Appl. Phys. B 83, 531–536 (2006)
DOI: 10.1007/s00340-006-2225-y

Applied Physics B
Lasers and Optics

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Vibration-insensitive reference cavity for an ultra-narrow-linewidth laser

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Received: 1 February 2006
Published online: 4 May 2006 •

ABSTRACT We report a novel mounting of the reference cavity used for stabilization of the clock laser in an optical frequency standard. The cavity axis is oriented horizontally and the cavity is supported in its horizontal symmetry plane on four support points. The positions of the points were optimized by finite-element analysis. A sensitivity to accelerations of 1.5 kHz/(m/s^2) in the vertical and 14 kHz/(m/s^2) in the horizontal direction was measured, which is a reduction in the vertical sensitivity by two orders of magnitude compared to the usual support from below.

PACS 42.60.Da; 46.70.Lk; 42.62.Fi

1 Introduction

Highly stable narrow-linewidth lasers [1–4] are essential as local oscillators in optical frequency standards [5–8], where they provide the necessary short-term stability between interrogations of the atomic transition. Furthermore, they are important in high-resolution spectroscopy [9], fundamental physics tests [10] as well as interferometric measurements like future space missions including the laser interferometer space antenna (LISA) [11]. Especially in the pulsed interrogation of narrow atomic transitions, the Dick effect [12–14] leads to degradation of the stability through the aliasing of high-frequency laser frequency noise. To achieve the ultimate stability close to the quantum projection limit [15], it is essential to reduce the spectral density of the laser's frequency noise in the low-frequency range.

To realize such a stable oscillator, a laser is usually locked to a narrow resonance of a high-finesse cavity [16]. Then, the stability of the locked laser is determined by the stability of the optical length of the reference cavity. The most severe fluctuations of the length are caused by low-frequency (below 100 Hz) seismic and acoustic accelerations resulting in quasi-static deformations of the cavity. These vibrations couple through the mounting to the cavity and lead to forces which deform the cavity and change its length.

In the widely used design where the reference cavity is supported from below this coupling typically leads to sensitivities to accelerations in the range of 100 kHz/ (m/s^2) [1]. Then, a sophisticated insulation of vibrations down to the range of $10^{-5}\,\mathrm{m/s^{-2}}$ is needed to achieve linewidths of one Hertz. In a different approach the cavity is designed in a such way that it already shows a low sensitivity to accelerations [17, 18]. In this paper we introduce a novel cavity mounting configuration [19], which allowed us to reduce the vibration sensitivity by two orders of magnitude in the vertical and one order of magnitude in the horizontal direction.

In this paper, we consider in Sect. 2 the basic general requirements of the stability of a cavity. Different ways of cavity mounting like asymmetric support from below and symmetric support are discussed and finite-element (FEM) simulations of these cases are shown. In Sect. 3 we present our experimental setup and the measurements showing the improvement of the vibration sensitivity. Finally, in Sect. 4 we show ways for future improvements.

2 Stability of the reference cavity

2.1 *General principles*

We first introduce a model which describes acceleration-induced deformations of the cavity. Depending on the frequency f of the vibrations compared to the mechanical eigenfrequencies f_i of the system (for 10-cm-sized cavities in the range above 10 kHz), two regimes can be distinguished. In the high-frequency regime $f \gtrsim f_i$ the external vibrations excite the eigenmodes and the response can be obtained by a decomposition of the individual modes and the response of the mode to the applied forces. In the low-frequency limit $f \ll f_i$ that is of primary interest here, the forces that are coupled to the solid accelerate the solid as a whole and lead to quasi-static deformation of the solid under the external force.

For the small forces acting here a linear dependence between the stress tensor σ_{ij} and the strain ε_{kl} can be assumed, given by Hooke's law [20]:

$$\sigma_{ij} = \sum_{kl} c_{ijkl} \times \varepsilon_{kl} \,, \tag{1}$$

with the strain tensor

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) , \tag{2}$$

where u_i is the displacement vector along the axis i ($i = x, y, z$). Thus, the changes of the dimension d of the cavity

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depend linearly on the forces:

$$\Delta d = \sum \alpha_k F_k \,, \tag{3}$$

with α_k the respective proportionality constant.

From symmetry considerations some initial hints for an optimized mount can be obtained. If the optical axis is invariant under a symmetry operation U of the body, i.e. $U(L) = L$, but if the forces reverse under the same symmetry operation, i.e. $U(\boldsymbol{F}_i) = -\boldsymbol{F}_i$, then from Hooke's law all deformations reverse sign under the symmetry operation $u(U(\boldsymbol{r})) = -u(\boldsymbol{r})$; therefore, $U(L) = -L$. But as L is invariant under the symmetry operation this implies that $\partial L = 0$. Thus, if the optical axis is included in the plane of symmetry and if all forces share the same symmetry with respect to that plane, acceleration-induced changes of the length are completely suppressed. For instance, for a horizontal mount of the cavity all support points have to lie in the symmetry plane or, if that is not possible, they have to act symmetrically to that plane. In the latter case, however, the requirement of symmetry in the forces is very difficult to obtain, as this requires that the elastic constants of symmetric supports are also exactly equal.

To illustrate the principles of an optimized mounting we first treat an example of a cuboid-shaped cavity of dimensions d_x, d_y, d_z. Only forces that accelerate the cavity are considered which cause quasi-static elastic deformations of the cavity. In the following we first assume that the cavity is held in a plane and therefore we do not need to take into account the bending which would appear if the cavity is held in discrete points. If the cavity is held asymmetrically from below (Fig. 1a) an acceleration a_y along the y direction leads to compression Δd_y in the same direction [20]:

$$\Delta d_y = \frac{F d_y}{EA} = \frac{\varrho d_y^2}{E} a_y \,, \tag{4}$$

where $F = \varrho d_x d_y d_z a_y$ is the accelerating force, ϱ the density, E Young's modulus, and A the cross section perpendicular to the acceleration. The compression along the direction of acceleration leads to an expansion in the perpendicular direction of

$$\Delta d_z = \nu \frac{\varrho d_z d_y}{E} a_y \,, \tag{5}$$

where ν denotes Poisson's ratio. In the case b when the cavity is fixed at the top the deformations are reversed: the cavity expands in the direction parallel to the acceleration and shortens along the perpendicular axis.

Since the cavity length change reverses its sign depending on the position of the supporting plane in cases a and b, there must be a point where the change amounts to zero. This situation is achieved when the cavity is supported in its symmetry plane (case c).

The length changes discussed here are due to vibration-induced accelerations, so they are independent of the orientation of the acceleration $\boldsymbol{a}$ with respect to the gravitational acceleration $\boldsymbol{g}$.

A support of an optical cavity in the symmetry plane was applied recently for a vertical orientation of the optical axis [18], which corresponds to Fig. 1c with the axis along the

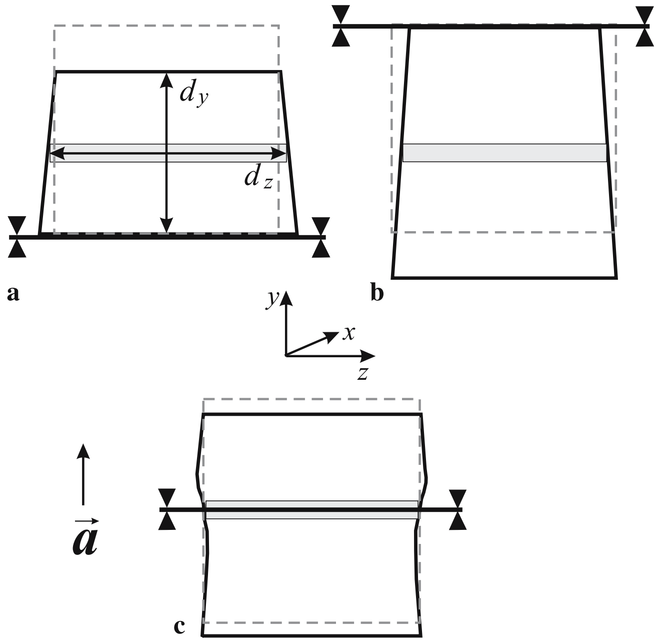

FIGURE 1 Influence of accelerations on a cuboid for different positions of the supporting plane. The original shape of the cube is indicated by *dashed lines*. The deformed shape is considered in the accelerated coordinate system

y direction. Due to the acceleration the upper half of the cavity is compressed while the lower half expands. The expansion of the upper half and the contraction of the lower half along the cavity axis are equal and compensate each other, which leads to a reduced vibration sensitivity of $10\,\text{kHz}/(\text{m/s}^2)$ [18]. In this case the two relative length variations which have to be compensated depend linearly on the length of the cavity (4), which makes it more difficult to apply this configuration for longer cavities that are less affected by thermal noise [21]. To reduce this effect we chose to put the optical axis in a horizontal symmetry plane. In this case the acceleration-induced length variation along the optical axis, i.e. in the z direction, is reduced by Poisson's ratio. In addition, the relative length change in the z direction does not depend on this length (5). Our novel cavity mounting where the cavity is supported symmetrically on four support points is shown in Fig. 2.

When the cavity is held in discrete points, in addition to the change of the distance between the centers of the mirrors shown in Fig. 1, accelerations also lead to a bending which

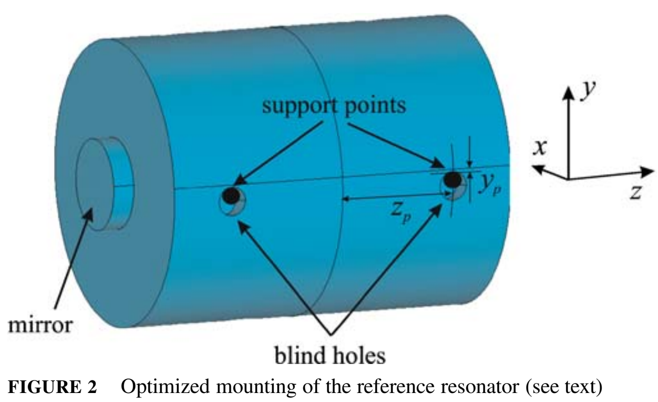

FIGURE 2 Optimized mounting of the reference resonator (see text)

causes a mirror tilt about the geometrical axis. As the laser frequency is determined by the length L_{opt} of the optical axis, which is characterized as being perpendicular to the mirror surfaces, the mirror tilt leads to a movement of the optical axis and a corresponding length change ΔL_{opt}. For small angles it amounts to [22]

$$\Delta L_{opt} = \frac{L\alpha^2(2+g_1+g_2)}{1-g_1g_2}, \tag{6}$$

where α is the tilt angle and g_1 and g_2 are the cavity g factors defined as $g_i = 1 - L/R_i$ with R_i the radius of curvature of the mirror. Since $\Delta L_{opt} \propto \alpha^2$ and the tilt angle is in the range of several nrad, this effect can be neglected.

If, however, the optical axis is not exactly centered in the middle of the mirrors but displaced by an amount Δr, then a tilt of the mirror with respect to its center will directly change the length of the optical axis by $\Delta r\alpha$, which is a more severe effect.

Thus, to minimize the sensitivity to the acceleration, the positions of the support points where the length change of the optical path of the cavity and the mirror tilt are minimal need to be determined. The optimization by finite-element analysis is described in Sect. 2.2.

2.2 *FEM simulations*

As in practice the spacer is not completely symmetric, the optimum positions of the support cannot be found analytically. Thus, in order to include all effects we performed finite-element analysis using a commercial program [23].

The cavity spacer and the optically contacted mirrors are made of ultra-low-expansion glass (ULE). The spacer has a length of 10 cm and a diameter of 8 cm. The Young's modulus of ULE is $E = 67.6 \times 10^9$ Pa; Poisson's ratio amounts to $\nu = 0.17$ [24]. We use a Cartesian coordinate system that has its origin in the center of the cylinder in such a way that the z axis is shown in the direction of the cylinder axis, the y axis in the vertical direction, and the x axis in the radial direction of the cylinder (Fig. 2). From the FEM calculations the displacement u_z is obtained, which denotes the displacement of a volume element in the direction of the z axis.

In our previous setup the reference cavity was supported at the Airy points (at distances $0.21 \times l$ from the ends) [25] from below by four small cylindrical elastomer pieces (Viton) under an angle $\alpha = 35°$ with respect to the vertical direction. We simulated this mounting configuration by setting the boundary condition such that no displacement normal to the surface at the support points was allowed. Since the problem is symmetric with respect to the xy plane and the yz plane, it is sufficient to simulate only a quarter of the cavity. The results of the simulation are shown in Fig. 3 in a cut at $x = 0$. In this case the change of the distance between the mirrors under an acceleration of $a_y = 10$ m/s amounts to 3.7×10^{-10} m leading to a calculated sensitivity to vertical acceleration of 170 kHz/(m/s^2), while the measured sensitivity is 120 kHz/(m/s^2) [4]. In this configuration the minimal change of length occurs in the lower part of the spacer (indicated by short arrows in Fig. 3).

In the next step we simulated the symmetrical mounting configuration shown in Fig. 2. The deformed shape and the

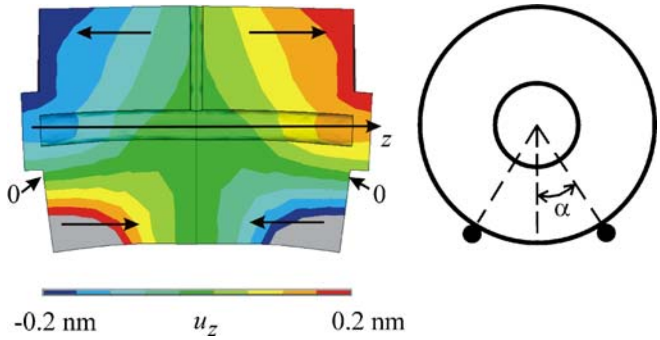

FIGURE 3 Calculated deformation u_z of a resonator supported from below ($\alpha = 35°$) under an acceleration of $a_y = 10\,\mathrm{m/s^2}$ in the y direction. The deformation of the cavity was magnified by a factor of 10^7 to make it visible. The *long arrows* show the direction of the displacement with respect to the z axis and the *short arrows* indicate the region where $u_z \approx 0$

displacement u_z in the z direction under an upward acceleration of $a_y = 10\,\mathrm{m/s^2}$ are shown in Fig. 4. Now the change of the distance is minimal between the mirrors, i.e. along the symmetry axis. The bending of the cavity in the middle amounts to 0.3 nm.

In order to find the positions of the supporting points where the length change between the mirrors and the mirror tilt both are minimized, the coordinates y_p and z_p of the support points are varied. We assumed a support on the top of a hole with diameter 8 mm and depth 10 mm (cf. Fig. 2). For the calculation, the support was described by setting the boundary condition $u_y = 0$ on the upper half of the inner surface of the blind hole. Due to the symmetry, only one quarter of the cavity was calculated using symmetric boundary conditions at the planes $x = 0$ and $z = 0$.

The results for different positions z_p and y_p are shown in Fig. 5. The vertical position y_p of the supporting points determines the mass distribution and therefore mostly affects the change in the length between the mirrors (Fig. 5a) with a minor influence on the tilt (Fig. 5b). The axial position z_p of the supporting points influences the acceleration-induced bending of the cavity and thus the tilt of the mirrors with respect to the geometrical axis (Fig. 5d). In this case the influence on the length is much smaller than in the variation of y_p. Thus, by optimization both the length change of the cavity and the mirror tilt can be zeroed at the same time at the optimal positions of the supporting points of $y_p = -0.9$ mm, $z_p = 35$ mm.

The influence of tolerances of the mechanical processing can be obtained from the same figure. For a 1-mm deviation of y_p from the optimum position, the acceleration sensitivity amounts to $k_y = 5.7\,\mathrm{kHz/(m/s^2)}$. The same deviation in z_p leads to a tilt of a few nrad. In combination with a deviation of the optical axis from the mirror center by 1 mm, this tilt would lead to a sensitivity of $k_y = 1.8\,\mathrm{kHz/(m/s^2)}$.

The sensitivity of the complete cavity to vibrations via the reaction forces on the four support points is given by (3). With the optimal design each coefficient α_k is equal to zero by virtue of the cavity symmetry. Thus, the sensitivity remains zero even when the forces are not balanced.

Up to now we discussed the sensitivity k_y to vertical vibrations, since in the laboratory vertical vibrations are more pronounced than horizontal ones. To also render the mounting insensitive to horizontal vibrations, the corresponding

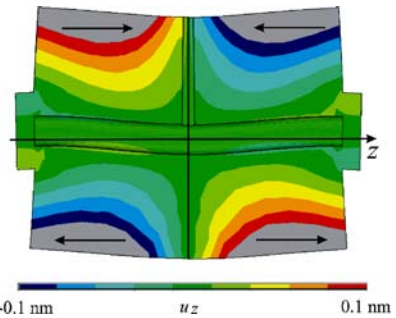

FIGURE 4 Calculated deformation of a resonator supported symmetrically in the horizontal plane. The displacement u_z is shown under an acceleration of $a_y = 10\,\mathrm{m/s^2}$ in the y direction. The bending of the cavity was magnified by a factor of 10^7 to make it easily visible. The *arrows* show the direction of the displacement with respect to the z axis

forces acting on the supporting points need to be perfectly balanced. In practice this is difficult to achieve, since the symmetry of the reaction forces depends on mechanical parameters of the mounting like its elasticity and spring constant. We performed simulations to find out how sensitive our mounting is to force imbalance. For this purpose we simulated an imbalance of 100%, i.e. the cavity is supported on one supporting point. For this case we obtain a sensitivity to vibrations along z of $k_z = 320\,\mathrm{kHz/(m/s^2)}$. This value was used to estimate the residual imbalance of the mounting from the measured sensitivity to the horizontal vibration (see Sect. 3).

The numerical uncertainty of the FEM simulations was estimated to be $1\,\mathrm{kHz/(m/s^2)}$ by comparing results for different meshings of the cavity and from the scatter of the results of Fig. 5, which amounts to $0.1\,\mathrm{kHz/(m/s^2)}$. The uncertainties of the values of Young's modulus and Poisson's ratio as well as their dependence on the temperature [24] lead to uncertainties in the simulations. Assuming an uncertainty of 2% in Young's modulus and Poisson's ratio, we calculated an uncertainty of $0.04\,\mathrm{kHz/(m/s^2)}$ and $0.08\,\mathrm{kHz/(m/s^2)}$, respectively, for the optimal position of the supporting points.

In addition, the mechanical tolerances of the cavity spacer should also be taken into account as the cavity is not a perfect body as it is assumed in the simulations. For instance, the parallelism of the end faces of the cylinder is specified by being less than 1.7 mrad. Further uncertainties up to 1 mm in the centering of the mirror can occur during the wringing (optical contacting) of the mirrors to the spacer. Thus, we assume a total uncertainty of the simulation of $2\,\mathrm{kHz/(m/s^2)}$.

3 Experimental results

Using the optimum positions from the simulations, the mounting of an existing reference cavity from an optical frequency standard was modified. The resonance linewidth of the cavity amounts to 19 kHz, corresponding to a finesse of 79 000. A diode laser at 657 nm can be locked to the reference cavity by the Pound–Drever–Hall technique [16] and a linewidth of 1 Hz was measured by comparison with a second independent system. Details of this setup are described in [4].

3.1 *Suspension of the cavity*

The cavity was prepared by drilling four blind holes with a diameter of 8 mm and a length of 10 mm at the positions determined by the simulations (Fig. 6). Since the cavity is supported on four points the support plane is overdetermined and thus the distribution of the forces would depend sensitively on mechanical tolerances. Thus, an elastic support on spring wires is used to compensate the unevenness and to balance the reaction forces. The spring constant of the wire of diameter 0.5 mm amounts to 4.3 N/mm. Viton cylinders of 3-mm diameter and 7-mm length were added to each spring wire to damp the oscillations of frequency 23 Hz. The spring wires were placed centrally in screws which were installed in the surrounding gold-plated copper cylinder that acts as

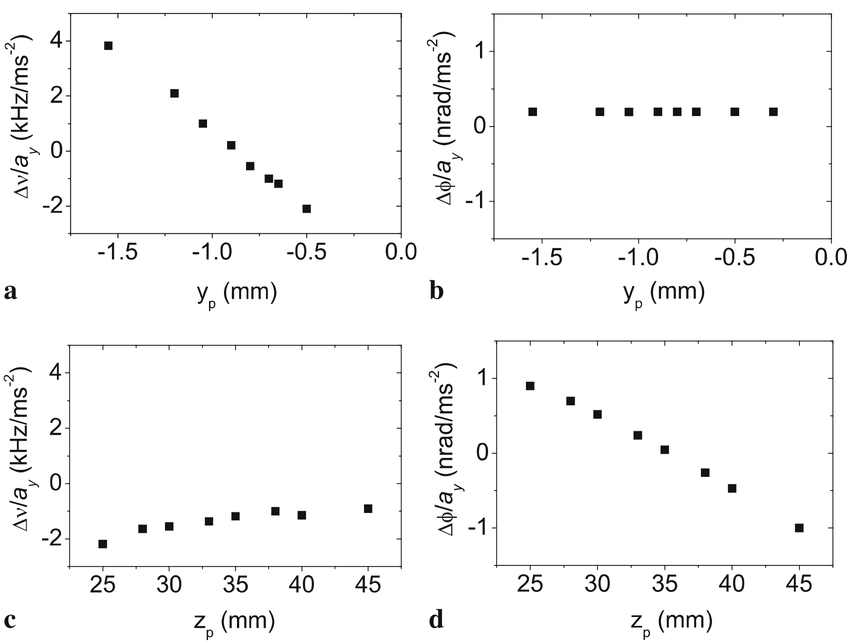

FIGURE 5 Calculated acceleration sensitivity due to the length change (**a**) and (**c**) and the mirror tilt (**b**) and (**d**) as a function of position of supporting points. During the variation of z_p (y_p) the other coordinate y_p (z_p) was held at $y_p = -0.9$ mm and $z_p = 35$ mm

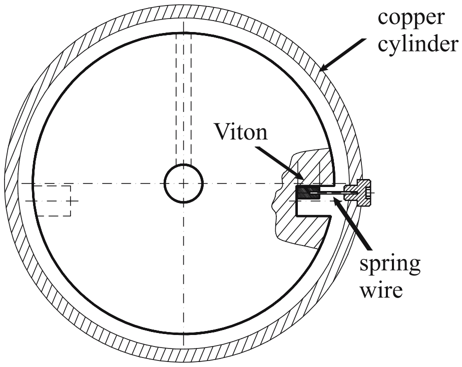

FIGURE 6 Novel mounting configuration

a heat shield. The setup is installed in a high-vacuum chamber ($p < 10^{-6}$ Pa) with an active temperature stabilization.

With this elastic mount the cavity can move with respect to the laser beam. This movement can be broaden the laser spectrum due to the Doppler effect. Under a horizontal acceleration of 10×10^{-5} m/s^2, which is a typical value in our laboratory, the cavity moves about 0.7 nm within one oscillation. As the amplitude of the oscillation is three orders of magnitude smaller than the laser wavelength of 657 nm, the movement will only introduce a phase modulation with modulation index $M \ll 1$. This will only introduce sidebands at the 10^{-6} level but no additional linewidth broadening due to the Doppler shift.

3.2 *Measurement of the vibration sensitivity*

To determine the acceleration-induced frequency change of the cavity, the beat between the laser locked to the cavity and a second independent laser system was analyzed. The cavity was placed on a platform hanging on four springs. The vibrations were measured with commercial seismometers: a_y with GS-1 (GeoSpace Technology, USA), a_z with L4-C (Marc Products, France), and a_x with an accelerometer (PCB, Piezotronics, USA). The movement of the platform was manually excited in three directions and the laser frequency and the acceleration of the platform in all three directions were recorded simultaneously. Even if the platform is oscillating mostly in one direction, there are still oscillations in the other two directions which are one order of magnitude weaker than the major one. To determine the transfer coefficient k_i for each direction, the results of the three measurements $l = 1, 2, 3$ had to be separated, taking into account the amplitude as well as the phase of the signals because the accelerations along the three axes need not be in phase depending on the excitation movement. The amplitude of the frequency variation in measurement l is then given by

$$f_l = \sum_{m=x,y,z} a_{lm} \mathrm{e}^{\mathrm{i}\varphi_{lm}} \, k_m \,, \tag{7}$$

where $a_{lm}\mathrm{e}^{\mathrm{i}\varphi_{lm}}$ are the complex acceleration components of measurement l in the three directions $m = x, y, z$ with amplitude a_{lm} and phase φ_{lm} relative to the observed frequency modulation. The amplitudes of the accelerations are summarized in Table 1. Combining three measurements along the x, y, and z axes and solving (7), we determined the acceleration sensitivity of $k_y = (1.5 \pm 0.3)\,\mathrm{kHz/(m/s^2)}$ in the vertical, $k_z = (14.5 \pm 1)\,\mathrm{kHz/(m/s^2)}$ in the axial, and $k_x = (11 \pm 0.8)\,\mathrm{kHz/(m/s^2)}$ in the radial x direction of the reference cavity. The coefficients k_x and k_z are in good agreement with the ones determined by tilting the resonator by a small angle β, which is equivalent to an acceleration of $a_x = g\beta$, where $g \approx 9.81\,\mathrm{m/s^2}$ denotes the gravitational acceleration.

The results of the measurements are shown in Fig. 7 for the vertical direction. On the left-hand side the acceleration (upper graph) and the corresponding beat frequency (lower graph) are presented in the case of the asymmetrical support where a high vibration sensitivity of $k_y = 120\,\mathrm{kHz/(m/s^2)}$ was determined. The analogous results for our new mounting on the right-hand side of Fig. 7 show an improvement of two orders of magnitude for the vertical direction. The measured acceleration sensitivity of $k_y = 1.5\,\mathrm{kHz/(m/s^2)}$ in the vertical direction is in agreement with the simulation $(0 \pm 2)\,\mathrm{kHz/(m/s^2)}$.

Since the vibration sensitivities for both horizontal directions (axial z and radial x) are an order of magnitude higher than for the vertical direction, we have analyzed our mounting in respect thereof. The symmetry of the forces in horizontal directions depends on the effective spring constant of the support points. It depends on the support area of the Viton cylinder as well as the accurate position and the spring constant of the spring wires. Using the results of our simulations in Sect. 2.2, we estimated that the measured vibration sensitivity of $k_z = 14.5\,\mathrm{kHz/(m/s^2)}$ is caused by an imbalance of 6% of the acting forces.

3.3 *Fully compensated cavities*

For a cavity mounting that is insensitive to acceleration in all directions, extensive vibration isolation suppression would not be necessary any more. In the vertical direction the achieved suppression of vibration-induced length fluctuations comes close to the limit set by mechanical tolerances and numerical errors of the calculation. For further improvement towards $k_y = 0$, the mass distribution can be adjusted by adding small masses to the top or bottom of the cavity spacer, depending on the measured sensitivity.

To further reduce the influence of horizontal vibrations, the symmetry of the reaction forces acting in these directions needs to be adjusted. To symmetrize the effective spring constants by modifying the springs seems to be quite challenging. A much simpler way would be to use nonlinear springs, i.e. springs where the spring constant depends on the working point. This can be achieved by using conical springs or elastomers where the area changes with load. If the nonlinearity of opposite supports has opposite sign, then the effect-

Measurement l	a_{lx} $(\mathrm{m/s^2})$	a_{ly} $(\mathrm{m/s^2})$	a_{lz} $(\mathrm{m/s^2})$	f_l (kHz)
1	0.08	0.004	0.015	0.68
2	0.018	0.3	0.025	0.2
3	0.008	0.001	0.1	1.32

TABLE 1 Amplitudes of accelerations and beat frequencies for three measurements

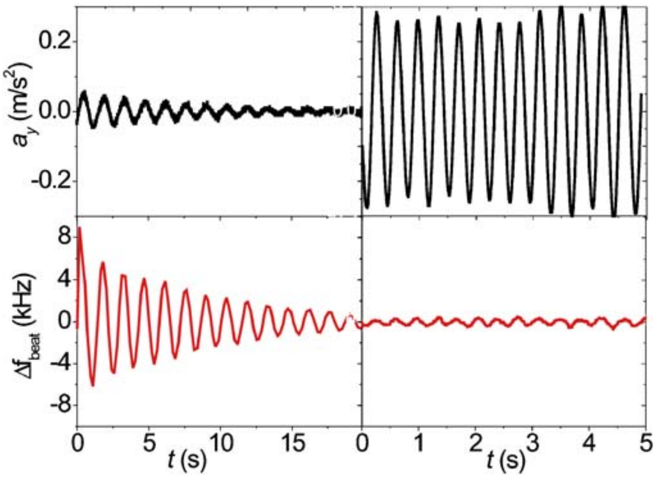

FIGURE 7 Comparison of the sensitivity of the laser frequency Δf_{beat} to vertical accelerations a_y for the asymmetrical (*left*) and the symmetrical (*right*) mounting configurations

ive spring constant can be symmetrized by varying the static load by tilting the setup, which can be performed even during operation.

4 Conclusions and outlook

We have demonstrated a considerable reduction of the vibration sensitivity of an optical reference cavity by using a symmetrical cavity support. We have measured a vibration sensitivity of $k_y = 1.5\,\text{kHz}/(\text{m/s}^2)$ in the vertical and $k_z = 14\,\text{kHz}(\text{m/s}^2)$ and $k_x = 11\,\text{kHz}/(\text{m/s}^2)$ in the two horizontal directions. With the reduced sensitivity and with a passive vibration-isolation system, the vibrations present in our laboratory will then only contribute 30 mHz to the linewidth at an optical frequency of 456 THz and are not the most dominant contributions any more. This mounting with a horizontal optical axis is more easily applicable to longer cavities than the configuration with a vertical optical axis, where the relative change of distance scales linearly with the length. This can be an advantage in terms of reducing the thermal noise [21], because the influence of the thermal noise of the cavity spacer on the spectral power density of frequency fluctuations $S_\nu(f)$ scales with the cavity length L as $S_\nu(f) \propto 1/L$ while keeping the cross section the same. The contributions from the mirror substrates and coatings even scale as $S_\nu(f) \propto 1/L^2$. Thus, with this novel mount a laser linewidth way below one Hertz seems reachable.

ACKNOWLEDGEMENTS We gratefully acknowledge the support by the Deutsche Forschungsgemeinschaft under SFB 407. We also thank A. Dettmer and V. Jäger from PTB's ultraprecision engineering division for the careful machining of the ULE cavity.

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