Appl Phys B (2010) 99: 809–815
DOI 10.1007/s00340-009-3892-2

Applied Physics B
Lasers and Optics

Propagation of spectral Stokes singularities of stochastic electromagnetic beams through atmospheric turbulence

H. Yan · B. Lü

Received: 30 October 2009 / Revised version: 14 December 2009 / Published online: 15 January 2010

Abstract The propagation of spectral Stokes singularities (vortices) of stochastic electromagnetic vortex beams through atmospheric turbulence is studied, where the electromagnetic Gaussian Schell-model (GSM) vortex beam is taken as an illustrative example. It is shown that the spectral Stokes vortices S_{12} (C-points), S_{23} and S_{31} introduced to describe the polarization singularities of stochastic electromagnetic beams appear in turbulence. The motion, creation, annihilation and polarization changes of S_{12}, S_{23} and S_{31} vortices, as well as the handedness inversion of S_{12} vortices may appear as the propagation distance or one beam parameter varies. In the process the topological relationship holds true. In comparison with the free-space propagation, the variation of the refractive index structure constant C_n^2 in atmospheric turbulence results in similar effects as above. The dependence of S_{12}, S_{23} and S_{31} vortices on the propagation distance and beam and turbulence parameters are illustrated by numerical examples.

1 Introduction

The propagation of laser beams through atmospheric turbulence has been studied extensively due to the importance for many applications such as remote sensing, optical communications, optical imaging and targeting systems etc. [1]. Recently, considerable interest has been shown in optical vortex beams propagating through turbulence and their potential applications. By using multiple phase screen simulations, Gbur and Tyson analyzed the propagation of vortex beams through weak-to-strong atmosphere turbulence and suggested that the topological charge of vortex beams could be used as information carrier in optical communication systems [2]. Based on the extended Huygens–Fresnel principle, Wang and Pu et al. demonstrated that the spreading of partially coherent vortex beams is less influenced by turbulence than that of partially coherent vortex-free beams. The larger the topological charge of partially coherent vortex beams is, the less the beam spreading induced by turbulence [3].

On the other hand, in addition to scalar phase singularities including the optical vortex, edge dislocation and hybrid dislocation, there exist a variety of polarization singularities in vector wave fields. Nye, Berry and Dennis made it clear that in the two-dimensional case the polarization pattern may contain singularities such as C-points (stars, monstars and lemons), which are isolated points of circular polarization, where the orientations of the major and minor axes of the polarization ellipse become undetermined, and L-lines, which are continuous lines of linear polarization, where the handedness of the polarization ellipse is undefined [4, 5]. Felde and Chernyshov et al. examined polarization singularities in partially polarized combined beams and pointed out the existence of the U (unpolarized) and P (completely polarized) singularities [6]. The Stokes parameters and complex Stokes scalar fields are often used to analyze polarization singularities [7–9]. More recently, we have extended the concept of the polarization singularities to stochastic electromagnetic beams [10] by using the spectral Stokes parameters introduced by Korotkova and Wolf [11].

The motivation of the present paper is to study the propagation of spectral Stokes singularities (vortices) of stochastic electromagnetic beams through atmospheric tur-

H. Yan · B. Lü (✉)
Institute of Laser Physics and Chemistry, Sichuan University,
Chengdu 610064, China
e-mail: baidalu0@tom.com
Fax: +86-28-85412322

H. Yan
e-mail: yanhongweidy@yahoo.com.cn

bulence. In Sect. 2, taking the electromagnetic Gaussian Schell-model (GSM) vortex beam as an illustrative example of stochastic electromagnetic beams, and using the extended Hyugens–Fresnel principle, the closed-form expression for the propagation of electromagnetic GSM vortex beams through atmospheric turbulence is derived and used to study their spectral Stokes singularities in turbulence. The influence of propagation distance and beam and turbulence parameters on the spectral Stokes singularities is illustrated in Sect. 3 by numerical examples. In Sect. 4 a brief summary of the main results concludes this paper.

2 Theoretical model

We consider a stochastic electromagnetic GSM vortex beam whose element W_{uv} of the 2×2 cross-spectral density in the plane $z=0$ is written as

$$\begin{aligned}
&W_{uv}(x_{01},y_{01},x_{02},y_{02},\omega,0)\\
&\quad = A_u A_v B_{uv}\frac{x_{01}-s_u-iy_{01}}{w_u}\frac{x_{02}-s_v+iy_{02}}{w_v}\\
&\qquad\times\exp\left[-\frac{(x_{01}-s_u)^2+y_{01}^2}{w_u^2}\right]\\
&\qquad\times\exp\left[-\frac{(x_{02}-s_v)^2+y_{02}^2}{w_v^2}\right]\\
&\qquad\times\exp\left[-\frac{(x_{01}-x_{02}-s_u+s_v)^2+(y_{01}-y_{02})^2}{2\delta_{uv}^2}\right]\\
&\quad (u,v=x,y,\ \text{unless otherwise stated}),
\end{aligned}\tag{1}$$

where the coefficients A_u, B_{uv} and the variances w_{0u}, δ_{uv} are independent of the position but may depend on the frequency ω [12], w_{0u} is the waist width in the u direction, s_u denotes the off-axis distance of the vortex embedded in W_{uv}, δ_{uv} is the spatial correlation length, $B_{uv}=B^*_{vu}$ are generally complex when $u\neq v$, and $B_{uv}=1$ when $u=v$. The asterisk $*$ denotes the complex conjugate. (x_{01},y_{01}), (x_{02},y_{02}) are position coordinates of two points in the plane $z=0$, the frequency ω is omitted later for the sake of simplicity.

In accordance with the extended Huygens–Fresnel principle [1], the propagation of the cross-spectral density matrix at the position $(x_1=x_2=x,\ y_1=y_2=y)$ in the z plane through atmospheric turbulence obeys

$$\begin{aligned}
&W_{uv}(x,y,z)\\
&\quad=\left(\frac{k}{2\pi z}\right)^2\iiiint W_{uv}(x_{01},y_{01},x_{02},y_{02},0)\\
&\qquad\times\exp\left[i\frac{k}{z}(xx_{01}+yy_{01}-xx_{02}-yy_{02})\right]\\
&\qquad\times\exp\left[-i\frac{k}{2z}(x_{01}^2+y_{01}^2-x_{02}^2-y_{02}^2)\right]\\
&\qquad\times\left\langle\exp\left[\psi^*(x,y,x_{01},y_{01},z)\right.\right.\\
&\qquad\left.\left.+\psi(x,y,x_{02},y_{02},z)\right]\right\rangle dx_{01}\,dy_{01}\,dx_{02}\,dy_{02},
\end{aligned}\tag{2}$$

where k is the wave number related to the wave length by $k=2\pi/\lambda$, $\langle\cdot\rangle$ denotes the ensemble average, and $\psi(\cdot)$ represents the random part of the complex phase of a spherical wave due to the turbulence, and under the quadratic approximation of the phase structure can be expressed as [13]

$$\begin{aligned}
&\left\langle\exp\left[\psi^*(x,y,x_{01},y_{01},z)+\psi(x,y,x_{02},y_{02},z)\right]\right\rangle\\
&\quad\cong\exp\left[-\frac{(x_{01}-x_{02})^2+(y_{01}-y_{02})^2}{\rho_0^2(z)}\right],
\end{aligned}\tag{3}$$

$$\text{with } \rho_0=\left(0.545C_n^2k^2z\right)^{-3/5}\tag{4}$$

is the spatial coherent radius of a spherical wave propagating through turbulence, and C_n^2 is the refractive index structure constant, which describes the strength of the turbulence.

On substituting from (1) into (2), and after tedious but straightforward diffraction integral calculations, we obtain

$$\begin{aligned}
&W_{uv}(x,y,z)\\
&\quad=\frac{Q_{uv}A_uA_vB_{uv}}{w_uw_v}\left\{\frac{\beta_xq_{uv(x)}}{p_{uv}}+\frac{\beta_yq_{uv(y)}}{p_{uv}}\right.\\
&\qquad+\left(\frac{1}{2\delta_{uv}^2}+\frac{1}{\rho_0^2(z)}\right)\frac{p_{uv}+q_{uv(x)}^2+q_{uv(y)}^2}{p_{uv}^2}\\
&\qquad+i\left[\left(\frac{1}{2\delta_{uv}^2}+\frac{1}{\rho_0^2(z)}\right)\frac{q_{uv(x)}}{p_{uv}}+\beta_x\right]\frac{q_{uv(y)}}{p_{uv}}\\
&\qquad\left.-i\left[\left(\frac{1}{2\delta_{uv}^2}+\frac{1}{\rho_0^2(z)}\right)\frac{q_{uv(y)}}{p_{uv}}+\beta_y\right]\frac{q_{uv(x)}}{p_{uv}}\right\},
\end{aligned}\tag{5}$$

where

$$\begin{aligned}
Q_{uv}&=\frac{k^2}{4z^2p_{uv}\alpha_{uv}^2}\exp\left[-\frac{ik}{2z}(s_u^2-s_v^2-2xs_u+2xs_v)\right.\\
&\qquad\left.-\frac{(s_u-s_v)^2}{\rho_0^2(z)}+\frac{\beta_x^2}{\alpha_{uv}}+\frac{\beta_y^2}{\alpha_{uv}}+\frac{q_{uv(x)}^2}{p_{uv}}+\frac{q_{uv(y)}^2}{p_{uv}}\right],\\
\beta_x&=\frac{ik(x-s_u)}{2z}-\frac{s_u-s_v}{\rho_0^2(z)},\qquad \beta_y=\frac{iky}{2B},\\
\alpha_{uv}&=\frac{1}{w_0^2}+\frac{ik}{2z}+\frac{1}{2\delta_{uv}^2}+\frac{1}{\rho_0^2(z)},
\end{aligned}\tag{6}$$

$$p_{uv}=\frac{1}{w_0^2}-\frac{ik}{2z}+\frac{1}{2\delta_{uv}^2}+\frac{1}{\rho_0^2(z)}-\frac{1}{\alpha_{uv}}\left[\frac{1}{2\delta_{uv}^2}+\frac{1}{\rho_0^2(z)}\right]^2,$$

$$q_{uv(x)} = -\frac{ik(x-s_v)}{2z} + \frac{s_u - s_v}{\rho_0^2(z)} + \frac{\beta_x}{\alpha_{uv}}\left[\frac{1}{2\delta_{uv}^2} + \frac{1}{\rho_0^2(z)}\right],$$

$$q_{uv(y)} = -\frac{iky}{2z}\left[1 - \frac{1}{\alpha_{uv}}\left(\frac{1}{2\delta_{uv}^2} + \frac{1}{\rho_0^2(z)}\right)\right].$$

The normalized spectral Stokes parameters are defined as [11]

$$\begin{aligned} S_1(x,y,z) &= s_0^{-1}\left[W_{xx}(x,y,z) - W_{yy}(x,y,z)\right], \\ S_2(x,y,z) &= s_0^{-1}\left[W_{xy}(x,y,z) + W_{yx}(x,y,z)\right], \\ S_3(x,y,z) &= is_0^{-1}\left[W_{yx}(x,y,z) - W_{xy}(x,y,z)\right], \end{aligned} \tag{7}$$

where

$$s_0(x,y,z) = W_{xx}(x,y,z) + W_{yy}(x,y,x,y,z). \tag{8}$$

From (7) the complex spectral Stokes fields turn out to be [14]

$$\begin{aligned} S_{12} &= S_1 + iS_2, \\ S_{23} &= S_2 + iS_3, \\ S_{31} &= S_3 + iS_1. \end{aligned} \tag{9}$$

The substitution from (5) into (7)–(9) yields the closed-form expressions for the normalized spectral Stokes parameters and complex Stokes fields. The spectral Stokes singularities of stochastic electromagnetic beams correspond to the zero points (phase singularities) of the complex Stokes fields $S_{ij} = 0$ ($i, j = 1, 2, 3$ unless otherwise stated), where S_{12} vortices ($S_{12} = 0$ i.e., $S_1 = S_2 = 0$) correspond to circular polarization (C-points), $S_3 > 0$ ($S_3 < 0$) means right-(left-) handedness. S_{23} vortices ($S_{23} = 0$ i.e., $S_2 = S_3 = 0$) and S_{31} vortices ($S_{31} = 0$ i.e., $S_3 = S_1 = 0$) must be located on L-lines ($S_3 = 0$ contours), which correspond to linear polarization [10, 14]. Equations (5)–(9) indicate that S_{12}, S_{23} and S_{31} vortices of stochastic electromagnetic GSM vortex beams in turbulence depend on the off-axis distance s_u, the waist width w_u, spatial correlation length δ_{uv}, structure constant C_n^2, and propagation distance z.

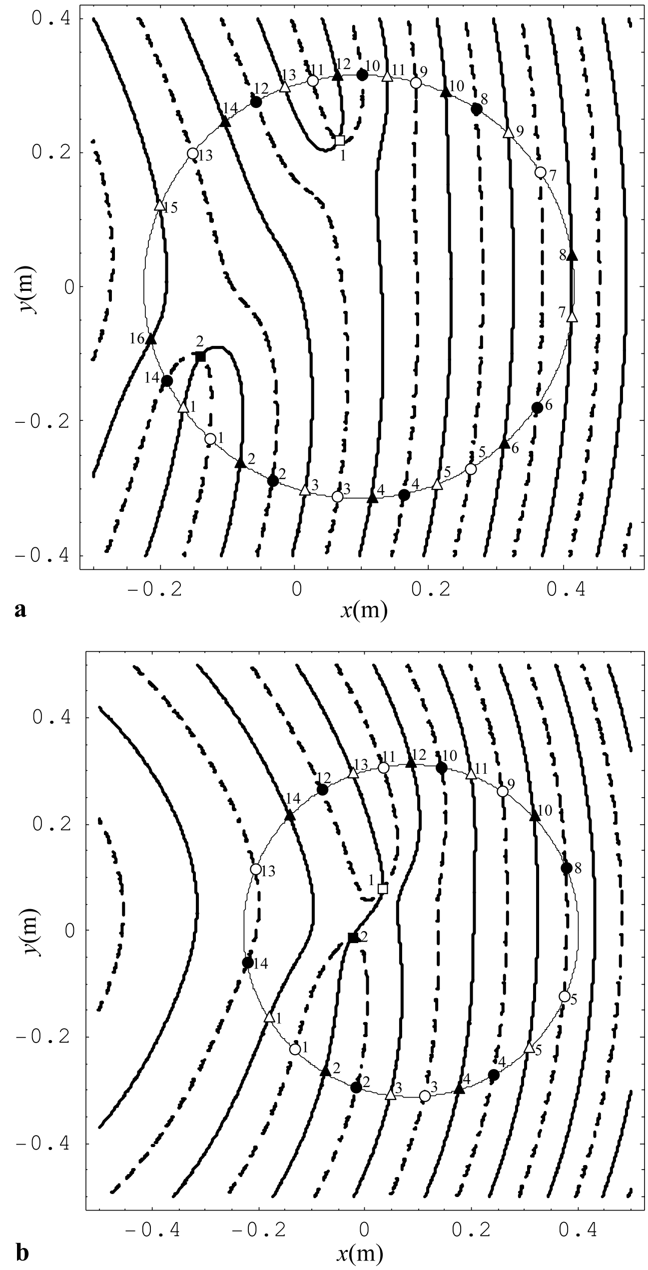


Fig. 1 The dependence of spectral Stokes vortices on the off-axis distance s_y. The calculation parameters are seen in the text

3 Dependence of spectral Stokes vortices of stochastic electromagnetic GSM vortex beams in turbulence on the propagation distance and beam and turbulence parameters

Numerical calculations were made to illustrate the dynamic behavior of stochastic electromagnetic GSM vortex beams propagating through atmospheric turbulence, when the propagation distance, beam parameter or structure constant varies. In the following calculations $B_{xx} = B_{yy} = 1$, $B_{xy} = 0.3 \times \exp(i\pi/6)$, $B_{yx} = 0.3 \times \exp(-i\pi/6)$, $A_x = 1.5$, $A_y = 1$, $\delta_{xx} = 2.5$ cm, $\delta_{yy} = 2$ cm, $\delta_{xy} = \delta_{yx} = 3$ cm, $s_x = 6$ cm, $w_x = 4$ cm, $\lambda = 632.8$ nm are kept fixed, $C_n^2 = 10^{-15}$ mm$^{-2/3}$ except for Fig. 3 and $z = 10$ km except for Fig. 4. The parameters chosen in the numerical calculations satisfy the realizability conditions for stochastic electromagnetic GSM beams [15].

Figure 1 gives the dependence of Stokes vortices of a stochastic electromagnetic GSM vortex beam on the off-axis distance s_y, where the calculation parameters are $s_y = -2$ cm in (a), $s_y = 0$ in (b), and $s_y = 5$ mm in (c), and $w_y = 1$ cm. $S_1 = 0$, $S_2 = 0$ and $S_3 = 0$ are shown by thin, dashed

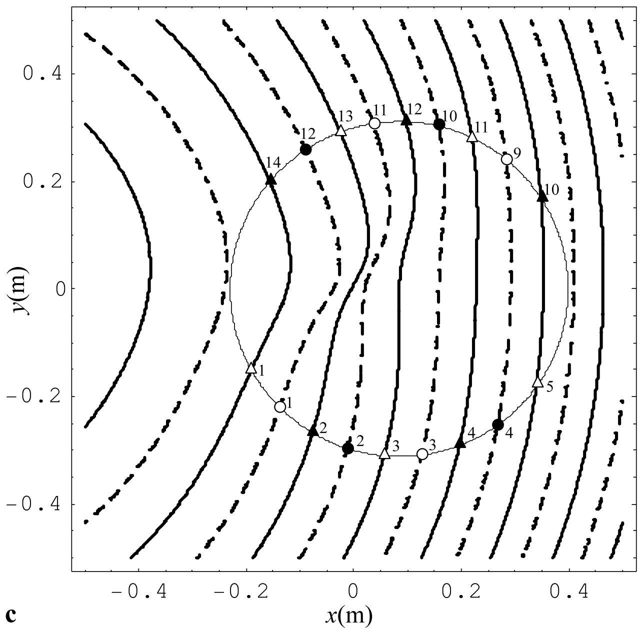


Fig. 1 (*Continued*)

and thick curves, S_{12}, S_{23} and S_{31} vortices which appear at the intersections of $S_i = 0$ and $S_j = 0$ are represented by circles, squares and triangles, the black and open ones denote the topological charges $m = +1$ and -1, respectively, determined by the topological sign principle [16]. As can be seen, there exist 14 S_{12} vortices (C-points), 2 S_{23} vortices and 16 S_{31} marked by 1, 2, ... In Fig. 1(a) the S_{12} vortices 1, 3, 5, 8, 10, 12 are left-handed, and the S_{12} vortices 2, 4, 6, 7, 9, 11, 13, 14 are right-handed. S_{23} vortices are located at (0.065 m, 0.216 m) and (−0.139 m, −0.102 m), and their degree of polarization is $P = 0.8570$ and 0.7523, respectively. The degree of polarization of S_{12} and S_{23} vortices is very small, for example, $P = 2.53 \times 10^{-4}$ and 1.70×10^{-4} for S_{12} vortices 1 and 14, $P = 1.98 \times 10^{-4}$ and 2.60×10^{-4} for S_{31} vortices 1 and 2, respectively, i.e., $P \approx 0$ similar to the U-singularities. With increasing s_y from $s_y = -0.2$ cm in Fig. 1(a) to $s_y = 0$ in Fig. 1(b), the motion and changes in the degree of polarization of spectral Stokes vortices take place; for example, S_{12} vortices 1 and 14 move from (−0.127 m, −0.226 m), and (−0.191 m, −0.139 m) to (−0.132 m, −0.224 m), (−0.222 m, −0.063 m), and their degree of polarization becomes $P = 1.72 \times 10^{-3}$, and 2.56×10^{-3}, the position of S_{23} vortices 1, 2 changes to (0.033 m, 0.077 m), and (−0.024 m, −0.013 m), and the degree of polarization of S_{23} vortices 1, 2 becomes $P = 0.9837$, and 0.9841, respectively. A pair of S_{12} vortices 6 and 7 with opposite topological charge $m = \pm 1$ but the same handedness, and three pairs of S_{31} vortices 6 and 9, 7 and 8, 15 and 16, with opposite topological charge $m = \pm 1$ annihilate each other. With a further increase of $s_y = 5$ mm in Fig. 1(c), the annihilation of two pairs of S_{12} vortices 5 and 8, 13 and 14 with opposite charge but the same handedness and a pair of S_{23} vortices 1, 2 with opposite charge $m = \pm 1$ appear.

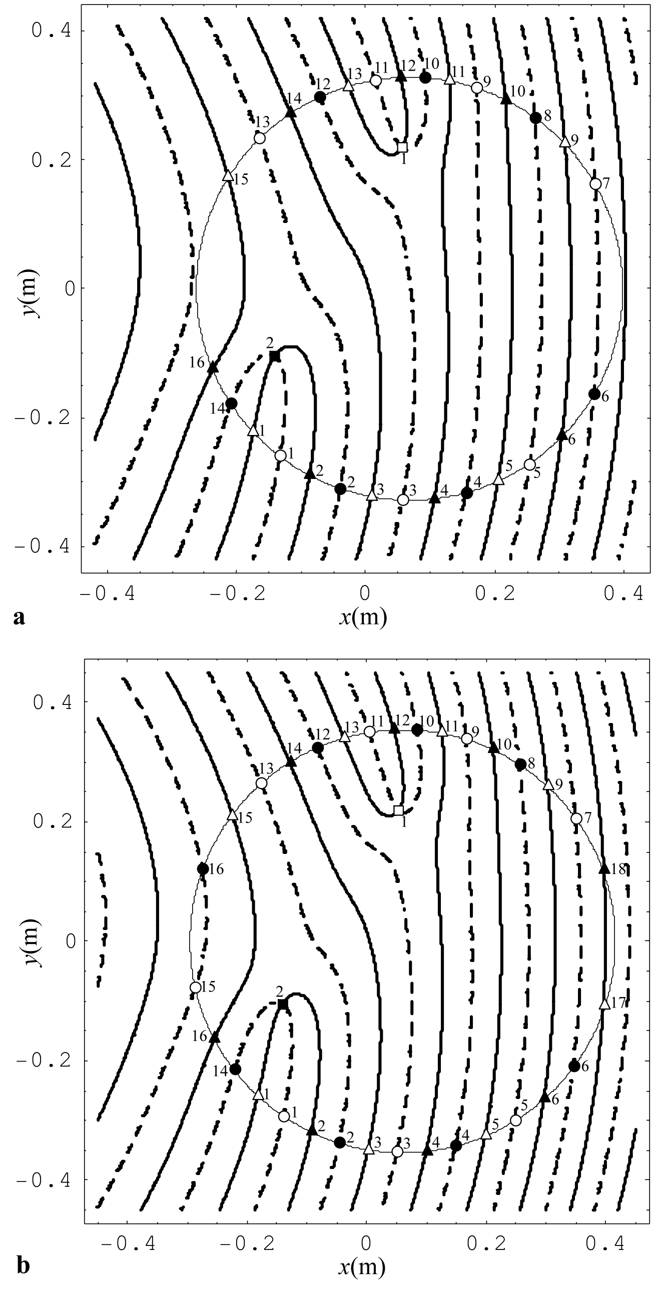
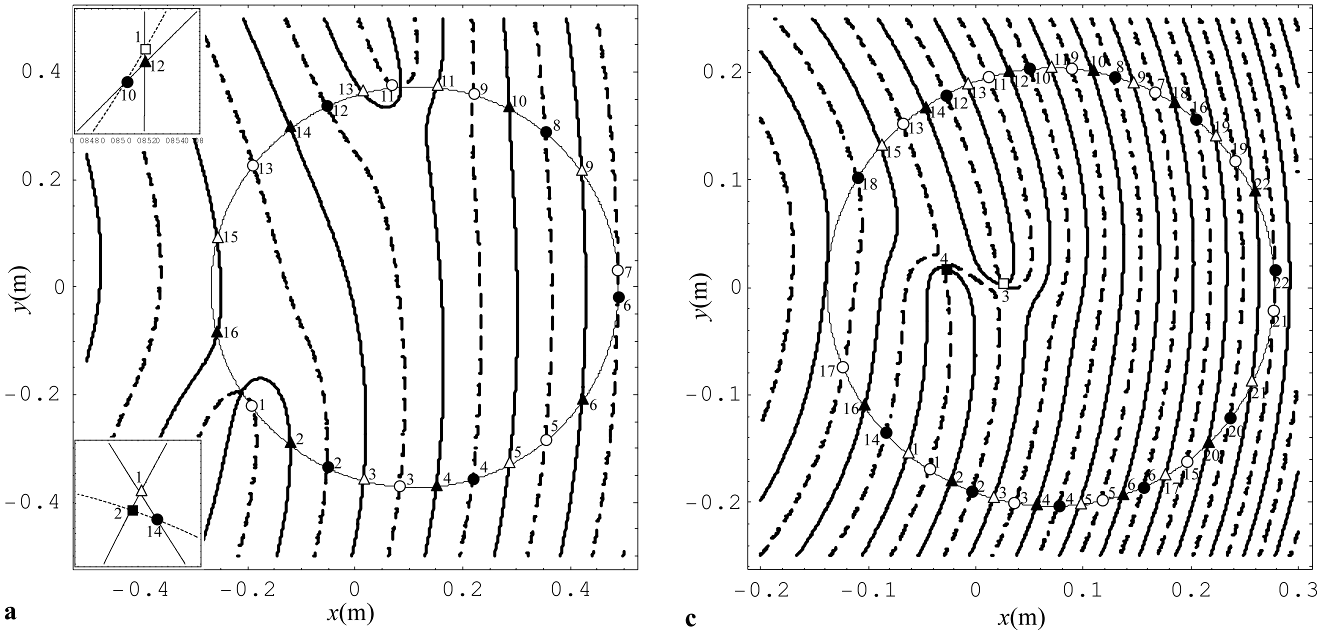


Fig. 2 The dependence of spectral Stokes vortices on the waist width w_y

The dependence of spectral Stokes vortices on the waist width w_y is plotted in Fig. 2: $w_y = 6$ mm in (a), $w_y = 4$ mm in (b), and $s_y = -2$ cm. It is seen that as the waist width is decreased from $w_y = 1$ cm in Fig. 1(a) to 6 mm in Fig. 2(a) S_{12} vortices 1 and 14 shift to (−0.132 m, −0.260 m), and (−0.209 m, −0.179 m), the degree of polarization becomes $P = 7.14 \times 10^{-5}$, 6.62×10^{-5}, and S_{23} vortices 1 and 2 shift to (0.056 m, 0.219 m), and (−0.141 m, −0.103 m), the degree of polarization of S_{23} vortices 1 and 2 becomes



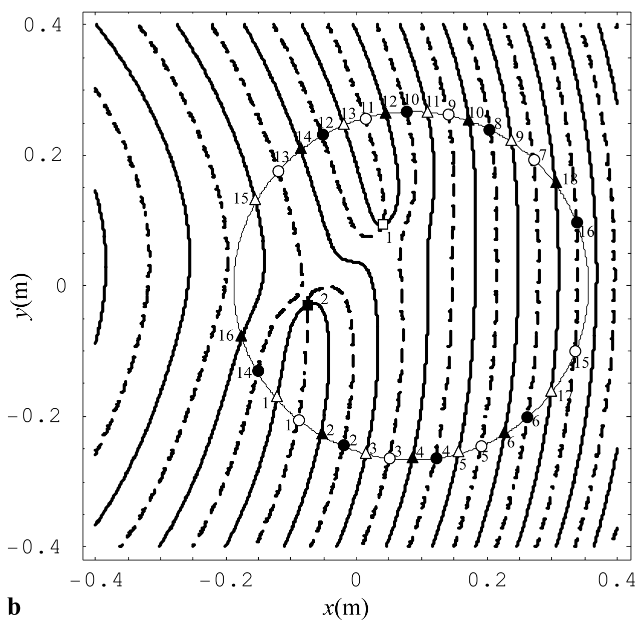


Fig. 3 The dependence of spectral Stokes vortices on the structure constant C_n^2

Fig. 3 (*Continued*)

$P = 0.9611$, and 0.9573, respectively. The pair of S_{31} vortices 7 and 8 annihilates. With a further decrease of $w_y = 4$ mm in Fig. 2(b), the creation of the pair of S_{12} vortices 15 and 16 with opposite charge $m = \pm 1$ and same left-handedness and the pair of S_{31} vortices 17 and 18 with opposite charge $m = \pm 1$ occurs. S_{23} vortices 1 and 2 shift to (0.053 m, 0.220 m), and (−0.141 m, −0.104 m), respectively, and their degree of polarization becomes $P = 0.9929$, and 0.9937, respectively.

The dependence of spectral Stokes vortices on the structure constant C_n^2 is depicted in Fig. 3; $C_n^2 = 1.4646 \times 10^{-15}$ mm$^{-2/3}$ in (a), $C_n^2 = 6 \times 10^{-16}$ mm$^{-2/3}$ in (b), and $C_n^2 = 0$ in (c), and $s_y = -2$ cm, and $w_y = 1$ cm. With increasing structure constant from $C_n^2 = 1 \times 10^{-15}$ mm$^{-2/3}$ in Fig. 1(a) to $C_n^2 = 1.4646 \times 10^{-15}$ mm$^{-2/3}$ in Fig. 3(a), the S_{12} vortices 1 and 14 shift to (−0.189 m, −0.226 m), and (−0.210 m, −0.197 m), and $P = 4.22 \times 10^{-5}$ and 1.18×10^{-6}, respectively. Specifically, the handedness inversion of S_{12} vortices 10 and 14 takes place, i.e., the handedness of S_{12} vortex 10 is changed from the left-handed polarization to the right-handed one, and the right-handed S_{12} vortex 14 becomes the left-handed one. The position of S_{23} vortices 1 and 2 is (0.085 m, 0.372 m), and (−0.211 m, −0.196 m), and their degree of polarization of decreases sharply, i.e., $P = 8.27 \times 10^{-5}$, 7.34×10^{-3}, respectively. The pair of S_{31} vortices 7 and 8 annihilates. In Fig. 3(b), where the $C_n^2 = 6 \times 10^{-16}$ mm$^{-2/3}$, the handedness inversion of S_{12} vortices 10 and 14 is present again, namely, the right-handed S_{12} vortex 10 and the left-handed S_{12} vortex 14 become the left-handed vortex 10 and the right-handed vortex 14. S_{23} vortices 1 and 2 move to (0.040 m, 0.093 m), and (−0.075 m, −0.029 m), and $P = 0.9816$, and 0.9619, respectively. The creation of the pair of S_{12} vortices 15 and 16 and the pair of S_{31} vortices 17 and 18 is observed in comparison with Fig. 3(a). When $C_n^2 = 0$ (free-space propagation) in Fig. 3(c), three pairs of S_{12} vortices 17 and 18, 19 and 20, 21 and 22, two pairs of S_{31} vortices 19 and 20, 21 and 22, a pair of S_{23} vortices 3 and 4 are present, and the pair of S_{23} vortices 1 and 2 disappear.

It can be shown that the variation of the spatial correlation length may lead to similar effects, and is omitted here for brevity.

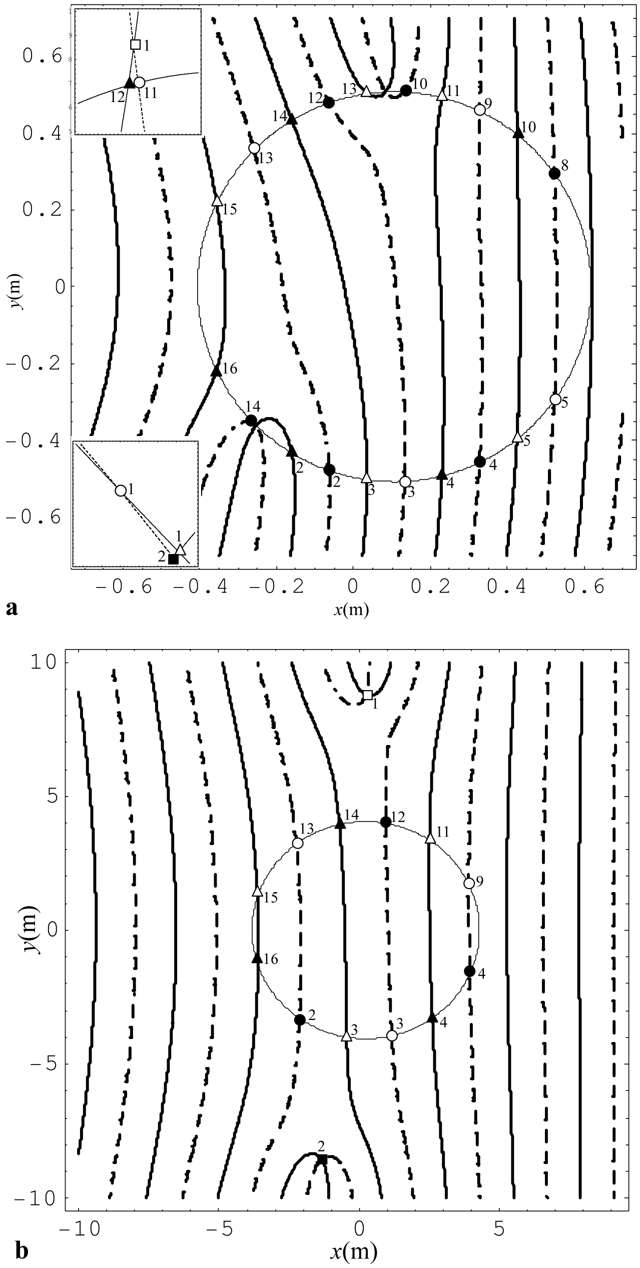


Fig. 4 The dependence of spectral Stokes vortices on the propagation distance z

The dynamic evolution of spectral Stokes vortices through atmospheric turbulence is represented in Fig. 4; $z = 14.31$ km in (a), and $z = 50$ km in (b), and $s_y = -2$ cm, and $w_y = 1$ cm. A comparison of Fig. 1(a) and Fig. 4(a) shows that with increasing propagation distance from $z = 10$ km to $z = 14.31$ km, a pair of S_{12} vortices 6 and 7, and the two pairs of S_{31} vortices 6 and 9, 7 and 8 annihilate. The handedness of the S_{12} vortex 1 is inverted from left-handed polarization to the right-handed one, and the right-handed S_{12} vortex 11 becomes left-handed. S_{23} vortices 1 and 2 move to (0.084 m, 0.509 m), and (−0.253 m, −0.361 m), the degree of polarization decreases sharply, i.e., $P = 0.01$, and 9.38×10^{-4}, respectively. When the propagation distance is increased to $z = 50$ km in Fig. 4(b), three pairs of S_{12} vortices 1 and 14, 5 and 8, 10 and 11 and three pairs of S_{31} vortices 1 and 2, 5 and 10, 12 and 13 disappear. The position of S_{23} vortices 1 and 2 is (0.236 m, 8.784 m), and (−1.341 m, −8.556 m), respectively, and their degree of polarization becomes $P = 1$, corresponding to the P-singularities.

The topological relationship between the topological charges of spectral Stokes vortices is expressed as [10, 14]

$$2\sigma_k \sum\nolimits_{(k)} m_{ij} = \sum\nolimits^{(k)} \sigma_i m_{jk} = \sum\nolimits^{(k)} \sigma_j m_{ik}, \tag{10}$$

where $\sum_{(k)}$ specifies summation over the spectral Stokes vortices contained within a closed $S_k = 0$ contour, and $\sum^{(k)}$ denotes summation over the spectral Stokes vortices on the contour. $\sigma_{i,j,k} = 1$ and -1 for $S_{i,j,k} > 0$ and $S_{i,j,k} < 0$, respectively, m_{ij} is the topological charge of S_{ij} vortices and the order of the indices of m_{ij} is a cyclic permutation of 1, 2 and 3. It can readily be shown that the topological relationship (10) holds true for the above cases. We take Fig. 1(b) as an example, where $S_1 = 0$ forms a closed contour. The term on the left-hand side of (10) reads $2\sigma_k \sum_{(k)} m_{ij} = 2 \times 1 \times (-1+1) = 0$, the middle term is equal to $\sum^{(k)} \sigma_i m_{jk} = -1 \times (-1+1-1+1-1+1) + 1 \times (-1+1-1+1) = 0$, and the term on the right-hand side is $\sum^{(k)} \sigma_j m_{ik} = -1 \times (-1+1-1+1-1+1) + 1 \times (-1+1-1+1-1+1) = 0$.

4 Concluding remarks

In this paper, by using the spectral Stokes parameters and the extended Huygens–Fresnel principle, the propagation of spectral Stokes singularities (vortices) of stochastic electromagnetic GSM vortex beams through atmospheric turbulence has been studied in detail. It has been found that there exist S_{12}, S_{23} and S_{31} spectral Stokes vortices in turbulence. By varying the propagation distance, or one beam parameter, such as the off-axis distance, waist width or spatial correlation length, the motion, creation, annihilation, and changes in the degree of polarization of S_{12}, S_{23} and S_{31} vortices, and the inversion of handedness of S_{12} vortices may appear. The creation and annihilation take place for a pair of S_{12} vortices with opposite topological charge but the same handedness, and for a pair of oppositely charged S_{23} or S_{31} vortices. In the process the topological relationship is valid. As compared with the free-space propagation [10], the variation of the structure constant C_n^2 in atmospheric turbulence results in similar effects such as the motion, creation, annihilation and polarization changes of S_{12}, S_{23} and S_{31} vortices and the handedness inversion of S_{12} vortices. The results obtained in this paper would be useful for studying propagation dynamics of stochastic electromagnetic vortex beams in atmospheric turbulence and for controlling their polarization singularities.

Acknowledgement This work was supported by the National Natural Science Foundation of China (NSFC) under grant No. 10874125.

References

1. L.C. Andrews, R.L. Phillips, *Laser Beam Propagation Through Random Media* (SPIE, Bellingham, 1998)
2. G. Gbur, R.K. Tyson, J. Opt. Soc. Am. A **25**, 225 (2008)
3. T. Wang, J. Pu, Z. Chen, Opt. Eng. **47**, 036002 (2008)
4. J.F. Nye, *Natural Focusing and Fine Structure of Light* (Institute of Physics Publishing, Bristol, 1999)
5. M.V. Berry, M.R. Dennis, Proc. R. Soc. Lond. Ser. A **457**, 141 (2001)
6. C.V. Felde, A.A. Chernyshov, G.V. Bogatyryova, P.V. Polyanskii, M.S. Soskin, JETP Lett. **88**, 418 (2008)
7. A.I. Mokhun, M.S. Soskin, I. Freund, Opt. Lett. **27**, 995 (2002)
8. I. Freund, Opt. Lett. **26**, 1996 (2001)
9. M.S. Soskin, V. Denisenko, I. Freund, Opt. Lett. **28**, 1475 (2003)
10. H.W. Yan, B.D. Lü, Opt. Lett. **34**, 1933 (2009)
11. O. Korotkova, E. Wolf, Opt. Lett. **30**, 198 (2005)
12. E. Wolf, *Introduction to the Theory of Coherence and Polarization of Light* (Cambridge University Press, Cambridge, 2007)
13. H.T. Yura, Appl. Opt. **11**, 1399 (1972)
14. I. Freund, A.I. Mokhun, M.S. Soskin, O.V. Angelsky, I.I. Mokhun, Opt. Lett. **27**, 545 (2002)
15. H. Roychowdhury, O. Korotkova, Opt. Commun. **249**, 379 (2005)
16. I. Freund, N. Shvartsman, Phys. Rev. A **50**, 5164 (1994)