laser whereas diffusional particles would broaden the frequency distribution of the laser output. Therefore, laser feedback interferometry is considered to work as an optical heterodyne interferometry (OHI). The similarity between the LDFI and OHI was proved by Zakian et al. [35, 36] with the combination of the Lang–Kobayashi rate equations and the dynamic light scattering theory, and hence the LDFI experimental results were explained using the OHI theory.

The OHI theory is based on the assumption of a planar incident field [38]. However, the measurements usually employ a lens to focus the incident beam to build a Gaussian beam zone in the sample cell so that an effective width of measurement zone is defined by the beam waist. In this case, the number of particles within the measurement volume varies randomly and the intensities of incident light illuminating these particles depend on the particles' locations. This leads to a random fluctuation of the weak backscattered field and hence introduces some additional effects on the signals, which has not been taken into consideration so far.

The paper describes a theoretical approach towards the case of the LDFI in a flowing Brownian motion system irradiated by a focused Gaussian incident field, under the adiabatic eliminations in the modified Lang–Kobayashi rate equations including a linewidth enhancement factor and the Langevin-type of temporal evolution for Brownian particles coupled to the modified Lang–Kobayashi rate equations. The back-emitted power of the LDFI and its time autocorrelation function are obtained, and special cases are studied.

It is worthwhile to note that the Lang–Kobayashi rate equations were also applied to the solid-state lasers with weak external feedback [39–43] and so were the modified Lang–Kobayashi rate equations [44, 45]. The differences of rate equations between diode lasers and solid-state lasers, which are the typical values of the character parameters and the used symbols in the above mentioned references, have no qualitative change on the scope of applicability of our approach. Therefore, though our theory is developed in the case of laser diodes, it may also be applied to solid-state lasers.

2 Theory

In this section, the modified Lang–Kobayashi rate equations are introduced firstly as a theory basis. Secondly, the back-emitted power from a laser diode is deduced for a flowing Brownian motion system, with a combination of the steady-state solutions of the modified Lang–Kobayashi rate equations and the dynamic light scattering theory. As a result, the expression of the time autocorrelation function of the back-emitted power is obtained.

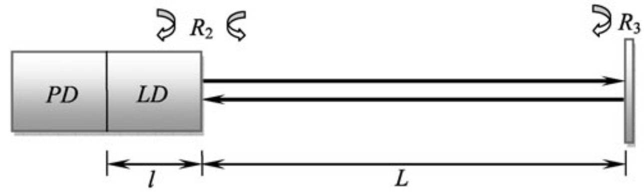

Fig. 1 Schematic diagram of LDFI in external reflector

2.1 Brief introduction of the modified Lang–Kobayashi rate equations

The schematic diagram of the interferometric measurement apparatus composed of a laser diode and an external reflector is shown in Fig. 1. The light emitted from a laser diode reflects off the external reflector, reenters the laser cavity after an external round-trip time and interferes with the original field. The configuration as Fig. 1 can be looked as a compound cavity composed of the laser diode and the external reflector. The complex electric field in the compound cavity is assumed to be of the form of the product of an envelope function $E(t)$ modulated by a rapidly oscillating optical field $e^{i\omega_0 t}$ with an assumed center-frequency ω_0. The modified rate equation for the complex electric field in the compound cavity is then given by: [6, 46]

$$\frac{dE(t)}{dt} = \left\{\frac{1}{2}(1+i\alpha)G_n\big(N(t)-N_{\text{th}}\big)\right\}E(t) + \tilde{\kappa}E(t-\tau)e^{-i\omega_0\tau}, \quad (1)$$

where τ is the external cavity round-trip time, given by $\tau = 2L/c$, L is the length of the external cavity, α is the linewidth enhancement factor. The last term in (1) is the important addition made by Lang and Kobayashi: the inclusion of the effect from the feedback of a time-delayed proportion of the optical field. The coupling coefficient $\tilde{\kappa}$ is given by $\tilde{\kappa} = (1-R_2)\sqrt{R_3}/\tau_c\sqrt{R_2}$ in which R_2 is the reflectivity of the optical power on the right laser-diode facet, R_3 is the reflectivity of the optical power on the external reflector. The meaning of the symbols used in the context is as follows:

$\lvert E(t)\rvert$	slowly varying envelope of the electric field, normalized so that $\lvert E(t)\rvert^2$ corresponds to photon density
$\phi(t)$	slowly varying electric field phase
ω_0	angular oscillation frequency of the unperturbed laser diode
G_n	modal gain coefficient (typical value $G_n = 8\times 10^{-13}\ \text{m}^3\,\text{s}^{-1}$)
$N(t)$	spatially averaged carrier density [m^{-3}]

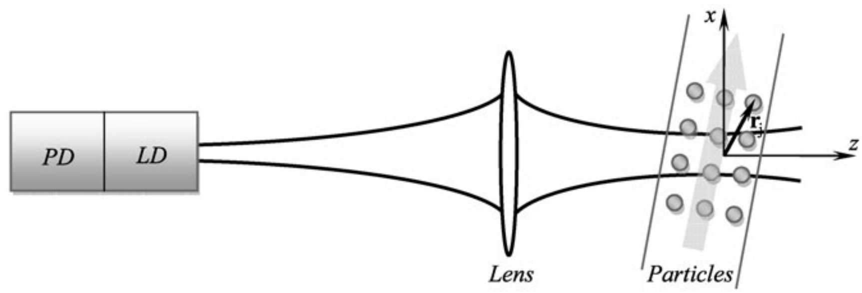

Fig. 2 Schematic diagram of LDFI in flowing Brownian motion system

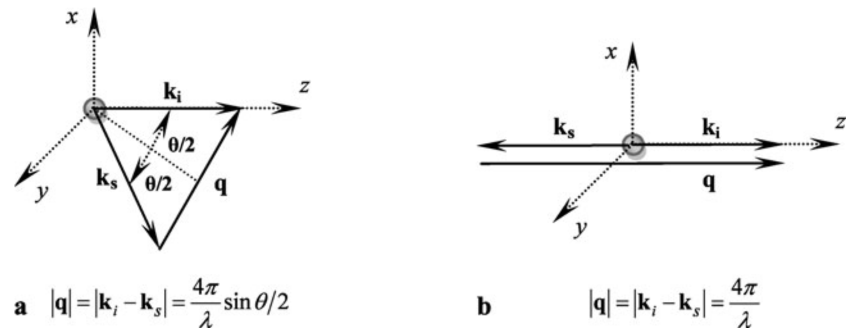

Fig. 3 Definition of the scattering vector ((**a**) is the general definition; (**b**) is the special definition in the case of LDFI, $\mathbf{k}_i$ is the incident wave vector in medium, $\mathbf{k}_s$ is the scattered wave vector in medium, λ is wavelength in medium, and θ is scattering angle which is equal to π in the case of LDFI)

N_0 carrier density at transparency (typical value $N_0 = 1.4 \times 10^{24}\ \text{m}^{-3}$)

N_{th} carrier density at threshold (typical value: $N_{\text{th}} = 2.3 \times 10^{24}\ \text{m}^{-3}$)

τ_p photon lifetime within the cavity; the typical value for a Fabry–Perot SL is $\tau_p = 1.6$ ps and the following equation holds $1/\tau_p = G_n(N_{\text{th}} - N_0)$

τ_e carrier lifetime (typical value $\tau_e = 2$ ns)

τ_c laser-diode cavity round-trip time $\tau_c = 2ln/c$ (typical value $\tau_c = 8$ ps), with l the length of the laser-diode cavity, n the laser-diode refractive index, c the velocity of light

G_{gen} electrical pumping term, given by $G_{\text{gen}} = J\eta/qd$, with J injection current density, η internal quantum efficiency, d active layer thickness, q electron charge

2.2 Emitted power of the laser diode feedback interferometry

Consider the case of the LDFI in a flowing Brownian motion system. The interferometric measurement apparatus composed of a laser diode and the flowing Brownian motion particles is schematically shown in Fig. 2. The external reflector in Fig. 1 is replaced by the flowing Brownian motion particles on which the light emitted from a laser cavity is focused by a convex lens. Some of the light scattered by the particles in the sample cell may be collected by the same lens into the laser cavity and interfere with the original field. When the Gaussian beam is focused on the flowing Brownian motion particle system, the square root of the reflectivity of the optical power on particles is given by $C_{\text{Mie}} \sum_{j=1}^{N_T} A[\mathbf{r}_j(t-\tau/2)] e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\tau/2)]}$, where C_{Mie} is a complex parameter depending on both the Mie coefficients in light scattering theory and the optical arrangement; $A[\mathbf{r}_j(t-\tau/2)]$ is an effective amplitude function that describes the incident Gaussian field amplitude as a function of particle position vector $\mathbf{r}_j(t-\tau/2)$ at a time instant $t-\tau/2$; N_T is the total number of particles in the sample cell; $\mathbf{q}$ is the scattering vector defined in Fig. 3, whose magnitude is given as $|\mathbf{q}| = \frac{4\pi}{\lambda}\sin\theta/2$ with λ wavelength in medium and θ scattering angle. For simplicity we assume that only the scattered light with those scattering angles very close to 180 deg can be collected by the lens and enter the laser cavity (i.e. $\theta \cong 180^{\circ}$). Thus, the rate equation for the complex electric field with the external feedback caused by the flowing Brownian motion particle system can be written as:

$$\frac{dE(t)}{dt} = \left\{\frac{1}{2}(1+i\alpha)G_n(N(t)-N_{\text{th}})\right\}E(t) + E(t-\tau)e^{-i\omega_0\tau}\frac{1-R_2}{\tau_c\sqrt{R_2}}C_{\text{Mie}} \times \sum_{j=1}^{N_T} A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right]e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})]}. \quad (2)$$

Since the rate equation includes the effect of a single round trip through the external cavity only, this model is inherently limited to the case of weak optical feedback where the contribution to the electric field rate equation from the second and subsequent round trips can be ignored.

By defining $E(t)$ in terms of a slowly varying amplitude and the phase, i.e. $E(t)=|E(t)|e^{i\phi(t)}$, the complex electric field can be separated into two purely real equations for the amplitude and the phase:

$$\frac{d|E(t)|}{dt}=\frac{1}{2}\left[G_n\big(N(t)-N_0\big)-\frac{1}{\tau_p}\right]\big|E(t)\big| + \mathrm{Re}\left\{\big|E(t-\tau)\big|e^{i[\phi(t-\tau)-\phi(t)-\omega_0\tau]}\frac{1-R_2}{\tau_c\sqrt{R_2}} \times C_{\mathrm{Mie}}\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right]e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})]}\right\}, \quad (3a)$$

$$\frac{d\phi(t)}{dt}=\frac{1}{2}\alpha G_n\big(N(t)-N_{\mathrm{th}}\big) + \mathrm{Im}\left\{\frac{|E(t-\tau)|}{|E(t)|}e^{i[\phi(t-\tau)-\phi(t)-\omega_0\tau]}\frac{1-R_2}{\tau_c\sqrt{R_2}} \times C_{\mathrm{Mie}}\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right]e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})]}\right\}. \quad (3b)$$

The above rate equations are coupled with another rate equation for the carrier density:

$$\frac{dN(t)}{dt}=G_{\mathrm{gen}}-\frac{N(t)}{\tau_e}-G_n\big[N(t)-N_0\big]\big|E(t)\big|^2. \quad (4)$$

If the laser-diode response frequency is much higher than the external modulation frequency, dynamic equations (3) and (4) of the LDFI can be rewritten as the steady-state equation by assuming that the time derivatives of both the electric field amplitude $|E(t)|$ and the carrier density $N(t)$ are small enough to be set to zero. In the present system, the response angular frequency is of the order of GHz because the characteristic time of system is a sum of the carrier lifetime 2 ns and the external cavity round-trip time 1 ns for a distance 15 cm from the laser diode to the target. However, the modulation frequency of the Brownian motion is a few decade KHz [38] estimated from the power spectrum maximum linewidth in the dynamic light scattering. Therefore, the scope of the analysis is to find the steady-state solutions by letting the time derivative of $|E(t)|$ and $N(t)$ equal zeros. This results in:

$$N(t)=N_{\mathrm{th}}-\frac{2}{G_n}\mathrm{Re}\left\{\frac{|E(t-\tau)|}{|E(t)|}\frac{1-R_2}{\tau_c\sqrt{R_2}}C_{\mathrm{Mie}} \times\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right] \times e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})+\phi(t-\tau)-\phi(t)-\omega_0\tau]}\right\}, \quad (5)$$

$$\big|E(t)\big|^2=\frac{G_{\mathrm{gen}}\tau_e-N(t)}{\tau_e G_n[N(t)-N_0]}. \quad (6)$$

Substitution of (5) into (6) yields the back-emitted power,

$$P(t)\propto\big|E(t)\big|^2 = \frac{\tau_p}{\tau_e}\left(G_{\mathrm{gen}}\tau_e-N_{\mathrm{th}}+\frac{2}{G_n}\mathrm{Re}\left\{\frac{|E(t-\tau)|}{|E(t)|}\frac{1-R_2}{\tau_c\sqrt{R_2}}C_{\mathrm{Mie}} \times\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right]e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})+\phi(t-\tau)-\phi(t)-\omega_0\tau]}\right\}\right) \times\left(1-\mathrm{Re}\left\{\frac{|E(t-\tau)|}{|E(t)|}\frac{2\tau_p}{\tau_c}\frac{1-R_2}{\sqrt{R_2}}C_{\mathrm{Mie}} \times\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right] \times e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})+\phi(t-\tau)-\phi(t)-\omega_0\tau]}\right\}\right)^{-1}. \quad (7)$$

Assuming the optical feedback strength be sufficiently weak

$$\mathrm{Re}\left\{\frac{|E(t-\tau)|}{|E(t)|}\frac{2\tau_p}{\tau_c}\frac{1-R_2}{\sqrt{R_2}}C_{\mathrm{Mie}}\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right] \times e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})+\phi(t-\tau)-\phi(t)-\omega_0\tau]}\right\}\ll 1 \quad (8)$$

we yield the approximation of (7)

$$P(t)\propto\big|E(t)\big|^2 \approx\frac{\tau_p}{\tau_e}(G_{\mathrm{gen}}\tau_e-N_{\mathrm{th}})\left\{1+\frac{|E(t-\tau)|}{|E(t)|}\frac{\tau_p}{\tau_c}\frac{1-R_2}{\sqrt{R_2}} \times\left[C_{\mathrm{Mie}}\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right] \times e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})+\phi(t-\tau)-\phi(t)-\omega_0\tau]} +C^*_{\mathrm{Mie}}\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right] \times e^{-i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})+\phi(t-\tau)-\phi(t)-\omega_0\tau]}\right]\right\} \quad (9)$$

in which the raised asterisk denotes the complex conjugate. Equation (9) shows that the optical feedback causes the emitted power modulation. The power emitted in the absence of feedback, given by $P_0=\tau_p(G_{\mathrm{gen}}-N_{\mathrm{th}}/\tau_e)$, is independent of time t.

Simplification of (9) can be done by considering the case of the short cavities to which the Lang–Kobayashi model was limited. In this case, the round-trip time τ outside the cavity is the order of nanoseconds when the distance from laser diode to target is 15 cm. The time is much shorter than the period of the modulation frequency with the order of 10^{-5} s which is the reciprocal of the modulation frequency of a few decade KHz. $\phi(t-\tau)$ in (9) can be expanded in the Taylor series as $\phi(t-\tau)\approx\phi(t)-\phi'(t)\tau$. $\phi'(t)$, the first derivative of phase $\phi(t)$ with respect to t, represents the instantaneous phase deviation. The instantaneous deviation which is caused by the fluctuation of Brownian motion has the same order of magnitude as the modulation frequency of Brownian motion. According to the calculation of the order of magnitude, we find $\phi'(t)\tau\approx 10^{-5}\ll 1$ and hence $\phi(t)\cong\phi(t-\tau)$. In addition, the assumption of weak optical feedback implies $E(t)\cong E(t-\tau)$. Thus, (9) can be simplified to

$$P(t)\propto\frac{\tau_p}{\tau_e}(G_{\text{gen}}\tau_e-N_{\text{th}})\left\{1+\frac{\tau_p}{\tau_c}\frac{1-R_2}{\sqrt{R_2}}\times\left[C_{\text{Mie}}\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right]e^{i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})-\omega_0\tau]}+C^*_{\text{Mie}}\sum_{j=1}^{N_T}A\left[\mathbf{r}_j\left(t-\frac{\tau}{2}\right)\right]e^{-i[\mathbf{q}\cdot\mathbf{r}_j(t-\frac{\tau}{2})-\omega_0\tau]}\right]\right\}. \tag{10}$$

For the convenience of the time autocorrelation function calculation, (10) can be abbreviated to

$$P(t)\propto\frac{\tau_p}{\tau_e}(G_{\text{gen}}\tau_e-N_{\text{th}})\left[1+mF(t)\right], \tag{11}$$

where m is the character parameter of LDFI and $F(t)$ is time-dependent and includes the interesting information about the particle size and flowing velocity, given as

$$m=\frac{\tau_p}{\tau_c}\frac{1-R_2}{\sqrt{R_2}}, \tag{12}$$

$$F(t)=C_{\text{Mie}}\sum_{j=1}^{N_T}A\left[\mathbf{r}_j(t')\right]e^{i[\mathbf{q}\cdot\mathbf{r}_j(t')+\varphi]}+C^*_{\text{Mie}}\sum_{j=1}^{N_T}A\left[\mathbf{r}_j(t')\right]e^{-i[\mathbf{q}\cdot\mathbf{r}_j(t')+\varphi]}. \tag{13}$$

In which $t'=t-\tau/2$ and $\varphi=-\omega_0\tau$ is time-independent.

2.3 Time autocorrelation function of the back emitted power

The time autocorrelation function for the back-emitted power is defined by

$$R(\Delta t)\equiv\lim_{T\to\infty}\frac{1}{T}\int_{t_0}^{t_0+T}P^*(t)P(t+\Delta t)\,dt. \tag{14}$$

Here T, t_0 and Δt denote the averaging time, the beginning of averaging and the delay time respectively. The raised asterisk denotes the complex conjugate. When $P(t)$ is a stationary and ergodic quantity, the time average in (14) is independent of t_0 and is often abbreviated into:

$$R(\Delta t)=\left\langle P^*(t)P(t+\Delta t)\right\rangle, \tag{15}$$

where the angle bracket indicates the time average given explicitly in the prior equation. Substituting (11) into (15) and noting that the average of a sum of terms is the sum of the average of each term, we obtain

$$\begin{aligned}R(\Delta t)=P_0^2\Bigg\{&1+mC_{\text{Mie}}\sum_{j=1}^{N_T}\left\langle A\left[\mathbf{r}_j(t)\right]e^{i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}\right\rangle\\&+mC^*_{\text{Mie}}\sum_{j=1}^{N_T}\left\langle A\left[\mathbf{r}_j(t)\right]e^{-i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}\right\rangle\\&+mC_{\text{Mie}}\sum_{j=1}^{N_T}\left\langle A\left[\mathbf{r}_j(t+\Delta t)\right]e^{i[\mathbf{q}\cdot\mathbf{r}_j(t+\Delta t)+\varphi]}\right\rangle\\&+mC^*_{\text{Mie}}\sum_{j=1}^{N_T}\left\langle A\left[\mathbf{r}_j(t+\Delta t)\right]e^{-i[\mathbf{q}\cdot\mathbf{r}_j(t+\Delta t)+\varphi]}\right\rangle\\&+m^2C^2_{\text{Mie}}\sum_{j=1}^{N_T}\sum_{k=1}^{N_T}\left\langle A\left[\mathbf{r}_j(t)\right]A\left[\mathbf{r}_k(t+\Delta t)\right]\times e^{i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}e^{i[\mathbf{q}\cdot\mathbf{r}_k(t+\Delta t)+\varphi]}\right\rangle\\&+m^2(C^*_{\text{Mie}})^2\sum_{j=1}^{N_T}\sum_{k=1}^{N_T}\left\langle A\left[\mathbf{r}_j(t)\right]A\left[\mathbf{r}_k(t+\Delta t)\right]\times e^{-i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}e^{-i[\mathbf{q}\cdot\mathbf{r}_k(t+\Delta t)+\varphi]}\right\rangle\\&+m^2|C_{\text{Mie}}|^2\sum_{j=1}^{N_T}\sum_{k=1}^{N_T}\left\langle A\left[\mathbf{r}_j(t)\right]A\left[\mathbf{r}_k(t+\Delta t)\right]\times e^{-i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}e^{i[\mathbf{q}\cdot\mathbf{r}_k(t+\Delta t)+\varphi]}\right\rangle\\&+m^2|C_{\text{Mie}}|^2\sum_{j=1}^{N_T}\sum_{k=1}^{N_T}\left\langle A\left[\mathbf{r}_j(t)\right]A\left[\mathbf{r}_k(t+\Delta t)\right]\times e^{i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}e^{-i[\mathbf{q}\cdot\mathbf{r}_k(t+\Delta t)+\varphi]}\right\rangle\Bigg\},\end{aligned} \tag{16}$$

where t' is replaced by t in the case of no confusion in the above equation for brevity; $\varphi=-\omega_0\tau$ is the time-independent phase.

For the further calculation of the time autocorrelation function, three reasonable rules should be discussed. Firstly, we shall assume that in the sample cell there are many particles with random separations. This implies incoherent scattering or independent scattering so that there is no systematic relation among the phases of the waves scattered by the individual particles. Hence, the motion of one particle is uncorrelated with that of any other particle. However, the motion of one particle at time instant t is correlated with the motion of the same particle at time instant $t + \Delta t$. These make the first rule that the average of the product of one term and any other term is the product of the average of each term, but this is not the case for the average of the product of one term at time t and the same term at time instant $t + \Delta t$. According to this rule, the double summation in (16) can be expanded to:

$$\begin{aligned}
&\sum_{j=1}^{N_T}\sum_{k=1}^{N_T}\Big\langle A\big[\mathbf{r}_j(t)\big]A\big[\mathbf{r}_k(t+\Delta t)\big]e^{\pm i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]} \\
&\quad\times e^{\pm i[\mathbf{q}\cdot\mathbf{r}_k(t+\Delta t)+\varphi]}\Big\rangle \\
&= \sum_{j\neq k=1}^{N_T}\sum_{k=1}^{N_T}\Big\langle A\big[\mathbf{r}_j(t)\big]e^{\pm i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}\Big\rangle \\
&\quad\times\Big\langle A\big[\mathbf{r}_k(t+\Delta t)\big]e^{\pm i[\mathbf{q}\cdot\mathbf{r}_k(t+\Delta t)+\varphi]}\Big\rangle \\
&\quad+\sum_{j=1}^{N_T}\Big\langle A\big[\mathbf{r}_j(t)\big]A\big[\mathbf{r}_j(t+\Delta t)\big] \\
&\quad\times e^{\pm i\{\mathbf{q}\cdot[\mathbf{r}_j(t)+\mathbf{r}_j(t+\Delta t)]+2\varphi\}}\Big\rangle,
\end{aligned}\tag{17a}$$

$$\begin{aligned}
&\sum_{j=1}^{N_T}\sum_{k=1}^{N_T}\Big\langle A\big[\mathbf{r}_j(t)\big]A\big[\mathbf{r}_k(t+\Delta t)\big] \\
&\quad\times e^{\pm i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}e^{\mp i[\mathbf{q}\cdot\mathbf{r}_k(t+\Delta t)+\varphi]}\Big\rangle \\
&= \sum_{j\neq k=1}^{N_T}\sum_{k=1}^{N_T}\Big\langle A\big[\mathbf{r}_j(t)\big]e^{\pm i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}\Big\rangle \\
&\quad\times\Big\langle A\big[\mathbf{r}_k(t+\Delta t)\big]e^{\mp i[\mathbf{q}\cdot\mathbf{r}_k(t+\Delta t)+\varphi]}\Big\rangle \\
&\quad+\sum_{j=1}^{N_T}\Big\langle A\big[\mathbf{r}_j(t)\big]A\big[\mathbf{r}_j(t+\Delta t)\big]e^{\pm i\mathbf{q}\cdot[\mathbf{r}_j(t)-\mathbf{r}_j(t+\Delta t)]}\Big\rangle,
\end{aligned}\tag{17b}$$

where $\mathbf{r}_j(t) + \mathbf{r}_j(t + \Delta t) = 2\mathbf{r}_j(t) + \mathbf{v}\Delta t + \Delta\mathbf{r}_j(t, \Delta t)$ is a sum of the position vectors at t and at $t + \Delta t$; $\mathbf{r}_j(t + \Delta t) - \mathbf{r}_j(t) = \mathbf{v}\Delta t + \Delta\mathbf{r}_j(t, \Delta t)$ is the change of the position vectors in the time interval from t to $t + \Delta t$ (i.e. the displacement of particles). $\mathbf{v}$ is the flow velocity which is assumed to be same for all particles in the small scattering volume of the laminar flow fluid-particle systems; $\mathbf{v}\Delta t$ is the translational displacement corresponding to flowing and $\Delta\mathbf{r}_j(t, \Delta t)$ is the diffusional displacement due to the Brownian motion.

Secondly, in (17), the magnitude of scattering vector $\mathbf{q}$ is equal to $4\pi/\lambda$. Within the averaging time interval, the particle moves through the beam waist, which is much larger than the light wavelength. The position vector $\mathbf{r}_j(t)$ moves through a distance of many wavelengths and hence $\mathbf{q}\cdot\mathbf{r}_j$ will vary through many cycles of 2π. Because $e^{\pm i\phi} = \cos\phi \pm i\sin\phi$, both the sine and cosine functions will vary through many oscillations. The amplitude function A is only responsible for the slow amplitude modulation. Thus the average of an exponential function over many oscillations is zero, that is:

$$\begin{cases}
\langle A[\mathbf{r}_j(t)]e^{\pm i[\mathbf{q}\cdot\mathbf{r}_j(t)+\varphi]}\rangle = 0, \\
\langle A[\mathbf{r}_j(t+\Delta t)]e^{\pm i[\mathbf{q}\cdot\mathbf{r}_j(t+\Delta t)+\varphi]}\rangle = 0, \\
\langle A[\mathbf{r}_j(t)]A[\mathbf{r}_j(t+\Delta t)]e^{\pm i[\mathbf{q}\cdot\mathbf{r}_j(t)+\mathbf{q}\cdot\mathbf{r}_j(t+\Delta t)+2\varphi]}\rangle = 0.
\end{cases}\tag{18}$$

Substitution of (17) and (18) into (16) yields:

$$\begin{aligned}
R(\Delta t) = P_0^2\bigg(&1 + m^2|C_{\mathrm{Mie}}|^2\sum_{j=1}^{N_T}\Big\langle A\big[\mathbf{r}_j(t)\big]A\big[\mathbf{r}_j(t+\Delta t)\big] \\
&\times\big[e^{-i[\mathbf{q}\cdot\mathbf{v}\Delta t+\mathbf{q}\cdot\Delta\mathbf{r}_j(t,\Delta t)]} \\
&+ e^{i[\mathbf{q}\cdot\mathbf{v}\Delta t+\mathbf{q}\cdot\Delta\mathbf{r}_j(t,\Delta t)]}\big]\Big\rangle\bigg).
\end{aligned}\tag{19}$$

Thirdly, the diffusional displacement is much less than the translational displacement within the averaging time interval. Hence the average of the right side of (19) can be separated into two averages independently. One corresponds to the diffusional motion and the other the translational motion [47]. So we have

$$\begin{aligned}
R(\Delta t) = P_0^2\bigg(&1 + m^2|C_{\mathrm{Mie}}|^2\sum_{j=1}^{N_T}\Big\langle A\big[\mathbf{r}_j(t)\big]A\big[\mathbf{r}_j(t)+\mathbf{v}\Delta t\big]\Big\rangle \\
&\times\big[e^{-i\mathbf{q}\cdot\mathbf{v}\Delta t}\big\langle e^{-i\mathbf{q}\cdot\Delta\mathbf{r}_j(t,\Delta t)}\big\rangle \\
&+ e^{i\mathbf{q}\cdot\mathbf{v}\Delta t}\big\langle e^{i\mathbf{q}\cdot\Delta\mathbf{r}_j(t,\Delta t)}\big\rangle\big]\bigg).
\end{aligned}\tag{20}$$

The time autocorrelation function measured in the LDFI is time-averaged, whereas in most theoretical calculations what is calculated is the ensemble-averaged autocorrelation function. These two autocorrelation functions will be identical if the mechanical system studied is ergodic according to Birkhoff's ergodic theorem. Fortunately, the flowing Brownian motion system can be looked upon as a semi-ergodic system, which assumes the equivalence between time-averaged and ensemble-averaged autocorrelation functions [38].

$\langle e^{\pm i\mathbf{q}\cdot\Delta\mathbf{r}_j(t,\Delta t)}\rangle$ is the well-known diffusional ensemble-average and is found by integrating over the normalized Gaussian probability distribution, the solution to the diffusion equation [38, 47]. Thus, we have

$$\big\langle\exp\big(\pm i\mathbf{q}\cdot\Delta\mathbf{r}_j(t,\Delta t)\big)\big\rangle = \exp\big(-Dq^2\Delta t\big),\tag{21}$$

where D is the diffusion coefficient, given as $D = k_B T / 3\pi\eta d$ with k_B Boltzmann's constant, T the temperature, η the viscosity of suspension and d the diameter of particle.

$\langle A[\mathbf{r}_j(t)]A[\mathbf{r}_j(t) + \mathbf{v}\Delta t]\rangle$ is the translational ensemble-average found by integrating over the probability of finding the jth particle initially in a volume element $dx\,dy\,dz$. If V_T is the total volume of the system, this probability is $V_T^{-1}\,dx\,dy\,dz$. Therefore, the translational ensemble-average can be written as

$$\begin{aligned}
&\big\langle A\big[\mathbf{r}(t)\big]A\big[\mathbf{r}(t) + \mathbf{v}\Delta t\big]\big\rangle \\
&\quad = \frac{1}{V_T}\iiint \exp\left(-\frac{x^2}{\omega_0^2} - \frac{y^2}{\omega_0^2}\right) \\
&\qquad \times \exp\left(-\frac{(x + v_x\Delta t)^2}{\omega_0^2} - \frac{(y + v_y\Delta t)^2}{\omega_0^2}\right) dx\,dy\,dz \\
&\quad = \frac{V_S}{V_T}\exp\left(-\frac{(v_x^2 + v_y^2)(\Delta t)^2}{2\omega_0^2}\right),
\end{aligned} \tag{22}$$

where v_x and v_y are the x and y components of the velocity vector $\mathbf{v}$; ω_0 is the beam waist radius; $V_S = \pi H \omega_0^2/2$ represents the effective scattering volume; $H = 2\pi\omega_0^2/\lambda$ represents the depth of the beam focus.

From the discussion above, the time autocorrelation function of the back-emitted power of the laser is given as:

$$\begin{aligned}
R(\Delta t) = P_0^2\Bigg\{1 + &\underbrace{2m^2|C_{\mathrm{Mie}}|^2\langle N\rangle}_{\text{coefficient}}\ \underbrace{\cos(\mathbf{q}\cdot\mathbf{v}\Delta t)}_{\text{Doppler term}} \\
&\times \underbrace{\exp\big(-Dq^2\Delta t\big)}_{\text{Brownian term}}\ \underbrace{\exp\left(-\frac{(v_x^2 + v_y^2)(\Delta t)^2}{2\omega_0^2}\right)}_{\text{Gaussian term}}\Bigg\}
\end{aligned} \tag{23}$$

where $\langle N\rangle = N_T V_S / V_T$ is the average particle number in the volume V_S. Equation (23) includes a constant term denoting the back-emitted power in the absence of external feedback and a perturbation term indicating the fluctuation of power due to the weak external feedback. The perturbation term is a product of a coefficient, a Doppler term, a Brownian term and a Gaussian term. The Doppler term is due to the translational flow of particles, dependent on the magnitudes of the flow velocity and the scattering vector and the angle between them. The Brownian term is due to the diffusional motion of particles and it decreases exponentially depending on the diffusion coefficient D and the magnitude of the scattering vector q. The Gaussian term is dependent on the transit time of the particles through the Gaussian beam waist due to the translational flow. The product of the Doppler, Brownian and Gaussian terms in (23) is same as that of Chowdhury's OHI theory [47] based on a focused Gaussian incident field. In fact, Chowdhury's OHI theory can also be applied to the interpretation of the feedback interferometry experiments as the OHI theory based on the planar incident field. However, the analysis of LDFI requires some amendments. The reason deeply involves the laser-diode physics, because the intrinsic nonlinear nature of the semiconductor active medium (in which both the optical gain and the refractive index are coupled to the effective optical power in the cavity through their dependence on the injected carrier density) leads to an amplitude modulation term that can be very different from the usual cosine function [46]. The nonlinear nature of the LDFI can be perfectly explained by the modified Lang–Kobayashi rate equations in the case of the short external cavity and the weak external feedback. Application of the Lang–Kobayashi rate equations produces a parameter m in (23), which is dependent on the nature of the laser diode including the laser-diode cavity round-trip time, the photon lifetime with the cavity, the reflectivity of the optical power on the right laser-diode facet (see (12)). The laser-diode cavity round-trip time is decided by the length of the laser-diode cavity, the laser-diode refractive index and the velocity of light. In addition, $|C_{\mathrm{Mie}}|^2$ is the peculiar parameter to our theory, which is because that Mie solution of Maxwell's equations for the scattering of electromagnetic radiation by spherical particles is introduced as the feedback term in the modified Lang–Kobayashi rate equations. Thus our theory is more suitable than the OHI theory for the interpretation of LDFI in the flowing Brownian motion system.

3 Case studies and discussions

With the aid of the Lang–Kobayashi rate equations, Karikh [48] has developed a theory for the LDFI in a flowing Brownian system irradiated by a planar incident field. Due to the assumption of the planar incident field, Karikh's theory cannot explain the case of the focused Gaussian incident field. In what follows, we shall discuss special cases for displaying the reliability and necessity of our theory. The reliability results from some cases, which can be explained by both our theory and Karikh's theory. The necessity results from other cases, which can be only explained by our theory.

3.1 Particle system with pure Brownian motion

In the case of no translational flow, the velocity in (23) equals zero. So both the Doppler term and the Gaussian term fade out and (23) can be simplified into

$$R(\Delta t) = P_0^2\Big(1 + 2m^2|C_{\mathrm{Mie}}|^2\langle N\rangle \underbrace{\exp\big(-Dq^2\Delta t\big)}_{\substack{\text{normalized}\\ \text{time autocorrelation function}}}\Big). \tag{24}$$

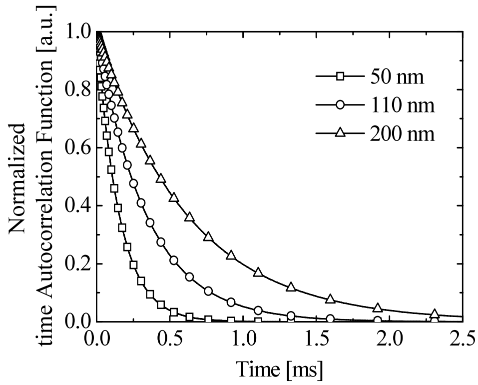

Fig. 4 Normalized time autocorrelation function for different particle diameters in the case of the pure Brownian motion system with 1.002 mPa s the viscosity of suspension and 293 K the temperature

The second term is similar to that of the planar incident field except for the coefficient. The corresponding normalized time autocorrelation function is given in Fig. 4, in which the particle diameters are respectively 50 nm, 110 nm and 200 nm. The viscosity of suspension η is 1.002 mPa s and the temperature T is 293 K. It can be seen that these time autocorrelation functions possess different exponential decays, showing the dependence of the Brownian motion on the particle size. The smaller the particle is, the quicker the time autocorrelation function damps down.

3.2 Particle system with planar incident field

In the limit of an infinite Gaussian beam waist radius, the Gaussian term equals one and (23) can be reduced to

$$R(\Delta t) = P_0^2\Big(1 + 2m^2|C_{\mathrm{Mie}}|^2\langle N\rangle \times \underbrace{\cos(\mathbf{q}\cdot\mathbf{v}\Delta t)\exp\big(-Dq^2\Delta t\big)}_{\substack{\text{normalized}\\ \text{time autocorrelation function}}}\Big). \quad (25)$$

The product of the Doppler and Brownian terms is same as that of a planar incident field. The corresponding normalized time autocorrelation function for different flow velocities is given in Fig. 5(a). The flow velocities are 3.0 mm s^{-1}, 6.0 mm s^{-1} and 10.0 mm s^{-1} respectively. The particle diameter is 100 nm and the angle between scattering wave vector and flow velocity is set to 85 deg. The parameters of the viscosity η and the temperature T are same with those in Fig. 4. It can be seen that the time autocorrelation functions undergo different periods of oscillation but their envelopes possess the same exponential decays. The period of oscillation is determined by the Doppler shift, which is a function of the flow velocity, the scattering vector and the angle between them. A high flow velocity corresponds to a big Doppler shift and a short oscillating period. The exponential decay is determined by the Brownian motion, which is relative to the particle size, the temperature and the viscosity of suspension. In the same temperature and viscosity of suspension, the same particle size corresponds to the same exponential decay. As a comparison, the normalized time autocorrelation function for different particle diameters and the same flow velocity are given in Fig. 5(b). The particle diameters are 20 nm, 110 nm and 200 nm respectively. The flow velocity is 6.0 mm s^{-1} and the angle between scattering wave vector and flow velocity is set to 85 deg. Due to the same reason, the time autocorrelation functions possess the same period of oscillation but their envelopes undergo different exponential decays.

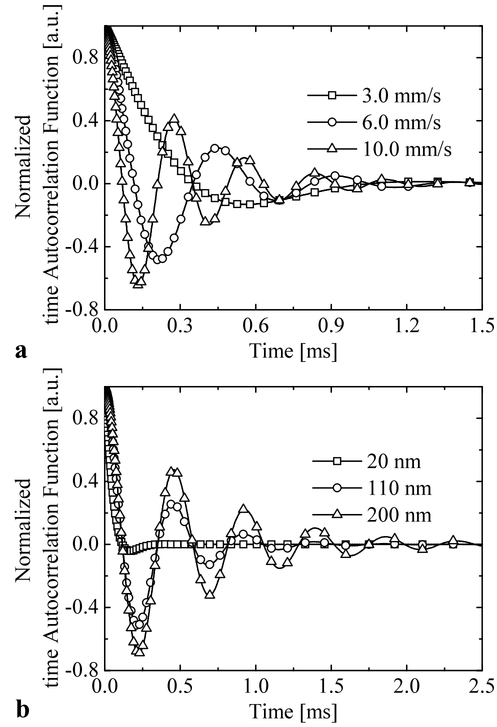

Fig. 5 Normalized time autocorrelation function in the case of the flowing Brownian motion system irradiated by planar incident field for (**a**) 1.002 mPa s the viscosity of suspension, 293 K the temperature, 85° incident angle, 100 nm particle diameter, different flow velocities and (**b**) 1.002 mPa s the viscosity of suspension, 293 K the temperature, 85° incident angle, 6.0 mm s^{-1} flow velocity, different particle diameters

3.3 Particle system with pure translational motion

When the velocity of the translational motion is fast enough or the particle size is sufficiently large, the Brownian motion can be ignored. Therefore, (23) can be transformed into the expression given by

$$R(\Delta t) = P_0^2\Bigg(1 + 2m^2|C_{\text{Mie}}|^2\langle N\rangle \underbrace{\times \cos(\mathbf{q}\cdot\mathbf{v}\Delta t)\exp\Bigg(-\frac{(v_x^2+v_y^2)(\Delta t)^2}{2\omega_0^2}\Bigg)}_{\substack{\text{normalized}\\ \text{time autocorrelation function}}}\Bigg). \quad (26)$$

The perturbation term describes the effects caused by the particles passing through the beam waist, including the Doppler shift in the frequency and the fluctuations of the amplitude of the back-scattered light. The perturbation term is very similar to the damped oscillation. However, the decay is dependent on the square of the delay time while the oscillation depends on the delay time itself. Figure 6 shows the corresponding normalized time autocorrelation function in the case of the scattering vector oblique to the flow direction. The beam waist radius is 25 µm. The flow velocities are 5 mm s^{-1} and 10 mm s^{-1} respectively. The angle between scattering wave vector and flow velocity is set to 85 deg. It can be found that the time autocorrelation functions, corresponding to different flow velocities, have different periods of oscillation and their envelopes have different Gaussian decays. The faster the flow velocity is, the quicker the time autocorrelation function oscillates and damps down.

In the case of the scattering vector perpendicular to the flow direction, the Doppler term cancels out and (26) can be reduced to

$$R(\Delta t) = P_0^2\Bigg(1 + 2m^2|C_{\text{Mie}}|^2\langle N\rangle \underbrace{\times \exp\Bigg(-\frac{(v_x^2+v_y^2)(\Delta t)^2}{2\omega_0^2}\Bigg)}_{\substack{\text{normalized}\\ \text{timeautocorrelationfunction}}}\Bigg). \quad (27)$$

The corresponding normalized time autocorrelation function for the different flow velocities are given in Fig. 7. The flow velocities are 10 mm s^{-1}, 15 mm s^{-1} and 30 mm s^{-1} respectively and the Gaussian beam waist radius is 25 µm. These time autocorrelation functions have different Gaussian decays but have no oscillations. The faster Gaussian decay is attributed to the faster translational flow of the illuminated target through the beam waist. The missing oscillation is attributed to the loss of Doppler shift because the scattering vector is perpendicular to the flow direction. Thus, the peculiar conclusion is drawn that the velocity of the translational flow can be extracted from the time autocorrelation function in the case of the scattering vector perpendicular to the flow direction.

According to the discussion above, the Gaussian term is the primary difference between our theory and Karikh's theory. In the case of no translational flow or infinite Gaussian beam waist, the Gaussian term disappears and our theory is equivalent to Karikh's theory.

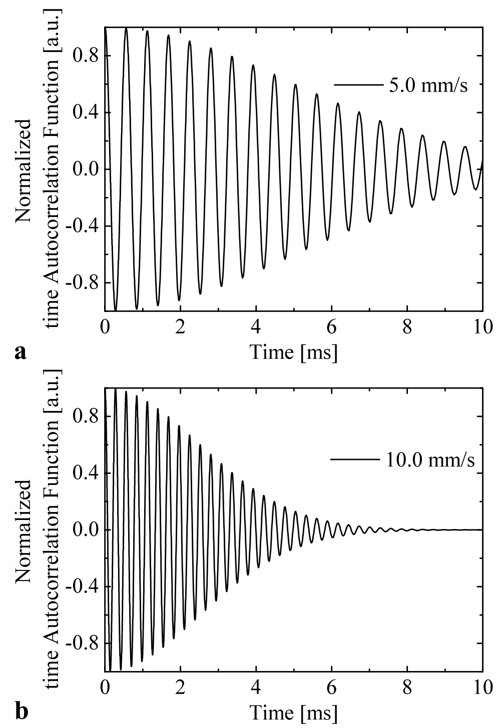

Fig. 6 Normalized time autocorrelation function in the case of the pure translational flow system irradiated by the focused Gaussian incident beam for 85° incident angle, 25 µm beam waist radius and different flow velocities ((**a**) flow velocity 5.0 mm s^{-1}; (**b**) flow velocity 10.0 mm s^{-1})

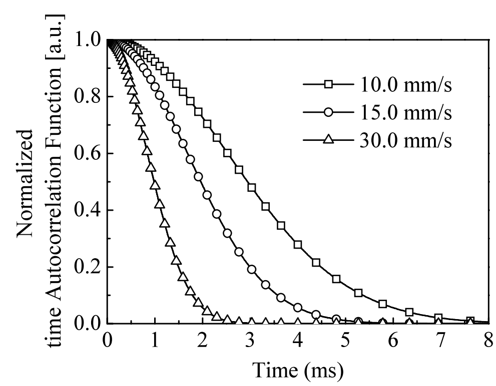

Fig. 7 Normalized time autocorrelation function for 25 µm beam waist radius, 90° incident angle and different flow velocities in the case of the pure translational flow system irradiated by the focused Gaussian incident beam

Finally, it is worthwhile to discuss that all the back-scattered light in the solid angle subtended by a convergent lens at a particle can be focused back into the laser cavity. However, in the solid angle the back-scattered light corresponds to different scattering angles. According to (23) these back-scattered lights may cause the time autocorrelation function to undergo different exponential decays and oscillations because of the dependence of the scattering wave vector **q** on the scattering angle. The present theoretical derivation is based on an assumption that the solid angle is so small that the back-scattered light with only 180 deg scattering angle can be focused back into the laser cavity. In the case of large solid angles, the situation is much more complicated. Besides, the strength of the perturbation is dependent on the parameter $|C_{\mathrm{Mie}}|^2$, which is also determined by the optical arrangement related with the solid angle. Therefore, further research on this is required.

4 Conclusion

In this work, a novel theory is proposed for LDFI in a flowing Brownian motion system irradiated by a focused Gaussian incident field. The back-scattered field from particles is introduced to the feedback term of the modified Lang–Kobayashi rate equations. With the aid of the steady-state solution of the modified Lang–Kobayashi rate equations, the back-emitted power of LDFI and the corresponding time autocorrelation function are derived. The time autocorrelation function is composed of one constant term and one perturbation term. The perturbation term includes the information on particle size and flow velocity.

The theory is based on the modified Lang–Kobayashi rate equations and produces the character parameter m, which is dependent on the nature of the semiconductor laser including the refractive index of the active medium, the length of the laser-diode cavity, the photon lifetime, the reflectivity of the optical power on the right laser-diode facet. Thus, our theory should be more suitable than the OHI theory for the interpretation of the LDFI in the flowing Brownian motion system.

The theory is based on the assumption of a focused Gaussian incident field and produces the Gaussian term which is dependent on the transit time of the illuminated target through the Gaussian beam waist due to the translational flow. The time autocorrelation function or its envelope has the Gaussian decay, the rate of which depends on the flow velocity. Hence the velocity of the translational flow may be extracted from either the Doppler term or the Gaussian term.

Finally, we should point out that, because of the consistency of rate equations between the diode lasers and other solid-state lasers, our theory can be applied to both diode lasers and solid-state lasers.

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