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Chaos in coupled lasers with low-frequency modulation

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Abstract The dynamics of the array consisting of two coupled solid-state lasers with frequency modulations was researched numerically. Array intensity's chaotic behavior is predicted when the modulation frequency is low. The physical mechanism of the chaos is discussed here qualitatively. It was found that not only a harmonic resonance itself but also the duration of the harmonic resonance long enough is needed for the appearance of the chaos.

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1 Introduction

Dynamics of the coupled nonlinear systems has attracted much attention for its possible practical applications, e.g., chaos communications [1, 2]. The array of coupled solid-state lasers is a representative paradigm of coupled nonlinear oscillators. It can provide a nice illustration of the complex dynamics of coupled oscillators. So, it is of great interest and many researches have been made on it. The coherence and chaotic behavior of two coupled solid-state lasers without modulations have already been discussed [3, 4]. The phase dynamics of the solid-state laser array with injection fields or loss and pump modulations have also been researched [5, 6]. Furthermore, the dynamics of the array of coupled multi-mode solid-state lasers have been discussed [7, 8]. Here, different from those mentioned above, the dynamics of the array with frequency modulations is being researched, and the temporal intensity behavior of the array corresponding to different modulation amplitudes is analyzed.

In this paper, we consider the dynamics of the array consisting of two coupled single-mode Nd:YAG (Neodymium doped Yttrium Aluminum Garnet) lasers with the length of one resonator being modulated. The temporal intensity behaviors of the array are researched numerically using different modulation amplitudes. It is found that synchronized chaos will appear for a certain regime of modulation amplitudes when the modulation frequency is low.

2 Model and parameters

The following equations can describe the temporal behavior of the complex electric field E and the gain G of two coupled single-mode class B lasers, which also assume that E and G vary slowly [3, 4].

$$\frac{d\mathbf{E}_1}{dt} = \frac{1}{\tau_c}\big[(G_1 - \alpha_1)\mathbf{E}_1 - \kappa \mathbf{E}_2\big] + i\omega_1 \mathbf{E}_1 \tag{1}$$

$$\frac{dG_1}{dt} = \frac{1}{\tau_f}\big(p_1 - G_1 - G_1|\mathbf{E}_1|^2\big) \tag{2}$$

$$\frac{d\mathbf{E}_2}{dt} = \frac{1}{\tau_c}\big[(G_2 - \alpha_2)\mathbf{E}_2 - \kappa \mathbf{E}_1\big] + i\omega_2 \mathbf{E}_2 \tag{3}$$

$$\frac{dG_2}{dt} = \frac{1}{\tau_f}\big(p_2 - G_2 - G_2|\mathbf{E}_2|^2\big) \tag{4}$$

In these equations, τ_c is the cavity round trip time, τ_f is the fluorescence time of the upper lasing level of the Nd^{3+} ion, p_n and α_n are the pump and cavity loss coefficients, and ω_n is the detuning of the lasers from a common cavity mode, where the subscript $n = 1, 2$ and stands for the two lasers, κ

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is the couple strength of the two lasers, and the minus sign of the couple term is chosen to account the anti-phase state observed in the experiment [3].

Assume that

$$\mathbf{E}_n(t) = \sqrt{I_n(t)}e^{i\varphi_n(t)} \tag{5}$$

where $I_n(t)$ and $\varphi_n(t)$ stand for the intensity and the phase of the field $\mathbf{E}_n(t)$, respectively. Substitute (5) into (1–4), and (1–4) can be rewritten as

$$\frac{dI_1}{dt} = \frac{2}{\tau_c}\left[(G_1 - \alpha_1)I_1 - \kappa\sqrt{I_1 I_2}\cos(\Phi)\right] \tag{6}$$

$$\frac{dG_1}{dt} = \frac{1}{\tau_f}(p_1 - G_1 - G_1 I_1) \tag{7}$$

$$\frac{dI_2}{dt} = \frac{2}{\tau_c}\left[(G_2 - \alpha_2)I_2 - \kappa\sqrt{I_1 I_2}\cos(\Phi)\right] \tag{8}$$

$$\frac{dG_2}{dt} = \frac{1}{\tau_f}(p_2 - G_2 - G_2 I_2) \tag{9}$$

$$\frac{d\Phi}{dt} = \Delta\omega + \frac{1}{\tau_c}\left(\sqrt{\frac{I_1}{I_2}} + \sqrt{\frac{I_2}{I_1}}\right)\kappa\sin(\Phi) \tag{10}$$

where $\Phi = \varphi_2 - \varphi_1$ and $\Delta\omega = \omega_2 - \omega_1$. Here, it is assumed that the two-laser pump and loss are identical, which is reasonable when the two lasers oscillate in a common cavity [4]. Then, the pairs of equations of intensities and gains are identical and have same solutions (i.e., $I_1 = I_2 = I$), which makes the time revolutions of I_1 and I_2 synchronized. This makes the differential equation of phase difference Φ to reduce to

$$\frac{d\Phi}{dt} = \Delta\omega + 2\tau_c^{-1}\kappa\sin(\Phi) \tag{11}$$

Furthermore, for small ratio of τ_c to τ_f, the laser relaxation oscillation frequency Ω_R for $\kappa = 0$ can be calculated as [4]

$$\Omega_R = \frac{1}{2\pi}\left[\frac{2(p-\alpha)}{\tau_c\tau_f}\right]^{\frac{1}{2}} \tag{12}$$

Taking into account the frequency modulation, the detuning difference will be time dependent, so the array will be nonautonomous [9]. The dynamics' behavior of the array will respond to the modulation. That is motivation of this work. Assuming that the modulation is sinusoidal, the detuning difference should be rewritten as

$$\Delta\omega(t) = \Delta\omega_0 + m\sin(2\pi\Omega t) \tag{13}$$

where $\Delta\omega_0$ is the detuning difference without modulation, m is the modulation amplitude and Ω is the modulation frequency. From (11) and (13) it can be found that the frequency modulation influences the system by alternating the phase difference of two lasers. Since the focus in this work is the effect of frequency modulation, $\Delta\omega_0$ is assumed to be zero. The parameters used in the simulation are shown in Table 1.

Table 1 Parameter values used in the simulation [4]. τ_c is for the cavity of 6 cm length, τ_f is for the 1064 nm transition, Ω_R is calculated using (12)

Parameters	α	p	κ	τ_c	τ_f	Ω	Ω_R
Values	0.04	1.33α	2×10^{-5}	0.45 ns	240 µs	1 kHz	78.7 kHz

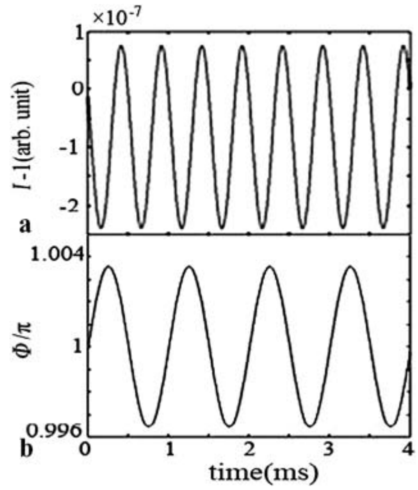

Fig. 1 Dynamics of the intensity (**a**) and the phase difference (**b**) with the modulation amplitude of $m = 10^3$ rad/s. To show the plot of intensity clearly, the y-axis is set as $(I-1)$

3 Results and discussion

It can be found that (11) will have no stationary solutions because of the time-dependent term $\Delta\omega(t)$, which is different from the array without modulations [4]. That means there will be no phase-locked state in the modulated array and the interferential fringe of the two-laser output will become unstable. Figure 1 displays the temporal plots of the phase difference and the intensity of the array with weak modulation. It can be found that the intensity frequency is twice as high as the phase frequency, which is the same as the modulation frequency.

With the increase of the modulation amplitude, the revolution of the intensity in a period will transfer from one peak to some smaller ones, as shown in Fig. 2. It should be noted that the temporal property of the phase difference is obviously asymmetric in a period, but the intensity revolution is periodic. This result implies that the asymmetry of the phase difference is not the key problem which makes the intensity chaotic.

With the further increase of the modulation amplitude, the chaotic behavior of the intensity will appear, as shown in

Fig. 3. From Figs. 3c and d one common thing can be found between two chaotic intensity behaviors, corresponding different modulation amplitudes, they both own the pulse sequences with the same repetition frequency. Estimated from numerical solutions by counting the number of the pulses of each sequence in a long time scale (10 ms), the repetition frequency is close to 78 kHz, which is almost the same as the laser relaxation oscillation frequency Ω_R (see Table 1). Therefore, it is reasonable to conclude that the pulse sequence is caused by the relaxation oscillation of the lasers.

The intensity chaos is really surprising since the modulation frequency is much smaller than the laser relaxation oscillation frequency. To explore the physical mechanism of the intensity chaos, the revolution of the phase difference is reviewed, because the phase difference is the bridge between the frequency modulation and the intensities of coupled lasers. One interesting thing is found that the frequency of the phase difference transferring 2π at the ascending (descending) edge (see the part marked with a frame in Fig. 3e, f, these frequencies are about 40 and 150 kHz separately) is close to the relaxation oscillation frequency. That implies the condition under which a harmonic resonance will possibly happen, which may finally cause the chaos of the intensity.

The condition is believed to be reasonable, because the phase difference affects the array intensities by the term $[\cos(\Phi)]$ (see (6) and (7)), which plays an important role in the dynamics of the coupled-laser system [4]. Note that the period of the term $[\cos(\Phi)]$ is 2π, so the frequency of the phase difference transferring 2π is actually the frequency of the term $[\cos(\Phi)]$. However, this condition is only a necessary condition, which is not sufficient for the appearance of chaos. Another needed condition is believed to be that the harmonic resonance, therefore the ascending (descending) edge, should last for a long enough. The second condition requires that the period of the phase difference should be long and the modulation frequency should be small. Actually, the similar chaotic behavior has not been found when the modulation frequency increases to 5 kHz (see Fig. 4), which may be linked to the necessity of the second condition.

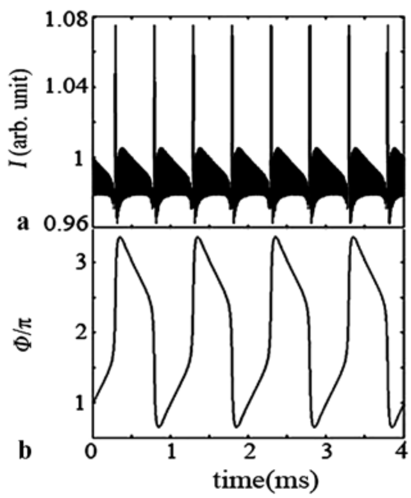

Fig. 2 Dynamics of the intensity (**a**) and the phase difference (**b**) with the modulation amplitude of $m = 10^5$ rad/s

4 Conclusion

In conclusion, the dynamics' behavior of the two-laser array with the frequency modulation is presented corresponding to the different modulation amplitudes. It has been demonstrated that the intensity chaos can happen when the modulation frequency is low. The physical mechanism of the chaos

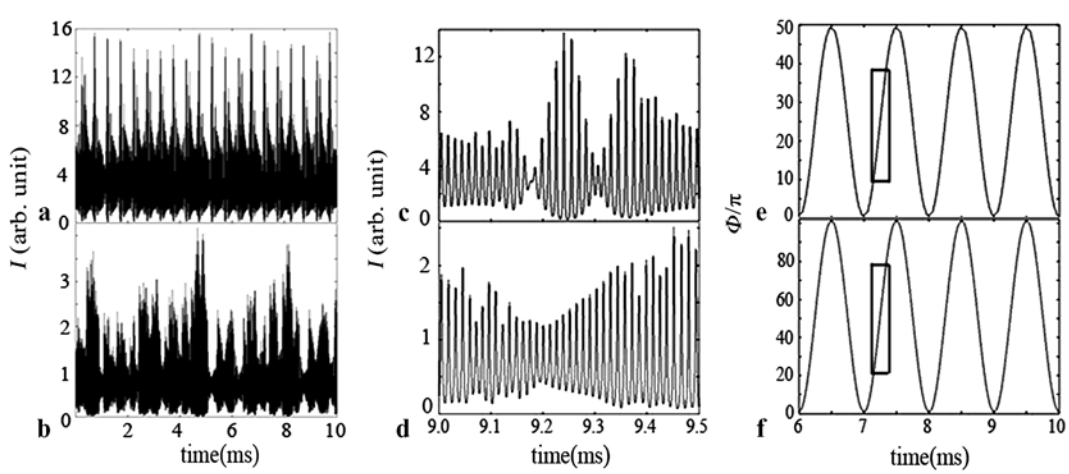

Fig. 3 Plots of the intensity chaos and the phase dynamics corresponding to $m = 5 \times 10^5$ rad/s (**a**, **c**, **e**) and 10^6 rad/s (**b**, **d**, **f**) separately. (**a**) and (**b**) are the plots with large time scale (10 ms); (**c**) and (**d**) are the plots with small time scale (0.5 ms). The ascending edges are labeled with frames in (**e**) and (**f**)

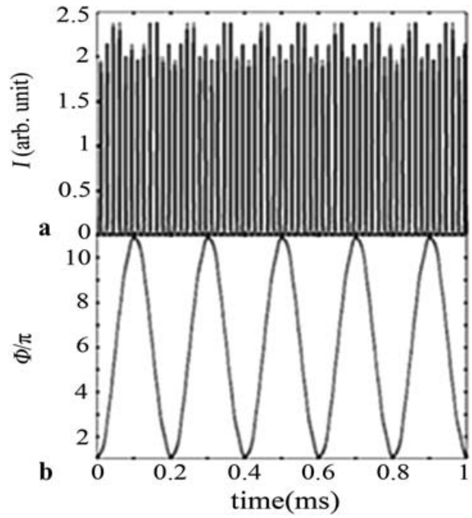

Fig. 4 Dynamics of the intensity (**a**) and the phase difference (**b**) with the frequency modulation ($m = 5 \times 10^5$ rad/s, $\Omega = 5$ kHz). Although the frequency of the phase difference transferring 2π at the ascending (descending) edge is about 70 kHz, which is close to the relaxation oscillation frequency, the intensity behavior is periodic rather than chaotic

is believed to be a harmonic resonance happening when the frequencies of the cosine of the phase difference is close to the relaxation oscillation frequency. The chaos will appear when the harmonic resonance last for a long enough. These results may have relevance to other coupled nonlinear oscillators with modulations.

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