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Changes in the spectral degree of coherence and spectral intensity of spatially and spectrally partially coherent cosh-Gaussian pulsed beams in free space

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Abstract The analytical expressions for the spectral degree of coherence and spectral intensity of spatially and spectrally partially coherent cosh-Gaussian pulsed (ChGP) beams propagating in free space are derived. It is shown that the spectral degree of coherence and spectral intensity of spatially and spectrally partially coherent ChGP beams depend on the spatial correlation parameter, decentered parameter and temporal coherence length. Depending on the decentered parameter, the effective width of the spectral degree of coherence can be larger than, smaller than or equal to that of the spectral intensity. The results are illustrated by numerical examples.

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1 Introduction

In recent years the spatial correlation properties of spatially partially coherent wave fields have attracted much attention [1–8]. Fischer and Visser analyzed the variation of the spectral degree of coherence and spectral intensity of focused spatially partially coherent Schell-model beams on the axis and in the focal plane [2]. Roychowdhury et al. dealt with electromagnetic Gaussian Schell-model (GSM) beams propagating through a gradient-index fiber, and the periodic variation of the spectral degree of coherence [3]. The spatial correlation properties of partially coherent flat-topped beams and Schell-model beams propagating through atmospheric turbulence were studied by Ji et al., who found that the oscillatory behavior of the spectral degree of coherence becomes weaker with increasing turbulence [5, 6]. Eyyuboglu et al. evaluated the spectral degree of coherence of partially coherent general beams including partially coherent cosh-Gaussian, cos-Gaussian, Gaussian, annular, and higher-order Gaussian beams in atmospheric turbulence and found that the modulus of the spectral degree of coherence for partially coherent general beams in free space approaches unity for large propagation distances [7].

On the other hand, much interest has been exhibited in ultra-short optical pulsed beams due to rapid advances in femtosecond pulsed laser technologies capable of generating few-cycle pulses in the laboratory [9–17]. As usual, the spectral components of ultra-short pulses are assumed to be fully correlated [12]. Recently, the concept of partially coherent pulses was introduced by Pääkkönen et al. [13], and a coherent-mode representation for spatially and spectrally partially coherent pulses was proposed by Lajunen et al. to describe some basic properties of such a type of pulses, and illustrated by spatially and spectrally partially coherent GSM pulses [12, 14].

In the present paper, taking the spatially and spectrally partially coherent cosh-Gaussian pulsed (ChGP) beam as another type of spatially and spectrally partially coherent pulsed beams, changes in the spectral degree of coherence and spectral intensity of ChGP beams are studied. In Sect. 2 analytical expressions for the spectral degree of coherence and spectral intensity of spatially and spectrally partially coherent ChGP beams propagating in free space are derived. The on-axis and transverse spectral degrees of coherence and spectral intensity of the ChGP beams are analyzed in

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Sects. 3 and 4, and illustrated by numerical examples. Finally, the main results obtained in this paper are summarized in Sect. 5.

2 Simulation model

In the space–time domain the mutual correlation function of a two-dimensional spatially and temporally partially coherent ChGP beam at the source plane $z = 0$ is written as [12, 18]

$$\begin{aligned}\Gamma(x_1', t_1, x_2', t_2) &= \Gamma_0 \cosh(\Omega x_1') \cosh(\Omega x_2') \\ &\quad \times \exp\left[-\frac{x_1'^2 + x_2'^2}{w_0^2} - \frac{(x_1' - x_2')^2}{2\sigma_0^2}\right] \\ &\quad \times \exp\left[-\frac{t_1^2 + t_2^2}{2T_0^2} - \frac{(t_1 - t_2)^2}{2T_c^2} + \mathrm{i}\omega_0(t_1 - t_2)\right], \end{aligned} \tag{1}$$

where w_0 and Ω are the waist width of the Gaussian part and the parameter associated with the cosh part, respectively, both of which are assumed to be independent of the frequency ω [18]. (x_1', x_2') and (t_1, t_2) are coordinates and times of two points at the source plane, respectively. ω_0 is the carrier frequency of the pulse and Γ_0 is a constant. σ_0 is the spatial correlation length, T_0 is the pulse duration and T_c describes the temporal coherence length of the pulse, which denotes the temporal correlation of the pulse.

Using the Fourier transform, the cross-spectral density function of the spatially and spectrally partially coherent ChGP beam at the source plane $z = 0$ in the space–frequency domain can be derived and is given by

$$\begin{aligned}W(x_1', \omega_1, x_2', \omega_2) &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma(x_1', t_1, x_2', t_2) \\ &\quad \times \exp\left[-\mathrm{i}(\omega_1 t_1 - \omega_2 t_2)\right] \mathrm{d}t_1 \, \mathrm{d}t_2 \\ &= W_0 \cosh(\Omega x_1') \cosh(\Omega x_2') \\ &\quad \times \exp\left[-\frac{x_1'^2 + x_2'^2}{w_0^2} - \frac{(x_1' - x_2')^2}{2\sigma_0^2}\right] \\ &\quad \times \exp\left[-\frac{(\omega_1 - \omega_0)^2 + (\omega_2 - \omega_0)^2}{2\Omega_0^2} - \frac{(\omega_1 - \omega_2)^2}{2\Omega_c^2}\right], \end{aligned} \tag{2}$$

where

$$\Omega_0 = \sqrt{\frac{1}{T_0^2} + \frac{2}{T_c^2}} \quad \text{(spectral width)}, \tag{3}$$

$$\Omega_c = \frac{T_c}{T_0} \Omega_0 \quad \text{(spectral coherence width)}, \tag{4}$$

$$W_0 = \frac{T_0}{2\pi \Omega_0} \Gamma_0. \tag{5}$$

The spectral coherence width Ω_c is a measure of the correlation between different frequency components of the pulse. Equations (3) and (4) give the relation between the pulse duration T_0, temporal coherence length T_c, spectral width Ω_0 and spectral coherence width Ω_c. It is readily seen that the spatially partially coherent and spectrally fully coherent ChGP beams can be regarded as a special case by letting $T_c \to \infty$ ($\Omega_c \to \infty$). In the limit $\sigma_0 \to \infty$ the spatially fully coherent and spectrally partially coherent ChGP beams are obtained. If $\sigma_0 \to \infty$ and $T_c \to \infty$, we obtain spatially and spectrally fully coherent ChGP beams.

The cross-spectral density function of the spatially and spectrally partially coherent ChGP beam propagating in free space is expressed as [19]

$$\begin{aligned}W(x_1, z_1, \omega_1, x_2, z_2, \omega_2) &= \frac{1}{2\pi} \sqrt{\frac{k_1 k_2}{z_1 z_2}} \exp\left[\mathrm{i}(z_2 k_2 - z_1 k_1)\right] \\ &\quad \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x_1', \omega_1, x_2', \omega_2) \\ &\quad \times \exp\left[\frac{\mathrm{i}k_2}{2z_2}(x_2' - x_2)^2 - \frac{\mathrm{i}k_1}{2z_1}(x_1' - x_1)^2\right] \mathrm{d}x_1' \, \mathrm{d}x_2', \end{aligned} \tag{6}$$

where k is the wave number related to the frequency ω by $k_i = \omega_i / c$ $(i = 1, 2)$, c being the speed of light in vacuum, (x_1, z_1) and (x_2, z_2) are two points in the half space $z > 0$ and * denotes the complex conjugate.

On substituting from (2) into (6), tedious but straightforward integral calculations yield

$$\begin{aligned}W(x_1, z_1, \omega_1, x_2, z_2, \omega_2) &= \frac{W_0 w_0^2}{8\beta_1 \beta_2} \sqrt{\frac{k_1 k_2}{z_1 z_2}} \exp\left[\mathrm{i}(z_2 k_2 - z_1 k_1)\right] \\ &\quad \times \exp\left[-\frac{(\omega_1 - \omega_0)^2 + (\omega_2 - \omega_0)^2}{2\Omega_0^2} - \frac{(\omega_1 - \omega_2)^2}{2\Omega_c^2}\right] \\ &\quad \times \exp\left[\frac{\mathrm{i}}{2}\left(\frac{k_2 x_2^2}{z_2} - \frac{k_1 x_1^2}{z_1}\right)\right] \\ &\quad \times \left\{\exp\left[\frac{\alpha_{11}^2}{4\beta_1^2} + \frac{1}{4\beta_2^2}\left(\alpha_{21} + \frac{\alpha_{11}\gamma^{-2} - \alpha_{11}}{2\beta_1^2}\right)^2\right]\right. \\ &\quad + \exp\left[\frac{\alpha_{11}^2}{4\beta_1^2} + \frac{1}{4\beta_2^2}\left(\alpha_{22} + \frac{\alpha_{11}\gamma^{-2} - \alpha_{11}}{2\beta_1^2}\right)^2\right] \\ &\quad + \exp\left[\frac{\alpha_{12}^2}{4\beta_1^2} + \frac{1}{4\beta_2^2}\left(\alpha_{21} + \frac{\alpha_{12}\gamma^{-2} - \alpha_{12}}{2\beta_1^2}\right)^2\right] \end{aligned}$$

$$+\exp\left[\frac{\alpha_{12}^2}{4\beta_1^2}+\frac{1}{4\beta_2^2}\left(\alpha_{22}+\frac{\alpha_{12}\gamma^{-2}-\alpha_{12}}{2\beta_1^2}\right)^2\right]\Bigg\}, \quad (7)$$

where

$$\alpha_{jk}=(-1)^{k+1}g+(-1)^{j+1}\mathrm{i}k_j x_j w_0/z_j \quad (j,k=1,2), \quad (8)$$

$$\beta_1^2=1+\frac{1}{2}\left(\gamma^{-2}-1\right)+\frac{\mathrm{i}k_1 w_0^2}{2z_1}, \quad (9)$$

$$\beta_2^2=1+\frac{1}{2}\left(\gamma^{-2}-1\right)-\frac{\mathrm{i}k_2 w_0^2}{2z_2}-\frac{(\gamma^{-2}-1)^2}{4\beta_1^2}, \quad (10)$$

$$\gamma=\left[1+(w_0/\sigma_0)^2\right]^{-1/2}$$
(spatial correlation parameter) (11)

and $g=\Omega w_0$ represents the decentered parameter. On placing $x_1=x_2=x$, $z_1=z_2=z$ and $\omega_1=\omega_2=\omega$ into (7), the spectral intensity of the ChGP beam at the position (x,z), at frequency ω, turns out to be

$$S(x,z,\omega)$$
$$=\frac{W_0 k w_0^2}{8\beta_1\beta_2 z}\exp\left[-\frac{(\omega-\omega_0)^2}{\Omega_0^2}\right]$$
$$\times\Bigg\{\exp\left[\frac{\alpha_{11}^2}{4\beta_1^2}+\frac{1}{4\beta_2^2}\left(\alpha_{21}+\frac{\alpha_{11}\gamma^{-2}-\alpha_{11}}{2\beta_1^2}\right)^2\right]$$
$$+\exp\left[\frac{\alpha_{11}^2}{4\beta_1^2}+\frac{1}{4\beta_2^2}\left(\alpha_{22}+\frac{\alpha_{11}\gamma^{-2}-\alpha_{11}}{2\beta_1^2}\right)^2\right]$$
$$+\exp\left[\frac{\alpha_{12}^2}{4\beta_1^2}+\frac{1}{4\beta_2^2}\left(\alpha_{21}+\frac{\alpha_{12}\gamma^{-2}-\alpha_{12}}{2\beta_1^2}\right)^2\right]$$
$$+\exp\left[\frac{\alpha_{12}^2}{4\beta_1^2}+\frac{1}{4\beta_2^2}\left(\alpha_{22}+\frac{\alpha_{12}\gamma^{-2}-\alpha_{12}}{2\beta_1^2}\right)^2\right]\Bigg\}. \quad (12)$$

The spectral degree of coherence is defined as [19]

$$\mu(x_1,z_1,\omega_1,x_2,z_2,\omega_2)$$
$$=\frac{W(x_1,z_1,\omega_1,x_2,z_2,\omega_2)}{\sqrt{S(x_1,z_1,\omega_1)S(x_2,z_2,\omega_2)}}, \quad (13)$$

where $W(x_1,z_1,\omega_1,x_2,z_2,\omega_2)$ and $S(x_i,z_i,\omega_i)$ $(i=1,2)$ are given by (7) and (12), respectively. It follows from (13) that the spectral degree of coherence $\mu(x_1,z_1,\omega_1,x_2,z_2,\omega_2)$ is dependent on the waist width, decentered parameter, spatial correlation parameter, pulse duration, temporal coherence length, frequency and position of two points.

Some special cases of (13) are of interest.

(a) For $\gamma=1$ (i.e. $\sigma_0\to\infty$), (13) reduces to the spectral degree of coherence of spatially fully coherent and spectrally partially coherent ChGP beams:

$$\mu_0=\exp\left[-\frac{(\omega_1-\omega_2)^2}{2\Omega_c^2}\right]. \quad (14)$$

Obviously, μ_0 depends on the temporal coherence length T_c, pulse duration T_0 and frequency difference $|\omega_1-\omega_2|$, and $\mu_0=1$ if $\omega_1=\omega_2$ or $T_c\to\infty$.

(b) For spatially and spectrally partially coherent ChGP beams at sufficiently long propagation distances ($z_1=z_2=z\to\infty$), we have

$$|\mu|=\mu_0=\exp\left[-\frac{(\omega_1-\omega_2)^2}{2\Omega_c^2}\right]. \quad (15)$$

For spectrally fully coherent ChGP beams ($T_c\to\infty$), we obtain $|\mu|=\mu_0=1$, which is consistent with the result in Ref. [7].

(c) For $x_1=x_2=0$, we obtain the spectral degree of coherence at a pair of points on the z axis:

$$\mu(0,z_1,\omega_1,0,z_2,\omega_2)$$
$$=\mu_0\frac{\sqrt{\beta_1\beta_{21}\beta_{12}\beta_{22}}}{\beta_1\beta_2}\exp\left[\mathrm{i}(z_2k_2-z_1k_1)\right]$$
$$\times\Bigg\{\exp\left[\frac{g^2}{4\beta_1^2}+\frac{g^2}{16\beta_2^2\beta_1^4}\left(2\beta_1^2+\gamma^{-2}-1\right)^2\right]$$
$$+\exp\left[\frac{g^2}{4\beta_1^2}+\frac{g^2}{16\beta_2^2\beta_1^4}\left(2\beta_1^2-\gamma^{-2}+1\right)^2\right]\Bigg\}\Bigg/$$
$$\Bigg(\Bigg\{\exp\left[\frac{g^2}{4\beta_1^2}+\frac{g^2}{16\beta_{21}^2\beta_1^4}\left(2\beta_1^2+\gamma^{-2}-1\right)^2\right]$$
$$+\exp\left[\frac{g^2}{4\beta_1^2}+\frac{g^2}{16\beta_{21}^2\beta_1^4}\left(2\beta_1^2-\gamma^{-2}+1\right)^2\right]\Bigg\}$$
$$\times\Bigg\{\exp\left[\frac{g^2}{4\beta_{12}^2}+\frac{g^2}{16\beta_{22}^2\beta_{12}^4}\left(2\beta_{12}^2+\gamma^{-2}-1\right)^2\right]$$
$$+\exp\left[\frac{g^2}{4\beta_{12}^2}+\frac{g^2}{16\beta_{22}^2\beta_{12}^4}\left(2\beta_{12}^2-\gamma^{-2}+1\right)^2\right]\Bigg\}\Bigg)^{1/2}, \quad (16)$$

$$\beta_{12}^2=1+\frac{1}{2}\left(\gamma^{-2}-1\right)+\frac{\mathrm{i}k_2 w_0^2}{2z_2}, \quad (17)$$

$$\beta_{21}^2=1+\frac{1}{2}\left(\gamma^{-2}-1\right)-\frac{\mathrm{i}k_1 w_0^2}{2z_1}-\frac{(\gamma^{-2}-1)^2}{4\beta_{11}^2}, \quad (18)$$

$$\beta_{22}^2=1+\frac{1}{2}\left(\gamma^{-2}-1\right)-\frac{\mathrm{i}k_2 w_0^2}{2z_2}-\frac{(\gamma^{-2}-1)^2}{4\beta_{12}^2}. \quad (19)$$

(d) For $g=0$, (13) reduces to the spectral degree of coherence of spatially and spectrally partially coherent Gaussian Schell-model pulsed (GSMP) beams:

$$\mu(x_1,z_1,\omega_1,x_2,z_2,\omega_2)$$
$$=\mu_0\frac{\sqrt{\beta_1\beta_{21}\beta_{12}\beta_{22}}}{\beta_1\beta_2}\exp\left[\mathrm{i}(z_2k_2-z_1k_1)\right]$$

$$\times \exp\left[\frac{i}{2}\left(\frac{k_2 x_2^2}{z_2} - \frac{k_1 x_1^2}{z_1}\right)\right]$$
$$\times \exp\left[-\frac{(k_1 x_1 w_0)^2}{4 z_1^2 \beta_1^2} + \frac{1}{4\beta_2^2}\left(\frac{-i k_2 x_2 w_0}{z_2} + \frac{i k_1 x_1 w_0}{z_1}\frac{(\gamma^{-2}-1)}{2\beta_1^2}\right)^2\right] \Big/$$
$$\left\{\exp\left[-\frac{(k_1 x_1 w_0)^2}{4 z_1^2 \beta_1^2} - \frac{(k_1 x_1 w_0)^2}{16 z_1^2 \beta_{21}^2 \beta_1^4}(2\beta_1^2 - \gamma^{-2} + 1)^2\right]\right.$$
$$\times \exp\left[-\frac{(k_2 x_2 w_0)^2}{4 z_2^2 \beta_{12}^2} \left.- \frac{(k_2 x_2 w_0)^2}{16 z_2^2 \beta_{22}^2 \beta_{12}^4}(2\beta_{12}^2 - \gamma^{-2} + 1)^2\right]\right\}^{1/2}. \tag{20}$$

3 On-axis spectral degree of coherence and spectral intensity

By using (12) and (16), the spectral degree of coherence and spectral intensity of spatially and spectrally partially coherent ChGP beams at a pair of points on the z axis can be studied, where $(x_1, z_1) = (0, 1\text{ m})$ for μ kept fixed, and our main attention is paid to the influence of the spatial correlation parameter γ, decentered parameter g and temporal coherence length T_c on the spectral degree of coherence and spectral intensity.

The modulus of the spectral degree of coherence $|\mu(0, z_1, \omega, 0, z_2, \omega)|$ and the normalized spectral intensity $S(0, z, \omega)/S_{max}(0, z, \omega)$ of a spatially and spectrally partially coherent ChGP beam on the z axis are shown in Fig. 1a and b, respectively, where the calculation parameters are $w_0 = 0.5$ mm, $g = 3$, $T_0 = 3$ fs, $\omega_0 = 2.979$ rad/fs, $\omega_1 = \omega_2 = \omega = 2.5$ rad/fs and $\gamma = 0.5, 0.7$ and 0.9. As can be seen from Fig. 1a, $|\mu(0, z_1, \omega, 0, z_2, \omega)|$ is dependent on the spatial correlation parameter γ and z_2; as γ decreases $|\mu|$ decreases. Figure 1b shows that the variation of $S(0, z, \omega)/S_{max}(0, z, \omega)$ with z_2 is non-monotonic in the region 0.5 m $\le z \le$ 5 m, and with increasing γ the position of the normalized maximum spectral intensity $S(0, z, \omega)/S_{max}(0, z, \omega)|_{max}$ increases, that is to say, the position of $S(0, z, \omega)/S_{max}(0, z, \omega)|_{max}$ moves away from the source plane.

Figure 2 represents (a) $|\mu(0, z_1, \omega, 0, z_2, \omega)|$ and (b) $S(0, z, \omega)/S_{max}(0, z, \omega)$ for different values of the decentered parameter $g = 0, 3$ and 5, where $\gamma = 0.5$, and the other calculation parameters are the same as those in Fig. 1. From Fig. 2a we see that g does not affect the maximum and the position of $|\mu|_{max}$, and $|\mu|$ decreases with an increase of g. It can be seen from Fig. 2b that, in the region 0.5 m $\le z \le$ 5 m, $S(0, z, \omega)/S_{max}(0, z, \omega)$ varies with

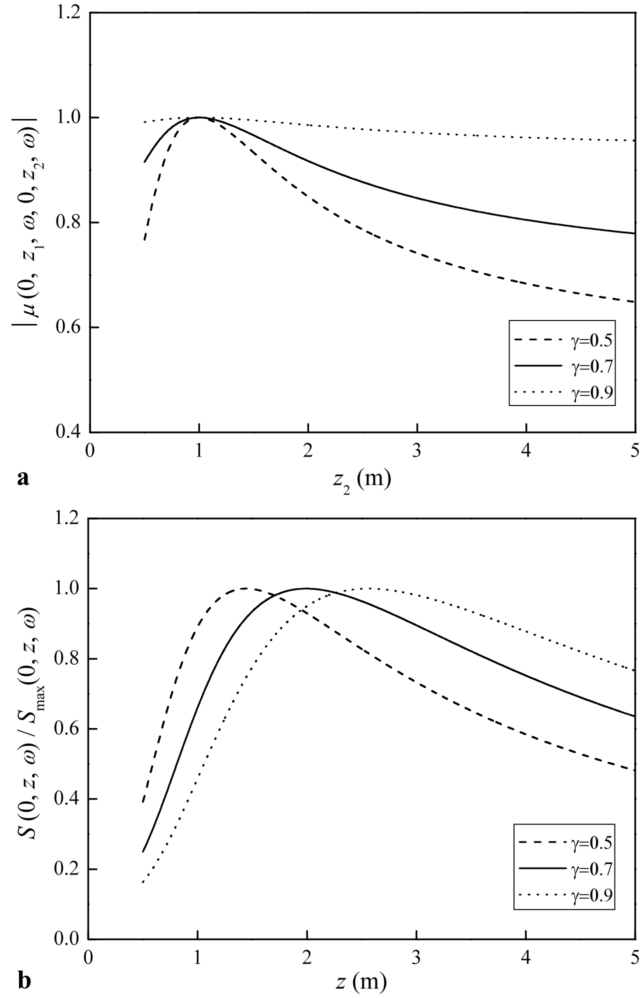

Fig. 1 (**a**) Modulus of the on-axis spectral degree of coherence $|\mu(0, z_1, \omega, 0, z_2, \omega)|$ versus z_2 and (**b**) normalized spectral intensity $S(0, z, \omega)/S_{max}(0, z, \omega)$ versus z for different values of the spatial correlation parameter γ

z non-monotonically if $g \neq 0$, and the larger the decentered parameter g, the more the maximum position moves away from the source plane, whereas, for the GSMP beam of $g = 0$, $S(0, z, \omega)/S_{max}(0, z, \omega)$ decreases with increasing z.

The spectral degree of coherence μ_0 of spatially fully coherent and spectrally partially coherent ChGP beams ($\gamma = 1$) as a function of T_c for different values of the pulse duration T_0 is given in Fig. 3, where $\omega_1 = 2.5$ rad/fs, $\omega_2 = 3.5$ rad/fs and $T_0 = 2, 3, 4$ and 5 fs. It follows from (14) that μ_0 is independent of the position of two points due to the spatially full coherence. As shown in Fig. 3, μ_0 increases with increasing T_c and decreasing T_0.

Figure 4a gives $|\mu(0, z_1, \omega_1, 0, z_2, \omega_2)|$ of a spatially and spectrally partially coherent ChGP beam for different values of the temporal coherence length T_c, where the calculation parameters are $\gamma = 0.5$, $\omega_1 = 2.5$ rad/fs, $\omega_2 = 3.5$ rad/fs and $T_c = 5$ fs, 10 fs and ∞; the other parameters are the same

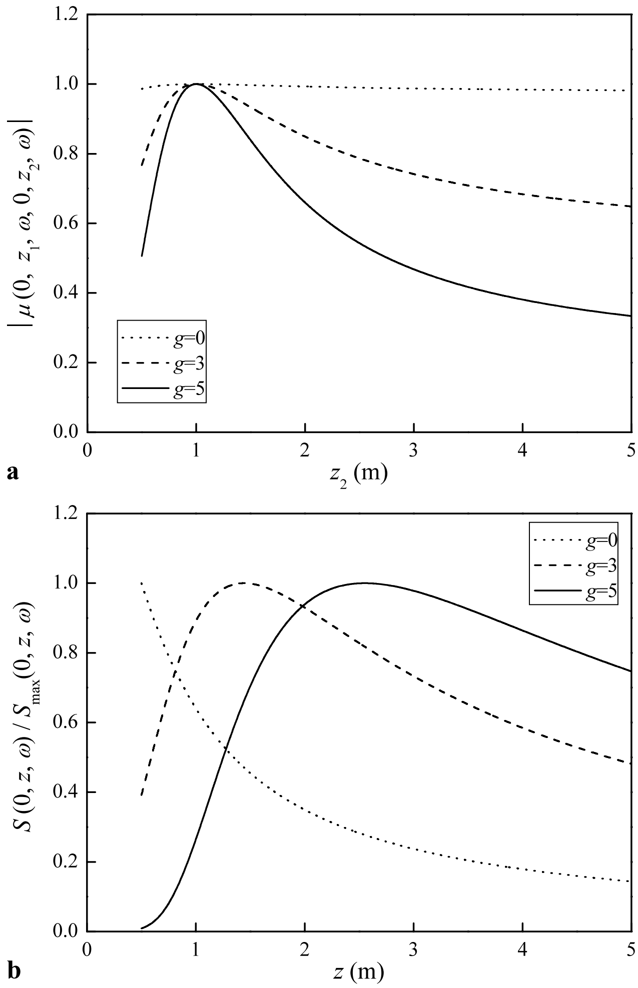

Fig. 2 (**a**) Modulus of the on-axis spectral degree of coherence $|\mu(0, z_1, \omega, 0, z_2, \omega)|$ and (**b**) normalized spectral intensity $S(0, z, \omega)/S_{\max}(0, z, \omega)$ for different values of the decentered parameter g

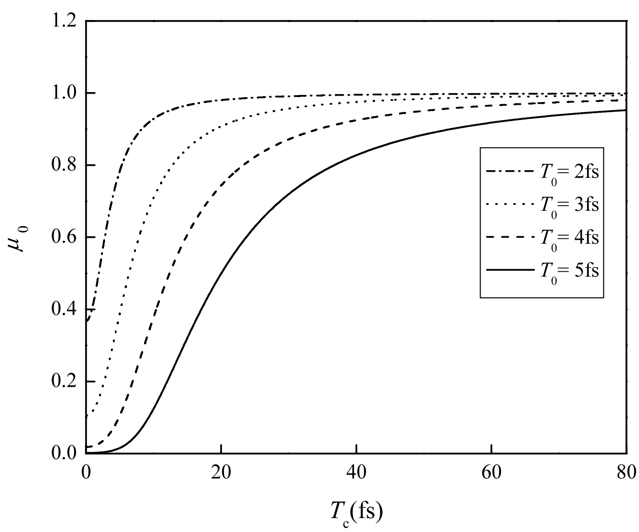

Fig. 3 The spectral degree of coherence μ_0 of spatially fully coherent and spectrally partially coherent ChGP beams versus T_c for different values of the pulse duration T_0

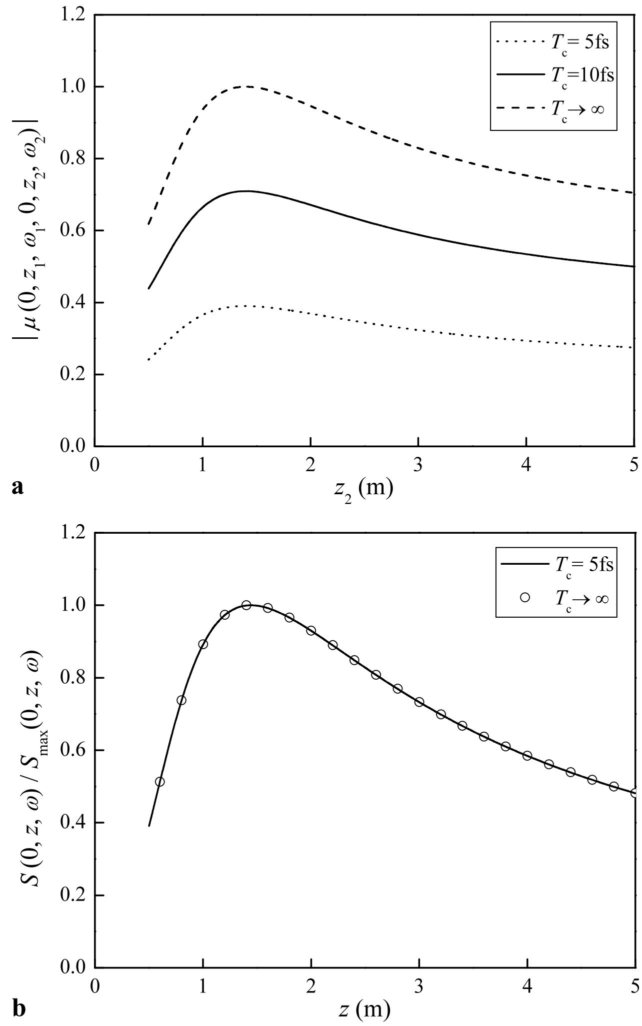

Fig. 4 (**a**) Modulus of the on-axis spectral degree of coherence $|\mu(0, z_1, \omega_1, 0, z_2, \omega_2)|$ for different values of T_c. (**b**) Normalized spectral intensity $S(0, z, \omega)/S_{\max}(0, z, \omega)$

as those in Fig. 1. From (16), it is seen that for $\omega_1 = \omega_2$ the spectral degree of coherence μ is independent of T_c and $|\mu|_{\max} = \mu_0 = 1$ (e.g. Figs. 1a and 2a). However, for $\omega_1 \neq \omega_2$, $|\mu|$ depends on T_c, as shown in Fig. 4a; a decrease of T_c results in a decrease of $|\mu|$ as well as of its maximum $|\mu|_{\max}$, namely, $|\mu|_{\max}$ is not equal to 1 owing to the spectrally partial coherence, because (16) implies that μ_0 modifies the spectral degree of coherence μ. μ_0 decreases with decreasing T_c, which leads to a decrease of $|\mu|$ as well as of its maximum $|\mu|_{\max}$. The normalized spectral intensity $S(0, z, \omega)/S_{\max}(0, z, \omega)$ is depicted in Fig. 4b with $\omega = 2.5$ rad/fs; two curves for $T_c = 5$ fs and $T_c \to \infty$ are shown to be coincident. The result can be interpreted by (12), for which we see that the term $\exp[-(\omega - \omega_0)^2/\Omega_0^2]$ depends on T_c, but does not affect the normalized spectral intensity $S(0, z, \omega)/S_{\max}(0, z, \omega)$.

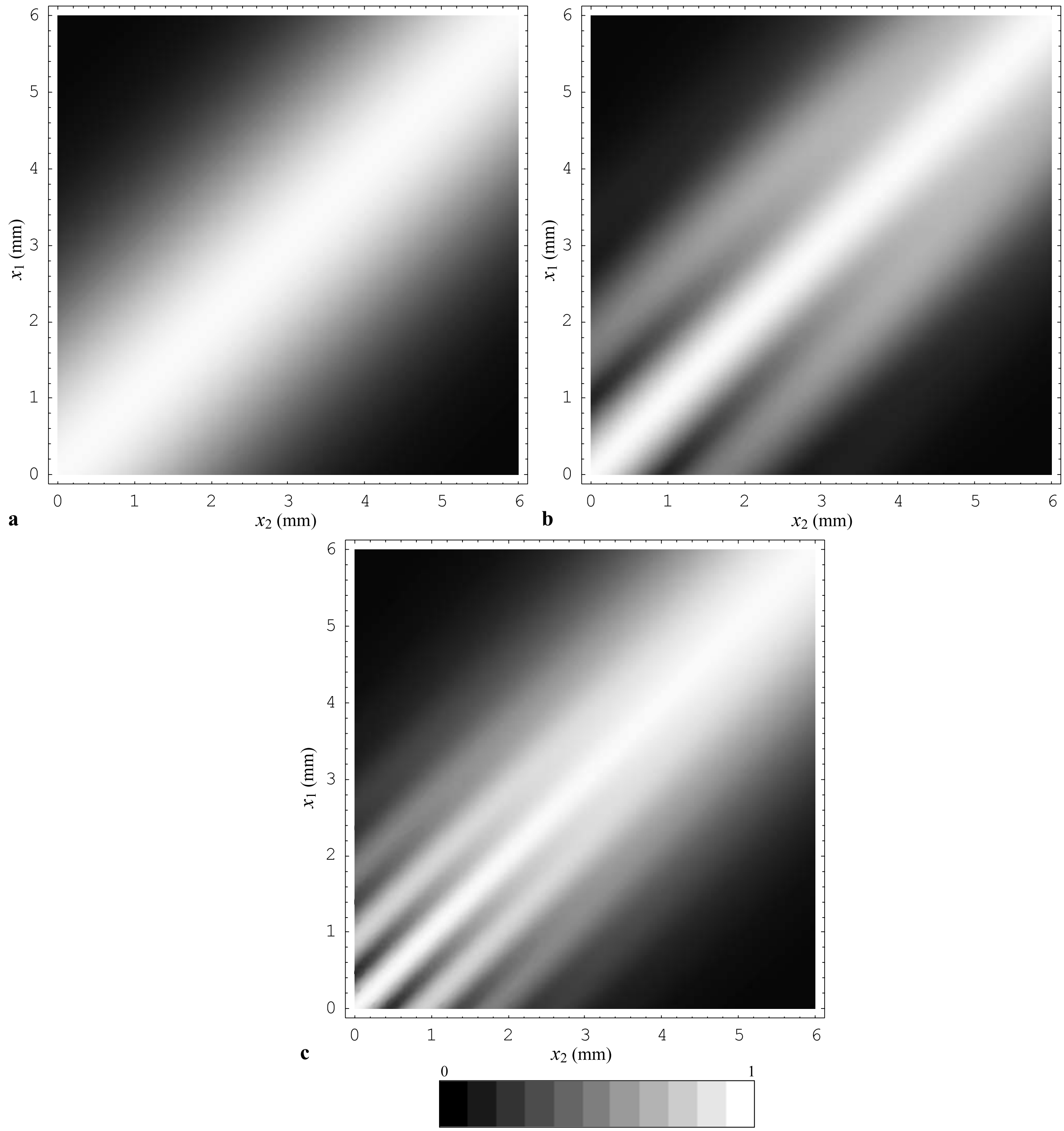

Fig. 5 Modulus of the spectral degree of coherence $|\mu(x_1, z, \omega, x_2, z, \omega)|$ for different values of g. (**a**) $g = 0$; (**b**) $g = 2.5$; (**c**) $g = 5$

4 Transverse spectral degree of coherence and spectral intensity

To illustrate the effect of the decentered parameter g on the transverse spectral degree of coherence, in Fig. 5 the modulus of the spectral degree of coherence $|\mu(x_1, z, \omega, x_2, z, \omega)|$ for a pair of points (x_1, z) and (x_2, z) is plotted, where the calculation parameters are $w_0 = 0.5$ mm, $\gamma = 0.5$, $T_0 = 3$ fs, $\omega_0 = 2.979$ rad/fs, $\omega_1 = \omega_2 = \omega = 2.5$ rad/fs, $z_1 = z_2 = z = 3$ m, (a) $g = 0$; (b) $g = 2.5$; (c) $g = 5$. It is observed that for $g = 0$, $|\mu|$ decreases gradually with increasing distance $|x_2 - x_1|$, whereas, for $g = 2.5$ and 5, $|\mu|$ exhibits an oscillatory behavior, which becomes stronger with increasing g. This is due to the oscillatory behavior of the cosh-Gaussian function for a large value of Ω. Therefore, besides the aperture diffraction [6], an oscillatory behavior of the spectral degree of coherence also appears by a suitable variation of the decentered parameter g.

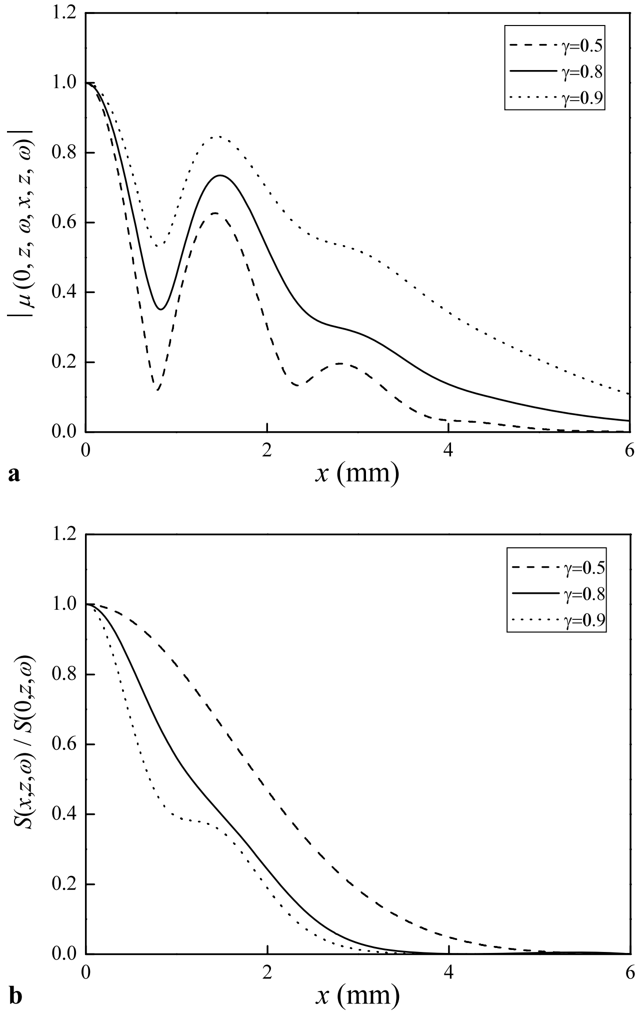

Fig. 6 (**a**) Modulus of the spectral degree of coherence $|\mu(0, z, \omega, x, z, \omega)|$ as a function of x for different values of γ. (**b**) Relative spectral intensity $S(x, z, \omega)/S(0, z, \omega)$

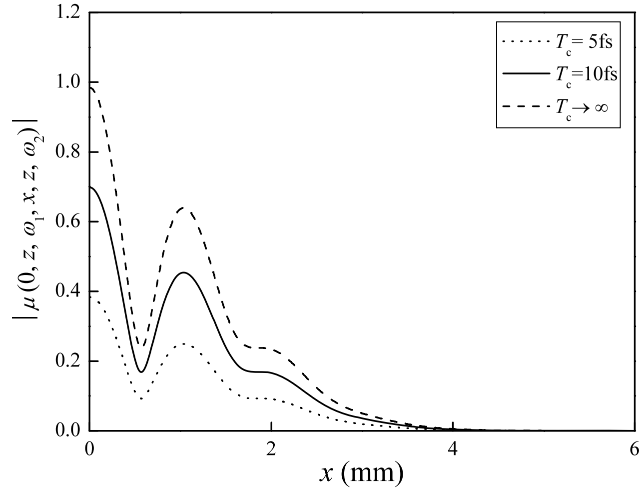

Fig. 7 Modulus of the spectral degree of coherence $|\mu(0, z, \omega_1, x, z, \omega_2)|$ as a function of x for different values of T_c

Figure 6 represents (a) the modulus of the spectral degree of coherence $|\mu(0, z, \omega, x, z, \omega)|$ and (b) the relative spectral intensity $S(x, z, \omega)/S(0, z, \omega)$ versus x for different values of the spatial correlation parameter $\gamma = 0.5, 0.8$ and 0.9, where $(x_1, z_1) = (0, 3\ \text{m})$ is kept fixed, $x_2 = x$, $g = 3$ and the other calculation parameters are the same as those in Fig. 5. From Fig. 6a we see that the oscillatory behavior of $|\mu|$ becomes smaller and $|\mu|$ increases with increasing γ. As can be seen from Fig. 6b, $S(x, z, \omega)/S(0, z, \omega)$ increases with decreasing γ. For $\gamma = 0.9$, $S(x, z, \omega)/S(0, z, \omega)$ shows an oscillatory behavior, which, however, disappears when γ becomes small (e.g. $\gamma \le 0.8$).

Figure 7 gives $|\mu(0, z, \omega_1, x, z, \omega_2)|$ versus x for different values of T_c, where the calculation parameters are $\gamma = 0.5$, $\omega_1 = 2.5$ rad/fs, $\omega_2 = 3.5$ rad/fs, $T_c = 5$ fs, 10 fs and ∞ and the other calculation parameters are the same as those in Fig. 6. It is seen that both $|\mu|$ and its maximum $|\mu|_{\max}$ decrease with decreasing T_c; the physical explanation of Fig. 7 is similar to Fig. 4a. Additionally, it can be shown that the temporal coherence length T_c does not affect the transverse spectral intensity distribution, and the illustrative numerical example is omitted.

It is interesting to compare the effective width of the spectral degree of coherence with that of the spectral intensity; Fig. 8a–d give $|\mu(0, z, \omega, x, z, \omega)|$ and $S(x, z, \omega)/S(0, z, \omega)$ for different values of the decentered parameter $g = 0, 1.3, 3$ and 6, respectively, where $\gamma = 0.6$, and the other calculation parameters are the same as those in Fig. 6. The effective width of the spectral degree of coherence is defined as the distance between two points for which $|\mu(0, z, \omega, x, z, \omega)|$ is decreased from its maximum value to one-half [1]. From Fig. 8a–d it is seen that the effective width of $|\mu(0, z, \omega, x, z, \omega)|$ is larger than that of $S(x, z, \omega)/S(0, z, \omega)$ with $g = 0$, but the effective width of $|\mu(0, z, \omega, x, z, \omega)|$ is less than that of $S(x, z, \omega)/S(0, z, \omega)$ with large values of g, e.g. $g \ge 3$, and, in particular, the effective width of $|\mu(0, z, \omega, x, z, \omega)|$ equals that of $S(x, z, \omega)/S(0, z, \omega)$ for $g = 1.3$.

5 Conclusions

In this paper, the analytical expressions for the spectral degree of coherence and spectral intensity of spatially and spectrally partially coherent ChGP beams propagating in free space have been derived and used to study changes in their spectral degree of coherence and spectral intensity. The spatially and spectrally partially coherent GSMP beam can be regarded as a special case by letting $g = 0$. It has been shown that, for the on-axis and transverse spectral degrees of coherence and spectral intensity, a decrease of the temporal coherence length T_c leads to a decrement of the modulus of the spectral degree of coherence $|\mu|$

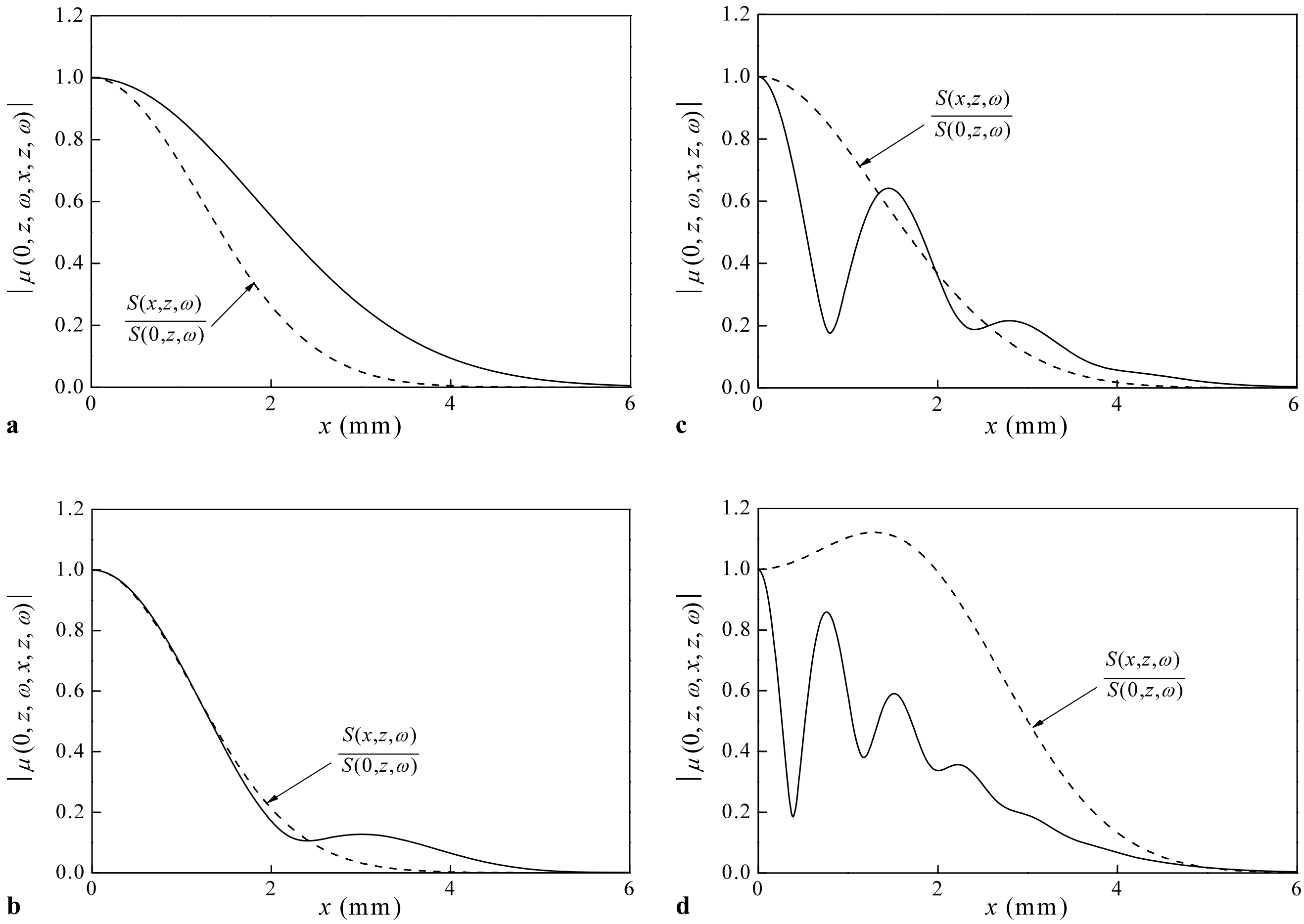

Fig. 8 Modulus of the spectral degree of coherence $|\mu(0, z, \omega, x, z, \omega)|$ and relative spectral intensity $S(x, z, \omega)/S(0, z, \omega)$ for different values of g. (**a**) $g = 0$; (**b**) $g = 1.3$; (**c**) $g = 3$; (**d**) $g = 6$

and its maximum $|\mu|_{\max}$, whereas T_c does not contribute to the spectral intensity distribution. As the decentered parameter g increases, the modulus of the on-axis spectral degree of coherence $|\mu(0, z_1, \omega, 0, z_2, \omega)|$ decreases and the position of the normalized maximum spectral intensity $S(0, z, \omega)/S_{\max}(0, z, \omega)|_{\max}$ moves away from the source plane. At a transverse plane an increase of g results in an oscillatory behavior of $|\mu(0, z, \omega, x, z, \omega)|$, which becomes more prominent with decreasing spatial correlation parameter γ. The relative spectral intensity $S(x, z, \omega)/S(0, z, \omega)$ increases with a decrease of γ. Depending on g, the effective width of the spectral degree of coherence can be larger than, smaller than or equal to that of the spectral intensity.

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References

1. J. Wu, A.D. Boardman, J. Mod. Opt. **38**, 1355 (1991)
2. D.G. Fischer, T.D. Visser, J. Opt. Soc. Am. A **21**, 2097 (2004)
3. H. Roychowdhury, G.P. Agrawal, E. Wolf, J. Opt. Soc. Am. A **23**, 940 (2006)
4. L. Rao, J. Pu, J. Opt. Soc. Am. A **24**, 2242 (2007)
5. X. Ji, X. Chen, S. Chen, X. Li, B. Lü, J. Opt. Soc. Am. A **24**, 3554 (2007)
6. X. Ji, G. Ji, Appl. Phys. B **92**, 111 (2008)
7. H.T. Eyyuboglu, Y. Baykal, Y. Cai, J. Opt. Soc. Am. A **24**, 2891 (2007)
8. G. Gbur, T.D. Visser, Opt. Commun. **222**, 117 (2003)
9. S.A. Akhmanov, V.A. Vysloukh, A.S. Chirkin, *Optics of Femtosecond Laser Pulses* (AIP, New York, 1992)
10. M. Nisoli, S.D. Silvestri, O. Svelto, R. Szipöcs, K. Ferencz, Ch. Spielmann, S. Sartania, F. Krausz, Opt. Lett. **22**, 522 (1997)
11. M.A. Porras, R. Borghi, M. Santarsiero, Phys. Rev. E **62**, 5729 (2000)
12. H. Lajunen, P. Vahimaa, J. Tervo, J. Opt. Soc. Am. A **22**, 1536 (2005)
13. P. Pääkkönen, J. Turunen, P. Vahimaa, A.T. Friberg, F. Wyrowski, Opt. Commun. **204**, 53 (2002)
14. H. Lajunen, J. Tervo, J. Turunen, P. Vahimaa, F. Wyrowski, Opt. Express **16**, 1894 (2003)
15. L. Wang, Q. Lin, H. Chen, S. Zhu, Phys. Rev. E **67**, 056613 (2003)
16. H. Lajunen, J. Turunen, P. Vahimaa, J. Tervo, F. Wyrowski, Opt. Commun. **255**, 12 (2005)
17. C. Ding, B. Lü, J. Opt. A, Pure Appl. Opt. **10**, 095006 (2008)
18. B. Lü, H. Ma, B. Zhang, Opt. Commun. **164**, 165 (1999)
19. L. Mandel, E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 2001)