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Scintillation properties of dark hollow beams in a weak turbulent atmosphere

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ABSTRACT The on-axis scintillation index for a circular dark hollow beam (DHB) propagating in a weak turbulent atmosphere is formulated, and the scintillation properties of a DHB are investigated in detail. The scintillation index for a DHB reduces to the scintillation index for a Gaussian beam, an annular beam and a flat-topped beam under certain conditions. It is found that the scintillation index of a DHB is closely related to the beam parameters and can be lower than that of a Gaussian beam, an annular beam and a flat-topped beam in a weak turbulent atmosphere at smaller waist sizes and longer propagation lengths.

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1 Introduction

In the past several years, dark hollow beams (DHBs) with zero central intensity have been widely investigated both theoretically and experimentally due to their unique physical properties and their important and extensive applications in laser optics, atomic optics, binary optics, optical trapping of particles and medical sciences [1]. DHBs can be used as optical pipes, optical tweezers and spanners, and for guiding, cooling and trapping of atoms [1–7], and they also provide a powerful tool to study the linear and nonlinear particle dynamics in a storage ring [8]. Various techniques, e.g. the transverse mode selection method, the geometrical optical method, the computer-generated hologram and spatial filtering, have been used to generate various dark hollow beams [9–13]. Several theoretical models, e.g. higher-order Bessel beams, high-order Mathieu beams, doughnut hollow beams and hollow Gaussian beams have been proposed to describe DHBs [2, 14–18]. The propagation properties of a DHB through free space or a paraxial optical system have been widely studied [14–21]. Recently, DHBs were extended to the partially coherent case [22, 23]. Higher-order partially coherent dark hollow beams have also been introduced recently [24].

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On the other hand, the investigation of the propagation properties of laser beams in a turbulent atmosphere becomes more and more important because of its wide applications in e.g. free-space optical communications, imaging systems and remote sensing and, up to now, much work has been carried out concerning the propagation of various coherent and partially coherent laser beams in a turbulent atmosphere [25–40]. Cai and He have investigated the irradiance and spreading properties of various DHBs in a turbulent atmosphere [40]. Scintillation properties (i.e. fluctuations of the intensity of a laser beam) of various laser beams, e.g. spherical and plane waves, Gaussian beams, elliptical Gaussian beams, annular beams, flattened Gaussian beams, cos-Gaussian beams and cosh-Gaussian beams, have been widely studied [41–50]. Konyaev et al. studied the effect of phase fluctuation on propagation of the vortex beams (one kind of DHBs) recently [51]. In the present paper, we study the on-axis scintillation properties of a DHB in a weak turbulent atmosphere, and we find that the on-axis scintillation index for a DHB can be smaller than that of a Gaussian beam, an annular beam and a flat-topped beam in a weak turbulent atmosphere under certain conditions, which will be useful in long-distance free-space optical communication.

2 Formulation

The electric field of a DHB of circular symmetry at $z=0$ can be expressed as the following finite sum of Gaussian modes [18]:

$$u_N(x,y,0)=\sum_{n=1}^{N}\frac{(-1)^{n-1}}{N}\binom{N}{n}\times\left[\exp\left(-\frac{x^2+y^2}{w_n^2}\right)-\exp\left(-\frac{x^2+y^2}{w_{np}^2}\right)\right], \tag{1}$$

where $\binom{N}{n}$ denotes a binomial coefficient, $w_n^2=w_0^2/n$, $w_{np}^2=pw_0^2/n$, w_0 is the beam waist size of the fundamental Gaussian mode, N is the order of a circular DHB and

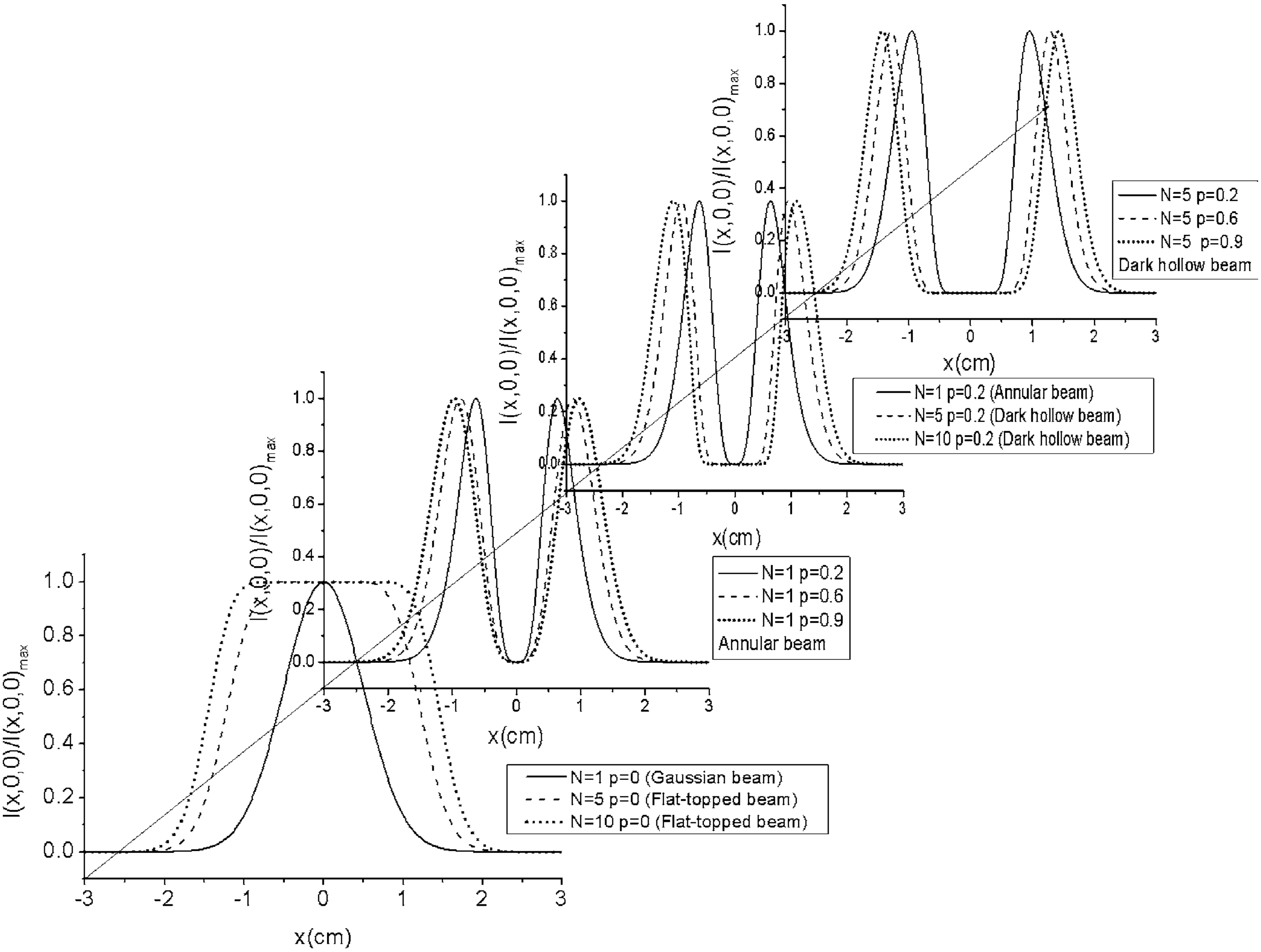

FIGURE 1 Cross line ($y = 0$) of the normalized irradiance distribution of a circular DHB for several different values of N and p with $w_0 = 1$ cm

$0 < p < 1$. When $N = 1$ and $p = 0$, (1) reduces to the expression for the electric field of a fundamental Gaussian beam. When $N > 1$ and $p = 0$, (1) reduces to the expression for the electric field of a flat-topped beam proposed by Li [52]. When $N = 1$ and $0 < p < 1$, (1) reduces to the expression for the electric field of an annular Gaussian beam as shown in [46]. Figure 1 shows the cross line ($y = 0$) of the normalized irradiance distribution of a circular DHB for several different values of N and p with $w_0 = 1$ cm. One sees that the central size across a DHB increases as N or p increases.

The propagation of a DHB in free space satisfies the following Huygens–Fresnel integral:

$$u_N^{\mathrm{FS}}(\boldsymbol{p}, z) = \frac{k \exp(\mathrm{i}kz)}{2\pi \mathrm{i}z} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_N(\boldsymbol{r}, 0) \times \exp\left[\frac{\mathrm{i}k}{2z}(\boldsymbol{p}-\boldsymbol{r})^2\right] \mathrm{d}x\,\mathrm{d}y\,, \tag{2}$$

where $\boldsymbol{p} = (p_x, p_y)$ and $\boldsymbol{r} = (x, y)$ are the transverse receiver and source coordinates, respectively. Substituting (1) into (2), we obtain (after some integration) the following expression for the electric field for a DHB at z in free space:

$$u^{\mathrm{FS}}(\boldsymbol{p}, z) = \exp(\mathrm{i}kz) \sum_{n=1}^{N} \frac{(-1)^{n-1}}{N} \binom{N}{n} \exp\left[\frac{\mathrm{i}k}{2z}\boldsymbol{p}^2\right] \times \left[\frac{1}{1+2\mathrm{i}z/kw_n^2} \exp\left(-\frac{k^2 w_n^2 \boldsymbol{p}^2}{4z^2 - 2\mathrm{i}kzw_n^2}\right) - \frac{1}{1+2\mathrm{i}z/kw_{np}^2} \exp\left(-\frac{k^2 w_{np}^2 \boldsymbol{p}^2}{4z^2 - 2\mathrm{i}kzw_{np}^2}\right)\right]. \tag{3}$$

In the range of weak turbulence, the on-axis scintillation index m^2 of a coherent laser beam on the output plane ($z = L$) can be expressed as follows [47, 49, 50, 53, 54]:

$$m^2 = 4B_x(\boldsymbol{p} = 0, L) = 4\pi \int_0^L \mathrm{d}z_1 \int_{-\infty}^{\infty} \mathrm{d}\kappa_x \int_{-\infty}^{\infty} \mathrm{d}\kappa_y \times \left[\mathrm{Re}(HH) + \mathrm{Re}(H^*H)\right] \varphi_n(\kappa)\,, \tag{4}$$

where ‘Re’ denotes the real part of a complex parameter and ‘H^*’denotes the conjugate of H. $B_x(\boldsymbol{p}, L)$ is the log-amplitude correlation function of the laser beam [53, 54], $\varphi_n(\kappa)$ is the spectral density for the index-of-refraction fluctuations, κ_x and κ_y are the x and y components of the spatial frequency and $\kappa = \left(\kappa_x^2+\kappa_y^2\right)^{1/2}$ and is expressed as follows:

$$H(\boldsymbol{p}=0, L, \kappa_x, \kappa_y, z_1) = \frac{k^2}{2\pi(L-z_1)u^{\mathrm{FS}}(0, L)} \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}p_{1x}\,\mathrm{d}p_{1y} u^{\mathrm{FS}}(\boldsymbol{p}_1, z_1)\exp(\mathrm{i}\kappa_x p_{1x}+\mathrm{i}\kappa_y p_{1y}) \times \exp\left[\mathrm{i}k(L-z_1)+\frac{\mathrm{i}k\boldsymbol{p}_1^2}{2(L-z_1)}\right]. \quad (5)$$

Here $u^{\mathrm{FS}}(\boldsymbol{p}, z)$ is the field of the laser beam at the propagation distance z in free space.

Applying (3), $u^{\mathrm{FS}}(0, L)$ in (5) is expressed as follows:

$$u^{\mathrm{FS}}(0, L) = \exp(\mathrm{i}kL)\sum_{n=1}^{N}\frac{(-1)^{n-1}}{N}\binom{N}{n} \times \left[\frac{1}{1+2\mathrm{i}L/kw_n^2}-\frac{1}{1+2\mathrm{i}L/kw_{np}^2}\right]. \quad (6)$$

Substituting (3) into (5), after some tedious but straightforward integration, we obtain the following expression for $H = (\boldsymbol{p}=0, L, \kappa_x, \kappa_y, z_1)$:

$$H(p_x=0, p_x=0, L, \kappa_x, \kappa_y, z_1) = \frac{-\mathrm{i}k}{u^{\mathrm{FS}}(0, L)}\exp(\mathrm{i}kL)\sum_{n=1}^{N}\frac{(-1)^{n-1}}{N}\binom{N}{n} \times \left(\frac{kw_n^2}{\left(2\mathrm{i}L+kw_n^2\right)}\exp\left[\frac{(L-z_1)\left(2z_1-\mathrm{i}kw_n^2\right)\kappa^2}{2k\left(2\mathrm{i}L+kw_n^2\right)}\right] - \frac{kw_{np}^2}{\left(2\mathrm{i}L+kw_{np}^2\right)}\exp\left[\frac{(L-z_1)\left(2z_1-\mathrm{i}kw_{np}^2\right)\kappa^2}{2k\left(2\mathrm{i}L+kw_{np}^2\right)}\right]\right). \quad (7)$$

By applying (7), we can express HH and H^*H as follows:

$$HH = -\frac{k^2}{[u^{\mathrm{FS}}(0,0,L)]^2}\exp(2\mathrm{i}kL) \times \sum_{n=1}^{N}\sum_{m=1}^{N}\frac{(-1)^{n+m}}{N^2}\binom{N}{n}\binom{N}{m} \times \left(A_1\exp\left[B_1\kappa^2\right]+A_2\exp\left[B_2\kappa^2\right]+A_3\exp\left[B_3\kappa^2\right] +A_4\exp\left[B_4\kappa^2\right]\right), \quad (8)$$

$$HH^* = \frac{k^2}{|u^{\mathrm{FS}}(0,0,L)|^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{(-1)^{n+m}}{N^2}\binom{N}{n}\binom{N}{m} \times \left(A_5\exp\left[B_5\kappa^2\right]+A_6\exp\left[B_6\kappa^2\right]+A_7\exp\left[B_7\kappa^2\right] +A_8\exp\left[B_8\kappa^2\right]\right), \quad (9)$$

where

$$A_1 = \frac{kw_n^2}{\left(2\mathrm{i}L+kw_n^2\right)}\frac{kw_m^2}{\left(2\mathrm{i}L+kw_m^2\right)},$$
$$B_1 = \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_n^2\right)}{2k\left(2\mathrm{i}L+kw_n^2\right)}+\frac{(L-z_1)\left(2z_1-\mathrm{i}kw_m^2\right)}{2k\left(2\mathrm{i}L+kw_m^2\right)},$$
$$A_2 = -\frac{kw_n^2}{\left(2\mathrm{i}L+kw_n^2\right)}\frac{kw_{mp}^2}{\left(2\mathrm{i}L+kw_{mp}^2\right)},$$
$$B_2 = \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_n^2\right)K^2}{2k\left(2\mathrm{i}L+kw_n^2\right)} + \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_{mp}^2\right)K^2}{2k\left(2\mathrm{i}L+kw_{mp}^2\right)},$$
$$A_3 = -\frac{kw_{np}^2}{\left(2\mathrm{i}L+kw_{np}^2\right)}\frac{kw_m^2}{\left(2\mathrm{i}L+kw_m^2\right)},$$
$$B_3 = \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_{np}^2\right)K^2}{2k\left(2\mathrm{i}L+kw_{np}^2\right)} + \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_m^2\right)K^2}{2k\left(2\mathrm{i}L+kw_m^2\right)},$$
$$A_4 = \frac{kw_{np}^2}{\left(2\mathrm{i}L+kw_{np}^2\right)}\frac{kw_{mp}^2}{\left(2\mathrm{i}L+kw_{mp}^2\right)},$$
$$B_4 = \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_{np}^2\right)}{2k\left(2\mathrm{i}L+kw_{np}^2\right)}+\frac{(L-z_1)\left(2z_1-\mathrm{i}kw_{mp}^2\right)}{2k\left(2\mathrm{i}L+kw_{mp}^2\right)},$$
$$A_5 = \frac{kw_n^2}{\left(2\mathrm{i}L+kw_n^2\right)}\frac{kw_m^2}{\left(-2\mathrm{i}L+kw_m^2\right)},$$
$$B_5 = \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_n^2\right)}{2k\left(2\mathrm{i}L+kw_n^2\right)}+\frac{(L-z_1)\left(2z_1+\mathrm{i}kw_m^2\right)}{2k\left(-2\mathrm{i}L+kw_m^2\right)},$$
$$A_6 = -\frac{kw_n^2}{\left(2\mathrm{i}L+kw_n^2\right)}\frac{kw_{mp}^2}{\left(-2\mathrm{i}L+kw_{mp}^2\right)},$$
$$B_6 = \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_n^2\right)}{2k\left(2\mathrm{i}L+kw_n^2\right)}+\frac{(L-z_1)\left(2z_1+\mathrm{i}kw_{mp}^2\right)}{2k\left(-2\mathrm{i}L+kw_{mp}^2\right)},$$
$$A_7 = -\frac{kw_{np}^2}{\left(2\mathrm{i}L+kw_{np}^2\right)}\frac{kw_m^2}{\left(-2\mathrm{i}L+kw_m^2\right)},$$
$$B_7 = \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_{np}^2\right)}{2k\left(2\mathrm{i}L+kw_{np}^2\right)}+\frac{(L-z_1)\left(2z_1+\mathrm{i}kw_m^2\right)}{2k\left(-2\mathrm{i}L+kw_m^2\right)},$$
$$A_8 = \frac{kw_{np}^2}{\left(2\mathrm{i}L+kw_{np}^2\right)}\frac{kw_{mp}^2}{\left(-2\mathrm{i}L+kw_{mp}^2\right)},$$
$$B_8 = \frac{(L-z_1)\left(2z_1-\mathrm{i}kw_{np}^2\right)}{2k\left(2\mathrm{i}L+kw_{np}^2\right)}+\frac{(L-z_1)\left(2z_1+\mathrm{i}kw_{mp}^2\right)}{2k\left(-2\mathrm{i}L+kw_{mp}^2\right)}. \quad (10)$$

We use the Kolmogorov spectrum for the spectral density of the index-of-refraction fluctuations, which is expressed as $\varphi_n(\kappa) = 0.033C_n^2\kappa^{-11/3}$ [25]; here C_n^2 is the structure constant of the turbulent atmosphere. By using the coordinate transformation $\mathrm{d}\kappa_x\,\mathrm{d}\kappa_y = \kappa\,\mathrm{d}\varphi\,\mathrm{d}\kappa$, $\kappa = \sqrt{\kappa_x^2+\kappa_y^2}$ in (4), and then substituting (8) and (9) into (4), we obtain (after some tedious integration) the following expression for the on-axis scintillation index of a DHB on the output plane ($z = L$):

$$m^2 = \mathrm{Re}\left(-4\pi^2\frac{k^2}{[u^{\mathrm{FS}}(0,0,L)]^2}\exp(2\mathrm{i}kL)\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{(-1)^{n+m}}{N^2} \times \binom{N}{n}\binom{N}{m}\left(m_1^2+m_2^2+m_3^2+m_4^2\right)\right)$$

$$+\mathrm{Re}\left(4\pi^2\frac{k^2}{[u^{\mathrm{FS}}(0,0,L)]^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{(-1)^{n+m}}{N^2}\binom{N}{n}\binom{N}{m}\right.$$
$$\left.\times\left(m_5^2+m_6^2+m_7^2+m_8^2\right)\right), \qquad (11)$$

where

$$m_q^2 = 0.033\Gamma\left(-\frac{5}{6}\right)C_n^2\int_0^L \mathrm{d}z_1 A_q(-B_q)^{5/6}$$
$$(q=1,2,3,4,5,6,7,8)\,. \qquad (12)$$

In the above derivations, we have applied the following integral formula [55]:

$$\int_0^\infty x^{v-1}\exp(-\alpha x^p)\,\mathrm{d}x = \frac{1}{p}\alpha^{-v/p}\Gamma\left(\frac{v}{p}\right)\,. \qquad (13)$$

Equations (11) and (12) are the main formulae derived in the present paper. Under the condition of $N=1$ and $p=0$, (11) reduces to the expression for the on-axis scintillation index of a fundamental Gaussian beam. Under the condition of $N>1$ and $p=0$, (11) reduces to the expression for the on-axis scintillation index of a flat-topped beam proposed in [47]. Under the condition of $N=1$ and $0<p<1$, (11) reduces to the expression for the on-axis scintillation index of an annular Gaussian beam [46]. Thus, (11) provides a convenient formula for studying the scintillation properties of a Gaussian beam, an annular beam, a flat-topped beam and a circular DHB in a weak turbulent atmosphere.

3 Results and discussion

In this section, we study the on-axis scintillation properties of a circular DHB in a weak turbulent atmosphere by using (11). We speak about the weak-turbulence regime when the scintillations of a plane-wave incidence are less than unity (i.e. $1.23C_n^2k^{7/6}L^{11/6}<1$) [26].

Figure 2 shows the variation of the scintillation index of a DHB against propagation length L for several different values of N and p in a weak turbulent atmosphere with $w_0 = 1$ cm, $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$ and $\lambda = 1.55\ \mu$m. For the convenience of comparison, the corresponding results of a Gaussian beam ($N=1$, $p=0$), an annular beam ($N=1$, $p=0.2$) and a flat-topped beam ($N=5$, $p=0$) are also shown in Fig. 2. One sees from Fig. 2 that the scintillation index of various laser beams in a weak turbulent atmosphere increases as the propagation length increases, the numerical results of the scintillation index of a Gaussian beam, an annular beam and a flat-topped beam agree well with the existing results reported in previous literature [45–47] and the scintillation index of a DHB beam is larger than that of a Gaussian beam and an annular beam at extremely short propagation distances, but is smaller than that of a Gaussian beam, an annular beam and a flat-topped beam at long propagation distances, which will be useful in long-distance free-space optical communication. One also finds from Fig. 2 that the scintillation index of a DHB decreases as its beam order N or p increases at long propagation distances. Figure 3 shows the variation of the scintillation index of a DHB against waist size w_0 for several different values of N and p in a weak turbulent atmosphere with $L = 1$ km, $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$ and $\lambda = 1.55\ \mu$m. The corresponding results of a Gaussian beam, an annular beam and a flat-topped beam are also shown in Fig. 3. One sees from Fig. 3 that the curve of the numerical result of the case $N=1$ and $p=0$ appropriately reflects the limiting case of the well-known scintillation behavior of a Gaussian beam while matching the scintillation index values obtained numerically from the well-known formulae $0.5C_n^2k^{7/6}L^{11/6}$ for a spherical wave and $1.23C_n^2k^{7/6}L^{11/6}$ for a plane wave [41]. The scintillation index for various laser beams will initially display a downward trend at small source sizes but, after reaching a dip, will start to increase, and the scintillation index of a DHB is smaller than that of a Gaussian beam, an annular beam and a flat-topped beam only for small source size w_0 ($w_0 < 2.5$ cm) when other conditions remain as given. The scintillation index of a DHB also is smaller than that of

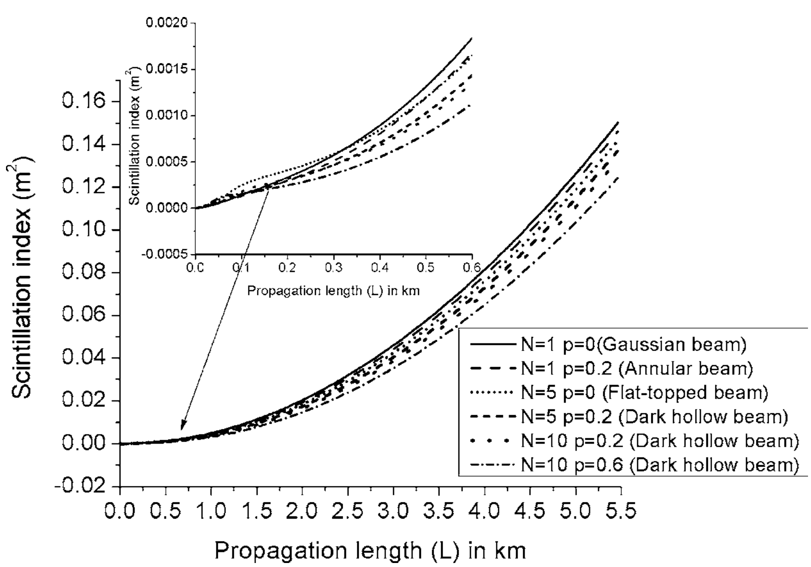

FIGURE 2 Variation of the scintillation index of a DHB against propagation length L for several different values of N and p in a weak turbulent atmosphere

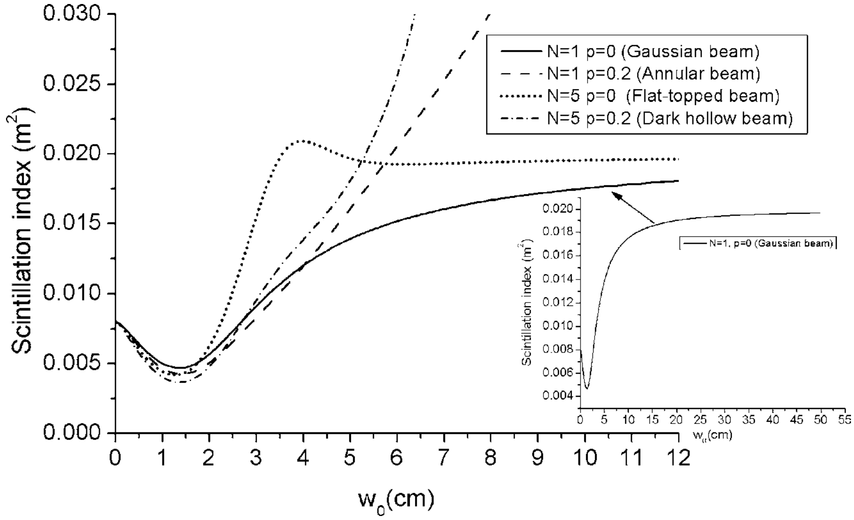

FIGURE 3 Variation of the scintillation index of a DHB against waist size w_0 for several different values of N and p in a weak turbulent atmosphere

a spherical wave and a plane wave for small source size as shown in Fig. 3. Note that both the scintillation index values of a Gaussian beam and a flat-topped beam will approach the plane-wave limit for larger beam waist size, while an annular beam and a dark hollow beam exhibit different characteristics for larger beam waist size. For an annular beam and a dark hollow beam, to approach the plane-wave limit, p should be chosen to go to zero. Here it is worth noting that DHBs, being annular-beam based, do not obey the plane-wave limit as explained in [49]. Additionally, the analysis of the intensity characteristics of DHBs using our previous results in [38] and [49] indicates that when the propagation length is kept constant at $L = 1$ km, the on-axis intensity tends to fall rapidly with increasing waist size of the source beam. This effect is essentially similar to measuring the scintillation index at the very outer edges of the beam, where the intensity has dropped to nearly zero. Thus, as discussed and illustrated in [50], in such cases the scintillation index will begin to rise sharply. This behavior is clearly visible from the curves of $N = 1$, $p = 0.2$ and $N = 5$, $p = 0.2$ of Fig. 3. Figure 4 shows the variation of the scintillation index of a DHB against beam order N for several different values of p in a weak turbulent atmosphere with $L = 1$ km, $w_0 = 1$ cm, $C_n^2 = 10^{-15}$ m$^{-2/3}$ and $\lambda = 1.55$ µm. The scintillation index decreases as its beam order N or p increases, and the results agree well with Fig. 2. It is also obvious from (12) and (13) that m^2 is linearly proportional to C_n^2.

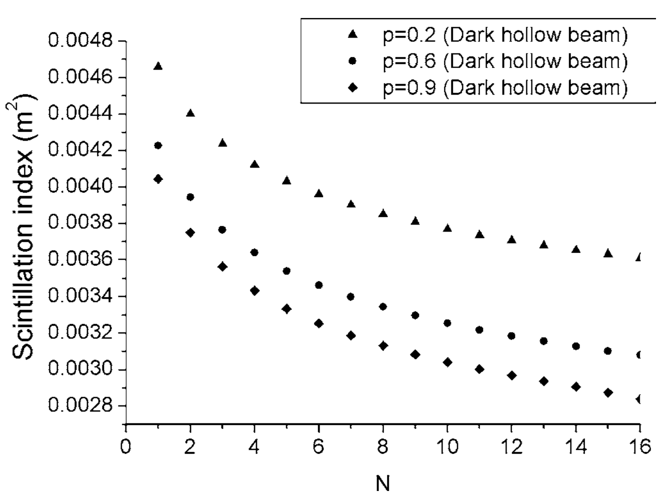

FIGURE 4 Variation of the scintillation index of a DHB against beam order N for several different values of p in a weak turbulent atmosphere

4 Conclusions

In conclusion, we have formulated the on-axis scintillation index of a circular DHB propagating in a weak turbulent atmosphere. We have investigated the on-axis scintillation properties of a DHB in a weak turbulent atmosphere, and have made some comparisons between the scintillation index of a Gaussian beam, an annular beam, a flat-topped beam and a DHB. As the limiting case solutions, the scintillation index for a DHB reduces to the scintillation index for a Gaussian beam or an annular beam or a flat-topped beam. We have found that the scintillation index of a DHB can be smaller than that of a Gaussian beam, an annular beam, a flat-topped beam, a spherical wave and a plane wave in a weak turbulent atmosphere particularly at small waist sizes, while the scintillation index of a DHB will be larger than those beams at large waist sizes and longer propagation lengths. We also found that the scintillation properties are closely related to the beam parameters of the DHB. Our results will be useful in long-distance free-space optical communication.

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REFERENCES

1 J. Yin, W. Gao, Y. Zhu, Generation of dark hollow beams and their applications, in *Progress in Optics*, vol. 44, ed. by E. Wolf (North-Holland, Amsterdam, 2003), pp. 119–204
2 T. Kuga, Y. Torii, N. Shiokawa, T. Hirano, Y. Shimizu, H. Sasada, Phys. Rev. Lett. **78**, 4713 (1997)
3 H. Ito, T. Nakata, K. Sakaki, M. Ohtsu, K.I. Lee, W. Jhe, Phys. Rev. Lett. **76**, 4500 (1996)
4 J. Yin, Y. Zhu, W. Jhe, Y. Wang, Phys. Rev. A **58**, 509 (1998)
5 M.J. Renn, D. Montgomery, O. Vdovin, D.Z. Anderson, C.E. Wieman, E.A. Cornell, Phys. Rev. Lett. **75**, 3253 (1995)

6 J. Yin, Y. Zhu, W. Wang, Y. Wang, W. Jhe, J. Opt. Soc. Am. B **15**, 25 (1998)
7 Z. Wang, J. Yin, Z. Wang, Opt. Express **14**, 9551 (2006)
8 Y.K. Wu, J. Li, J. Wu, Phys. Rev. Lett. **94**, 134 802 (2005)
9 N.R. Heckenberg, R. McDuff, C.P. Smith, A.G. White, Opt. Lett. **17**, 221 (1992)
10 X. Wang, M.G. Littman, Opt. Lett. **18**, 767 (1993)
11 J. Arlt, K. Dholakia, Opt. Commun. **177**, 297 (2000)
12 R. Chakraborty, A. Ghosh, J. Opt. Soc. Am. A **23**, 2278 (2006)
13 Z. Liu, H. Zhao, J. Liu, J. Lin, M.A. Ahmad, S. Liu, Opt. Lett. **32**, 2076 (2007)
14 F. Gori, G. Guattari, C. Padovani, Opt. Commun. **64**, 491 (1987)
15 J.C. Gutierrez-Vega, M.D. Iturbe-Castillo, S. Chavez-Cerda, Opt. Lett. **25**, 1493 (2000)
16 Y. Cai, X. Lu, Q. Lin, Opt. Lett. **28**, 1084 (2003)
17 Y. Cai, Q. Lin, J. Opt. Soc. Am. A **21**, 1058 (2004)
18 Z. Mei, D. Zhao, J. Opt. Soc. Am. A **22**, 1898 (2005)
19 Y. Cai, C. Chen, F. Wang, Opt. Commun. **278**, 34 (2007)
20 Y. Cai, L. Zhang, Opt. Commun. **265**, 607 (2006)
21 Y. Cai, S. He, J. Opt. Soc. Am. A **23**, 1410 (2006)
22 Y. Cai, L. Zhang, J. Opt. Soc. Am. B **23**, 1398 (2006)
23 X. Lu, Y. Cai, Phys. Lett. A **369**, 157 (2007)
24 H.T. Eyyuboğlu, Opt. Laser Technol. **40**, 156 (2008)
25 A. Ishimaru, *Wave Propagation and Scattering in Random Media*, vol. 2 (Academic, New York, 1978)
26 L.C. Andrews, R.L. Phillips, *Laser Beam Propagation Through Random Media* (SPIE, Bellingham, WA, 2005)
27 H.T. Yura, Appl. Opt. **11**, 1399 (1972)
28 V.A. Banakh, V.L. Mironov, Opt. Lett. **1**, 172 (1977)
29 V.A. Banakh, V.L. Mironov, Opt. Lett. **4**, 259 (1979)
30 S.C.H. Wang, M.A. Plonus, J. Opt. Soc. Am. **69**, 1297 (1979)
31 C.Y. Young, Y.V. Gilchrest, B.R. Macon, Opt. Eng. **41**, 1097 (2002)
32 T. Shirai, A. Dogariu, E. Wolf, J. Opt. Soc. Am. A **20**, 1094 (2003)
33 H.T. Eyyuboğlu, Y. Baykal, Opt. Express **12**, 4659 (2004)
34 H.T. Eyyuboğlu, A. Arpali, Y. Baykal, Opt. Express **14**, 4196 (2006)
35 C. Arpali, C. Yazicioglu, H.T. Eyyuboğlu, S.A. Arpali, Y. Baykal, Opt. Express **14**, 8919 (2006)
36 Y. Cai, S. He, Opt. Lett. **31**, 568 (2006)
37 Y. Cai, S. He, Appl. Phys. Lett. **89**, 041 117 (2006)
38 Y. Cai, J. Opt. A Pure Appl. Opt. **8**, 537 (2006)
39 Y. Cai, Y. Chen, H.T. Eyyuboğlu, Y. Baykal, Appl. Phys. B **88**, 467 (2007)
40 Y. Cai, S. He, Opt. Express **14**, 1353 (2006)
41 L.C. Andrews, R.L. Phillips, C.Y. Hopen, *Laser Beam Scintillation with Applications* (SPIE, Bellingham, WA, 2001)
42 V.A. Banakh, G.M. Krekov, V.L. Mironov, S.S. Khmelevtsov, R.S. Tsvik, J. Opt. Soc. Am. **64**, 516 (1974)
43 Y. Cai, Y. Chen, H.T. Eyyuboğlu, Y. Baykal, Opt. Lett. **32**, 2405 (2007)
44 O. Korotkova, Proc. SPIE **6105**, 61 050V (2006)
45 L.C. Andrews, M.A. Al-Habash, C.Y. Hopen, R.L. Phillips, Waves Random Media **11**, 271 (2001)
46 F.E.S. Vetelino, L.C. Andrews, Proc. SPIE **5160**, 86 (2004)
47 Y. Baykal, H.T. Eyyuboğlu, Appl. Opt. **45**, 3793 (2006)
48 D.C. Cowan, J. Recolons, L.C. Andrews, C.Y. Young, Proc. SPIE **6215**, 62 150B (2006)
49 H.T. Eyyuboğlu, Y. Baykal, J. Opt. Soc. Am. A **24**, 156 (2007)
50 H.T. Eyyuboğlu, Y. Baykal, Appl. Opt. **46**, 1099 (2007)
51 P.A. Konyaev, V.P. Lukin, V.A. Sennikov, in *ICONO/LAT* (Minsk, 2007), p. 112
52 Y. Li, Opt. Lett. **27**, 1007 (2002)
53 Y. Baykal, J. Opt. Soc. Am. A **23**, 889 (2006)
54 Y. Baykal, J. Opt. Soc. Am. A **21**, 1290 (2004)
55 I.S. Gradshteyn, I.M. Ryzhik, *Tables of Integrals, Series and Products* (Academic, San Diego, CA, 2000)