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Enhancement of stimulated emission in 12-fold symmetric quasi-crystals

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ABSTRACT In this paper, we investigate the stimulated emission in a 12-fold symmetric quasiperiodic photonic crystal. The stimulated emission peaks in the quasiperiodic photonic crystal are more abundant and stronger than those in a periodic crystal. Also, more stimulated emission peaks appear as the crystal size and the gain increase, and some frequencies of the peaks are independent of the incident direction. These phenomena may be due to wave localization in the quasiperiodic photonic crystal.

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1 Introduction

The localization of classical waves in disordered media has been studied [1, 2], and wave propagation in amplifying random media has been pursued intensively [3–5]. However, certain periodically correlated amplifying nonperiodic systems with a gain medium, for example a quasiperiodic photonic crystal (QPC) with a gain medium, have not been studied adequately. The QPC possesses a rotation period but loses translation symmetry. The interaction of light with gain in such a structure is an interesting topic [6]. In this paper, we studied the stimulated emissions in a QPC with a gain medium using the multiple-scattering method [7, 8]. Simulation results revealed that there are abundant lasing modes in the QPC, and some phenomena are different from those in a periodic crystal. Sharp lasing peaks appear when the gain or size of the system is greater than a well-defined threshold value [9]. The appearance of the lasing mode is due to the feedback of the quasiperiodic long-range order [6]. In our calculations, we assumed that the imaginary part of the dielectric constant is no longer equal to zero, but is a negative value, which means that the impurity atoms (gain medium) with population inversion are uniformly distributed in the crystal. With impurity atoms, the calculated transmittance can be greater than unity. This indicates that stimulated emission takes place in the crystal [10].

2 Photonic crystal structure and its transmission spectra

We study the stimulated emission in a two-dimensional 12-fold symmetric QPC that consists of dielectric rods with a gain medium. Figure 1 shows the schematic structure of a 12-fold symmetric QPC, which is formed by placing dielectric rods with a circular cross section in the vertices of two-dimensional 12-fold symmetric quasiperiodic lattices. The pattern is tiled with squares and rhombuses (with an acute angle of 36°) of equal side lengths $a = 330$ nm, and the crystal consists of 189 dielectric rods with a rod radius of 70 nm. The calculation process of the transmission spectrum of the QPC is described as follows.

The incident light is a plane wave. A slit with a width of $4.5a$ is put in front of the sample with a distance of $6.5a$ between the center of the slit and the surface of the sample, and then the incident beam passes through the slit and illuminates the sample, which can be looked upon as a slit light source. Here a is the side length of the square unit in the QPC pattern, as shown in Fig. 1. The light wave arising from a slit centered at the origin with width w in the y direction is given by

$$u(x, y) = \left(\frac{k_0}{4}\right) \int_{-w/2}^{w/2} \mathrm{d}y' [H_0(k\varrho) + \mathrm{i}\frac{x}{\varrho} H_1(k_0\varrho)] \,, \qquad (1)$$

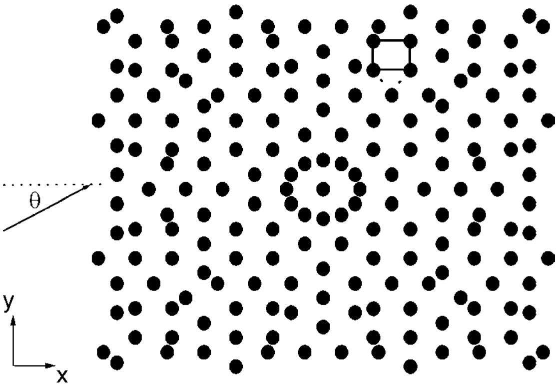

FIGURE 1 Schematic figure of a two-dimensional 12-fold symmetric quasiperiodic photonic crystal

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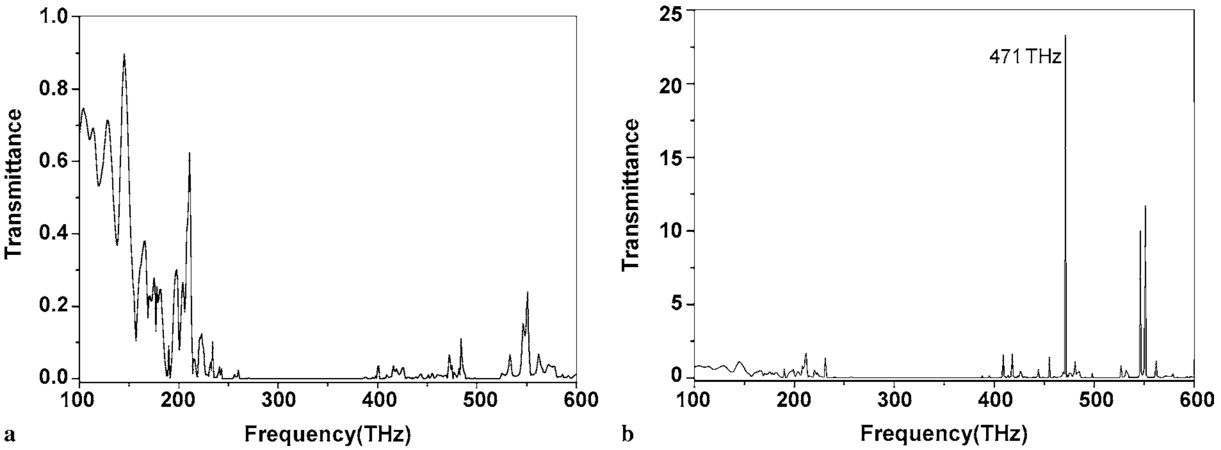

FIGURE 2 Transmittances for TM polarization wave as a function of frequency calculated for a crystal with 189 dielectric rods with dielectric constants of (**a**) 11.4 and (**b**) 11.4 − 0.08i

where $\varrho = \sqrt{x^2 + (y - y')^2}$, and H_m is a Hankel function of the first kind. A generalized transmission coefficient is defined as the ratio of energy flux to that of the incident wave at $\theta = 0$:

$$T = \left|1 + \sqrt{2\pi/k_0 w^2}\,\mathrm{e}^{\mathrm{i}\pi/4} f_\mathrm{s}(0)\right|^2, \tag{2}$$

where $f_\mathrm{s}(0)$ is the total scattering amplitude at $\theta = 0$ in the far field. Then the transmission coefficient is calculated using the multiple-scattering method [7, 8].

Figure 2a plots the transmittance as a function of frequency for a transverse magnetic (TM) polarized (E field parallel to the cylindrical rods) incident wave for the QPC with a dielectric constant of 11.4, where the incident light propagates perpendicular to the two-dimensional QPC. Figure 2b shows the transmittance of the crystal with a dielectric constant of 11.4 − 0.08i, and the other parameters are the same as those used in Fig. 2a. The negative imaginary part of the dielectric constant represents gain in the material [10, 11]. The existence of calculated transmittance with a value greater than unity means that the stimulated emission occurred in the crystal. The sharp stimulated peaks at 471, 546, and 551 THz coincide with the frequencies in the photonic band, where the group velocities v_g are small [10].

3 Stimulated emission in quasiperiodic crystal

We find that there are some interesting phenomena when the size of the photonic crystal varies. As the number varies from 101 (from Fig. 3a) to 189 (Fig. 3b), the intensity of the strongest stimulated emission rises from 27 to 1812, where the dielectric rods have a dielectric constant of 11.4 − 0.1i. The frequency difference between these two peaks (at 488 THz in Fig. 3a and 480 THz in Fig. 3b) is only 8 THz. Both of these peaks coincide with the band gap edge where the group velocity is small. As shown in Fig. 3c and d, the peaks in the crystals with 261 and 334 rods are not very strong. This means that the peak intensity is strong if the size of the crystal is adequate at a defined frequency. The enhancement factor, which is estimated as the ratio of the transmittance in a QPC with impurity atoms (Fig. 3b) and that in a QPC without impurity atoms (Fig. 2a) is as large as 51 994. By comparing the stimulated emission in a quasiperiodic crystal with that in a periodic crystal with similar size and similar parameters, we found that the strongest peak intensity in a square lattice periodic crystal (Fig. 4) is only 30, while that in a quasiperiodic crystal is 1812 (see Fig. 3b), so the enhancement factor reaches 60. Compared with the periodic crystal, the quasiperiodic structure has a lower lasing threshold, which might result in the localization in quasicrystal, and it makes the optical paths of the waves much longer than that in a periodic medium [5]. As the crystal size increases, some original peaks remain, other peaks disappear and some new peaks appear, although most of them are weak. For example, three peaks appear at 234, 457, and 476 THz with respective intensities of 13, 17 and 18 in Fig. 3c. In Fig. 3d, two strong peaks appear at 242 THz with an intensity of 114 and 245 THz with an intensity of 92.5, and some weak peaks appeared at 437, 479, 481, and 557 THz with intensities of only about 10. This means that the number of lasing points satisfying the standing wave condition increases as the size of the quasicrystal increases.

Figure 5 shows the transmission spectra of the crystal with 261 dielectric rods with dielectric constants of 11.4 − 0.2i under various incident angles. Several stimulated emission peaks are found to be independent of the light incident angles, especially the peak at 225 THz. It has already been demonstrated that the band gap of a quasiperiodic structure is independent of the incident angle [12], therefore, the corresponding localized modes in the QPC are independent of the incident angle of light. Consequently, the stimulated emission that occurs at the frequency of localized mode should also be independent of the incident angle.

4 Local density of states in quasiperiodic crystal

To explain the stimulated emission described above, we calculate the local density of states (LDOS) in the 12-fold symmetric quasiperiodic crystals with gain medium by using the multiple-scattering method and Greens function [13]. The LDOS can be calculated via

$$\varrho(r, \omega) = -\frac{2\omega}{\pi c^2}\,\mathrm{Im}[G(r, r_\mathrm{s}, \omega)]\,. \tag{3}$$

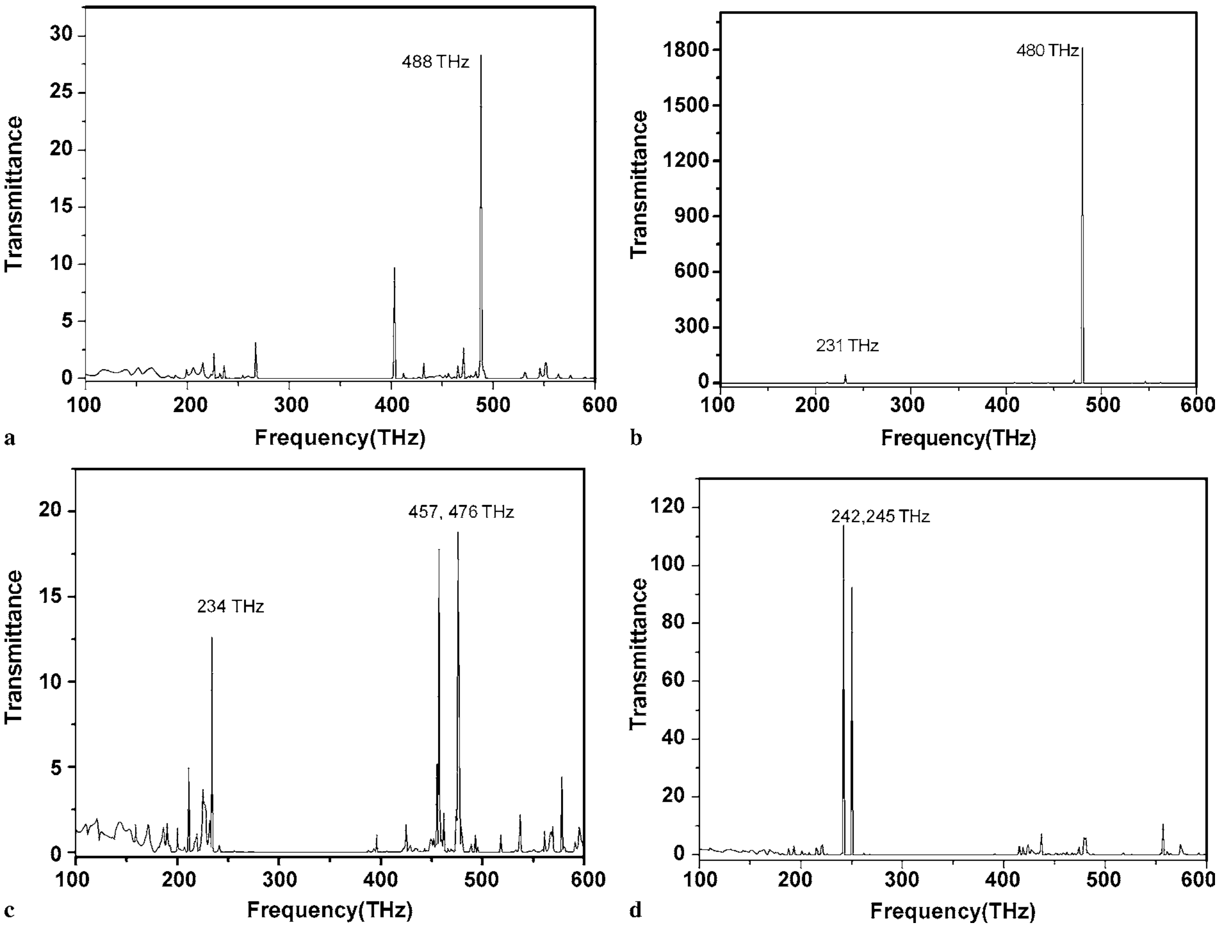

FIGURE 3 Transmittance for TM polarization as a function of frequency calculated for a crystal with (**a**) 101, (**b**) 189, (**c**) 261, and (**d**) 334 dielectric rods with a dielectric constant of 11.4 − 0.1i

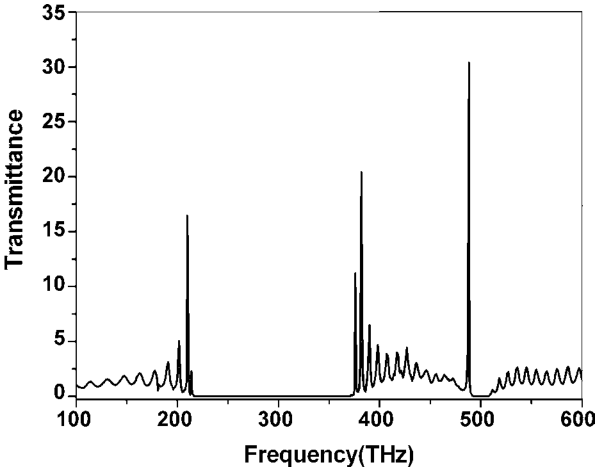

FIGURE 4 Transmittance for a periodic crystal with 195 dielectric rods with a dielectric constant of 11.4 − 0.1i (similar size and same dielectric constant as that of quasiperiodic PC in Fig. 3b)

It shows that the LDOS is expressed by the imaginary part of Greens function. Where $G(r, r_s, \omega)$ is the electromagnetic Greens function for a source at location r_s and an observation point at r. The source is an infinite line antenna parallel to the cylinder axes.

The calculated LDOS for the stimulated peaks at 480 and 489 THz for a QPC with 189 dielectric rods with dielectric constants of 11.4 − 0.1i are shown in Fig. 6a and b, respectively. The LDOS at frequency of 225 THz for a QPC with 261 dielectric rods with dielectric constants of 11.4 − 0.2i and 11.4 are shown in Fig. 6c and d, respectively. It can be found in Fig. 6a–d that the distributions of LDOS do not possess translation symmetry but extend along their axes of 12-fold symmetry. Similar to the explanation from M. Notomi et al. [6], these stimulated emission occurs due to the feedback of the quasiperiodic long-range order, which is different from that in both periodic crystals and random media. Furthermore, the stimulated emission peaks and corresponding LDOS are determined by standing wave modes in the quasiperiodic structure. The standing wave condition is satisfied only at symmetry points in the first Brillouin zone for periodic photonic crystals, but there are a large number of lasing points that satisfy the standing wave condition for quasiperiodic crystals [6], which explains why the lasing modes in QPC are more abundant than those in periodic crystals. The LDOS patterns in Fig. 6a–c are different from each other, which means that the peaks at different frequencies are determined by different standing wave conditions in the QPC. For some frequencies, the light is strongly localized in quasiperiodic crystals, which can be clearly demonstrated by the LDOS distribution in Fig. 6c. For comparison, we also calculate LDOS in periodic square photonic crystals for TM wave at a frequency of 564 THz in the pho-

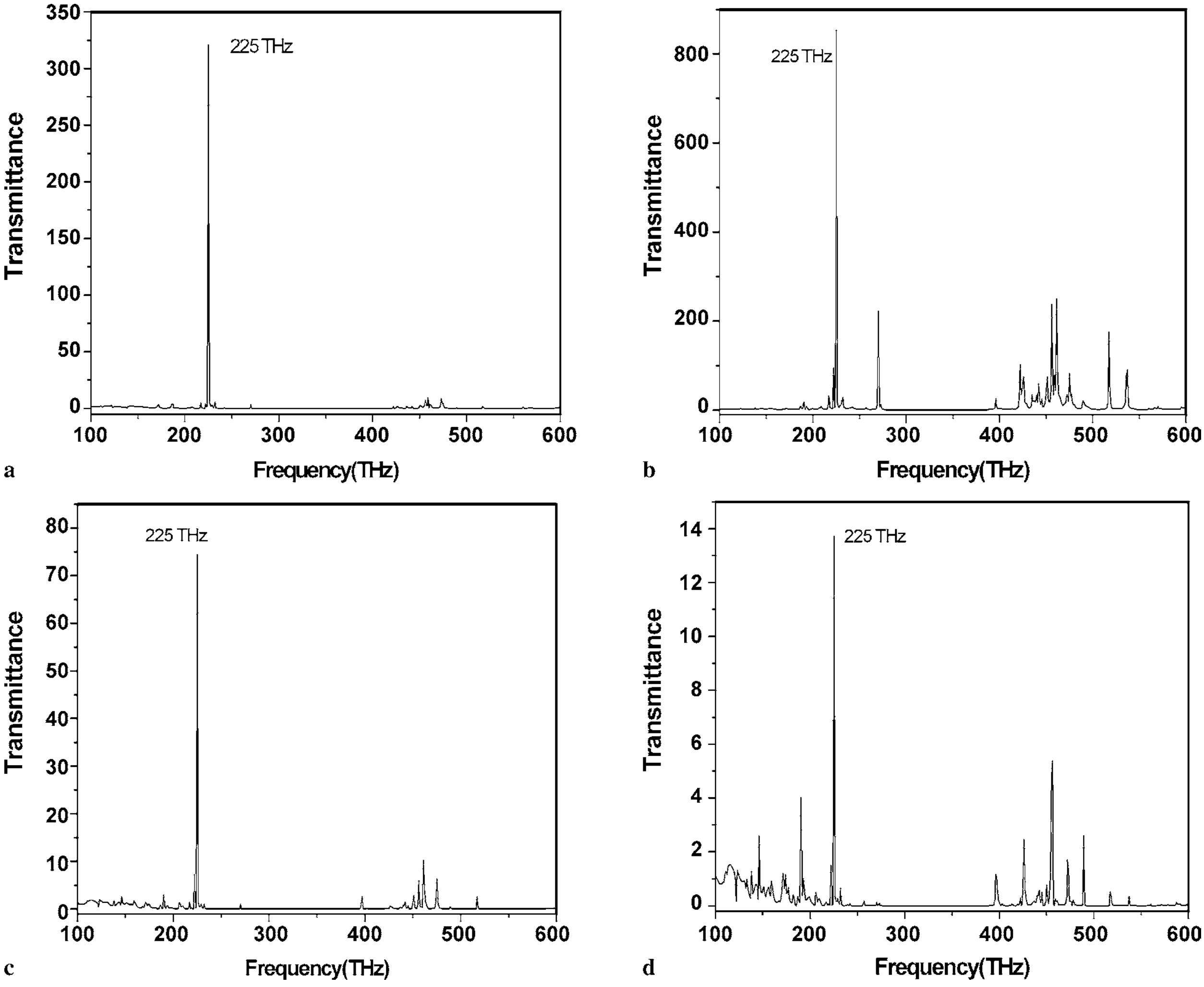

FIGURE 5 Transmittance for quasiperiodic crystal under various incident angles of (**a**) 5°, (**b**) 10°, (**c**) 15°, and (**d**) 20°, with 261 dielectric rods with a dielectric constant of $11.4-0.2i$

tonic band, where the number of the dielectric rods is 169, the lattice constant 330 nm, the radius of the rod 100 nm, and the dielectric constant $11.4-0.1i$, which is same as that used in Fig. 6a. From Fig. 6e and f, it can be found that the local density of states in square photonic crystal are periodically distributed in the square lattice, and the large LDOS locates in the dielectric rods. The largest LDOS are 2.35 and 1.0 in the square photonic crystal with gain and without gain, respectively. The largest LDOS is smaller than that in a quasiperiodic crystal with the same dielectric constant. Compared with a periodic crystal, a quasiperiodic crystal is a special regular structure without translation symmetry, so the localized mode may be determined by the structure of quasiperiodic crystals without adding defects [14]. Localization induced by the structure of quasiperiodic crystals will enhance the optical path due to multiple scattering and will then reduce the lasing threshold, which is similar to the case in random systems [5].

It should be noted that the LDOS in a QPC with gain is stronger than that without gain. The LDOS in vacuum is only 0.25. The largest LDOS is up to 21.3 for the crystal with gain in Fig. 6c, while it is only 2.6 for the crystal without gain in Fig. 6d. The enhanced LDOS in QPC seems to be evidence that optical gain is beneficial to wave localization, which is similar to the case in which optical gain helps spatial confinement of light in a random medium [15]. Figure 7 plots the sections through Fig. 6c (solid line) and 6d (dotted line) at $x=0$. For the crystal with gain, there is oscillatory character in the LDOS, and it is likely to be associated with Fabry–Pérot interference effects between the dielectric rods, or the boundaries of the crystal. However, for the medium without gain in Fig. 6d, the oscillatory character is not as obvious near the boundaries of the crystal, and the oscillatory ripple is something different from that in Fig. 6c. Moreover, the LDOS patterns in Fig. 6c and d are largely different from those in Fig. 6a and b for the different size of crystal. The Fabry–Pérot interference is the effect of finite sizes and effective refractive index of the crystal. These facts are indications that the stimulated emission peak at 225 THz in Fig. 6c of the QPC with gain is a cooperative effect of both the Fabry–Pérot interference and the long-range order of the quasiperiodic crystal. We know that the stimulated emission due to the long-range order in quasiperiodic crystals is independent of the incident angle, but the com-

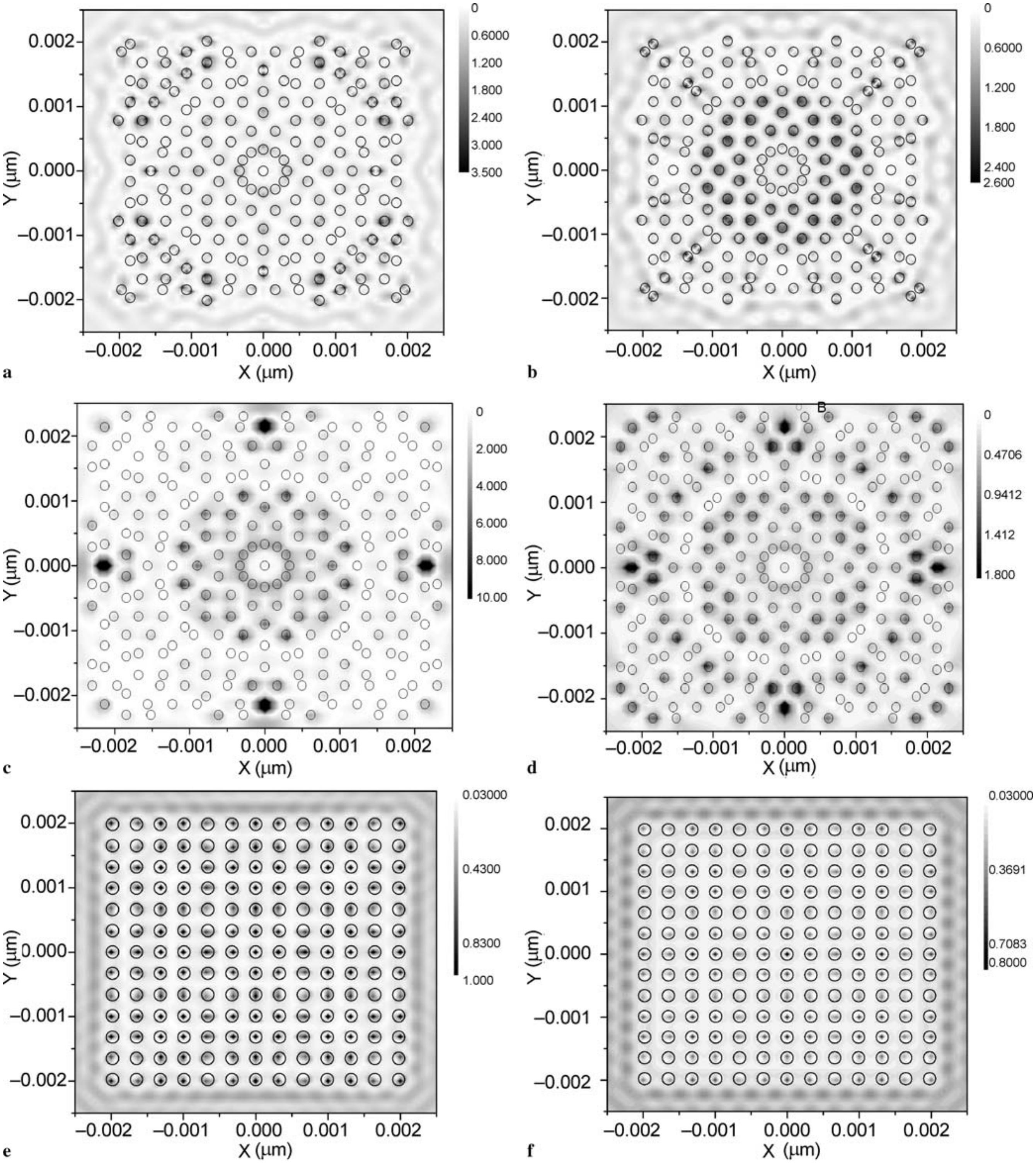

FIGURE 6 Calculated local density of states in a 12-fold quasiperiodic crystal (**a**) at 480 THz frequency with 189 dielectric rods with a dielectric constant of $11.4-0.1i$; (**b**) at 489 THz with 189 dielectric rods with a dielectric constant of $11.4-0.1i$; (**c**) at 225 THz with 261 dielectric rods with a dielectric constant of $11.4-0.2i$; and (**d**) at 225 THz with 261 dielectric rods with a dielectric constants of 11.4; (**e**) LDOS at 564 THz in a square photonic crystal with 169 dielectric rods with a dielectric constant of $11.4-0.1i$; and (**f**) LDOS at 564 THz in a square photonic crystal with 169 dielectric rods with a dielectric constants of 11.4. The *solid circles* represent the dielectric rods composed of photonic crystal

ponent due to Fabry–Pérot interference does depend on the incident angle. Therefore, as the incident angle varies, the emission intensity changes, though the frequency of 225 THz does not vary.

5 Conclusions

In summary, the stimulated emission in a 12-fold symmetric quasiperiodic photonic crystal was investigated

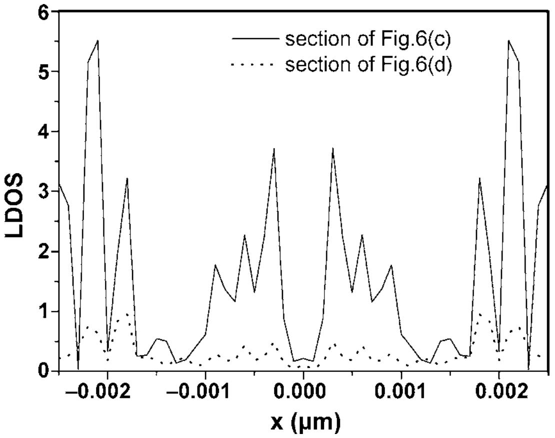

FIGURE 7 Sections through Fig. 6c (*solid line*) and Fig. 6d (*dotted line*) at $x = 0$

theoretically by using the multiple-scattering method. The stimulated emission peaks are 60-fold stronger and more abundant than those in periodic crystals, and the frequencies of some peaks are independent of the incident angle. The stimulated emission stems from the structure of quasiperiodic long-range order in quasiperiodic crystals. Some strongly stimulated peaks are determined by the quasiperiodic structure and Fabry–Pérot interference effect.

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