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Photon-emission properties of quantum-dot-based single-photon sources under different excitations

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ABSTRACT The photon-emission efficiencies and photon indistinguishabilities of a single-photon source, which employs a cavity coupled with a quantum dot, are studied under above-band and resonant excitations. The results are obtained by solving master equations and by applying the quantum regression theorem. According to the study, the photon indistinguishability increases with the Purcell factor under resonant excitation, which is consistent with the increase in emission efficiency; however, these two figures of merit are inconsistent for the above-band excitation scheme. Moreover, the efficiencies, defined as the average photon number emitted in one excitation cycle, are almost the same for the two different excitation schemes, whereas the excitation power needed to reach that efficiency is much lower under resonant excitation than that for above-band excitation. These results will be helpful in improving the performances of the applications concerning indistinguishability and efficiency.

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1 Introduction

Single-photon sources are essential to the applications of quantum information technologies, such as quantum cryptography [1] and linear optics quantum computation (LOQC) [2]. In addition to the investigations of atomic [3] and molecular [4] single-photon sources, recent progresses in semiconductor single-photon sources [5, 6] are shedding light on practical applications of such devices. Previous efforts have been largely devoted to improving the photon-emission efficiencies of the light source. After the suggestion of LOQC by Knill et al. [2], more emphasis is being focused on the indistinguishability of the emitted photons [6, 7], for LOQC is based on the quantum interference of two indistinguishable photons. In the two-photon interference, both identical photons impinging simultaneously on different ports of a 50:50 beam splitter will leave at the same output port. This phenomenon was first observed by Hong et al. using photons from a parametric down-conversion source [8], and then studied extensively by Legero et al. [9], but the microscopic properties of the emitters were not considered in Ref. [9].

It was shown in Ref. [7] that, due to the time jitter induced by relaxation, a quantum-dot (QD)-based single-photon source under above-band excitation could not achieve high efficiency and high indistinguishability simultaneously. One way out of this problem, as pointed out in Ref. [7], is to employ the cavity-assisted spin-flip Raman transition, which requires a magnetic field as strong as 10 T.

Our analysis, however, suggests that using resonant excitation can also achieve the same effect: the increases in efficiency and in indistinguishability are consistent with each other. Moreover, a more accurate model has been used to analyze the efficiencies under two different excitation schemes. Our results show that although the emission efficiencies under π excitation are almost the same in the two cases, the required excitation power is much lower in resonant excitation than in above-band excitation. Since higher power will induce a higher temperature of the device, we conclude that resonant excitation is superior to above-band excitation even if we consider only the issue of efficiency.

2 Indistinguishabilities

2.1 *Above-band excitation*

It is one of the main results of Ref. [7] that there is a trade-off between collection efficiency and indistinguishability if the quantum dot is excited by laser pulses of which the frequency is larger than the band gap of the surrounding bulk materials, i.e. the above-band excitation. For the purpose of comparison, we will re-state some of the details of this result.

When a quantum dot is excited in an above-band manner, three quantum-dot states will be involved, namely, the ground state $|g\rangle$, the pumped state $|p\rangle$, and the exciton state $|e\rangle$. Immediately after excitation, generated carriers will relax from the pumped state $|p\rangle$ to the exciton state $|e\rangle$, and this process is characterized by a relaxation rate, Γ_{relax}, which is typically around 100 GHz [10]. The spontaneous transition between $|e\rangle$ and $|g\rangle$ is characterized by Γ_0, which is typically 1 GHz for a quantum dot. For a single-photon

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source employing a cavity coupled with a quantum dot, the effect of the cavity is considered by replacing Γ_0 with $\Gamma_0 + F_p\Gamma_0$, where F_p is the Purcell factor.

If the quantum dot is pumped by a classical pump Ω, then the coherent part of the system Hamiltonian in the interaction picture is

$$H_{int} = i\hbar\Omega(\sigma_{pg} - \sigma_{gp}), \tag{1}$$

and the master equation considering dissipation and relaxation becomes [7, 11]

$$\frac{d}{dt}\rho = \frac{1}{i\hbar}[H_{int}, \rho] + \frac{\Gamma_{relax}}{2}(2\sigma_{ep}\rho\sigma_{pe} - \sigma_{pp}\rho - \rho\sigma_{pp}) + \frac{\Gamma_0}{2}(2\sigma_{ge}\rho\sigma_{eg} - \sigma_{ee}\rho - \rho\sigma_{ee}), \tag{2}$$

where $\sigma_{\mu\nu} = |\mu\rangle\langle\nu|$ $(\mu, \nu = g, p, e)$ are projection operators. Equation (2) is used to derive the optical Bloch equations, which are then integrated numerically. We used the methods developed by Kiraz et al. [7] to calculate the photon indistinguishability. The corresponding results are presented in Fig. 1a, in which the indistinguishability and the emission efficiency, estimated as $F_p/(F_p + 1)$, are plotted versus the Purcell factor.

Clearly, as the Purcell factor increases, the emission efficiency increases, but the photon indistinguishability suffers an obvious decrease, which is undesirable for applications involving two photon interferences. To understand this, let us examine the two parameters Γ_0 and Γ_{relax}. When the Purcell factor is relatively small, say 1, the temporal duration of each photon wave packet is roughly the reciprocal of Γ_0, which is about 1 ns. On the other hand, the time jitter induced by the relaxation process is around 10^{-2} ns. Therefore, the time jitter is negligible compared with the relatively long wave packet. However, when the emission is enhanced by the coupling between the quantum dot and a cavity, the time duration of the photon wave packet is prominently shortened by a factor roughly equaling the Purcell factor. If the wave packet is shortened to the extent that it is comparable with the reciprocal of Γ_{relax}, the time jitter corresponding to the relaxation process will randomly delay the photons that could otherwise arrive at the surface of the beam splitter at exactly the same time, thus reducing the indistinguishability.

We note that the experimental results in Ref. [6] do not fully conform to the prediction of Ref. [7] as stated above. That is because several factors, other than arrival time, are influencing the indistinguishability, making an actual result much more complicated. However, we will still focus on the arrival time as the factor in the following discussion.

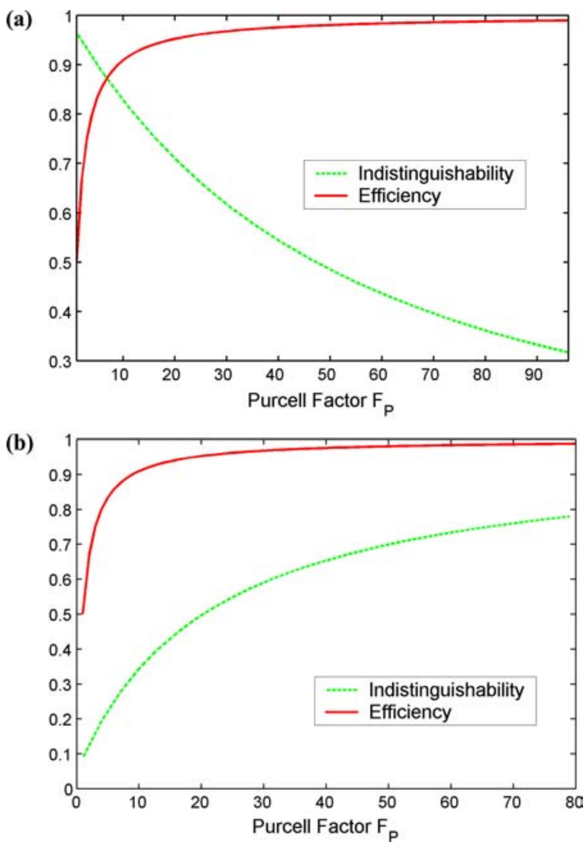

FIGURE 1 The dependence of indistinguishability and efficiency on the Purcell factor F_p, where **a** describes the above-band excitation case and **b** describes the resonant excitation case. The parameters are $\Gamma_0 = 1$ GHz, excitation pulse width $0.05/\Gamma_0$, $\Gamma_{relax} = 100$ GHz for (**a**), $\Gamma_{deph} = 10$ GHz for (**b**)

2.2 *Resonant excitation*

Having understood how the relaxation process affects the indistinguishability, we think presumably that if the relaxation is fast enough such that any possibly enhanced emission within current technology is slow compared with this relaxation, the time jitter induced by relaxation will no longer play the main role in affecting the photon indistinguishability; thus, the consistency between high indistinguishability and efficiency may be achieved. One of the feasible ways to realize this is to excite the quantum dot with pumping lasers tuned close to the radiation wavelength; hence, the quantum dot can be described by a two-level model. Here, only two quantum-dot states will be considered, i.e. the ground state $|g\rangle$ and the exciton state $|e\rangle$. The master equation of this system becomes

$$\frac{d}{dt}\rho = \frac{1}{i\hbar}[H_{int}, \rho] + \frac{\Gamma_0}{2}(2\sigma_{ge}\rho\sigma_{eg} - \sigma_{ee}\rho - \rho\sigma_{ee}), \tag{3}$$

where $H_{int} = i\hbar\Omega(\sigma_{eg} - \sigma_{ge})$. Equation (3) is used to derive the optical Bloch equations. By applying the quantum regression theorem, we arrive at a complete set of equations

$$\begin{aligned} \frac{d}{d\tau}O(t,\tau) &= -\Omega P(t,\tau) + \Omega Q(t,\tau) - \frac{\Gamma_0}{2}O(t,\tau), \\ \frac{d}{d\tau}P(t,\tau) &= 2\Omega O(t,\tau) - \Gamma_0 P(t,\tau), \\ \frac{d}{d\tau}Q(t,\tau) &= -2\Omega O(t,\tau) + \Gamma_0 P(t,\tau), \end{aligned} \tag{4}$$

where $O(t,\tau) = \langle\sigma_{eg}(t+\tau)\sigma_{ge}(t)\rangle$, $P(t,\tau) = \langle\sigma_{ee}(t+\tau)\sigma_{ge}(t)\rangle$, and $Q(t,\tau) = \langle\sigma_{gg}(t+\tau)\sigma_{ge}(t)\rangle$, with initial conditions $O(t,0) = \langle\sigma_{ee}(t)\rangle$, $P(t,0) = 0$ and $Q(t,0) = \langle\sigma_{ge}(t)\rangle$. Other methods used in this section are the same as stated in previous analysis of above-band excitation. When performing numerical simulations, we set $\Gamma_0 = 1$ GHz and the dephasing between $|g\rangle$ and $|e\rangle \Gamma_{\text{deph}} = 10\,\text{GHz}$. The result is shown in Fig. 1b.

Obviously, the increase in efficiency is consistent with the increase in indistinguishability, and this result confirms our prediction. Therefore, a quantum-dot-based single-photon source with resonant excitation can achieve both high indistinguishability and high emission efficiency by employing a cavity. It is noted that the absolute value of the indistinguishability in Fig. 1b is relatively low compared with that in Fig. 1a. This is simply because of the dephasing rate we have employed in the resonant excitation scheme, whereas we omit it when obtaining Fig. 1a. Since the only effect is the shift of the curve of the indistinguishability as a whole when we take account of the dephasing rate, the absolute values shown in these figures do not have much significance, while the consistency properties between efficiency and indistinguishability do.

3 Efficiencies

Section 2 is devoted to studying the consistency between efficiency and indistinguishability, where the efficiencies are estimated in an approximate way. In this section, however, we will proceed to analyze in detail the efficiencies in two different excitation schemes.

Again, we start with the case of above-band excitation. In this situation, we will consider altogether four states constituting the Hilbert space: $|1\rangle = |g, 0\rangle$, $|2\rangle = |p, 0\rangle$, $|3\rangle = |e, 0\rangle$, and $|4\rangle = |g, 1\rangle$, where the notation $|\mu, n\rangle$ ($\mu = g, p, e$; $n = 0, 1$) represents the state in which the quantum dot is in state $|\mu\rangle$ and the photon number within the cavity is n. The coherent part of the system in the interaction picture (under rotating wave approximation) is

$$H_{\text{int}} = \mathrm{i}\hbar g(\sigma_{34} - \sigma_{43}) + \mathrm{i}\hbar\Omega(\sigma_{21} - \sigma_{12}), \tag{5}$$

where g is the coupling between the quantum dot and the cavity mode. Considering the dissipation induced by both the cavity and spontaneous emission and considering the relaxation as well, the master equation becomes

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\rho = &\frac{1}{\mathrm{i}\hbar}[H_{\text{int}}, \rho] + \frac{\Gamma_{\text{relax}}}{2}(2\sigma_{32}\rho\sigma_{23} - \sigma_{22}\rho - \rho\sigma_{22})\\ &+ \frac{\Gamma_0}{2}(2\sigma_{13}\rho\sigma_{31} - \sigma_{33}\rho - \rho\sigma_{33})\\ &+ \kappa(2\sigma_{14}\rho\sigma_{41} - \sigma_{44}\rho - \rho\sigma_{44}),\end{aligned} \tag{6}$$

where κ represents the cavity decay rate, while other symbols and parameters have the same meaning as described in Sect. 2. Equation (6) is used to derive the optical Bloch equations, which are then integrated numerically with respect to time. To examine the efficiency of the system, we examine the parameter $P(t)$ [12]:

$$P(t) = 2\kappa \int_0^t \langle a^\dagger(t')a(t')\rangle \mathrm{d}t' = 2\kappa \int_0^t \langle\sigma_{44}(t')\rangle \mathrm{d}t', \tag{7}$$

which can be interpreted as the average photon number detected during the time interval from 0 to t, provided that an ideal photodetector is used. If t is large enough for the system to decay completely and still small enough so that it is earlier than the occurrence of the second excitation pulse, this parameter $P(t)$ is a good measure of the efficiency.

For resonant excitation, on the other hand, we should consider three states: $|1\rangle = |g, 0\rangle$, $|2\rangle = |e, 0\rangle$, and $|3\rangle = |g, 1\rangle$, where the notation has the same meaning as in the previous paragraphs. Similar to the case of above-band excitation, the coherent part of the Hamiltonian in the present situation is

$$H_{\text{int}} = \mathrm{i}\hbar g(\sigma_{23} - \sigma_{32}) + \mathrm{i}\hbar\Omega(\sigma_{21} - \sigma_{12}), \tag{8}$$

and the master equation of the system is

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\rho = &\frac{1}{\mathrm{i}\hbar}[H_{\text{int}}, \rho] + \frac{\Gamma_0}{2}(2\sigma_{12}\rho\sigma_{21} - \sigma_{22}\rho - \rho\sigma_{22})\\ &+ \kappa(2\sigma_{13}\rho\sigma_{31} - \sigma_{33}\rho - \rho\sigma_{33}).\end{aligned} \tag{9}$$

Again, Eq. (9) is used to derive the optical Bloch equations. The parameter $P(t)$ we considered is now defined as

$$P(t) = 2\kappa \int_0^t \langle\sigma_{33}(t')\rangle \mathrm{d}t'. \tag{10}$$

By assuming the parameters $\Gamma_0 = 1\,\text{GHz}$, $\Gamma_{\text{relax}} = 100\,\text{GHz}$, and $\kappa = 10\,\text{GHz}$, and tuning the peak excitation rate to achieve π excitation, we attain the result shown in Fig. 2. Two points should be noted: first, as the coupling between the quantum dot and the cavity increases, the average emitted photon number will be greater than one, due to the possibility that the quantum dot can be re-excited after a complete decay cycle; second, under π excitations, and hence optimal excitations, the efficiencies of the resonant excitation case and the above-band excitation case do not differ much from each other. However, this does not mean that these two excitation schemes can give the same performances if the efficiencies are the figures of merit to be concerned. The excitation rates to gain optimal excitations are different in the two cases, as shown in Fig. 3. Under resonant excitation, the required peak excitation rate is almost half of what is required in the above-band case. If we recall that the excitation rate is proportional to the amplitude of the electric field of the classical pump field, i.e. the square root of the pump field power, we will find that the required

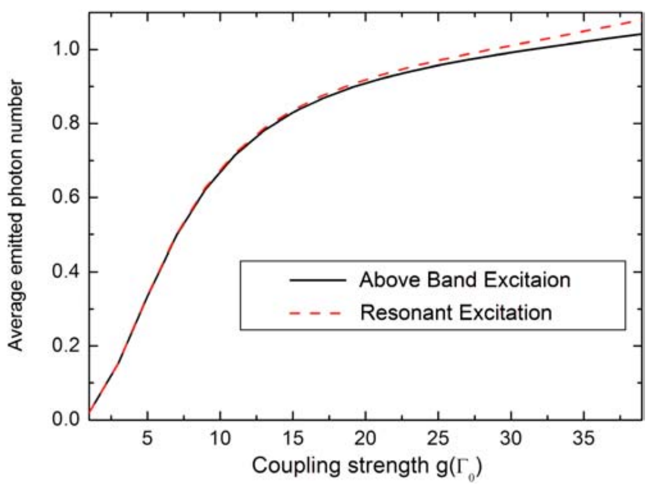

FIGURE 2 The dependence of efficiencies on coupling strength g between quantum dot and cavity mode. The parameters are $\Gamma_0 = 1\,\text{GHz}$, $\Gamma_{\text{relax}} = 100\,\text{GHz}$, and the excitation pulse width $0.05/\Gamma_0$

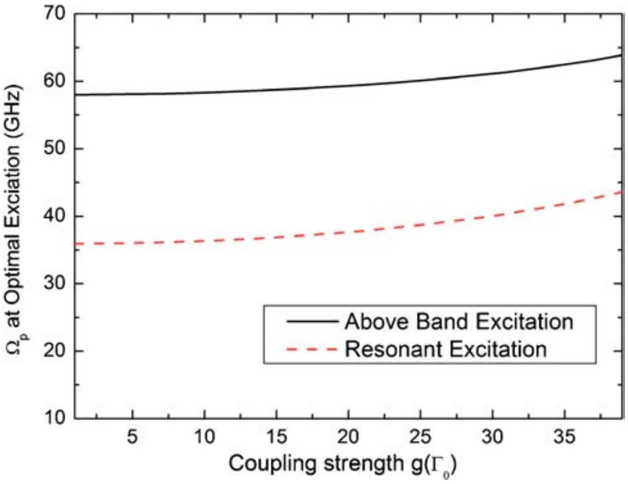

FIGURE 3 The required peak excitation rates to achieve optimal excitation under different coupling strengths *g*

power for above-band excitation to reach the optimal situation is much higher, say, approximately four times, than the resonant case. As we all know, high working power will inevitably give rise to high working temperature, which is a particularly undesirable condition for semiconductor devices. Therefore, even if only efficiency is what we are concerned about, resonant excitation is a better choice according to our study. Furthermore, it should be noted that as the excitation frequency moves up from the resonant frequency, the absorption of the QD will increase accordingly [13]. So, the required power in above-band excitation is expected to be even higher than the value predicted here, thus further supporting our conclusion.

4 Conclusions

In summary, we have compared in detail the photon indistinguishabilities and efficiencies of a quantum-dot-based single-photon source under resonant excitation and above-band excitation. We found that the increases in indistinguishability and efficiency will be consistent under resonant excitation, while the two parameters cannot be boosted simultaneously in above-band excitation. Pumping the quantum dot with a resonant laser source will provide an easier way to generate indistinguishable photons with high efficiency. Our study also shows that the efficiencies in the two excitation schemes are close to each other if optimal excitations are provided, but the required pumping power in above-band excitation is much greater than that in resonant excitation; thus, more heat will be produced in the former case. Therefore, resonant excitation is a better choice if a device with high efficiency and reliability is expected.

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