# Large negative and positive lateral shift on reflection from a left-handed prism coated with a weakly absorbing dielectric film

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Received: 7 September 2009 / Revised version: 19 November 2009 / Published online: 14 January 2010 © Springer-Verlag 2010

**Abstract** A large negative or positive lateral shift of the beam reflected from a left-handed prism coated with a weakly absorbing dielectric film is investigated theoretically. It is shown that the large lateral shift can be negative as well as positive due to the formation of the unusual guided modes in the weakly absorbing film near the resonant condition, which acts the same effect as the surface wave. This unusual guided wave can travel a longer distance than the surface wave. These giant negative and positive lateral shifts are almost fifty times larger than that from the lossless structure [Wang in Appl. Phys. Lett. 87:221102, 2005]. The sign of this large lateral shift could be controlled easily by adjusting the parameters of the present structure.

# 1 Introduction

The Goos–Hänchen (GH) effect [1, 2] is an optical phenomenon, in which a light beam undergoes a lateral shift away from the position predicted by geometrical optics when totally reflected. This effect, dependent on the polarization state of light [3], was first explained theoretically in terms of a stationary-phase approach [4]. Usually the GH shift is on the scale of the wavelength at a single dielectric interface. It was very difficult to observe directly in a single reflection, until Bretenaker [5] proposed a method based on the high sensitivity of the laser eigenstate to small perturbation. In the past decades, various systems have been proposed to enhance the lateral shift in many papers [6–11]. Recently, the GH shifts related with the left-handed materials

H. Wang (⊠) · Z. Zhou · H. Tian Department of Physics, Harbin Institute of Technology, Harbin 150001, China e-mail: wanghf@live.cn Fax: +860-451-86414130 (a material with both negative permittivity and negative permeability, which is also written as LHM, which was introduced theoretically by Veselago [12] and demonstrated experimentally by Shelby [13]) have been extensively studied for potential device applications. Berman [14], Lakhtakia [15], and Qing [16] intensively studied the lateral shift of the reflected beam from an interface between the left- and right-handed materials. Meanwhile, a large lateral shift was found on reflection from layered configurations with weakly absorbing media [17, 18] and resonant artificial structures [19] etc. Chen and Li [20] stated that the lateral shift can be negative as well as positive from a lossless left-handed slab. Wang and Zhu [18] presented the result that a weak absorption has an important influence on the lateral shift of a LHM slab in vacuum. In this paper, we analyze the large lateral shift from a left-handed prism coated with a weakly absorbing dielectric film due to the formation of the unusual guided modes; predict the conditions under which a large positive or negative lateral shift occurs in the present structure; and study the dependence of the lateral shift on the parameters of the present structure.

#### 2 Theory and calculation of the lateral beam shift

Figure 1 shows a structure consisting of a left-handed prism coated with a weakly absorbing dielectric film. The dielectric permittivity and magnetic permeability of medium 1, 2, 3 are  $\varepsilon_1$ ,  $\mu_1$ ,  $\varepsilon_2$ ,  $\mu_2$ ,  $\varepsilon_3$  and  $\mu_3$ , respectively. Only  $\varepsilon_2$ is a complex permittivity, which can be expressed as  $\varepsilon_2 = \varepsilon_r + i\varepsilon_i$ , where  $\varepsilon_r$  and  $\varepsilon_i$  are the real part and imaginary part of the permittivity  $\varepsilon_2$ , respectively. For the simplicity of analysis, three media are assumed to satisfy the relations of  $\varepsilon_1\mu_1 > \varepsilon_r\mu_2 > \varepsilon_3\mu_3 > 0$  and  $-\varepsilon_r \gg \varepsilon_i > 0$ . A light beam is incident on the interface between medium 1 and medium 2



Fig. 1 Schematic diagram of the left-handed prism coated with a weakly absorbing dielectric film. The *dashed line* is the path of the reflected beam predicted by geometrical optics

from the optically thick medium 1 at the angle of incidence  $\theta$  between  $\theta_1$  and  $\theta_2$ , where  $\theta_1 = \arcsin(\sqrt{\varepsilon_3\mu_3}/\sqrt{\varepsilon_1\mu_1})$  and  $\theta_2 = \arcsin(\sqrt{\varepsilon_r\mu_2}/\sqrt{\varepsilon_1\mu_1})$ , respectively. Thus the electromagnetic wave is a traveling wave in medium 2 and becomes an evanescent wave in medium 3. Furthermore, when the parameters of the present structure satisfy a certain resonant condition, unusual guided modes are formed in the *x* direction in medium 2, and the guided wave results in enhancement of the lateral shift.

For the incident beam with a large beam waist (i.e., a beam with a narrow angle spectrum), the lateral shift  $\Delta$  can be calculated analytically by  $\Delta = -(1/k) d\Phi/d\theta$ , where  $\Phi$  is the phase of the reflection coefficient,  $k = 2\pi n_1/\lambda$  is the wave vector in medium 1;  $\lambda$  is the wavelength of light in vacuum and the refractive index  $n_1$  of medium 1 can be written as  $n_1 = -(\varepsilon_1 \mu_1)^{1/2}$ . The reflection coefficient can be written as  $r = \rho \exp(i\Phi)$ ; thus the lateral shift also can be expressed by

$$\Delta = \frac{i\lambda}{2\pi n_1} \left( \frac{1}{r} \frac{dr}{d\theta} - \frac{1}{\rho} \frac{d\rho}{d\theta} \right). \tag{1}$$

In our configuration, for the monochromatic plane wave of angular frequency  $\omega$ , the reflection coefficients can be expressed by [21, 22]

$$r(\theta) = \frac{(1-\alpha_1)(1+\alpha_2) + (1+\alpha_1)(1-\alpha_2)\exp(2ik_{z2}d)}{(1+\alpha_1)(1+\alpha_2) + (1-\alpha_1)(1-\alpha_2)\exp(2ik_{z2}d)},$$
(2)

where  $k_{zi} = \eta(\varepsilon_i \mu_i - \varepsilon_1 \mu_1 \sin^2 \theta)^{1/2} \omega/c$  (i = 1, 2, 3),  $\alpha_1 = k_{z2}\mu_1/k_{z1}\mu_2$ ,  $\alpha_2 = k_{z3}\mu_2/k_{z2}\mu_3$  for TE polarization,  $\alpha_1 = k_{z2}\varepsilon_1/k_{z1}\varepsilon_2$ ,  $\alpha_2 = k_{z3}\varepsilon_2/k_{z2}\varepsilon_3$  for TM polarization, and *c* is the speed of light in vacuum;  $\eta = 1$  for right-handed material and  $\eta = -1$  for left-handed material.

For the present structure, the dispersion equation of the guided modes [23]:

$$\xi(\theta, d) = \operatorname{Re}[k_{z2}d - \arctan(|k_{z3}|\mu_2/k_{z2}\mu_3)] + \pi/2 = m\pi,$$
(3)

where *m* is an integer. From this equation, the resonant angle can be attained for certain values of the parameters in the structure, and is corresponding to an effective value of m. Because the absolute value of refractive index in medium 1 is larger than that of medium 2, this structure is a leaky wave guide. When d and  $\theta$  satisfy the resonant condition of guided modes, the guide wave is formed inside medium 2 along the x direction. As a result, the energy also travels in the weakly absorbing film along the x direction, which leads to a great enhancement of the lateral shift. But due to the leak of energy from medium 2 to medium 1, the intensity of the energy gradually decreases to zero and the propagating length is limited. We only consider the TE polarization, and the results are qualitatively similar for the TM polarization. In the following discussion, we chose the parameters of the system:  $\varepsilon_1 = -12$ ,  $\mu_1 = -1$  (for the LHM in Ref. [13]),  $\varepsilon_r = 1.96$ ,  $\mu_2 = 1$  (for Teflon),  $\varepsilon_3 = 1$  and  $\mu_3 = 1$  (for air), respectively.

### 3 Results and discussion

Figure 2 shows the dependence of the reflectivity  $\rho$  and lateral shift  $\Delta$  on the angle of incidence  $\theta$  for various  $\varepsilon_i$ . In Fig. 2(a), there are two dips (peaks) corresponding to the angles which satisfy the resonant conditions of  $\xi = m\pi$ (m = 1, 2). The larger m is, the smaller the corresponding resonant angle is. The angles of resonance are 29.60 and 24.56 degrees, respectively. It is also clear that as  $\varepsilon_i$ increases from zero, the reflectivity  $\rho$  decreases gradually; and  $\rho$  ultimately approaches zero when  $\varepsilon_i$  reaches the optimal value  $\varepsilon_{io}$  at the resonant angle of incidence corresponding to m = 1. But the reflectivity begins to increase when the increasing  $\varepsilon_i$  is larger than the optimal value, which can be seen from Fig. 2(b) and (c). From Fig. 2(d), we can see that the lateral shift is large negative near the resonant angle when  $\varepsilon_i$  is smaller than the optimal value  $\varepsilon_{io}$ , and large positive when  $\varepsilon_i$  is larger than the optimal value. That is to say, the lateral shift has an abrupt change over the optimal value of  $\varepsilon_i$ . Wang and Zhu showed that the lateral shift always is negative for a Kretschmann configuration without absorption [24] at any angle of incidence. However, for the structure with a weakly absorbing dielectric film, the lateral shifts can also be positive for a specified  $\varepsilon_i$  (i.e.,  $\varepsilon_i = 0.024$ ) near the resonant condition.

Figure 3 shows the dependence of the reflectivity  $\rho$ , phase  $\Phi$  and lateral shift  $\Delta$  on the imaginary part of the



Fig. 2 Dependence of the reflectivity (a), (b), (c) and lateral shift (d) on the angle of incidence for various  $\varepsilon_i$  for a TE-polarized reflected beam. The thickness of the weakly absorbing film is  $d = \lambda$ 

permittivity  $\varepsilon_i$ . The thickness of the weakly absorbing film is  $d = \lambda$ . When the resolution of  $\varepsilon_i$  is sufficiently high, negative and positive shifts of about 1500 $\lambda$  can be obtained. It is found from Fig. 3(b) that the lateral shift quickly transfers from the large negative value to the large positive value at the fixed thickness *d* and angle of incidence. Furthermore, we also found that the maximum of the lateral shift quickly damps with the deviation of the incident angle from the resonant angle.

Figure 4 shows a typical dependence of the reflectivity and lateral shift on the thickness d for various  $\varepsilon_i$  at the fixed incident angle  $\theta = 29.59^{\circ}$ . The lateral shift becomes large and the reflectivity becomes very small when the thickness d approaches the condition of resonance. For given  $\varepsilon_i$  and  $\theta$ , both large negative and positive lateral shifts can be attained by varying the thickness d of the weakly absorbing film. It can be seen from Figs. 2(a), (b), (c), 3(a) and 4(a) that for the structure with a weakly absorbing film the reflectivity is remarkably lowered when the resonant conditions are satisfied. This is to say, when the resonance takes place, the energy of the incident beam is almost completely coupled to the weakly absorbing film in which a gradually damped guided wave is formed, being able to propagate a long distance. Thus large positive and negative lateral shifts occur near the resonant condition, which is shown in Figs. 2(d), 3(b) and 4(b).

The above results show that loss parameter  $\varepsilon_i$  plays an important role in the lateral shift. For the case of without loss, the enhanced lateral shift is caused by the formation of leaky guide wave, thus the lateral shift is only tens of wavelengths even at resonance and always negative. As a result of the effect of  $\varepsilon_i$ , a guide wave without leakage is formed in the weakly absorbing film at resonance. For the wave guide, the internal damping is proportional to  $\varepsilon_i$ , and the radiation damping is almost independent on  $\varepsilon_i$ . With the increasing of  $\varepsilon_i$ , the internal damping increases from zero,



Fig. 3 Dependence of the reflectivity, phase (a) at the resonant angle of incidence 29.60° and lateral shift (b) on  $\varepsilon_i$ . The thickness of the weakly absorbing film is  $d = \lambda$ 

the lateral shifts are negative and enhanced gradually. At resonance ( $\varepsilon_i = \varepsilon_{io}$ ), null reflection indicates that no energy leaks out of the wave guide. Furthermore, as  $\varepsilon_i$  exceeds the optimal value  $\varepsilon_i$ , the internal damping is larger than the radiation damping and the lateral shifts become positive, which implies the excitation of the forward propagating wave. The effect of  $\varepsilon_i$  on the lateral shift is similar to that of the thickness of a metal film in the Kretschmann configuration at surface plasmon resonance [7].

## 4 Conclusions

In conclusion, a large negative or positive lateral shift from the left-handed prism coated with a weakly absorbing dielectric film has been analyzed, which is due to the formation of the unusual guided modes in a weakly absorbing film. When  $\theta$ ,  $\varepsilon_i$  and d satisfy the resonant condition of  $\xi = m$ , an unusual guided wave is formed in the second medium. In



Fig. 4 Dependence of the reflectivity (a) and lateral shift (b) on the thickness d for various values of  $\varepsilon_i$ . The angle of incidence is 29.59°

fact, the electromagnetic field in the *x* component acts as the surface wave, which decays more slowly, which leads to a large lateral shift. The sign of this large lateral shift could be controlled easily by adjusting the parameters (i.e.  $\theta$ ,  $\varepsilon_i$  and *d*) of the present structure. If the imaginary part of the permittivity in medium 2 sufficiently approaches the optimal value, the negative or positive lateral shift can reach about 1500 $\lambda$  near the resonant condition with weak absorption.

Acknowledgements The research has been financially supported by the Science Fund for Distinguished Young Scholars of Heilongjiang Province (Grant No. JC200710), Natural Science Foundation of China (Grant No. 50902034), National Science Foundation for Post-doctoral Scientists of China (Grant No. 20080440134), and Heilongjiang Postdoctoral Fund (Grant No. LBHZ-08115).

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