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Analysis of a loss-compensated recirculating delayed self-heterodyne interferometer for laser linewidth measurement

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ABSTRACT A loss-compensated recirculating delayed self-heterodyne interferometer (LC-RDSHI) for laser linewidth measurement is theoretically analyzed. An analytical result for the output spectrum of the LC-RDSHI is obtained. It is found that the spectrum from a LC-RDSHI is equivalent to a spectrum from a conventional delayed self-heterodyne interferometer with equivalent time delay and frequency shift, but modified by a periodical function, which could significantly influence the laser linewidth measurement. The parameters of a LC-RDSHI must be optimized to permit an accurate and direct measurement of laser linewidth from the output spectrum.

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1 Introduction

Since it was first proposed by Okoshi et al. [1] in 1980, delayed self-heterodyne interferometer (DSHI) has been extensively used for narrow laser linewidth measurement due to its high frequency resolution and simplicity. In a conventional DSHI, the laser beam is split to travel two different path-lengths so that one is delayed by a time τ_d compared to the other. The linewidth measurement is performed by recording the rf beating signal from the laser beam and its time delayed version. However, the requirement on the time delay has limited the usefulness of this technique. Theoretical simulations indicate that the time delay τ_d should be several times longer than the laser coherent time τ_c to permit a direct measurement from the rf beating spectrum [2, 3]. Subcoherence delay length maybe used with a more complicated signal processing method of fitting the recorded rf spectrum to the theoretical results [4]. However, the measurement accuracy was rather limited.

This problem was alleviated by a recirculating DSHI first proposed by Tsuchida [5] in which a fiber loop, composed by a coupler, a fiber delay line, and an accousto-optic modulator (AOM), was used as the time delay line. Each pass of the recirculator delayed the optical beam by a time τ_0 and introduced a frequency shift Ω by the AOM. Thus multiple time delays were obtained by recirculating the light in the loop and each of them could be determined by counting the frequency shift. Due to the fiber loop loss, Tsuchida could only measure three orders of the rf beat notes. Park et al. [6, 7] proposed a loss compensated recirculating DSHI (LC-RDSHI), in which an erbium-doped fiber amplifier (EDFA) was included in the loop to partially compensate the loss. Park was able to observe as many as 30 orders of beat-notes, which were used to measure the linewidth of an erbium-doped fiber ring laser. Using the same method, Fatemi et al. [8], measured the rf spectra up to 20 orders to determine the linewidth of a frequency comb. In all these measurements, it has been assumed that the rf spectral line shape for the kth order beat-note is equivalent to a conventional DSHI with the same time delay ($k\tau_0$) and frequency shift ($k\Omega$). However, this fundamental assumption has not been proved by strict theoretical analysis. In fact, in this paper we show that this assumption and the direct linewidth measurement are valid only when the system parameters are properly chosen to remove the effect of the multi-interferences from the recirculations.

The remainder of the paper is organized as follows. In Sect. 2, the mathematical description of laser phase noise and the model of a conventional DSHI are reviewed. In Sect. 3, the theoretical analysis of a LC-RDSHI is presented and an analytical result is obtained to describe the effect of muliple recirculations on the rf spectrum. In Sect. 4, based on the analytical result, the rf spectrum generated under different system parameters of the LC-RDSHI is discussed. Finally conclusions are given in Sect. 5.

2 Model of laser phase noise and a conventional DSHI

Theoretical analysis of conventional DSHIs together with mathematical descriptions of the laser phase noise has been considered in [2, 3]. The analysis is simplified and more explicitly presented here by using the complex amplitude representation of a quasi-monochromatic wave. We start the analysis from modeling the laser field as a sinusoidal wave with random fluctuations of both the amplitude and

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phase,

$$E(t) = [A + \Delta A(t)] \exp[j\phi(t)]. \quad (1)$$

where A is a stationary value for the amplitude, and $\Delta A(t)$ and $\phi(t)$ are random processes corresponding to the amplitude modulation noise and the phase noise of the laser, respectively. We assume $\Delta A(t) \ll A$, which is true for most practical cases. In (1), the optical center angular frequency ω_0 has been ignored thus $E(t)$ represents the complex amplitude of the laser field. From the Wiener–Khinchin theorem [9], the power spectral density can be obtained by the Fourier transform of the field autocorrelation function which is defined by

$$R_e(\tau) = \langle E(t)E^*(t-\tau)\rangle, \quad (2)$$

where $*$ denotes the complex conjugate, the angle brackets denote an ensemble average. By substituting (1) into (2), one arrives at the following result [10],

$$R_e(\tau) = A^2\langle \exp[j\Delta\phi(t,\tau)]\rangle + A\langle[\Delta A(t) + \Delta A(t-\tau)] \exp[j\Delta\phi(t,\tau)]\rangle + O((\Delta A)^2) \quad (3)$$

where

$$\Delta\phi(t,\tau) = \phi(t) - \phi(t-\tau) \quad (4)$$

is the phase jitter of the laser. It is clear that the last term in the right side of (3) is negligible and the strongest contribution to the laser spectrum comes from the first term consisting of pure phase fluctuation. For simplicity, we ignore the amplitude fluctuation in the discuss by assuming $\Delta A(t) = 0$, and (3) reduces to

$$R_e(\tau) = A^2\langle \exp[j\Delta\phi(t,\tau)]\rangle \quad (5)$$

The lasers considered here are semiconductor lasers that operate far above threshold. The corresponding frequency noise of such lasers has a constant spectral density (white noise) [11–13]. Under these conditions, the phase jitter $\Delta\phi(t,\tau)$ is a zero-mean, stationary and Gaussian random process with variance increasing linearly with the time delay

$$\langle\Delta\phi^2(t,\tau)\rangle = \langle\Delta\phi^2(\tau)\rangle = \Delta\omega|\tau| \quad (6)$$

where $\Delta\omega$ is the full width at half maxima (FWHM) of the Lorentzian spectral line shape. Using the well-known condition [14]

$$\langle \exp[\pm j\Delta\phi(t,\tau)]\rangle = \exp[-\langle\Delta\phi^2(\tau)\rangle/2] \quad (7)$$

together with (6), the Fourier transform of (2) leads to the Lorentzian line shape of the laser

$$S_e = \mathbb{F}[R_e(\tau)] \propto 2\Delta\omega/\left[(\Delta\omega/2)^2 + \omega_d^2\right], \quad (8)$$

where ω_d is the frequency deviated from the optical center frequency and $\mathbb{F}$ denotes the Fourier transform, which is defined by

$$H(\omega) = \mathbb{F}[h(t)] = \int_{-\infty}^{+\infty} h(t)\exp(-i\omega t)\,dt. \quad (9)$$

In a conventional DSHI as shown in Fig. 1, the detected optical field is the sum of the laser beam and a time delayed and frequency shifted version of itself

$$E_T(t) = E(t) + E(t-\tau_d)\exp(j\Omega t), \quad (10)$$

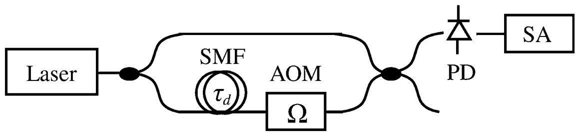

FIGURE 1 Schematic of a conventional delayed self-heterodyne interferometer. SMF, single mode fiber; SA, spectrum analyzer; PD, photodetector

where τ_d and Ω are the time delay and angular frequency shift respectively. For simplicity, we have assumed the two beams have equal amplitude. The photocurrent $I(t)$ is proportional to the optical intensity because of the square law of the photodetector,

$$I(t) \propto E_T(t)E_T^*(t) \propto 2 + \exp\{-j[\phi(t) - \phi(t-\tau_d)]\}\exp(j\Omega t) + \text{c.c.} \quad (11)$$

where c.c. denotes the complex conjugate of the preceding term. In (11) the constant coefficient related to the laser field intensity and the photodetector sensitive [3] has been ignored. The photocurrent contains a dc and a quasi-monochromatic signal centered at angular frequency Ω with a constant amplitude and random phase fluctuations equivalent to the laser field phase jitter. Here we are only interested in and consider the quasi-monochromatic term that contains the information of the laser phase noise. Similar to the case of the laser field, by ignoring the center angular frequency Ω, we can determine the complex amplitude of this term, which is given by

$$I_\Omega(t) = \exp\{-j[\phi(t) - \phi(t-\tau_d)]\}. \quad (12)$$

The autocorrelation function of $I_\Omega(t)$ is thereby calculated by

$$R_\Omega(\tau) = \langle I_\Omega(t)I_\Omega^*(t-\tau)\rangle = \langle \exp\{-j[\phi(t) - \phi(t-\tau_d) - \phi(t-\tau) + \phi(t-\tau_d-\tau)]\}\rangle. \quad (13)$$

Using (7) in (13) and after some straightforward algebra, (13) we can express this as a function of phase jitter variance, which is given by [3]

$$R_\Omega(\tau) = \exp[B(\tau,\tau_d)], \quad (14)$$

where

$$B(\tau,\tau_d) = -\langle\Delta\phi^2(\tau_d)\rangle - \langle\Delta\phi^2(\tau)\rangle + \frac{1}{2}\langle\Delta\phi^2(\tau+\tau_d)\rangle + \frac{1}{2}\langle\Delta\phi^2(\tau-\tau_d)\rangle. \quad (15)$$

The spectrum of the corresponding photocurrent component is thereby obtained by the Fourier transform of (14)

$$S_0(\omega,\tau_d,\Omega) = \mathbb{F}[R_\Omega(\tau)] = \mathbb{F}\{\exp[B(\tau,\tau_d)]\}, \quad (16)$$

In deriving (16), the carrier angle frequency Ω has been implied, thus ω is defined as the angular frequency deviated from the carrier angular frequency Ω. For the Lorentzian spectral line shape of the laser, (6) is satisfied and with the help of (15), (16) leads to

$$S_0(\omega) = \exp(-\Delta\omega\tau_d)\delta(\omega) + \frac{2\Delta\omega}{\Delta\omega^2 + \omega^2} \times \left\{1 - \exp(-\Delta\omega\tau_d)\left[\cos(\omega\tau_d) + \frac{\Delta\omega}{\omega}\sin(\omega\tau_d)\right]\right\}. \quad (17)$$

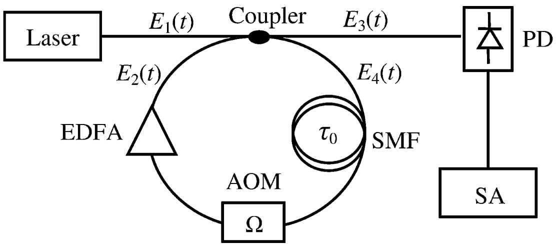

FIGURE 2 Schematic of a loss-compensated recirculating delayed self-heterodyne interferometer

In the limit of large delay times, the spectrum becomes exactly Lorentzian with width equal to twice of the laser optical spectral width. A time delay of at least 5 times longer than the laser coherence time was suggested for a direct laser linewidth measurement from the spectrum [2, 3]. For relatively small delay times that are comparable with or shorter than the coherence time of the laser, the quasi-Lorentzian part of the spectrum is broadened and shows sidelobe structures and the spectral power is shifted to the delta function at the modulation frequency.

3 Model of a LC-RDSHI

The schematic of a LC-RDSHI is shown in Fig. 2 in which a loss-compensated fiber loop is used as a fiber delay line. The fiber delay line contains a fiber coupler to couple the light into the loop, a span of single-mode fiber to provide the time delay to the field, an AOM to introduce a frequency shift to the field and an EDFA to partially compensate the loss of the fiber span and other components involved in the loop. A practical fiber loop may also include a polarization controller, which has been ignored here, by assuming the field polarization is maintained during its propagation in the system. We start the analysis of such a LC-RDSHI with the equation that describes the relationship of the fields between the input and output ports of the coupler, which is written as [15]

$$\begin{bmatrix} E_3(t) \\ E_4(t) \end{bmatrix} = \begin{bmatrix} \sqrt{\alpha} & j\sqrt{1-\alpha} \\ j\sqrt{1-\alpha} & \sqrt{\alpha} \end{bmatrix} \begin{bmatrix} E_1(t) \\ E_2(t) \end{bmatrix}, \tag{18}$$

where $E_1(t)$, $E_2(t)$, $E_3(t)$ and $E_4(t)$ are the optical fields at input ports 1 and 2 and output ports 3 and 4 of the coupler as shown in Fig. 2 and α is a real number between 0 and 1, i.e., a fraction α of the input power at port 1 appears at output port 3 of the coupler. The laser output is directly connected to input port 1 of the coupler, therefore

$$E_1(t) = E(t) = A\exp[j\phi(t)]. \tag{19}$$

Part of the field output from the laser is coupled into the fiber loop by the coupler and recirculates inside the loop. Each pass of the loop introduces a time delay τ_0 and an angular frequency shift Ω to the laser field. Thus the electrical field $E_2(t)$ at port 2 of the coupler contains components of discrete angular frequency shift of $k\Omega$ ($k = 1, 2, \dots$). We denote the component with frequency shift $k\Omega$ by $E_{2,k}(t)$. Thereby the field $E_{2,1}(t)$ has an angular frequency shift of Ω and must be from the laser output field after one pass of the loop and is given by

$$\begin{aligned} E_{2,1}(t) &= j(1-\alpha)^{1/2}E_1(t-\tau_0)\exp(j\Omega t)\beta^{1/2} \\ &= j(1-\alpha)^{1/2}A\exp[j\phi(t-\tau_0)]\exp(j\Omega t)\beta^{1/2}, \end{aligned} \tag{20}$$

where β is the overall effective gain of the components from port 4 to port 2 of the coupler in the fiber loop. Here we assume β is real. (18) has been used in deriving (20). Similarly, the component $E_{2,n}(t)$ ($n = 1, 2, \dots$) has an angular frequency shift of $n\Omega$ and must be from the component $E_{2,n-1}(t)$ after one pass of the loop and is thus expressed as

$$E_{2,n}(t) = E_{2,n-1}(t-\tau_0)\exp(j\Omega t)\alpha^{1/2}\beta^{1/2}. \tag{21}$$

Using the recursion (21) together with its initial condition of (20), we can determine a general expression for $E_{2,n}(t)$, which is given by

$$\begin{aligned} E_{2,n}(t) &= j[(1-\alpha)/\alpha]^{1/2}A\exp[j\phi(t-n\tau_0)] \\ &\quad\times \exp[jn(n-1)\Omega\tau_0/2]\exp(jn\Omega t)(\alpha\beta)^{n/2}. \end{aligned} \tag{22}$$

The total field at port 2 of the coupler is the sum of the components of all possible frequencies and is given by

$$E_2(t) = \sum_{n=1}^{\infty} E_{2,n}(t). \tag{23}$$

Using (18), the total laser field at the photodetector is given by

$$E_3(t) = \alpha^{1/2}E_1(t) + j(1-\alpha)^{1/2}E_2(t). \tag{24}$$

Substituting (22) into (23), and then substituting (19) and (23) into (24), we find

$$E_3(t) \propto \sum_{n=0}^{\infty} c_n \exp[j\phi(t-n\tau_0)]\exp(jn\Omega t), \tag{25}$$

where the coefficient c_n is defined by

$$\begin{aligned} c_0 &= \alpha/(\alpha-1) \\ c_n &= \gamma^{n/2}\exp[jn(n-1)\Omega\tau_0/2], \quad n = 1, 2, \dots . \\ \gamma &= \alpha\beta \end{aligned} \tag{26}$$

The coefficient γ is the effective overall gain of the fiber loop including the coupling loss from port 2 to port 4 of the coupler. Obviously, γ is real and must have a value between 0 and 1 to permit a stable light propagation in the loop. The photocurrent from the photo detector is proportional to the optical intensity at output port 3 of the coupler and is given by

$$\begin{aligned} I(t) &\propto E_3(t)E_3^*(t) \\ &= \left\{\sum_{k=0}^{\infty} c_k \exp[j\phi(t-k\tau_0)]\exp(jk\Omega t)\right\} \\ &\quad\times \left\{\sum_{l=0}^{\infty} c_l \exp[j\phi(t-l\tau_0)]\exp(jl\Omega t)\right\}^* \\ &= \sum_{l=0}^{\infty} c_l^* c_l + \sum_{m=1}^{\infty}\left\{\sum_{l=0}^{\infty} c_l{}^* c_{l+m}\exp\{-j[\phi(t-l\tau_0)\right. \\ &\quad \left. -\phi(t-(l+m)\tau_0)]\}\exp(jm\Omega t) + \text{c.c.}\right\}. \end{aligned} \tag{27}$$

Equation (27) indicates that the photocurrent contains a series of beat-notes at discrete angular frequencies. Without loss of generality, the photocurrent component with center angular frequency $m\Omega$ (the mth order beat-note) for $m \geq 1$ is considered, and its complex amplitude is given by

$$I_m(t)=\sum_{l=0}^{\infty} c_l^* c_{l+m} \exp\{-j\,[\phi\,(t-l\tau_0)-\phi\,(t-(l+m)\tau_0)]\}. \tag{28}$$

The autocorrelation function of $I_m(t)$ is thus given by

$$\begin{aligned}R_m(\tau) &= \langle I_m(t) I_m^*(t-\tau)\rangle \\ &= \sum_{l=0}^{\infty}\sum_{k=0}^{\infty} c_l^* c_{l+m} c_k c_{k+m}^* \\ &\quad\times \langle \exp\{-j[\phi(t-l\tau_0)-\phi(t-(l+m)\tau_0) \\ &\qquad -\phi(t-k\tau_0-\tau)+\phi(t-(k+m)\tau_0-\tau)]\}\rangle.\end{aligned} \tag{29}$$

Following the same approach in deriving (14), we can reduce (29) to a function of phase jitter of the laser field and write it as

$$\begin{aligned}R_m(\tau) &= \sum_{l=0}^{\infty}\sum_{k=0}^{\infty} c_l^* c_{l+m} c_k c_{k+m}^* \exp\Big[-\langle\Delta\phi^2(m\tau_0)\rangle \\ &\quad -\langle\Delta\phi^2(\tau+(l-k)\tau_0)\rangle \\ &\quad +\frac{1}{2}\langle\Delta\phi^2(\tau+(l-k+m)\tau_0)\rangle \\ &\quad +\frac{1}{2}\langle\Delta\phi^2(\tau-(l-k-m)\tau_0)\rangle\Big] \\ &= \sum_{l=0}^{\infty}\sum_{k=0}^{\infty} c_l^* c_{l+m} c_k c_{k+m}^* \exp[B(\tau+(l-k)\tau_0, m\tau_0)].\end{aligned} \tag{30}$$

The power spectrum is thereby calculated by the Fourier transform of (30) and is expressed as

$$\begin{aligned}S_m(\omega) &= \mathbb{F}[R_m(\tau)] \\ &= \sum_{l=0}^{\infty}\sum_{k=0}^{\infty} c_l^* c_{l+m} c_k c_{k+m}^* \exp[j(l-k)\omega\tau_0] \\ &\quad\times \mathbb{F}\{\exp[B(\tau, m\tau_0)]\} \\ &= S_0(\omega, m\tau_0, m\Omega)\left|\sum_{l=0}^{\infty} c_l^* c_{l+m} \exp(jl\omega\tau_0)\right|^2.\end{aligned} \tag{31}$$

Here we have used the relation

$$\mathbb{F}(\tau+\tau_0) = \exp(j\omega\tau_0)\mathbb{F}(\tau). \tag{32}$$

Substituting (26) into (31) and after some straightforward algebra, we find the line shape of the mth order beat-note from the LC-RDSHI

$$S_m(\omega) = \frac{\gamma^m}{(1-\alpha)^2} P(\omega) S_0(\omega, m\tau_0, m\Omega), \tag{33}$$

where

$$P(\omega) = \alpha + \frac{(1-\alpha)(\gamma^2-\alpha)}{1+\gamma^2-2\gamma\cos[(\omega+m\Omega)\tau_0]} \tag{34}$$

is a periodical function of angular frequency ω, resulting from the multi-interferences of the laser field after multi-passing through the loop. It is worth recalling that $S_0(\omega, m\tau_0, m\Omega)$ is the rf spectrum from a conventional DSHI with time delay $m\tau_0$ and angular frequency shift $m\Omega$. Noting that in (33), the carrier frequency has been implied; thereby ω is the angular frequency deviated from center angular frequency $m\Omega$. (33) is the main result of this paper and shows that the spectral component at frequency $m\Omega$ of a LC-RDSHI is a modified version of the spectrum from a conventional DSHI with equivalent time delay and frequency shift, which indicates that a direct measurement of the laser linewidth from the spectrum may not be appropriate. The effect of the modification function $P(\omega)$ on the spectrum from the LC-RDSHI is discussed in the next section.

It is worth noting that in the analysis we have ignored the amplified spontaneous emission (ASE) noise from the EDFA. This is reasonable because the EDFA only introduces an additive noise to the electrical field, and the beating spectrum corresponding to the ASE noise is a delta function at the modulation frequency [16, 17]. The power level of the delta function depends on the power of the ASE within the detection bandwidth, which is in general very weak and below the ground noise level of the detecting equipment.

4 Discussions on the performance of a LC-RDSHI

The modification function $P(\omega)$ given by (34) is periodical with a period determined by the time delay τ_0 introduced by one pass of the fiber loop. Figure 3 shows the spectrum of order $m = 10$ from a LC-RDSHI for different values of τ_0. The frequency f is defined by $f = \omega/2\pi$, which has been normalized by the laser linewidth $\Delta\nu$, where $\Delta\nu = \Delta\omega/2\pi$. In plotting Fig. 3, the delta function at the modulation frequency in (17) has been ignored because the power of the delta function is negligible when $m\Delta\nu\tau_0 \gg 1$, which is the case in Fig. 3. Illustrative values of parameters $\alpha = 0.5$ and $\gamma = 0.9$ are used. The corresponding spectrum from a conventional DSHI is also plotted (dashed line). It is evident that the spectrum from a LC-RDSHI exhibits an oscillating structure with period dependent on time delay τ_0. When $\tau_0$is comparable with or smaller than the laser coherence time, such as the cases shown in Fig. 3a and b, it is difficult to recover the information of the laser linewidth from the spectrum because of the oscillations. For longer τ_0 such as $\tau_0 = 5/\Delta\nu$ as shown in Fig. 3c, the linewidth could be possibly obtained by measuring the envelope of spectrum, which is equivalent to the output from a conventional DSHI. However this is practically less meaningful because a conventional DSHI would be adequate in such a case.

In order to permit a direct measurement of the linewidth, the modification function must be invariant of angular frequency ω. From (34) this is the case when the parameters α and γ satisfy

$$\gamma = \gamma_0 = \alpha^{1/2} \tag{35}$$

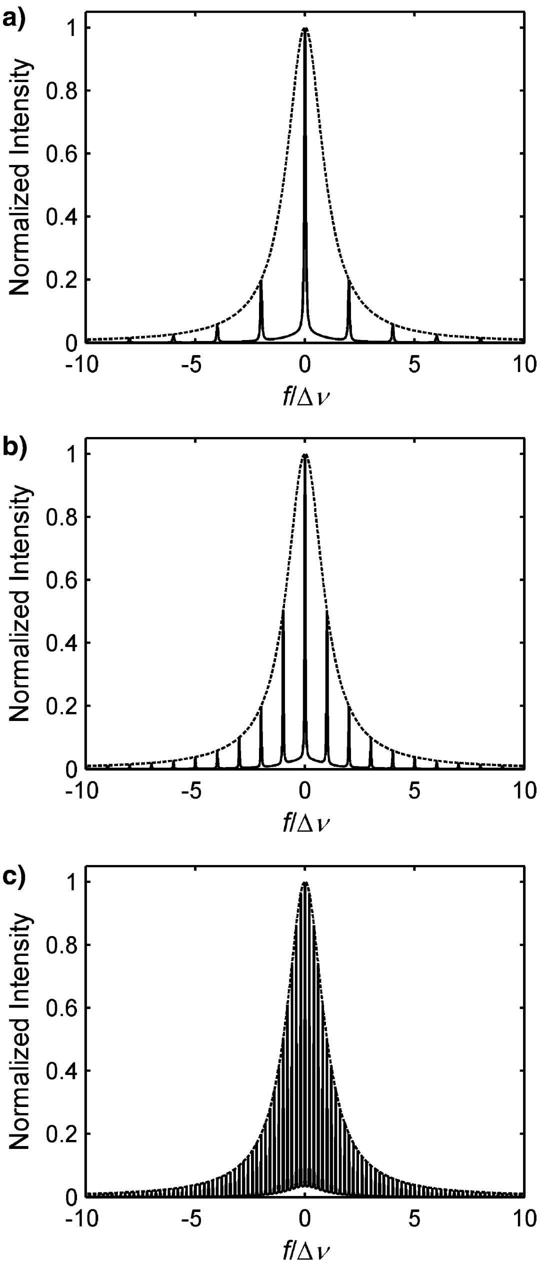

FIGURE 3 Normalized spectral intensity from a LC-RDSHI with $\alpha = 0.5$, $\gamma = 0.9$ and $m = 10$ (*solid line*) and from a conventional DSHI (*dashed line*) for time delay $m\tau_0$ and frequency $m\Omega$. **a**, $\Delta\nu\tau_0 = 0.5$, **b**, $\Delta\nu\tau_0 = 1$, **c**, $\Delta\nu\tau_0 = 5$

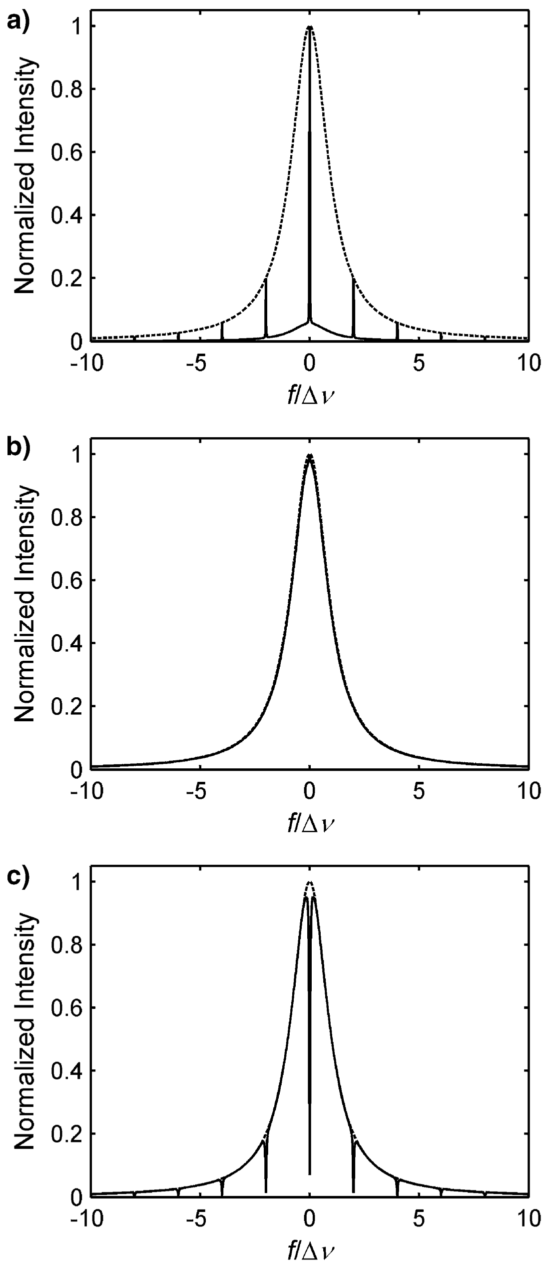

FIGURE 4 Normalized spectral intensity from a LC-RDSHI with $\alpha = 0.9$, $\Delta\nu\tau_0 = 0.5$ and $m = 10$ (*solid line*) and from a conventional DSHI (*dashed line*) for time delay $m\tau_0$ and frequency $m\Omega$. **a** $\gamma = 0.98$, **b**, $\gamma = 0.949$, **c**, $\gamma = 0.92$

The corresponding spectrum from the LC-RDSHI thereby reduces to

$$S_m(\omega) = \frac{\alpha}{(1-\alpha)^2}\gamma^m S_0(\omega, m\tau_0, m\Omega), \tag{36}$$

which has exactly the same line shape as from a conventional DSHI. By choosing the order m of beat-note such that $m\tau_0$ is several times larger than the coherent time, an accurate and direct measurement of the Lorentzian linewidth from the corresponding beating spectrum is possible. Figure 4 shows the $m = 10$ beat note for different values of parameter γ. Other parameters used here are $\alpha = 0.9$ and $\Delta\nu\tau_0 = 0.5$. The delta function in (17) has also been ignored. Figure 4b is the case in which $\gamma = 0.949$, and (35) is satisfied. The spectrum from the LC-RDSHI is overlapped with that from the conventional DSHI. However, a slight increase of γ value to 0.98 significantly deviates the spectrum line shape from the preferred Lorentzian one (as shown in Fig. 4a); while the spectrum line shape is less sensitive to the decrease of γ value from $\gamma_0 = 0.949$ as shown in Fig. 4c, in which $\gamma = 0.92$, and the Lorentizian shape is somewhat maintained except for the introduction of some narrow holes on the spectrum. The effective gain of the loop must be carefully controlled in the measurement of the laser line shape using a LC-RDSHI.

Provided (35) is satisfied, (36) is valid and the spectral peak power of different beat-notes is a function of parameter γ and exponentially decreases as the order number increases,

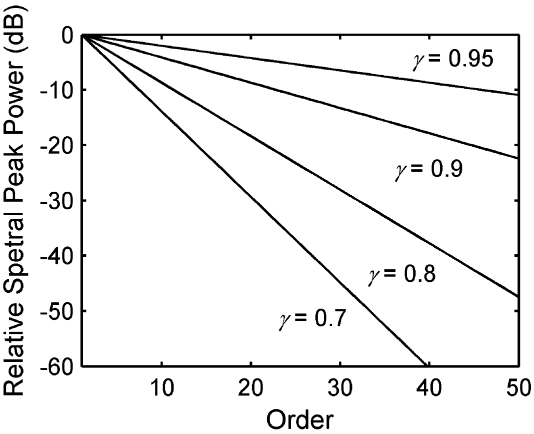

FIGURE 5 Relative spectral peak power (dB) vs. order number for different values of parameter γ

which fundamentally limits the number of beat notes that can be detected by a LC-RDSHI. Figure 5 shows the spectral peak power as a function of order number for different values of γ. The peak power has been normalized by the power of the 1st order of the beat-note. For $\gamma = 0.95$, which corresponds to $\alpha = 0.9$, the relative spectral peak power is decreased by only about 11 dB for m as many as 50. The relative spectral peak power (dB) is a linear function of order number. More beat-notes could be measured by increasing the value of γ. Thus, a coupler with relatively large value of parameters α is preferred for a LC-RDSHI. However, with larger value of γ, the requirement on the accuracy of gain control becomes more stringent.

We note that in [6–8] which used the LC-RDSHIs to measure the linewidths of different kinds of lasers, the operation conditions of the fiber delay loops in their experiments were not fully described and the effect on the beating spectrum of multi-recirculation of the laser field in the fiber loop was not addressed. Based on the analysis of this paper, the results of these measurements must be validated by eliminating the multi-circulation effect through an appropriate control of the operation conditions of the fiber delay loop.

5 Conclusions

An analytical theory to describe the autocorrelation function and power spectrum from a LC-RDSHI is presented. The theory indicates that a specific beat note of the spectrum can not be simply taken as the equivalence from a conventional DSHI with an equivalent time delay and frequency shift. A periodic modulation function must be included in the conventional DSHI spectrum to correctly represent the LC-RDSHI spectrum.

In order to accurately and directly measure the Lorentzian laser linewidth from the LC-RDSHI spectrum, the overall gain of the fiber delay loop must be carefully controlled according to the couplers used in the fiber delay loop, such that the modification function is invariant to frequency and the spectrum becomes exactly Lorentzian.

The relative spectral peak power of the beat-notes from an LC-RDSHI always decreases exponentially with the increase of the order number of a beat-note. More beat notes could be detected with larger overall gain of the fiber delay loop. However, the spectral line shape is more sensitive to the overall gain fluctuations of the fiber loop and more stringent control accuracy is required.

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