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Changes in the state of polarization of apertured stochastic electromagnetic modified Bessel–Gauss beams in free-space propagation

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Abstract By using the generalized Huygens–Fresnel diffraction integral, the analytical expressions for the cross-spectral density matrix, spectral degree of polarization, orientation angle and degree of ellipticity of polarization ellipse of apertured stochastic electromagnetic modified Bessel–Gauss beams (MBGBs) through a paraxial optical *ABCD* system are derived, and used to study the changes in the state of polarization of apertured stochastic electromagnetic MBGBs propagating in free space. The invariance of the on-axis state of polarization of unapertured stochastic electromagnetic MBGBs propagating through paraxial optical *ABCD* systems is illustrated analytically and numerically. For apertured stochastic electromagnetic MBGBs, the on-axis spectral degree of polarization, orientation angle and degree of ellipticity of polarization ellipse increase with increasing propagation distance, and approach asymptotic values when the propagation distance is large enough. There is a uniform distribution region of the state of polarization around the center of the beams whose range decreases with increasing truncation parameter. In addition, the state of polarization of apertured stochastic electromagnetic MBGBs upon propagation can be modulated by controlling the truncation parameter.

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1 Introduction

The polarization properties of stochastic electromagnetic wavefields are important when one takes into account their vectorial nature in various applications, e.g. in connection with optical communications, with imaging by laser radar and optical coherence tomography, etc. [1–3]. It is usually assumed that the state of polarization of a beam is invariant as the beam propagates in free space. This assumption is true only for spatially fully coherent beams. As usual, the state of polarization of a spatially partially coherent beam will change in free-space propagation [4]. Up to now, the changes in the spectral degree of polarization of partially coherent electromagnetic beams in dielectric media, through turbulent atmosphere, gradient-index fiber and through various complex optical systems have been studied extensively [5–10]. Most studies restricted to the spectral degree of polarization, and few papers explored the changes in the state of polarization represented by the polarization ellipse of unapertured electromagnetic Gaussian Schell-model (GSM) beams [11–13].

On the other hand, a class of partially coherent modified Bessel–Gauss beam (MBGBs) with a separable phase introduced by Ponomarenko [14] possess some remarkable properties, for example, it can carry optical vortices and its spectral degree of coherence does not depend on the relative orientation of a pair of points at the transversal plane. This kind of beams could be utilized for the case where highly isotropic coherence properties are need. Recently, Seshadri studied the propagation properties of spatially fully coherent scalar modified Bessel–Gauss beams [15]. The scintillation index of spatially fully coherent scalar modified

Bessel–Gaussian beams propagating in turbulent media was reported by Eyyuboğlu et al. [16]. However, the polarization properties of spatially partially coherent electromagnetic MBGBs have not been studied.

In this paper, by using the generalized Huygens–Fresnel diffraction integral, we study the changes in the state of polarization of apertured stochastic electromagnetic MBGBs in free-space propagation. In Sect. 2, the analytical expressions for the cross-spectral density matrix, spectral degree of polarization, orientation angle and degree of ellipticity of polarization ellipse of apertured stochastic electromagnetic MBGBs through a paraxial optical *ABCD* system are derived, and used to study the changes in the state of polarization of apertured stochastic electromagnetic MBGBs propagating in free space. In Sect. 3 a numerical calculation example is presented, and the results are interpreted physically. Section 4 concludes the main results obtained in this paper.

2 Cross-spectral density matrix of apertured stochastic electromagnetic MBGBs propagating through a paraxial optical *ABCD* system

Consider a stochastic, statistically stationary electromagnetic MBGBs, located at the plane $z=0$. The second-order correlation properties of the beam are characterized by a 2×2 electric cross-spectral density matrix [17]

$$\overleftrightarrow{W}^{0}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\left[W^{0}_{uv}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\right] \quad (u=x,y;\ v=x,y,\ \text{unless otherwise stated}), \tag{1a}$$

where $\boldsymbol{\rho}_l=(\rho_l,\varphi_l)$ $(l=1,2)$ is the two-dimensional position vector at the source plane $z=0$. The matrix element is

$$\begin{aligned}W^{0}_{uv}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)&=A_uA_vB_{uv}\frac{\xi_{uv}^{-m/2}}{1-\xi_{uv}}\exp\left[-im(\varphi_1-\varphi_2)\right]\\&\quad\times\exp\left[-\frac{1+\xi_{uv}}{1-\xi_{uv}}\frac{(\rho_1^2+\rho_2^2)}{w_0^2}\right]I_m\left(\frac{4\sqrt{\xi_{uv}}}{1-\xi_{uv}}\frac{\rho_1\rho_2}{w_0^2}\right),\end{aligned} \tag{1b}$$

where, $I_m(\bullet)$ denotes the modified Bessel function of order m, w_0 stands for the waist width of the Gaussian part ($m=0, \xi_{uv}=0$), ξ_{uv} is the coherence parameter, $0<\xi_{uv}<1$, and two limiting cases $\xi_{uv}\to 0$ and $\xi_{uv}\to 1$ correspond to the fully coherent and incoherent cases, respectively. The coefficients A_u, B_{uv}, ξ_{uv} and w_0 are independent of the position but may depend on the frequency ω [1]. The coefficients B_{uv} are generally complex when $u\neq v$ and $B_{uv}=1$ when $u=v$.

Within the framework of the paraxial approximation, the propagation of the cross-spectral density matrix through a paraxial optical system parameterized by transfer matrix $\left(\begin{smallmatrix}A&B\\C&D\end{smallmatrix}\right)$ obeys [18]

$$\begin{aligned}\overleftrightarrow{W}(\boldsymbol{r}_1,\boldsymbol{r}_2,z,\omega)&=\left(\frac{k}{2\pi B}\right)^2\int_0^{+\infty}\int_0^{+\infty}\int_0^{2\pi}\int_0^{2\pi}\overleftrightarrow{W}^{0}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\\&\quad\times H(\rho_1)H^*(\rho_2)\exp\bigg(-\frac{ik}{2B}\Big\{A\big(\rho_1^2-\rho_2^2\big)\\&\quad-2\big[r_1\rho_1\cos(\theta_1-\varphi_1)-r_2\rho_2\cos(\theta_2-\varphi_2)\big]\\&\quad+D\big(r_1^2-r_2^2\big)\Big\}\bigg)\rho_1\rho_2\,d\rho_1\,d\rho_2\,d\varphi_1\,d\varphi_2,\end{aligned} \tag{2}$$

where k is the wave number related to the wavelength λ by $k=2\pi/\lambda$. $\boldsymbol{r}_l=(r_l,\theta_l)$ $(l=1,2)$ is the position vector at the plane z.

The hard-edged aperture function is written as

$$H(\rho)=\begin{cases}1, & |\rho|\le a,\\ 0, & |\rho|>a.\end{cases} \tag{3}$$

$H(\rho)$ can be expressed as a finite sum of complex Gaussian functions

$$H(\rho)=\sum_{n=1}^{N}F_n\exp\left(-\frac{G_n\rho^2}{a^2}\right), \tag{4}$$

where a is the half width of circular aperture, the coefficients F_n, G_n and N are given in [19].

By substituting (1), (3) and (4) into (2), and performing integral calculations, the cross-spectral density matrix of apertured stochastic electromagnetic MBGBs propagating through a paraxial optical *ABCD* system at the plane z is given by

$$\overleftrightarrow{W}(\boldsymbol{r}_1,\boldsymbol{r}_2,z,\omega)=\left[W_{uv}(\boldsymbol{r}_1,\boldsymbol{r}_2,z,\omega)\right] \tag{5a}$$

where

$$\begin{aligned}W_{uv}(\boldsymbol{r}_1,\boldsymbol{r}_2,z,\omega)&=\frac{w_0^2}{w_{uv}^2}\sum_{s=1}^{N}\sum_{n=1}^{N}F_sF_n^*A_uA_vB_{uv}\frac{\xi_{uv}^{-m/2}}{1-\xi_{uv}}\\&\quad\times\exp\left[-im(\theta_1-\theta_2)\right]\\&\quad\times\exp\left[-\left(\frac{1+\xi_{uv}}{1-\xi_{uv}}+\frac{G_n^*}{\delta^2}\right)\frac{r_1^2}{w_{uv}^2}\right.\\&\quad\left.-\left(\frac{1+\xi_{uv}}{1-\xi_{uv}}+\frac{G_s}{\delta^2}\right)\frac{r_2^2}{w_{uv}^2}\right]\\&\quad\times I_m\left(\frac{4\sqrt{\xi_{uv}}}{1-\xi_{uv}}\frac{r_1r_2}{w_{uv}^2}\right)\exp\left[-ik\frac{(r_1^2-r_2^2)}{2R_{uv}}\right],\end{aligned} \tag{5b}$$

$$w_{uv}^2 = w_0^2\frac{B^2}{z_0^2}\left[1+\left(\frac{G_s}{\delta^2}+\frac{iAz_0}{B}\right)\left(\frac{G_n^*}{\delta^2}-\frac{iAz_0}{B}\right)\right.$$
$$\left.+\frac{1+\xi_{uv}}{1-\xi_{uv}}\left(\frac{G_s}{\delta^2}+\frac{G_n^*}{\delta^2}\right)\right], \tag{5c}$$

$$R_{uv} = \frac{Bw_{uv}^2}{Dw_{uv}^2 - Aw_0^2}, \tag{5d}$$

$$\delta = \frac{a}{w_0} \quad \text{(truncation parameter)}, \tag{5e}$$

$$z_0 = \frac{\pi w_0^2}{\lambda} \quad \text{(Rayleigh length)}. \tag{5f}$$

In the derivation of (5) the following integral formulae were used [20]:

$$\int_0^{2\pi} \exp\left[ix\cos(\varphi_1-\varphi_2)-in\varphi_1\right]d\varphi_1$$
$$= 2\pi i^n \exp(-in\varphi_2)J_n(x), \tag{6a}$$

$$\int_0^{\infty} x\exp\left[-ex^2\right]J_n(fx)I_n(gx)\,dx$$
$$= \frac{1}{2e}\exp\left(\frac{g^2-f^2}{4e}\right)J_n\left(\frac{gf}{2e}\right), \tag{6b}$$

$$\int_0^{\infty} x\exp\left[-\beta x^2\right]J_n(\alpha x)J_n(\gamma x)\,dx$$
$$= \frac{1}{2\beta}\exp\left(-\frac{\alpha^2+\gamma^2}{4\beta}\right)I_n\left(\frac{\alpha\gamma}{2\beta}\right). \tag{6c}$$

Equation (5) indicates that the cross-spectral density matrix of apertured stochastic electromagnetic MBGBs propagating through a paraxial optical *ABCD* system is dependent on the truncation parameter δ, beam parameters m, ξ_{uv}, coefficients A_u, A_v, B_{uv} and optical *ABCD* system. For the unapertured case ($\delta \to \infty$), the matrix element reduces to

$$W_{uv}(\boldsymbol{r}_1,\boldsymbol{r}_2,z,\omega)$$
$$= \frac{w_0^2}{w^2}\frac{\xi_{uv}^{-m/2}}{1-\xi_{uv}}A_uA_vB_{uv}\exp\left[-im(\theta_1-\theta_2)\right]$$
$$\times\exp\left[-\frac{1+\xi_{uv}}{1-\xi_{uv}}\frac{(r_1^2+r_2^2)}{w^2}\right]$$
$$\times I_m\left(\frac{4\sqrt{\xi_{uv}}}{1-\xi_{uv}}\frac{r_1r_2}{w^2}\right)\exp\left[-ik\frac{(r_1^2-r_2^2)}{2R}\right], \tag{7a}$$

where

$$w^2 = w_0^2\left[A^2+(B/z_0)^2\right], \tag{7b}$$

$$R = \frac{Bw^2}{Dw^2-Aw_0^2}. \tag{7c}$$

The spectral degree of polarization $P(\boldsymbol{r},z,\omega)$, orientation angle $\phi(\boldsymbol{r},z,\omega)$ and degree of ellipticity $\varepsilon(\boldsymbol{r},z,\omega)$ of polarization ellipse of apertured stochastic electromagnetic MBGBs through a paraxial optical *ABCD* system are expressed as [1, 11, 13]

$$P(\boldsymbol{r},z,\omega)$$
$$= \sqrt{1-\frac{4\mathrm{Det}[\overleftrightarrow{\mathrm{W}}(\boldsymbol{r},\boldsymbol{r},z,\omega)]}{\{\mathrm{Tr}[\overleftrightarrow{\mathrm{W}}(\boldsymbol{r},\boldsymbol{r},z,\omega)]\}^2}}$$
$$= \sqrt{\frac{[W_{xx}(\boldsymbol{r},\boldsymbol{r},z,\omega)-W_{yy}(\boldsymbol{r},\boldsymbol{r},z,\omega)]^2+4|W_{xy}(\boldsymbol{r},\boldsymbol{r},z,\omega)|^2}{[W_{xx}(\boldsymbol{r},\boldsymbol{r},z,\omega)+W_{yy}(\boldsymbol{r},\boldsymbol{r},z,\omega)]^2}}, \tag{8}$$

$$\phi(\boldsymbol{r},z,\omega) = \frac{1}{2}\arctan\left\{\frac{2\mathrm{Re}[W_{xy}(\boldsymbol{r},\boldsymbol{r},z,\omega)]}{W_{xx}(\boldsymbol{r},\boldsymbol{r},z,\omega)-W_{yy}(\boldsymbol{r},\boldsymbol{r},z,\omega)}\right\}, \tag{9}$$

$$\varepsilon(\boldsymbol{r},z,\omega) = \frac{A_{\mathrm{minor}}}{A_{\mathrm{major}}}, \tag{10}$$

where the "degree of ellipticity" characterizes the shape of polarization ellipse and W_{uv}, A_{minor} and A_{major} are given by

$$W_{uv}(\boldsymbol{r},\boldsymbol{r},z,\omega) = \frac{w_0^2}{w_{uv}^2}\sum_{s=1}^{N}\sum_{n=1}^{N}F_sF_n^*A_uA_vB_{uv}\frac{\xi_{uv}^{-m/2}}{1-\xi_{uv}}\exp\left(-\frac{1+\xi_{uv}}{1-\xi_{uv}}\frac{2r^2}{w_{uv}^2}\right)$$
$$\times\exp\left[-\left(\frac{G_s}{\delta^2}+\frac{G_n^*}{\delta^2}\right)\frac{r^2}{w_{uv}^2}\right]I_m\left(\frac{4\sqrt{\xi_{uv}}}{1-\xi_{uv}}\frac{r^2}{w_{uv}^2}\right), \tag{11}$$

$$A_{\mathrm{minor}}^2(\boldsymbol{r},z,\omega) = \left(\sqrt{[W_{xx}(\boldsymbol{r},\boldsymbol{r},z,\omega)-W_{yy}(\boldsymbol{r},\boldsymbol{r},z,\omega)]^2+4|W_{xy}(\boldsymbol{r},\boldsymbol{r},z,\omega)|^2}\right.$$
$$\left.-\sqrt{[W_{xx}(\boldsymbol{r},\boldsymbol{r},z,\omega)-W_{yy}(\boldsymbol{r},\boldsymbol{r},z,\omega)]^2+4[\mathrm{Re}W_{xy}(\boldsymbol{r},\boldsymbol{r},z,\omega)]^2}\right)/2, \tag{12a}$$

$$A_{\mathrm{major}}^2(\boldsymbol{r},z,\omega) = \left(\sqrt{[W_{xx}(\boldsymbol{r},\boldsymbol{r},z,\omega)-W_{yy}(\boldsymbol{r},\boldsymbol{r},z,\omega)]^2+4|W_{xy}(\boldsymbol{r},\boldsymbol{r},z,\omega)|^2}\right.$$
$$\left.+\sqrt{[W_{xx}(\boldsymbol{r},\boldsymbol{r},z,\omega)-W_{yy}(\boldsymbol{r},\boldsymbol{r},z,\omega)]^2+4[\mathrm{Re}W_{xy}(\boldsymbol{r},\boldsymbol{r},z,\omega)]^2}\right)/2. \tag{12b}$$

The on-axis spectral degree of polarization P, orientation angle ϕ and degree of ellipticity of polarization ellipse ε of apertured stochastic electromagnetic MBGBs are expressed as

$$P(0,z,\omega)=\sqrt{\frac{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{w_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{w_{yy}^2(1-\xi_{yy})}\right]^2+4\left|\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{w_{xy}^2(1-\xi_{xy})}\right|^2}{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{w_{xx}^2(1-\xi_{xx})}+\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{w_{yy}^2(1-\xi_{yy})}\right]^2}}, \tag{13}$$

$$\phi(0,z,\omega)=\frac{1}{2}\arctan\left\{\frac{2\mathrm{Re}\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{w_{xy}^2(1-\xi_{xy})}\right]}{\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{w_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{w_{yy}^2(1-\xi_{yy})}}\right\}, \tag{14}$$

$$\begin{aligned}\varepsilon(0,z,\omega)=\Bigg[\Bigg(&\sqrt{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{w_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{w_{yy}^2(1-\xi_{yy})}\right]^2+4\left|\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{w_{xy}^2(1-\xi_{xy})}\right|^2}\\
&-\sqrt{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{w_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{w_{yy}^2(1-\xi_{yy})}\right]^2+4\mathrm{Re}\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{w_{xy}^2(1-\xi_{xy})}\right]^2}\Bigg)\Bigg/\\
\Bigg(&\sqrt{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{w_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{w_{yy}^2(1-\xi_{yy})}\right]^2+4\left|\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{w_{xy}^2(1-\xi_{xy})}\right|^2}\\
&+\sqrt{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{w_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{w_{yy}^2(1-\xi_{yy})}\right]^2+4\mathrm{Re}\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{w_{xy}^2(1-\xi_{xy})}\right]^2}\Bigg)\Bigg]^{\frac{1}{2}}.\end{aligned} \tag{15}$$

When $\delta\to\infty$,

$$P(0,z,\omega)=\sqrt{\frac{\left(\frac{A_x^2}{1-\xi_{xx}}-\frac{A_y^2}{1-\xi_{yy}}\right)^2+4\left|\frac{A_xA_yB_{xy}}{1-\xi_{xy}}\right|^2}{\left(\frac{A_x^2}{1-\xi_{xx}}+\frac{A_y^2}{1-\xi_{yy}}\right)^2}}, \tag{16}$$

$$\phi(0,z,\omega)=\frac{1}{2}\arctan\left[\frac{2\mathrm{Re}\left(\frac{A_xA_yB_{xy}}{1-\xi_{xy}}\right)}{\frac{A_x^2}{1-\xi_{xx}}+\frac{A_y^2}{1-\xi_{yy}}}\right], \tag{17}$$

$$\varepsilon(0,z,\omega)=\sqrt{\frac{\sqrt{\left(\frac{A_x^2}{1-\xi_{xx}}-\frac{A_y^2}{1-\xi_{yy}}\right)^2+4\left|\frac{A_xA_yB_{xy}}{1-\xi_{xy}}\right|^2}-\sqrt{\left(\frac{A_x^2}{1-\xi_{xx}}-\frac{A_y^2}{1-\xi_{yy}}\right)^2+4\left[\mathrm{Re}\left(\frac{A_xA_yB_{xy}}{1-\xi_{xy}}\right)\right]^2}}{\sqrt{\left(\frac{A_x^2}{1-\xi_{xx}}-\frac{A_y^2}{1-\xi_{yy}}\right)^2+4\left|\frac{A_xA_yB_{xy}}{1-\xi_{xy}}\right|^2}+\sqrt{\left(\frac{A_x^2}{1-\xi_{xx}}-\frac{A_y^2}{1-\xi_{yy}}\right)^2+4\left[\mathrm{Re}\left(\frac{A_xA_yB_{xy}}{1-\xi_{xy}}\right)\right]^2}}}. \tag{18}$$

Equations (16)–(18) indicate the invariance of the on-axis state of polarization of unapertured stochastic electromagnetic MBGBs propagating through paraxial optical *ABCD* systems.

3 Illustrative example

In free-space propagation, the *ABCD* matrix reduces to

$$\begin{pmatrix}A & B\\ C & D\end{pmatrix}=\begin{pmatrix}1 & z\\ 0 & 1\end{pmatrix}. \tag{19}$$

On placing (19) into (5c), we obtain

$$w_{uv}^2=w_0^2\frac{z^2}{z_0^2}\left[1+\left(\frac{G_s}{\delta^2}+i\frac{z_0}{z}\right)\left(\frac{G_n^*}{\delta^2}-i\frac{z_0}{z}\right)+\frac{1+\xi_{uv}}{1-\xi_{uv}}\left(\frac{G_s}{\delta^2}+\frac{G_n^*}{\delta^2}\right)\right]. \tag{20}$$

In the following calculations, $m=0, \xi_{xx}=0.5, \xi_{yy}=0.6, \xi_{xy}=0.4, A_x=2, A_y=1, B_{xy}=0.3\exp(i\pi/6)$, and $N=10$ are kept fixed, and the selected parameters satisfy the non-negative definiteness of the cross-spectral density matrix [21].

Figure 1(a)–(c) gives the on-axis spectral degree of polarization P (a), orientation angle ϕ (b) and degree of ellipticity of polarization ellipse ε (c) of an apertured stochastic electromagnetic MBGB versus the propagation distance $z/z_0 (z_0 = \pi w_0^2/\lambda)$ for different values of the truncation parameter $\delta = 2, 2.5$ and 4.8. For the unapertured case ($\delta \to \infty$) the on-axis P, ϕ and ε remain unchanged upon propagation, which is consistent with (16)–(18). It is shown that for the apertured case with increasing $z/z_0 P, \phi$ and ε increase and approach asymptotic values when z/z_0 is large enough. For example, depending on δ, the asymptotic value of P is equal to 0.62, 0.60 and 0.56 for $\delta = 2$, 2.5 and 4.8, respectively. In addition, the calculation results indicate that there is nearly no difference between the results at $\delta = 4.8$ and $\delta \to \infty$, i.e. the aperture diffraction effect can be ignored when $\delta > 4.8$. The above results can be interpreted as follows.

Equation (13) can be rewritten as

$$P(0,z,\omega) = \sqrt{\left[\frac{\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{w_{yy}^2 F_s F_n^* A_x^2}{w_{xx}^2(1-\xi_{xx})} - \sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_s F_n^* A_y^2}{1-\xi_{yy}}}{\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{w_{yy}^2 F_s F_n^* A_x^2}{w_{xx}^2(1-\xi_{xx})} + \sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_s F_n^* A_y^2}{1-\xi_{yy}}}\right]^2 + 4\frac{\left|\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_s F_n^* A_x A_y B_{xy}}{1-\xi_{xy}}\right|^2}{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{w_{xy}^2 F_s F_n^* A_x^2}{w_{xx}^2(1-\xi_{xx})} + \sum_{s=1}^{N}\sum_{n=1}^{N}\frac{w_{xy}^2 F_s F_n^* A_y^2}{w_{yy}^2(1-\xi_{yy})}\right]^2}}. \quad (21)$$

As can been seen from (21), $P(0,z,\omega)$ increases with increasing z/z_0 if w_{yy}^2/w_{xx}^2 is an increasing function of z/z_0, whereas w_{xy}^2/w_{xx}^2 and w_{xy}^2/w_{yy}^2 are decreasing functions of z/z_0.

Because the term w_{yy}^2/w_{xx}^2 can be expressed as

$$\frac{w_{yy}^2}{w_{xx}^2} = \frac{1 + \left(\frac{G_s}{\delta^2} + i\frac{z_0}{z}\right)\left(\frac{G_n^*}{\delta^2} - i\frac{z_0}{z}\right) + \frac{1+\xi_{yy}}{1-\xi_{yy}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right)}{1 + \left(\frac{G_s}{\delta^2} + i\frac{z_0}{z}\right)\left(\frac{G_n^*}{\delta^2} - i\frac{z_0}{z}\right) + \frac{1+\xi_{xx}}{1-\xi_{xx}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right)}, \quad (22)$$

and the selected parameters satisfy the condition

$$\frac{1+\xi_{yy}}{1-\xi_{yy}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right) > \frac{1+\xi_{xx}}{1-\xi_{xx}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right), \quad (23)$$

thus w_{yy}^2/w_{xx}^2 increases with increasing z/z_0. The terms w_{xy}^2/w_{xx}^2 and w_{xy}^2/w_{yy}^2 are given by

$$\frac{w_{xy}^2}{w_{xx}^2} = \frac{1 + \left(\frac{G_s}{\delta^2} + i\frac{z_0}{z}\right)\left(\frac{G_n^*}{\delta^2} - i\frac{z_0}{z}\right) + \frac{1+\xi_{xy}}{1-\xi_{xy}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right)}{1 + \left(\frac{G_s}{\delta^2} + i\frac{z_0}{z}\right)\left(\frac{G_n^*}{\delta^2} - i\frac{z_0}{z}\right) + \frac{1+\xi_{xx}}{1-\xi_{xx}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right)}, \quad (24)$$

$$\frac{w_{xy}^2}{w_{yy}^2} = \frac{1 + \left(\frac{G_s}{\delta^2} + i\frac{z_0}{z}\right)\left(\frac{G_n^*}{\delta^2} - i\frac{z_0}{z}\right) + \frac{1+\xi_{xy}}{1-\xi_{xy}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right)}{1 + \left(\frac{G_s}{\delta^2} + i\frac{z_0}{z}\right)\left(\frac{G_n^*}{\delta^2} - i\frac{z_0}{z}\right) + \frac{1+\xi_{yy}}{1-\xi_{yy}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right)}, \quad (25)$$

respectively, where

$$\frac{1+\xi_{xy}}{1-\xi_{xy}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right) < \frac{1+\xi_{xx}}{1-\xi_{xx}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right), \quad (26)$$

$$\frac{1+\xi_{xy}}{1-\xi_{xy}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right) < \frac{1+\xi_{xx}}{1-\xi_{xx}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right). \quad (27)$$

Therefore, w_{xy}^2/w_{xx}^2 and w_{xy}^2/w_{yy}^2 decrease with increasing z/z_0. A similar procedure can be used to explain the evolution of ϕ and ε upon propagation with increasing z/z_0.

For large enough z/z_0, (20) can be approximately expressed as

$$w_{uv}^2 \cong w_0^2\frac{z^2}{z_0^2}\left[1 + \frac{G_s G_n^*}{\delta^4} + \frac{1+\xi_{uv}}{1-\xi_{uv}}\left(\frac{G_s}{\delta^2} + \frac{G_n^*}{\delta^2}\right)\right], \quad (28)$$

thus (13)–(15) are written as

$$P(0,z,\omega)=\sqrt{\frac{[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{H_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{H_{yy}^2(1-\xi_{yy})}]^2+4|\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{H_{xy}^2(1-\xi_{xy})}|^2}{[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{H_{xx}^2(1-\xi_{xx})}+\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{H_{yy}^2(1-\xi_{yy})}]^2}}, \tag{29}$$

$$\phi(0,z,\omega)=\frac{1}{2}\arctan\left\{\frac{2\mathrm{Re}[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{H_{xy}^2(1-\xi_{xy})}]}{\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{H_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{H_{yy}^2(1-\xi_{yy})}}\right\}, \tag{30}$$

$$\begin{aligned}\varepsilon(0,z,\omega)=\Bigg[\Bigg(&\sqrt{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{H_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{H_{yy}^2(1-\xi_{yy})}\right]^2+4\left|\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{H_{xy}^2(1-\xi_{xy})}\right|^2}\\ &-\sqrt{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{H_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{H_{yy}^2(1-\xi_{yy})}\right]^2+4\mathrm{Re}\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{H_{xy}^2(1-\xi_{xy})}\right]^2}\Bigg)\Bigg/\\ \Bigg(&\sqrt{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{H_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{H_{yy}^2(1-\xi_{yy})}\right]^2+4\left|\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{H_{xy}^2(1-\xi_{xy})}\right|^2}\\ &+\sqrt{\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_x^2}{H_{xx}^2(1-\xi_{xx})}-\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_y^2}{H_{yy}^2(1-\xi_{yy})}\right]^2+4\mathrm{Re}\left[\sum_{s=1}^{N}\sum_{n=1}^{N}\frac{F_sF_n^*A_xA_yB_{xy}}{H_{xy}^2(1-\xi_{xy})}\right]^2}\Bigg)\Bigg]^{\frac{1}{2}},\end{aligned} \tag{31}$$

where

$$H_{uv}^2=1+\frac{G_sG_n^*}{\delta^4}+\frac{1+\xi_{uv}}{1-\xi_{uv}}\left(\frac{G_s}{\delta^2}+\frac{G_n^*}{\delta^2}\right). \tag{32}$$

We see that P,ϕ and ε are independent of z/z_0, and on substituting $\delta=2,2.5$ and 4.8 into (29), we obtain $P=0.62$, 0.60 and 0.56, respectively, which is consistent with the above results in Fig. 1(a).

In Fig. 2(a)–(c), we plot the off-axis spectral degree of polarization P (a), orientation angle ϕ (b) and degree of ellipticity of polarization ellipse ε (c) of an apertured stochastic electromagnetic MBGB versus the propagation distance z/z_0 for different values of the truncation parameter $\delta=0.5$, 1.2 and ∞ with $r/w_0=10$. It is shown that the truncation parameter δ affects the state of polarization. The range of the off-axis ΔP, $\Delta\phi$ and $\Delta\varepsilon$ defined as the difference between the maximum and minimum values of the P,ϕ and ε increases with increasing δ.

The color-coded plots of the spectral degree of polarization P ((a)–(c)), orientation angle ϕ ((d)–(f)) and degree of ellipticity of polarization ellipse ε ((g)–(i)) of an apertured stochastic electromagnetic MBGB at the plane $z/z_0=10$ for different values of the truncation parameter δ are given in Fig. 3(a)–(i). In Fig. 3(a), (d) and (g), i.e., for the case of $\delta=0.5$, we see that there is a uniform distribution region of the state of polarization around the center of the beams. With increasing δ, the range of the uniform distribution region decreases.

The spectral degree of polarization P (a), orientation angle ϕ (b) and degree of ellipticity of polarization ellipse ε (c) of an apertured stochastic electromagnetic MBGB versus the truncation parameter δ at the plane $z/z_0=10$ for different values of the relative transversal distance $r/w_0=10$, 20 and 30 are plotted in Fig. 4(a)–(c). As can be seen, P,ϕ and ε change with increasing truncation parameter δ, and exhibit an oscillatory behavior when $\delta<3$ at $r/w_0=30$. This phenomenon is induced by the aperture diffraction and can be used to modulate the state of polarization of apertured stochastic electromagnetic MBGBs upon propagation. For example, $(P,\phi,\varepsilon)=(0.484, 0.183\text{ rad}, 0.078)$ and $(P,\phi,\varepsilon)=(0.347, 0.115\text{ rad}, 0.065)$ are obtained at $\delta=1$ and 2, respectively.

4 Conclusion

In this paper, based on the generalized Huygens–Fresnel diffraction integral, the analytical expressions for the cross-spectral density matrix, spectral degree of polarization,

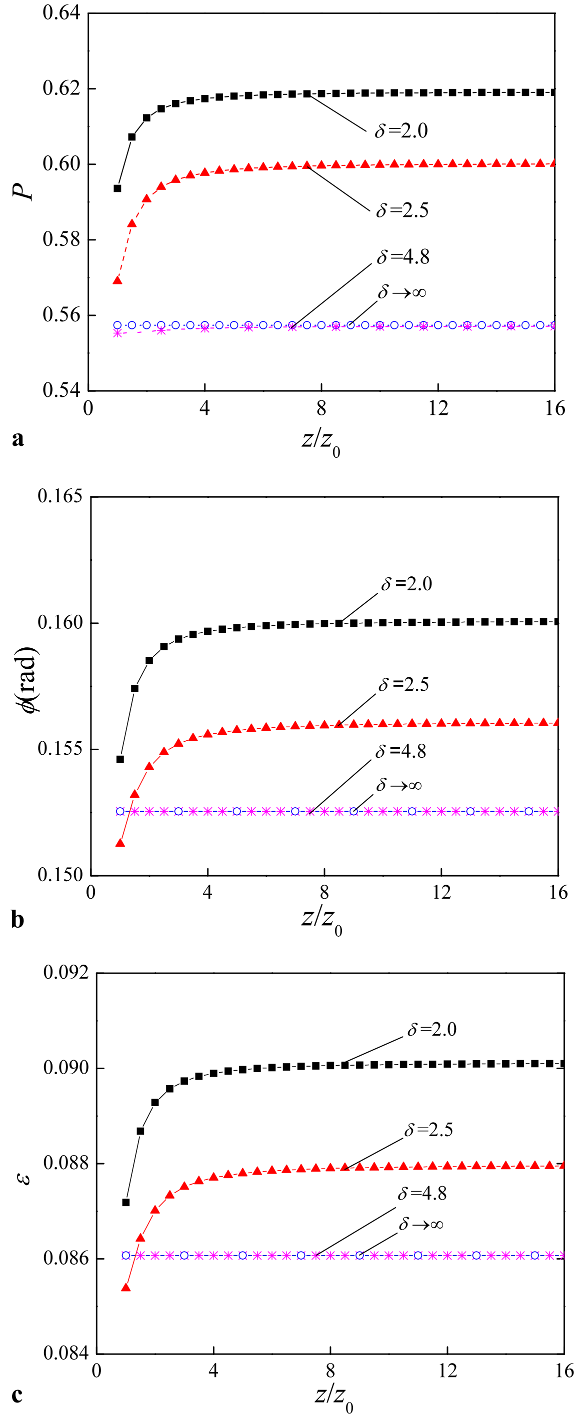

Fig. 1 On-axis spectral degree of polarization P (**a**), orientation angle ϕ (**b**) and degree of ellipticity of polarization ellipse ε (**c**) of an apertured stochastic electromagnetic MBGB versus the propagation distance z/z_0 for different values of the truncation parameter $\delta = 2, 2.5, 4.8$ and ∞

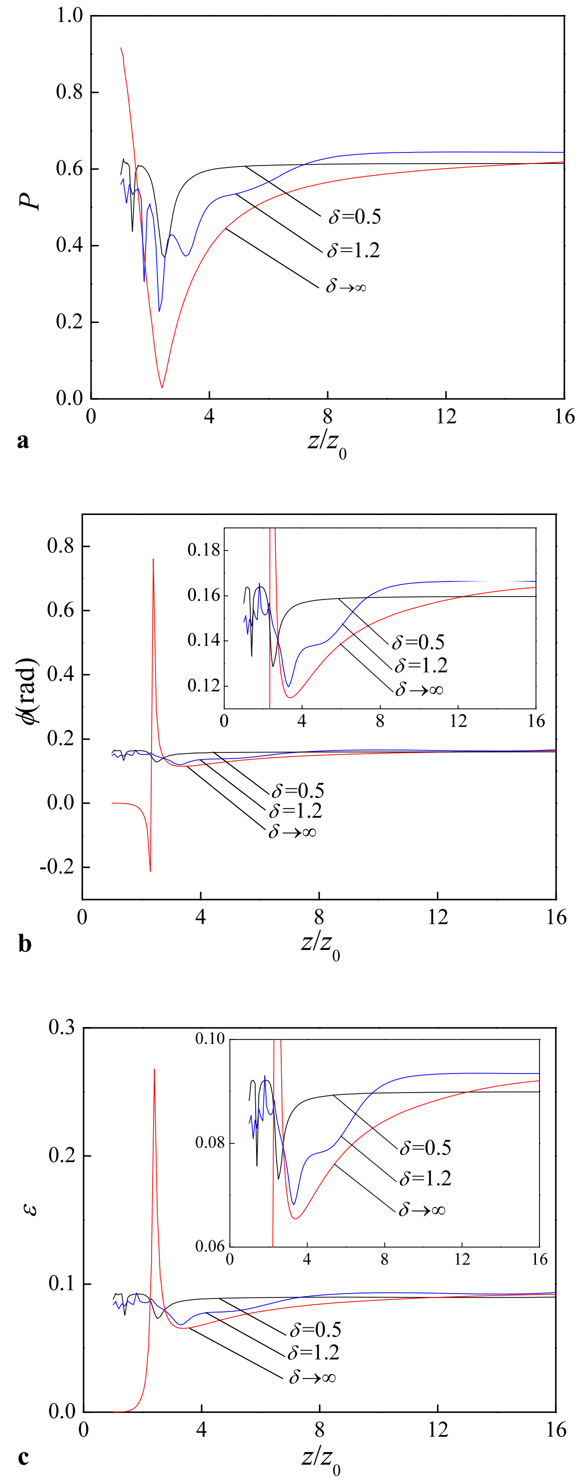

Fig. 2 Off-axis spectral degree of polarization P (**a**), orientation angle ϕ (**b**) and degree of ellipticity of polarization ellipse ε (**c**) of an apertured stochastic electromagnetic MBGB versus the propagation distance z/z_0 for different values of the truncation parameter $\delta = 0.5, 1.2$ and ∞

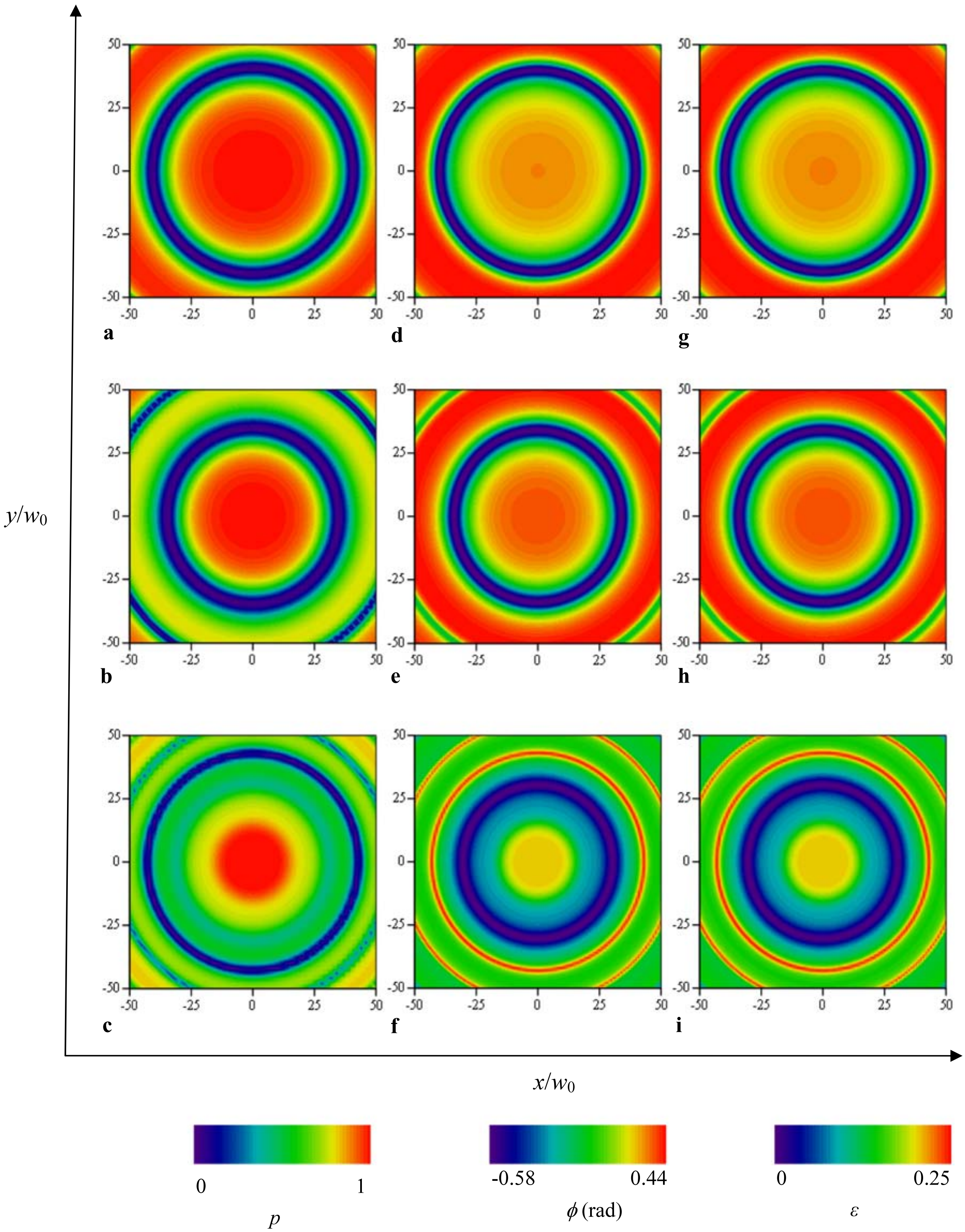

Fig. 3 Color-coded plot of the spectral degree of polarization P ((**a**)–(**c**)), orientation angle ϕ ((**d**)–(**f**)) and degree of ellipticity of polarization ellipse ε ((**g**)–(**i**)) of an apertured stochastic electromagnetic MBGB at the plane $z/z_0 = 10$ for different values of the truncation parameter δ. (**a**), (**d**), (**g**): $\delta = 0.5$, (**b**), (**e**), (**h**): $\delta = 0.6$, (**c**), (**f**), (**i**): $\delta = 1.2$

orientation angle and degree of ellipticity of polarization ellipse of apertured stochastic electromagnetic MBGBs through a paraxial optical *ABCD* system have been derived. We have shown analytically and numerically the invariance of the on-axis state of polarization of unapertured stochastic electromagnetic MBGBs through paraxial optical *ABCD* systems, which is different from stochastic electromagnetic GSM beams, where the on-axis spectral degree of polariza-

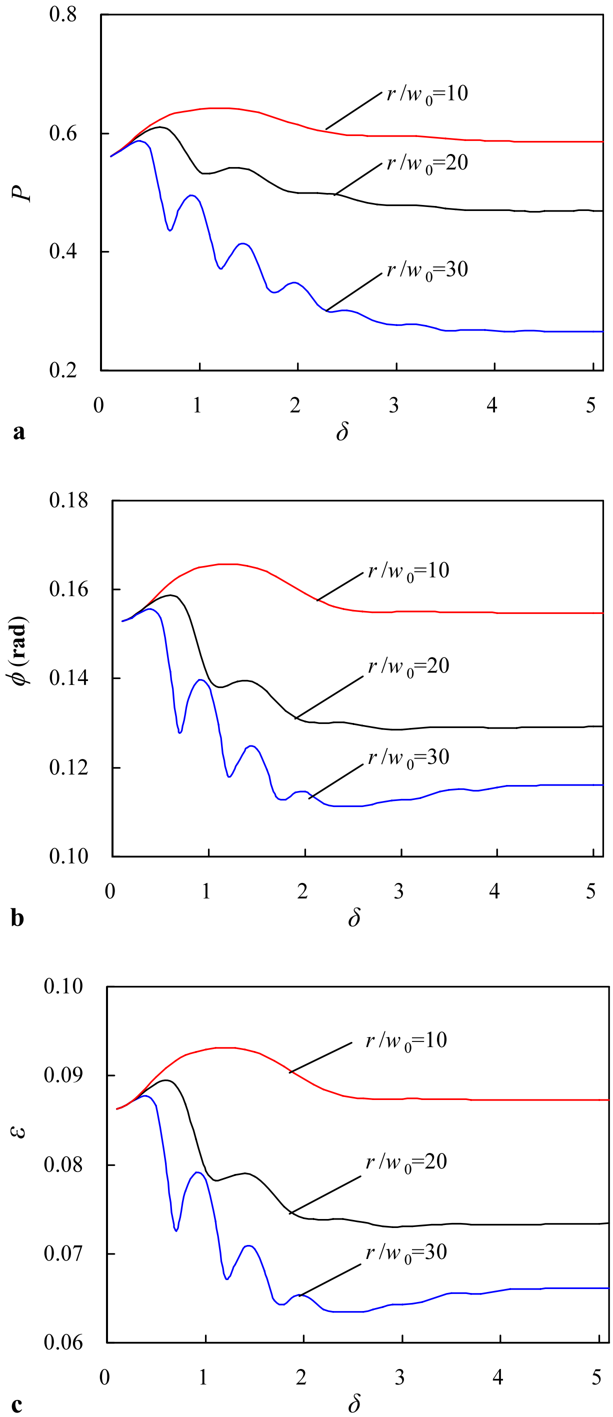

Fig. 4 Spectral degree of polarization P (**a**), orientation angle ϕ (**b**) and degree of ellipticity of polarization ellipse ε (**c**) of an apertured stochastic electromagnetic MBGB versus truncation parameter δ at the plane $z/z_0 = 10$ for different values of the relative transversal distance $r/w_0 = 10$, 20 and 30

tion, orientation angle and degree of ellipticity of polarization ellipse change upon propagation [11]. For apertured stochastic electromagnetic MBGBs the on-axis spectral degree of polarization, orientation angle and degree of ellipticity of polarization ellipse increase with increasing propagation distance, and approach asymptotic values when the propagation distance is large enough. The range of off-axis ΔP, $\Delta\phi$ and $\Delta\varepsilon$ defined as the difference between the maximum and minimum values of the P, ϕ and ε increases with increasing δ. There is a uniform distribution region of the state of polarization around the center of the beams whose range decreases with increasing δ. The state of polarization of apertured stochastic electromagnetic MBGBs can be modulated by controlling truncation parameter δ. The results obtained may provide additional information about the sample under test in optical diagnostic application such as polarization sensitive optical coherence tomography, etc. [2, 3].

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