Appl Phys B (2008) 92: 513–518
DOI 10.1007/s00340-008-3111-6

Applied Physics B
Lasers and Optics

Optical frequency counter based on two mode-locked fiber laser combs

J.-L. Peng · T.-A. Liu · R.-H. Shu

Received: 2 May 2008 / Published online: 29 July 2008

Abstract This work demonstrates a semi-automatic optical frequency counter based on two mode-locked fiber laser combs. The mode number of the comb line involved in the optical frequency measurement is determined by operating the two laser combs at three different repetition rates, with two of them similar enough to have the same mode number of the beating comb lines. The determination of the mode number is independent of the frequency fluctuation of the laser under measurement. The whole measurement process was automated, except for the frequency stabilization of the laser combs and the optimization of the beat signal–to–noise ratio.

PACS 06.30.Ft · 42.55.Wd · 42.62.Eh

1 Introduction

Radio frequency (RF) counter based on the modern electronic devices allowing frequency counting up to about 60 GHz have been widely commercialized. Wavemeters utilizing interference techniques by comparison with a reference light source are also well implemented for optical frequency (or wavelength) measurements. However, the measurement uncertainty is about 10–100 MHz. A commercial optical frequency counter has been developed with a minimum resolution of 100 kHz, based on the optical frequency comb generated by modulating a continuous-wave of laser light with an electro–optical modulator inside a Fabry–Perot cavity.[1] However, the measurable wavelength range is only from 1530–1565 nm. Mode-locked (ML) femtosecond lasers, which can serve as ultraboradband optical frequency combs, have been used in optical frequency metrology over the past several years [1]. The frequency of a laser under measurement (LUM) when employing a ML laser comb to measure the beat frequency can be expressed as $f_L = nf_r \pm f_o \pm f_b$, where n is the mode number (ordinal number) of the beating comb line, f_r denotes the repetition rate of the pulse train, and f_o denotes the carrier–envelope offset (CEO) frequency. To realize an optical frequency counter based on a ML laser, both the signs of the CEO frequency and the beat frequency and the value of n need to be determined. The sign of the CEO frequency and beat frequency can be determined uniquely by observing the corresponding beat frequency variations while changing the repetition frequency or the CEO frequency of the laser comb [2, 3].

Ma et al. presented a method for determining the mode number without using a wavemeter [4]. They changed the repetition rate smoothly while counting the total number of shifts in mode. The mode number was then determined by measuring the beat frequencies at different repetition rates. For low repetition rate and noisy LUM the shift in mode number can be larger than 100, which means that counting the total mode number shift is not practical. Peng et al. noted that the mode number shift is linear to the difference in repetition rate for small difference of some tens of kHz [3]. Therefore, they only needed to measure the repetition rate change required to shift one comb mode while slowly scanning the repetition rate, and did not need to count the total mode number shifted. Based on this technique, a semi-automatic optical frequency counter using two fiber laser

J.-L. Peng (✉) · T.-A. Liu · R.-H. Shu
Center for Measurement Standards, 321, Section 2, Kuang Fu Road, Hsinchu 30042, Taiwan
e-mail: jlpeng@itri.org.tw

[1] http://www.optocomb.com/eng/index.html

combs has been demonstrated [5]. However, the process for the determination of one comb mode shifted is still complicated. Zhang et al. derived the mode number of a laser comb with comb spacing of 200 MHz in two steps [6]. First, an estimated mode number with certain accuracy is obtained by operating the laser comb at two different repetition rates with the difference being small enough to maintain the same mode number. The laser comb is then operated at larger different repetition rate. The mode number change is then derived from the estimated mode number, and the exact mode number is calculated from the mode number difference. No mode number change is required in the first step only if the frequency instability of the LUM was less than about 10 kHz. Otherwise, the method processed by changing the repetition rate, and counting the mode number change by monitoring the RF spectrum analyzer, which is still not practical for an LUM with larger instability. Additionally, this approach is not succinct enough to realize an optical frequency counter based on low repetition rate laser combs.

This work presents a semi-automatic optical frequency counter based on two mode-locked fiber laser combs without the need to scan the repetition rate. Similar to Zhang's method, the mode number was determined by operating the two laser combs at three different repetition rates with two of them similar enough to have the same mode number of the beating comb lines. A measurement process is developed to determine the mode number, which is independent of the frequency fluctuation of the LUM. All other measurement processes are automated, except the frequency stabilization of the ML lasers and the optimization of the beat signal–to–noise ratio (S/N), making the optical frequency counter semi-automatic. The following sections describe the theory of the technique and the experimental setup and results for the frequency measurement of an iodine-stabilized Nd:YAG laser using the optical frequency counter.

2 Theory

The laser under measurement (LUM) is first beaten with the two laser combs operated at different repetition rates, f_{r1} and f_{r2}, and the corresponding beat frequencies are f_{b1} and f_{b2}. For simplicity, the signs of the CEO frequencies of the two laser combs and the beat frequencies are assumed to be determined to be positive by observing the beat frequency variation while slightly changing the repetition rate and the CEO frequency, and the two CEO frequencies are equal to f_o. Let the mode numbers of the beating comb lines be n and $n+m$ for f_{r1} and f_{r2}, respectively. The frequency of the LUM can then be expressed as

$$f_L = n f_{r1} + f_o + f_{b1}, \tag{1}$$

$$f_L = (n+m) f_{r2} + f_o + f_{b2}. \tag{2}$$

The repetition rate of one laser is then changed slightly. This change is small enough to keep the same mode number of the beating comb line and the same beat frequency sign, e.g., f_{r2} is slightly changed to f_{r3} and the measured beat frequency is f_{b3}. Thus, the frequency of the LUM can be expressed as

$$f_L = (n+m) f_{r3} + f_o + f_{b3}. \tag{3}$$

From (1), (2), and (3), the mode number n and the mode number difference m can be derived as

$$m = \frac{(f_{b2}-f_{b1})(f_{r1}-f_{r3}) - (f_{b3}-f_{b1})(f_{r1}-f_{r2})}{f_{r1}(f_{r3}-f_{r2})}, \tag{4}$$

$$n = \frac{m f_{r2} + f_{b2} - f_{b1}}{f_{r1}-f_{r2}}. \tag{5}$$

The frequency fluctuation of the LUM can be cancelled when f_{b1} and f_{b2}, as well as another set f_{b1} and f_{b3}, are measured simultaneously. Thus, the determination of n or m is independent of the frequency fluctuation of the LUM. Equation (4) can also be written as

$$m = \frac{\frac{(f_{b2}-f_{b3})}{f_{r3}-f_{r2}}(f_{r1}-f_{r2}) - (f_{b2}-f_{b1})}{f_{r1}}. \tag{6}$$

Although both (6) and (4) evaluate to m, the measurement processes to ensure m to be independent of the frequency fluctuation of the LUM are different. For (6), f_{b1} and $f_{b2,}$ as well as another set f_{b2} and $f_{b3,}$ should be measured simultaneously. Thus, the repetition rates of the two laser combs should be operated at f_{r1} and f_{r2} first, and f_{r1} is then changed to f_{r3} while f_{r2} remains the same. This measurement process is different from the one for (4) as described above.

The criteria of the required repetition rate change are determined by the requirement that the uncertainty of the measured n and m should be much less than 1. The uncertainty of n can be derived from (5), and be expressed as

$$\delta n = \sqrt{\left(\frac{\delta(f_{b2}-f_{b1})}{f_{r1}-f_{r2}}\right)^2 + \left(n\frac{\delta(f_{r1}-f_{r2})}{f_{r1}-f_{r2}}\right)^2}, \tag{7}$$

where $\delta()$ indicates the uncertainty. The uncertainty of $m f_{r2}$ is neglected since f_{r2} is usually phase-locked to a highly stable RF standard and $m \ll n$ ($m \approx 30, n \approx 10^6$ in this experiment). Because the frequency fluctuation of the LUM can be subtracted for the two-comb technique, the uncertainty of the beat frequency difference $f_{b2} - f_{b1}$ depends only on the relative instability between the comb lines, i.e., $\delta(f_{b2}-f_{b1}) = \delta((n+m)f_{r2} - n f_{r1}) \cong n\delta(f_{r2}-f_{r1})$, where the uncertainty of the CEO frequency is neglected. Additionally, this derivation does not consider the measurement uncertainty coming from the microwave counter. Since f_{r2}

and f_{r1} are phase-locked to the same source, they are correlated and $\delta(f_{r2} - f_{r1}) \cong \sqrt{2}\sigma(\tau) f_r$, where $\sigma(\tau)$ is the tracking instability of the repetition rate f_r at an integration time of τ, and $f_r = f_{r1}$ or f_{r2}. Therefore,

$$\delta n = \frac{2n\sigma(\tau) f_r}{f_{r1} - f_{r2}}. \quad (8)$$

The uncertainty of m can be derived from (6) more easily than from (4). The main uncertainty comes from the first term in the numerator of (6), and can be expressed as:

$$\begin{aligned}\delta m &\cong \sqrt{\left(\frac{\delta(f_{b2} - f_{b3})}{f_{r3} - f_{r2}} \frac{f_{r1} - f_{r2}}{f_{r1}}\right)^2 + \left(\frac{f_{b2} - f_{b3}}{f_{r3} - f_{r2}} \frac{f_{r1} - f_{r2}}{f_{r1}} \frac{\delta(f_{r3} - f_{r2})}{f_{r3} - f_{r2}}\right)^2} \\ &= \sqrt{\left(\frac{n\delta(f_{r2} - f_{r3})}{f_{r3} - f_{r2}} \frac{f_{r1} - f_{r2}}{f_{r1}}\right)^2 + \left((n+m) \frac{f_{r1} - f_{r2}}{f_{r1}} \frac{\delta(f_{r3} - f_{r2})}{f_{r3} - f_{r2}}\right)^2} \\ &\cong \frac{2n\sigma(\tau)(f_{r1} - f_{r2})}{f_{r3} - f_{r2}}. \end{aligned} \quad (9)$$

For a repetition rate of 100 MHz with a tracking instability of 2×10^{-13}@1 s and $n \approx 3 \times 10^6$ used in this experiment, $f_{r1} - f_{r2} \gg 120$ Hz is required to satisfy $\delta n \ll 1$, and $f_{r3} - f_{r2} \gg 1.2 \times 10^{-6}(f_{r1} - f_{r2})$ is required to satisfy $\delta m \ll 1$. This experiment set $f_{r1} - f_{r2} \approx 1$ kHz, which can be easily achieved by tuning a piezoelectric transducer (PZT) mounted on the laser cavity, and requires $f_{r3} - f_{r2} \gg 1.2$ mHz. The maximum value of $f_{r3} - f_{r2}$ is limited by the condition that the mode number of the beating comb line must not change when the repetition rate is varied from f_{r2} to f_{r3}. Two beat signals were generated between 0 and f_r when the LUM was beating with the neighboring comb lines. To prevent the beat signals from crossing each other at $f_r/2$ while the repetition rate changed, which may confuse the judgment of whether the mode number has changed, this experiment set a small value of 0.1 Hz for $f_{r3} - f_{r2}$. The frequency of the beating comb line was thus shifted by only about 300 kHz, enabling the mode number and the sign of the beat frequency to be maintained easily without ambiguity.

Equation (6) has a similar form to that derived by Zhang et al. for one laser comb operating at three different repetition rates successively [6]. Equations (4) and (6) yield the same value of m if only one laser comb is used, since the measured beat frequencies have no correlation. In the one-comb case, $\delta(f_{b2} - f_{b1}) = \sqrt{2}\delta\nu$, where $\delta\nu$ represents the frequency instability of the LUM. The uncertainties of n and m are mainly dominated by the first term inside the square root of (7) and (9), respectively. Therefore, $f_{r1} - f_{r2} \gg \sqrt{2}\delta\nu$ and $f_{r3} - f_{r2} \gg \sqrt{2}\delta\nu(f_{r1} - f_{r2})/f_{r1}$ are required for $\delta n \ll 1$ and $\delta m \ll 1$. To keep the mode number the same, the difference between f_{r2} and f_{r3} should be minimized. Therefore, one-comb technique restricts the measurable instability of the LUM. Zhang et al. discussed the one-comb technique in detail and showed that the mode number can be kept the same only for an LUM with an instability of less than 10 kHz for a laser comb with 200 MHz comb spacing [6]. Increasing fluctuation requires scanning the repetition rate, and counting the mode number shifted.

3 Experimental setup and results

Figure 1 schematically shows the structure of the semi-automatic optical frequency counter based on two ML Erbium-doped fiber laser combs. The two ML Er:fiber lasers were home-made ring lasers based on polarization additive pulse mode-locking (P-APM) [7]. A PZT that could tune the repetition rate by about 3.5 kHz was mounted onto each laser cavity. The detailed construction of the two fiber laser combs is described elsewhere [3]. Each laser comb had two branches of octave-spanning supercontinua ranging from 1050 nm to 2100 nm. One branch was adopted to detect the CEO frequency, and the other was adopted to generate a beat signal with the LUM. Both the repetition rate and the offset frequency were stabilized to synthesizers with the time base referenced to a 10 MHz low-noise oven-controlled quartz oscillator, which was phase-locked to a global positioning system receiver-disciplined Rb clock. The 10 MHz reference signal had an instability of less than 2×10^{-12} for an integration time of over 1 s and a relative uncertainty of 2×10^{-12}. The stabilized repetition frequency had an out-of-loop tracking instability of 2×10^{-13}@1 s, and the fluctuation of the CEO frequency was of the order of mHz. Details of the frequency stabilization of the repetition rate and the CEO frequency have been presented elsewhere [8].

The LUM was coupled into a single-mode fiber and split into two beams by a 3 dB coupler. Each beam line was then combined with the two laser combs by two 3 dB couplers. The combined laser beams were collimated to monochromators to filter out the required laser light for the beat signal detection. Polarization controllers were employed to ensure that the polarization of the fiber laser combs matched that of the LUM. The beat signals were detected by InGaAS photodiodes, and filtered by RF bandpass filters. Two frequency counters were externally triggered to measure the beat frequencies between the LUM and the two laser combs simultaneously. The gate times of the counters were set to 1 s. An iodine-stabilized Nd:YAG laser at wavelength near 1064 nm was utilized as the LUM for testing the optical frequency counter. The second harmonic of the Nd:YAG laser was locked to the a_{10} component of the R(56)32-0 transition in the iodine molecule.

All controlling and measurement processes were fulfilled by a computer, except that the frequency stabilization of the laser combs and the optimization of the beat signal–to–noise ratio were manually operated. The repetition rates of

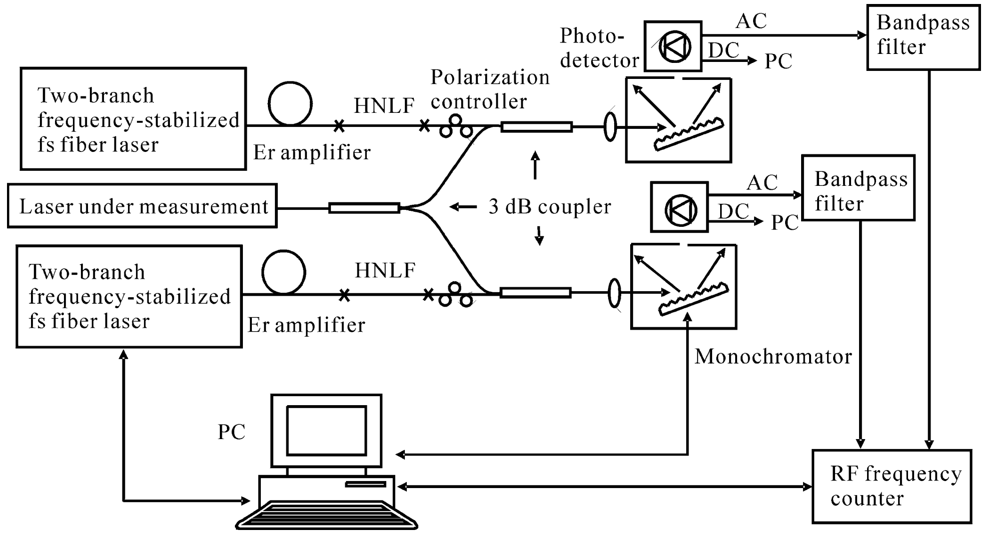
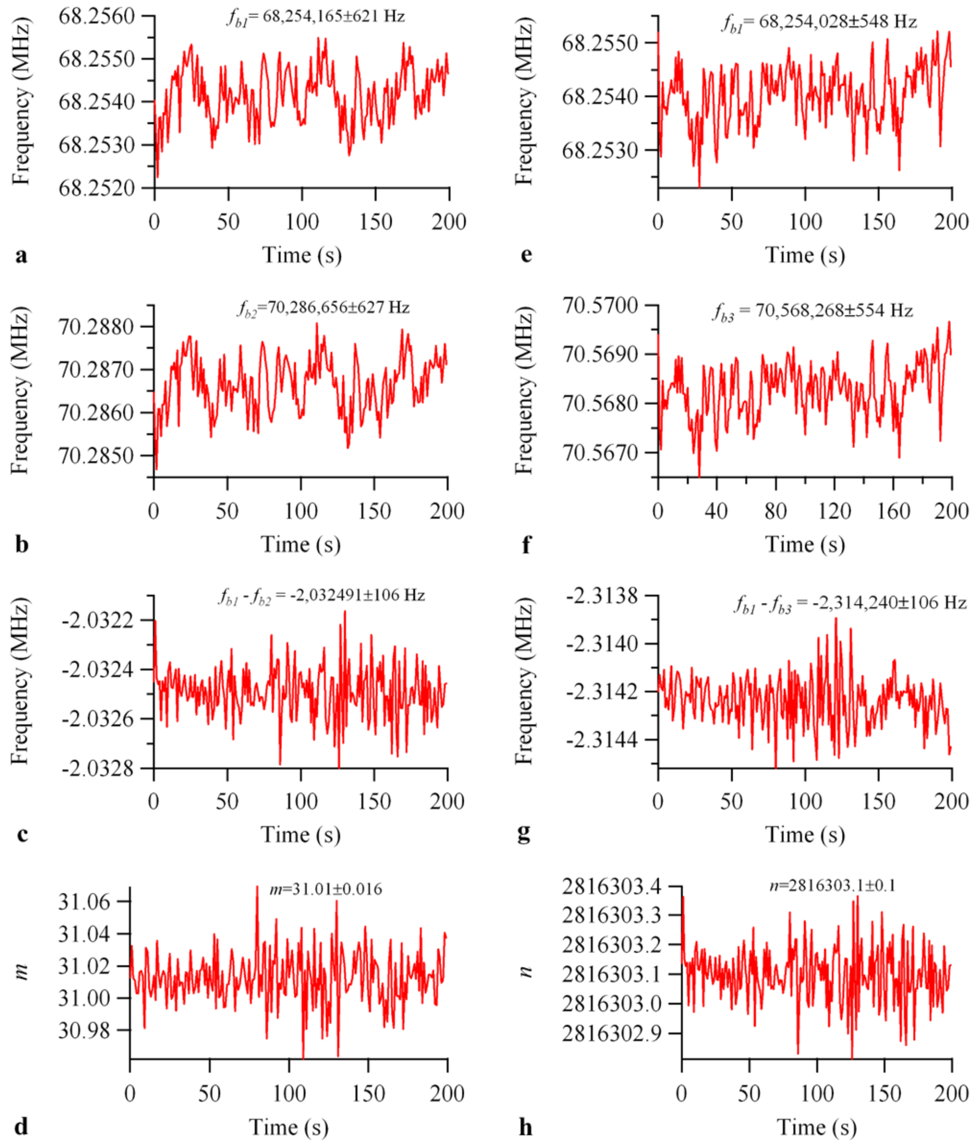


Fig. 1 Schematic diagram of the optical frequency measurement using two mode-locked Er:fiber combs. Each laser has two branches of octave-spanning supercontinuum. One branch is for the frequency stabilization of the repetition rate and the CEO frequency (not shown), and the other branch is for beating with the laser under measurement. HNLF: highly nonlinear fiber, PC: personal computer, DC, AC: direct, alternating current

the laser combs, comb1 and comb2, were first manually stabilized to 100 MHz and 99.9989 MHz setting by two synthesizers, respectively. Another synthesizer was adopted to stabilize both the CEO frequencies to 140 MHz. Defining f_o to be larger than f_r affects the counting of the mode number, but does not change the result of the frequency measurement. The light of the two laser combs for the frequency measurement was then turned off by shutting down the erbium amplifiers. The computer was used to control the monochromators based on the detected DC level to let the LUM shine on the photodetectors. The light of the two laser combs were later turned on again for the beat signal detection. The detected beat signals were manually optimized by adjusting the polarization controller and the beat frequencies were measured simultaneously by the externally triggered RF counters. Figures 2a and 2b show the measured beat frequencies f_{b1} and f_{b2} for $f_{r1} = 100$ MHz and $f_{r2} = 99.9989$ MHz. Figure 2c shows the difference between f_{b1} and f_{b2}. The repetition rate of laser comb2 was then increased by 0.1 Hz. Figures 2e and 2f show the measured beat frequencies f_{b1} and f_{b3} for $f_{r1} = 100$ MHz and $f_{r3} = 99.9989001$ MHz. Figure 2g shows the difference between f_{b1} and f_{b3}. The fact that the fluctuations of the difference signals are smaller than those of the individual beat frequencies indicates that the frequency variations of the Nd:YAG laser were subtracted in the difference of the beat frequencies. Since the repetition rate changed from f_{r2} to f_{r3} was only 0.1 Hz, the frequency of the beating comb line changed by only about 300 kHz, thus ensuring that the beating comb line would remain the same. By comparing the beat frequency variation from Figs. 2b to 2f, the signs of f_{b2} and f_{b3} for the calculation of the frequency of the LUM were determined to be negative. The sign of f_{b1} was also determined to be negative by further varying the repetition rate of comb1 by 0.1 Hz. The sign of the CEO frequencies of 140 MHz were both determined to be negative by measuring the beat frequencies while varying the CEO frequencies by 0.1 MHz. The mode number difference m was then calculated according to (4) using the data of Figs. 2a, 2b, 2e, and 2f; this is shown in Fig. 2d. Clearly, $m = 31$. Figure 2h shows the calculated mode number according to (5), which has an uncertainty of only 0.1. Clearly, $n =$ 2816303. The frequencies of the LUM measured according to (1) and (2) are equal to 281630091745.8(0.9) kHz and 281630091746.0(0.9) kHz, respectively, which agrees with the previously measured value within the uncertainty [3].

The two-comb results show that the mode number can be accurately determined within a few seconds. If $f_{r1} - f_{r2}$ and $f_{r3} - f_{r2}$ are multiplied by 10, i.e., $f_{r1} - f_{r2} = 10$ kHz, $f_{r3} - f_{r2} = 1$ Hz, then the measured uncertainty of m remains the same, i.e., less than 0.1 for each point as shown in Fig. 2d, but the uncertainty of n falls to one-tenth of the original value, as can be seen from (8) and (9). The measured uncertainty of n is then less than 0.1 for each measurement point, and n can be also determined with one second integration time. When f_{r2} changes to f_{r3} by 1 Hz, the comb lines shift less than 3 MHz, and the mode number of the beating comb line remains the same. Moreover, the sign of the beat frequency does not change either if the beat signal is 3 MHz far away from 0, $f_r/2$ and f_r. This signal can be achieved by choosing proper repetition rate or CEO frequency. Although the PZT used can tune the repetition rate by only about 3.5 kHz, a more different repetition rate can be achieved by changing the base plate temperature of the fiber laser comb, or by first constructing of the laser combs. Therefore, an optical frequency counter based on the two-comb technique can have features of immunity to the frequency instability of the LUM and short measurement time.

Fig. 2 Simultaneously measured beat frequencies for $f_{r1} = 100$ MHz (**a**) and $f_{r2} = 99.9989$ MHz (**b**), and that for $f_{r1} = 100$ MHz (**e**) and $f_{r3} = 99.9989001$ MHz (**f**). The gate time of the counter was 1 s. Parts (**c**) and (**g**) are the beat frequency differences. Parts (**d**) and (**h**) are the calculated m and n values, respectively



4 Conclusions

This work has demonstrated a semi-automatic optical frequency counter based on two fiber laser combs operated at three different repetition rates, with two of them being close enough to each other for the mode numbers of the beating comb lines to be the same. Simple measurement processes were presented to determine the difference in mode number between the two laser combs, and to determine the absolute mode number involved in the optical frequency measurement, which is independent of the frequency fluctuation of the LUM. An integration of 1 s is long enough to determine the mode number. All measurement processes were automated, except the frequency stabilization of the ML lasers and the optimization the S/N of the beat signal between the LUM and the laser comb. The frequency stabilization and the optimization of the beat signal S/N can be fulfilled automatically in principle. We believe that a benchtop fully automatic optical frequency counter based on the two-comb technique with octave-spanning counting capability is feasible in the near future.

Acknowledgements The authors would like to thank the Bureau of Standards, Metrology and Inspection of the Republic of China, Taiwan for financially supporting this research. Y.-C. Cheng is appreciated for contributing to the electronics used in this study.

References

1. Th. Udem, R. Holzwarth, T.W. Hänsch, Nature **416**, 233 (2002)
2. H. Inaba, Y. Daimon, F.-L. Hong, A. Onae, K. Minoshima, T.R. Schibli, H. Matsumoto, M. Hirano, T. Okuno, M. Onishi, M. Nakazawa, Opt. Express **14**, 5223 (2006)
3. J.-L. Peng, R.-H. Shu, Opt. Express **15**, 4485 (2007)

4. L.-S. Ma, M. Zucco, S. Picard, L. Robertsson, R.S. Windeler, IEEE J. Sel. Top. Quantum Electron. **9**, 1066 (2003)
5. T.-A. Liu, R.-H. Shu, J.-L. Peng, in *Pacific Rim Conference on Lasers and Electro-Optics*, ThG2-2 (2007)
6. J. Zhang, Z.H. Lu, Y.H. Wang, T. Liu, A. Stejskal, Y.N. Zhao, R. Dumke, Q.H. Gong, L.J. Wang, Laser Phys. **17**, 1025 (2007)
7. L.E. Nelson, D.J. Jones, K. Tamura, H.A. Haus, E.P. Ippen, Appl. Phys. B **65**, 277 (1997)
8. J.-L. Peng, H. Ahn, R.-H. Shu, H.-C. Chui, J.W. Nicholson, Appl. Phys. B **86**, 49 (2007)