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High-Q filters based on one-dimensional photonic crystals using epsilon-negative materials

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ABSTRACT A new type of filter based on one-dimensional photonic crystals is presented. A photonic crystal consists of a periodic repetition of air layers and epsilon-negative (ENG) material layers. This type of filter has high-Q values without defects. The Q values and the filtering frequency can be precisely adjusted by modulating the thicknesses of the ENG material layers and the air layers, respectively. A method of deciding the position of the transmission band for the photonic crystals is presented. The effect of absorption of ENG materials on the filter is also considered.

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1 Introduction

Photonic crystals (PCs) have attracted intense study over the last decade due to their unique electromagnetic properties and potential applications [1]. It has been proven that a photonic band gap (PBG) can be formed as a result of the interference of Bragg scattering in a periodical dielectric structure. In a conventional PC (with positive indices), such a Bragg gap depends strongly on the details of the interference process. For a one-dimensional (1D) photonic crystal with a defect, the two Bragg stacks neighboring the defect can be considered as photonic barriers since the propagation of EM waves whose frequencies lie in the PBG of the Bragg stacks is forbidden. Due to the confinement effect of photonic barriers, discrete (localized) defect modes will appear inside the PBG, leading to the filtering phenomenon. The stronger the confinement effect of the barrier, the smaller the full width at half maximum (FWHM) of the defect mode, and the larger the quality factor (Q) of the filter since Q is inversely proportional to the FWHM.

There are two main ways to enhance the confinement effect of photonic barriers in a conventional 1D PC. One way is to enlarge the thicknesses of photonic barriers by increasing the period number of the Bragg stacks [2, 3]. The higher the period number, the stronger the confinement effect of the barrier. However, such a confinement effect comes at the cost of increasing the volume of the structure. The other way is to enlarge the depth of photonic barriers by changing material parameters, e.g., by increasing the ratio of the refractive indices of the two media. A bigger ratio will lead to a wider gap (deeper barrier) and a stronger confinement effect. Nevertheless, this method is limited by the availability of the refractive indices of candidate materials. Therefore, it is hard to obtain a high-Q filter for a conventional 1D photonic crystal. Such difficulties originate from the fact that the PBG in a conventional 1D photonic crystal comes from wave (Bragg) scattering.

Recently, double negative refraction (DNG) materials, i.e., left-handed materials (LHM) with simultaneous negative permittivity and negative permeability have attracted considerable attention because of their peculiar properties, such as the reversal of Doppler shift and their famous negative refraction property [4–14]. It has been demonstrated that stacking alternating layers of positive-index and double negative-index media leads to a type of PBG corresponding to a zero-averaged refractive index. A number of unique properties of the zero-n gap on the beam shaping effect have been studied [15–17]. In addition to the DNG materials, other material, called single-negative (SNG) material, has also been studied. The SNG materials consist of mu-negative (MNG) materials with negative μ but positive ε, and epsilon-negative (ENG) materials with negative ε but positive μ. It has been found that one-dimensional photonic crystal (1DPC) consisting of a periodic repetition of MNG and ENG layers can possess another type of photonic gap with an effective phase φ_{eff} of zero called the SNG gap or the zero-φ_{eff} gap [17–19]. In this paper, we present a new type of 1DPC structure. This structure consists of a periodic repetition of positive-index layers and ENG layers. Without any defect, this structure can be used as a high-Q filter with a small period number. Consider for example the 1DPC with the a finite periodic structure of $(\text{AB})^N$, where A represents ENG materials and B represents air, and N is the number of periods. Metamaterials with effective negative permittivity over a frequency band have been fabricated by using wire elements [18]. Metamaterials with effective negative permeability in a particular

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frequency range have also been obtained by utilizing split ring resonators. However, it is necessary to combine both the methods of fabricating ENG media and MNG media, to form metamaterials with simultaneously negative permittivity and permeability [18]. Since only ENG media is needed in the structure of $(AB)^N$, from the point view of fabrication techniques, fabrication of this structure may be less intricate than that for DNG materials.

2 Methods and results

The thickness of A and B are first supposed to be $d_a = d_b = d/2$, and $d = 10$ mm. We choose a basic frequency $\omega_0 = \pi c/d$ (c is the velocity of light in free space). In the following numerical studies, all frequencies and lengths are in units of ω_0 and d, respectively. Corresponding to light with the basic frequency, the period length of the 1DPC is just half of the wavelength. For the A layers, the relative permittivity and permeability in the ENG materials are given by [19]

$$\varepsilon_a = 1 - \omega_{ep}^2/\omega^2, \quad \mu_a = 1\,, \tag{1}$$

where ω_{ep} is the electronic plasma frequency. The ENG frequency is determined by $\omega < \omega_{ep}$. When $\omega > \omega_{ep}$, the A layers are turned into positive-index materials. We also assume the values of the relative permittivity and permeability in B layers to be $\varepsilon_b = \mu_b = 1$. In our calculation, we take $\omega_{ep} = \sqrt{2}\omega_0$.

For an infinite periodic structure, according to Bloch's theorem, the dispersion at any incident angle follows the relation [21]

$$\cos\beta_z(d_a + d_b) = \cos\left[k_z^{(A)} d_a\right]\cos\left[k_b^{(B)} d_b\right] - \frac{1}{2}\left(\frac{q_B}{q_A} + \frac{q_A}{q_B}\right)\sin\left[k_z^{(A)} d_a\right]\sin\left[k_b^{(B)} d_b\right], \tag{2}$$

where β_z is the z component of the Bloch wave vector, and $k_z^j = \omega/c\sqrt{\varepsilon_j}\sqrt{\mu_j}\sqrt{1-(\sin^2\theta/\varepsilon_j\mu_j)}$ is the z component of wave vector $\boldsymbol{k}^j$ in the jth layer (c is the velocity of light in a vacuum and θ is the incident angle). For a TE wave, $q_j = \sqrt{\varepsilon_j}/\sqrt{\mu_j}\sqrt{1-(\sin^2\theta/\varepsilon_j\mu_j)}$; for a TM wave, $q_j = \sqrt{\mu_j}/\sqrt{\varepsilon_j}\sqrt{1-(\sin^2\theta/\varepsilon_j\mu_j)}$. The condition of (2) having no real solution for β_z is $|\cos\beta_z(d_a+d_b)| > 1$, which corresponds to the band gap of 1DPC and is well-known as the Bragg condition. In the range of ENG frequencies, $k_z^{(A)}$ is an imaginary number due to the negative value of ε_a, thus $\cos\left[k_z^{(A)} d_a\right] = \cosh\left[|k_z^{(A)} d_a|\right]$, and $\sin\left[k_z^{(A)} d_a\right] = \sinh\left[|k_z^{(A)} d_a|\right]$. For normal incidence ($\theta = 0$) and a TE wave, $q_A = \sqrt{1 - 2\frac{\omega_0^2}{\omega^2}}$ and $q_B = 1$. If we define $x = \omega/\omega_0 = \omega/(\pi c/d)$, the right side of (2) can be regarded as a function of x with the form

$$\begin{aligned} F(x) = {} & \cosh\left[\left(\sqrt{2/x^2-1}\right)(x\omega_0/c)d_a\right]\cos[(x\omega_0/c)d_b] \\ & - 1/2\left(\frac{1}{\sqrt{1-2/x^2}} + \sqrt{1-2/x^2}\right) \\ & \times \sinh\left[\left(\sqrt{2/x^2-1}\right)(x\omega_0/c)d_a\right]\sin[(x\omega_0/c)d_b]\,. \end{aligned} \tag{3}$$

If $F(x)$ is real number and $|F(x)| \le 1$, the corresponding range of ω values become a transmission band, otherwise it becomes a gap. Given definite values of d_a and d_b, we can decide the band structure by means of (3).

Figure 1 shows the range of ω values that satisfy the condition of $F(x)$ being a real number, and $|F(x)| \le 1$ for different values of d_a and d_b , which are indicated by black areas. For the case of $d_b = 0.5d$, the black area takes $\omega/\omega_0 = 1$ as its center axis. Although the black area (transmission band) decreases with an increase of the value of d_a, the position of

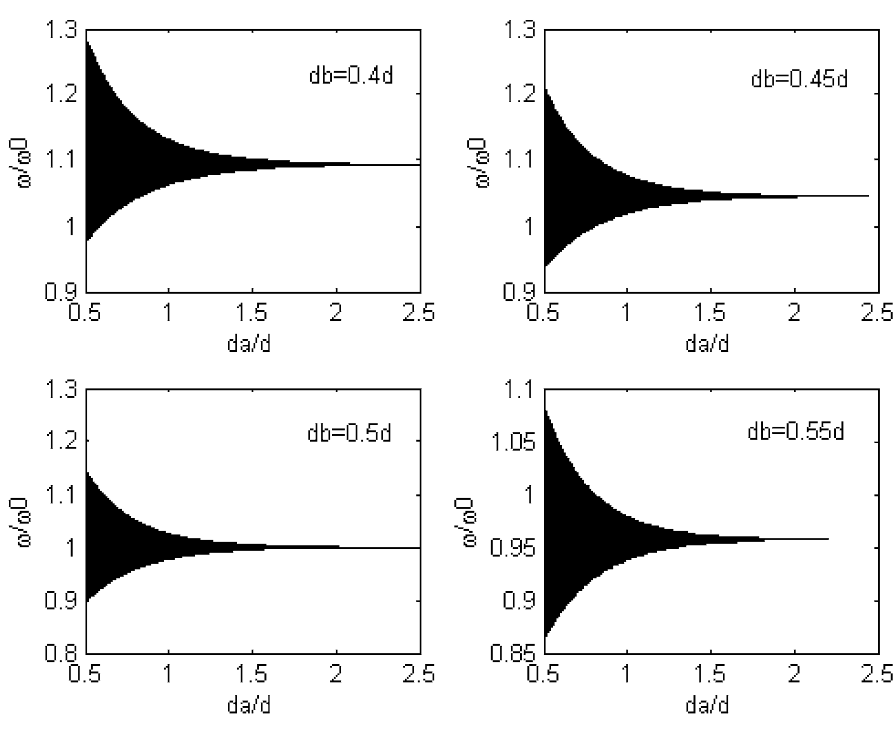

FIGURE 1 The range of ω values that satisfies the condition of $F(x)$ being a real number and $|F(x)| \le 1$ for different values of d_a and d_b

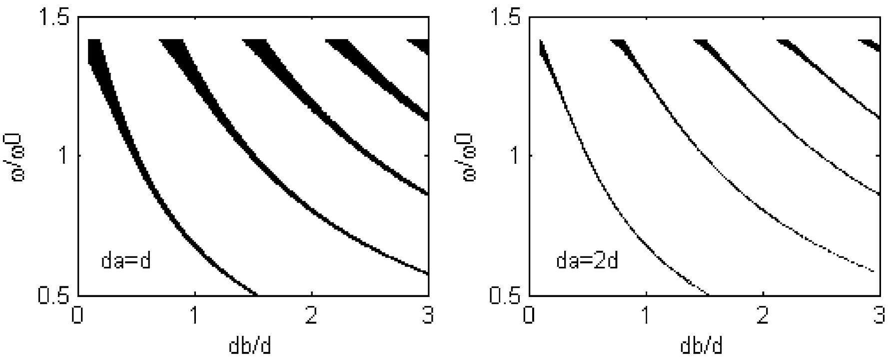

FIGURE 2 The range of transmission bands for different values of d_b for a fixed value of d_a

the center axis of the black area remains invariant. Moreover, after $d_a > 2d$, the transmission band almost becomes a line corresponding to $\omega/\omega_0 = 1$, which means that the structure can be used as a high-Q filter based on the current conditions. In contrast to conventional filters, there is no defect in it. The cases of $d_b = 0.4d$, $d_b = 0.45d$ and $d_b = 0.55d$ are similar to the case of $d_b = 0.5d$ except that the positions of the center of the transmission band are different. The higher the value of d_b, the less the value of the center axis of the transmission band. In addition, when d_a surpasses a critical value, the transmission band will fade away, which can be seen for the cases of $d_b = 0.45d$ and $d_b = 0.55d$. Figure 2 also plots the range of the transmission band for different values of d_b for a fixed value of d_a. Comparing the cases of $d_a = d$ and $d_a = 2d$, we still find that the values of d_a only influence the width of the transmission band while the values of d_b decide the position of the transmission band. For the high-Q filtering characters we can give a qualitative explaination. Due to the negative value of ε_a in the ENG frequency range, the field in A layers are evanescent waves. However, the evanescent waves in A layers can still be coupled into propagation waves in the B layers due to the small thickness of the A layers. In addition, due to the sudden change of impedance from the air to the ENG layer, there is a large reflection on the interface between two layers. All of the reflected light will interact each other. The intensity of the reflection light increases with an increase of the thickness of the ENG layer. Only when the phase difference between two adjacent-reflection beams is near to π, can the reflection light beams will cancel each other out by interaction, and light can then pass through this structure, otherwise light is prohibited. The phase difference between two adjacent-reflection beams is only dependent on the thickness of B the layers. For $d_b = 0.5d$, only if $\omega/\omega_0 = 1$, the phase difference between two adjacent-reflection beams is just π. If d_b deviates from $0.5d$, the value of ω/ω_0 must deviate from 1 simultaneously in order to keep the phase difference of π between two adjacent-reflection beams invariant. Therefore, the position of the center axis of the transmission band is sensitive to the value of d_b, while the width of the transmission band is only dependent on the value of d_a. In order to exactly observe the high-Q filtering function of the PC structure, Fig. 3 plots $F(\omega/\omega_0)$ with different structural parameters, from which we can exactly decide the range of transmission band indicated by gray areas. If we regard ω_0 as the filtering frequency, ac-

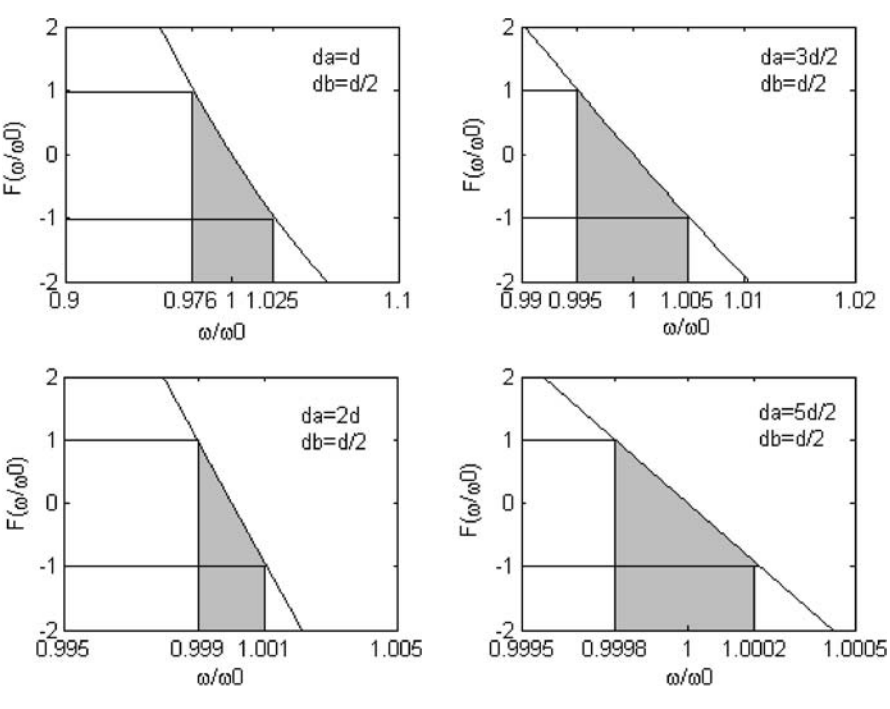

FIGURE 3 The values of $F(\omega/\omega_0)$ with different structural parameters, from which we can exactly decide the range of the transmission band as indicated by *gray areas*

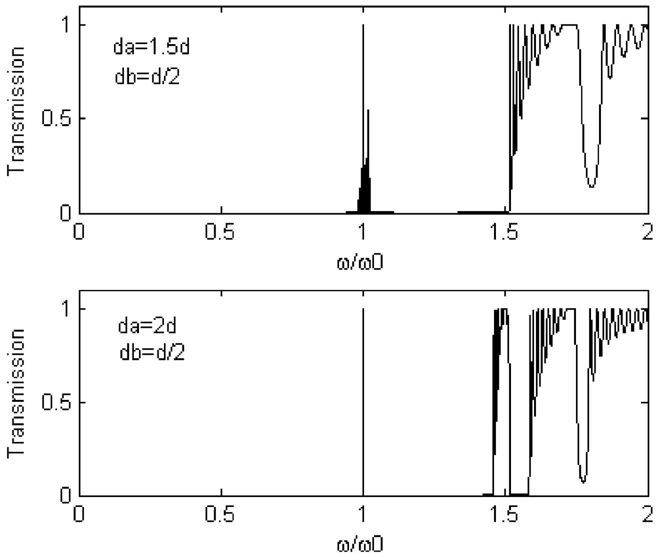

FIGURE 4 The transmission ratio $t(\omega)$ for different thicknesses of pair layers for the same $N = 20$

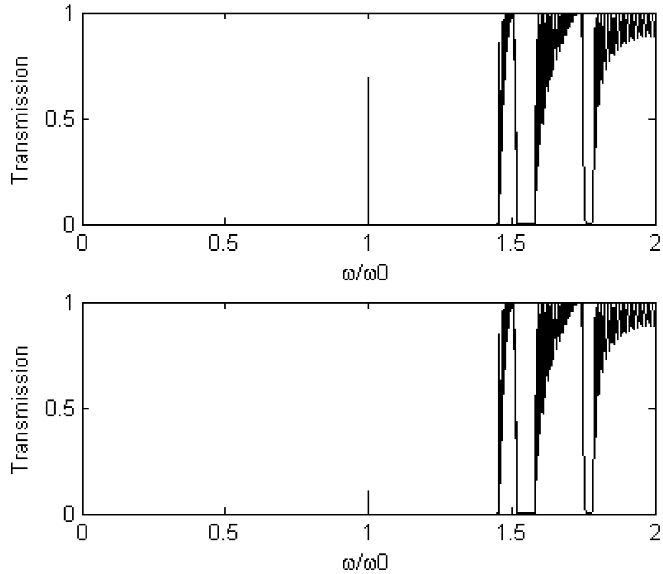

FIGURE 6 The transmission ratio $t(\omega)$ for two values of Γ and structural parameters of $d_a = 2d$, $d_b = 0.5d$ and $N = 20$

cording to the Q definition of $\frac{2\omega_0}{\Delta\omega}$, we obtain the Q values as follow: 40.8 for $d_a = d$ and $d_b = 0.5d$; 200 for $d_a = 1.5d$ and $d_b = 0.5d$; 1000 for $d_a = 2d$ and $d_b = 0.5d$; 5000 for $d_a = 2.5d$ and $d_b = 0.5d$.

As an example, we can study the transmission properties for the finite periodic structure of $(AB)^{20}$. Let a plane wave be injected from vacuum into the 1DPC at an incident angle θ, the transmission ratio $t(\omega)$ for both TE and TM waves and the field distribution inside the structure can be obtained from the transfer matrix method [19]. In the case of normal incidence, TE and TM waves have the same results. Figure 4 shows the transmission ratio $t(\omega)$ for different thicknesses of pair layers for the same $N = 20$. For the case of $d_a = 2d$ and $d_b = 0.5d$, there is only one narrow peak in the wide low-frequency gap. From (1), when $\omega < \sqrt{2}\omega_0$, $\varepsilon_a < 0$, thus most of the wide gap results in the PC structure with ENG materials. Figure 5 further plots the relative field distribution (in units of incident field intensity) inside the structure with two light frequencies of $\omega = \omega_0$ and $\omega = 0.99\omega_0$, and structural parameters of $d_a = 2d$ and $d_b = 0.5d$, respectively. For the case of $\omega = \omega_0$, the light passes through the structure without any loss, while for the case of $\omega = 0.99\omega_0$, the light intensity almost decreases to 10^{-30} times that of the incident field. Therefore the PC structure can filter a light with high-pure-frequency.

Up to here, all the above results are based on (1) which is just an idealistic model. For ENG materials, the absorption is unavoidable, no matter how tiny. In this case, the relative permittivity and permeability in the ENG materials should be rewritten as [22],

$$\varepsilon_a = 1 - \omega_{ep}^2 \big/ (\omega^2 - i\omega\Gamma)\,, \qquad \mu_a = 1\,, \tag{4}$$

where Γ is collision frequency or damping factor which contributes to the absorption and loss. Based on (4), we can calculate the transmission ratio $t(\omega)$ again by transfer matrix with structural parameters of $d_a = 2d$, $d_b = 0.5d$ and $N = 20$. The frequency dependence of the dielectric properties of ENG materials is included in the calculations. Figure 6 shows the results for two values of 18 850 Hz and 188 500 Hz for Γ. The two values of Γ are 10^{-7} and 10^{-6} times of the value of ω_0, respectively. From this we find that the absorption only decreases the transmission intensity, but has little effect on the Q value of this filter.

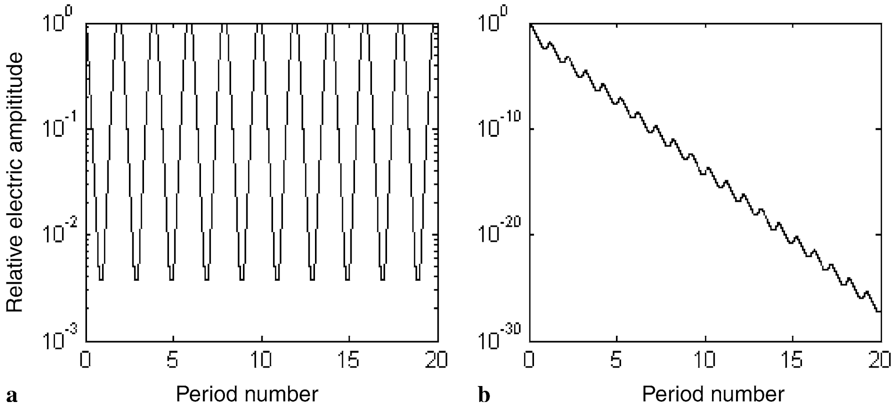

FIGURE 5 The relative field distribution inside the structure ($d_a = 2d$ and $d_b = 0.5d$) with two light frequencies of $\omega = \omega_0$ and $\omega = 0.99\omega_0$

3 Conclusion

In conclusion, we suggest a new type of PC structure with ENG materials. The PC structure can be used as a high-Q filter. Compared with conventional filters, it shows high-Q values and does not need a large ratio of the refractive indices of the two media or any defects. Furthermore, the Q values and the filtering frequency can be easily and precisely adjusted. The absorption of ENG materials can decrease the transmission intensity. Our study may be helpful in devising of high-Q filters.

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