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From laser-induced line narrowing to electromagnetically induced transparency: closed system analysis

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ABSTRACT The laser-induced line-narrowing effect, discovered more than thirty years ago, can also be applied to recent studies in high resolution spectroscopy based on electromagnetically induced transparency. In this paper we first present a general form of the transmission width of electromagnetically induced transparency in a homogeneously broadened medium. We then analyze a Doppler broadened medium by using a lorentzian function as the atomic velocity distribution. The dependence of the transmission linewidth on the driving field intensity is discussed and compared to the laser-induced line-narrowing effect. This dependence can be characterized by a parameter which can be regarded as "the degree of optical pumping."

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1 Introduction

Over the last decade, considerable attention has been paid to studies of the atomic coherence effects and their applications [1, 2]. The technique of electromagnetically induced transparency (EIT) which makes an opaque medium become transparent by applying an external coherent radiation field [3, 4], yields various applications from the enhancement of nonlinear optical processes [5–7], to slow light [8–14]. In addition to the elimination of absorption, the absorption profile reveals a narrow transmission line, which has been applied to high resolution spectroscopy and high sensitivity magnetometry [15–18].

Since many of these experiments are performed in an atomic cell configuration, the Doppler broadening effect on EIT is an important concern. Recent theoretical investigations of Doppler broadening effects on EIT, however, have been focused mainly on the existence of EIT for certain configurations [19–21]. The issue of EIT linewidth for a Doppler broadened medium has been lately addressed by Taichenachev and coworkers [22]. As the width of transmission line is directly related to the dispersion near the EIT resonance, it is also a key issue in dispersive measurements.

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In a three-level Λ-type system if the system is homogeneously broadened, as is well known, EIT can be achieved when the intensity of the driving field (Ω^2) is larger than the product of the decay rate of the coherence between the lower levels (γ_{bc}) and the homogeneous linewidth (γ). Then, if the system is inhomogeneously broadened (say, with the width W_{D}), one might guess that EIT can be achieved when Ω^2 is larger than $\gamma_{bc} W_{\mathrm{D}}$ instead of $\gamma_{bc}\gamma$. This is not so. We show that one can still have EIT when $\Omega^2 \gg \gamma_{bc}\gamma$, even in the case of inhomogeneous broadening.

For the spectral width of EIT, if the system is homogeneously broadened, the two absorption lines are separated approximately by the Rabi frequency of the driving field Ω when Ω is larger than the homogeneous linewidth γ. When $\Omega \ll \gamma$, it becomes Ω^2/γ. Then, if the system is inhomogeneously broadened, it might be inferred that the EIT width goes as Ω when Ω is larger than the inhomogeneous linewidth W_{D}, and becomes Ω^2/W_{D} as $\Omega \ll W_{\mathrm{D}}$.

In the literature, however, we find that the narrow feature superimposed on the Doppler broadened profile has been studied more than thirty years ago. The laser-induced line-narrowing effect was discovered by Feld and Javan [23] and the spectral width of the narrow line was shown to be linearly proportional to the driving field Rabi frequency. Various aspects of this effect have been investigated by Hänsch and Toschek [24]. These effects were also called nonlinear interference effects [25]. In a recent article [26], it has been proposed that this laser-induced line narrowing can be applied to the recent experiments based on EIT, and the spectral line of the EIT resonance can be narrower in a Doppler broadened system than in a homogeneously broadened system. Here we analyze these ideas in detail and demonstrate the power broadening of the linewidth of the EIT resonance in a Doppler broadened system.

Under the condition of $\Omega \ll W_{\mathrm{D}}$, there are again two different regimes of the EIT width: In one limit it is proportional to the Rabi frequency of the driving field, which obeys the same expression as the spectral width shown in the study of laser-induced line narrowing [23]. As the driving field gets strong, it becomes power broadened and indeed has a form proportional to the intensity of the driving field (as Ω^2/W_{D}).

This paper is organized as follows: In Sect. 2 we set up our model scheme of the three-level system, and the transmission width of EIT in a homogeneously broadened medium is discussed. In Sect. 3 the Doppler averaged susceptibility is obtained by using a lorentzian function for the velocity distribution and the absorption profile, the EIT condition, and the linewidth of EIT are discussed. A comparison of closed and open systems is briefly given in Sect. 4. Section 5 contains an abstract of the present paper.

2 Homogeneously broadened system

We consider the model scheme depicted in Fig. 1. The transition $a \leftrightarrow c$ is coupled to a coherent driving field and the transition $a \leftrightarrow b$ is coupled to a weak probe field. The atom–field interaction hamiltonian can be written

$$\nu = -\hbar\alpha e^{-i\nu t}\,|a\rangle\,\langle b| - \hbar\Omega e^{-i\nu_0 t}\,|a\rangle\,\langle c| + \text{H.c.}, \tag{1}$$

where α is the Rabi frequency of the probe field, Ω is the Rabi frequency of the driving field. In this model we take the decay rate from the level a to b (c) to be γ (γ'). The relaxation between the lower levels is denoted γ_{bc}; the decay rate of the off-diagonal density matrix element (ϱ_{bc}) is defined by γ_{bc}.

The equations of motion for the density matrix elements in a rotating frame are then given by

$$\dot{\varrho}_{ab} = -\Gamma_{ab}\varrho_{ab} - i\alpha(\varrho_{aa} - \varrho_{bb}) + i\Omega\varrho_{cb} \tag{2a}$$
$$\dot{\varrho}_{cb} = -\Gamma_{cb}\varrho_{cb} - i\alpha\varrho_{ca} + i\Omega\varrho_{ab} \tag{2b}$$
$$\dot{\varrho}_{ac} = -\Gamma_{ac}\varrho_{ac} - i\alpha\varrho_{bc} - i\Omega(\varrho_{aa} - \varrho_{cc}) \tag{2c}$$
$$\dot{\varrho}_{cc} = -\gamma_{bc}\varrho_{cc} + \gamma'\varrho_{aa} + \gamma_{bc}\varrho_{bb} - i\Omega(\varrho_{ca} - \varrho_{ac}) \tag{2d}$$
$$\dot{\varrho}_{aa} = -(\gamma+\gamma')\varrho_{aa} - i\alpha(\varrho_{ab} - \varrho_{ba}) - i\Omega(\varrho_{ac} - \varrho_{ca}). \tag{2e}$$

Here we assume that the Rabi frequencies are real, the Γ_{ij} are defined by $\gamma_{ij} + i\Delta_{ij}$, where

$$\gamma_{ab} = \gamma_{ac} = \frac{1}{2}(\gamma + \gamma' + \gamma_{bc}), \qquad \gamma_{cb} = \gamma_{bc}, \tag{3}$$

and the Δ_{ij}´s are given by $\Delta_{ab} = \omega_{ab} - \nu$, $\Delta_{ac} = \omega_{ac} - \nu_0$ and $\Delta_{cb} = \Delta_{ab} - \Delta_{ac}$.

For a weak probe field, a first-order solution for the off-diagonal density matrix element ϱ_{ab} (which governs the absorption of the probe field) can be found in the steady state:

$$\varrho_{ab}^{(1)} = \frac{-i\alpha}{\Gamma_{ab}\Gamma_{cb} + \Omega^2} \times \left[\Gamma_{cb}\left(\varrho_{aa}^{(0)} - \varrho_{bb}^{(0)}\right) + \frac{\Omega^2}{\Gamma_{ca}}\left(\varrho_{cc}^{(0)} - \varrho_{aa}^{(0)}\right)\right], \tag{4}$$

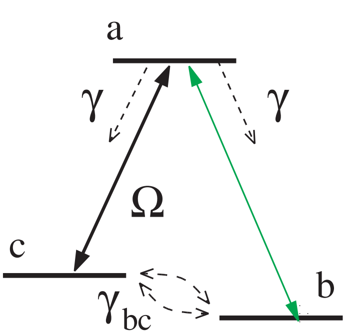

FIGURE 1 Three-level model scheme. The upper level a decays to b and c with decay rate γ. The relaxation rate between levels b and c is denoted by γ_{bc} which is assumed to be small compared with γ

where $\varrho_{ll}^{(0)}$ is the population in level l in the absence of the probe field. The susceptibility is then written as

$$\chi = \eta\left\{\frac{\varrho_{ab}^{(1)}}{\alpha}\right\}, \tag{5}$$

where η is given by $\eta \equiv (3/8\pi)N\gamma\lambda^3$ for the atomic number density N and the wavelength λ. The effect of the probe field intensity on the susceptibility is ignored by using the linear approximation [22].

2.1 *Optical pumping and population distribution*

Let us find the population of each level in the absence of the probe field (i.e. the zeroth-order population). Obviously, if the driving field is not turned on, we have $\varrho_{aa} = 0$ and $\varrho_{bb} = \varrho_{cc} = 1/2$ from (2). Now as the driving field is turned on, in the steady state, we have from (2c,e)

$$\varrho_{ac}^{(0)} = -\frac{i\Omega}{\Gamma_{ac}}(\varrho_{aa}^{(0)} - \varrho_{cc}^{(0)}),$$
$$\varrho_{aa}^{(0)} = -\frac{i\Omega}{\gamma+\gamma'}(\varrho_{ac}^{(0)} - \varrho_{ca}^{(0)}). \tag{6}$$

Let us now assume, for the sake of simplicity, that the decay rate from the level a to c is the same as the decay rate from the level a to b, i.e. $\gamma' = \gamma$ and the driving field detuning is denoted as Δ_0. Then, we have $\Gamma_{ac} = \gamma_{ac} + i\Delta_{ac} = (2\gamma + \gamma_{bc})/2 + i\Delta_0$, and

$$\varrho_{ac}^{(0)} - \varrho_{ca}^{(0)} = \frac{-i2\Omega(\gamma + \gamma_{bc}/2)}{(\gamma+\gamma_{bc}/2)^2 + \Delta_0^2}\left(\varrho_{aa}^{(0)} - \varrho_{cc}^{(0)}\right). \tag{7}$$

By (6,7) we obtain

$$\left[2\gamma + \frac{\Omega^2}{X}\right]\varrho_{aa}^{(0)} = \frac{\Omega^2}{X}\varrho_{cc}^{(0)}, \tag{8}$$

where

$$X \equiv \frac{[(\gamma+\gamma_{bc}/2)^2 + \Delta_0^2]}{2(\gamma+\gamma_{bc}/2)}. \tag{9}$$

Note that (2d) can be written as

$$\dot{\varrho}_{cc} = -\left(\gamma_{bc} + \frac{\Omega^2}{X}\right)\varrho_{cc} + \left(\gamma + \frac{\Omega^2}{X}\right)\varrho_{aa} + \gamma_{bc}\varrho_{bb}. \tag{10}$$

Hence using $\varrho_{bb} = 1 - \varrho_{aa} - \varrho_{cc}$, we obtain the zeroth-order population

$$\varrho_{aa}^{(0)} = \frac{2\gamma_{bc}\Omega^2}{2D}, \tag{11a}$$
$$\varrho_{bb}^{(0)} = \frac{4\gamma X\gamma_{bc} + 2\gamma_{bc}\Omega^2 + 2\Omega^2\gamma}{2D}, \tag{11b}$$
$$\varrho_{cc}^{(0)} = \frac{4\gamma X\gamma_{bc} + 2\gamma_{bc}\Omega^2}{2D}, \tag{11c}$$

where $D \equiv 4\gamma_{bc}\gamma X + 3\gamma_{bc}\Omega^2 + \Omega^2\gamma$. For $\gamma \gg \gamma_{bc}$, these can be simplified as

$$\varrho_{aa}^{(0)} - \varrho_{cc}^{(0)} \approx -\frac{4\gamma X \gamma_{bc}}{2D}, \quad \varrho_{bb}^{(0)} \approx \frac{4\gamma X \gamma_{bc} + 2\Omega^2 \gamma}{2D}, \tag{12}$$

where

$$X \approx \frac{\gamma^2 + \Delta_0^2}{2\gamma}, \quad D \approx 4\gamma_{bc}\gamma X + \Omega^2 \gamma. \tag{13}$$

Note that when the driving field is in resonance, the usual EIT condition $\Omega^2 \gg \gamma_{bc}\gamma$ is equivalent to $\varrho_{bb} \approx 1$ in (12); i.e. complete optical pumping to the level b is required to achieve EIT.

2.2 *Transmission width of EIT*

Now let us consider the transmission width under the condition of a resonant driving field. When we have a resonant driving field, i.e. $\Delta_0 = 0$, from (13) we find $X \approx \gamma/2$ and $D \approx \Omega^2\gamma$. Therefore, $\varrho_{aa}^{(0)} \approx \varrho_{cc}^{(0)} \approx 0$ and $\varrho_{bb}^{(0)} \approx 1$, i.e. all the populations are in level b. As is discussed in the previous section, the condition $\Omega^2 \gg \gamma_{bc}\gamma$ leads to complete optical pumping in the homogeneously broadened case.

Equations (4,5) then yield

$$\chi = \frac{\eta(-\mathrm{i})\Gamma_{cb}(-1)}{\Gamma_{ab}\Gamma_{cb} + \Omega^2}. \tag{14}$$

Since $\Gamma_{ab} \approx \gamma + \mathrm{i}\Delta$ and $\Gamma_{cb} = \gamma_{bc} + \mathrm{i}\Delta$, we have

$$\chi = \frac{\eta \mathrm{i}}{Z}(\gamma_{bc} + \mathrm{i}\Delta)\left[(\Omega^2 - \Delta^2) - \mathrm{i}\Delta\gamma\right], \tag{15}$$

where $Z = (\Omega^2 - \Delta^2) + \Delta^2\gamma^2$. Hence, the imaginary part is found to be

$$\chi'' = \frac{\eta}{Z}\left[\gamma_{bc}(\Omega^2 - \Delta^2) + \Delta^2\gamma\right]. \tag{16}$$

Since the maximum of χ'' is $1/\gamma$ at $\Delta \approx \Omega$, we may define Γ_{EIT}, the half width of EIT by $\chi''(\Delta = \Gamma_{\mathrm{EIT}}) = 1/2\gamma$, which gives

$$\Delta^4 - \Delta^2(2\Omega^2 + \gamma^2) + \Omega^4 = 0, \tag{17}$$

and the solution is

$$\Delta^2 = \frac{\gamma^2}{2}\left[2s + 1 \pm \sqrt{4s+1}\right], \tag{18}$$

where $s = \Omega^2/\gamma^2$. Hence for $s \gg 1$ we have

$$\Delta^2 \approx \gamma^2(s + \sqrt{s})$$
$$\Rightarrow \Delta \approx \pm\,\Omega \pm \frac{\gamma}{2}, \tag{19}$$

which shows that the absorption peaks are at $\pm\Omega$ with full width γ and the half width of transmission is obtained:

$$\Gamma_{\mathrm{EIT}} \approx \Omega - \frac{\gamma}{2}. \tag{20}$$

On the other hand, for $s \ll 1$ we have

$$\Delta^2 \approx \frac{\gamma^2}{2}\left[2s + 1 \pm \left(1 + 2s - 2s^2\right)\right]. \tag{21}$$

Therefore,

$$\Longrightarrow \quad \Delta \approx \pm\left(\gamma + \frac{\Omega^2}{\gamma}\right), \quad \pm\frac{\Omega^2}{\gamma}. \tag{22}$$

Hence, when $\Omega \ll \gamma$, we have the absorption profile showing a whole envelope with half width $\gamma + \Omega^2/\gamma$, and at the center there exists a transmission line with half width

$$\Gamma_{\mathrm{EIT}} \sim \Omega^2/\gamma. \tag{23}$$

We note that under the EIT condition, $\Omega^2 \gg \gamma\gamma_{bc}$, Γ_{EIT} cannot be smaller than γ_{bc}.

3 Inhomogeneously broadened system

Now if the system is Doppler broadened, for the atoms with velocity v the radiation fields are Doppler shifted by $\nu \to \nu(1 - v/c) = \nu - kv$ for the probe field with k the component of the wavevector on the propagation axis, and $\nu_0 \to \nu_0(1 - v/c) = \nu_0 - k'v$ for the driving field. Hence, for a Doppler broadened system, we replace Δ_{ij} by $\Delta_{ab} \to \Delta_{ab} + kv$, $\Delta_{ac} \to \Delta_{ac} + k'v$, and $\Delta_{cb} \to \Delta_{cb} + (k - k')v$. In the present analysis we assume that the energy difference between level b and c is small enough so that we have $k' \approx k$ and the probe field and the driving field are copropagating such that the $(k - k')v$ term can be neglected. Hence the atomic polarization should be averaged over the entire velocity distribution such that

$$\chi = \int \mathrm{d}(kv)\ f(kv)\ \eta \left\{\frac{\varrho_{ab}(kv)}{\alpha}\right\}, \tag{24}$$

where $f(kv)$ is the velocity distribution function, and again η is given by (5). We now consider the case where the inhomogeneous line is larger than any other of the quantities involved, so that $W_{\mathrm{D}} \gg \Omega, \gamma \gg \gamma_{bc}$, and the condition $\Omega^2 \gg \gamma_{bc}\gamma$ is still satisfied.

The population distribution in (12) is now different for atoms with different velocities. As we mentioned in Sect. 2, we then need to replace Δ_0 with $\Delta_0 + k'v \approx \Delta_0 + kv$ for the expression of X in (13) such that for a resonant driving field ($\Delta_0 = 0$) we have

$$X \approx \frac{\gamma^2 + (kv)^2}{2\gamma}, \quad D \approx 2\gamma_{bc}[\gamma^2 + (kv)^2] + \Omega^2\gamma. \tag{25}$$

Hence, for the atom with velocity v, $\varrho_{ab}(kv)$ can be written

$$\varrho_{ab}(kv) = \frac{\mathrm{i}\alpha}{Y}\frac{1}{2D} \times \left[\Gamma_{cb}(4\gamma X\gamma_{bc} + 2\Omega^2\gamma) - \frac{\Omega^2 4\gamma X \gamma_{bc}}{\gamma + \gamma_{bc}/2 + \mathrm{i}kv}\right], \tag{26}$$

where $Y = (\gamma + \gamma_{bc}/2 + \mathrm{i}\Delta + \mathrm{i}kv)(\gamma_{bc} + \mathrm{i}\Delta) + \Omega^2$. Here we have assumed that the $k' \approx k$ and $(k - k')v$ terms can be neglected for the copropagating fields.

3.1 *Doppler average using a lorentzian distribution*

We now need to evaluate the expression for the susceptibility given in (24). Normally, the velocity distribution is described by a gaussian function given by

$$f(kv)=\frac{1}{\sqrt{\pi}ku}\exp\left[-\frac{(kv)^2}{(ku)^2}\right], \tag{27}$$

where $u=(2k_{\mathrm{B}}T/M)^{1/2}$ is the most probable speed of the atom given for temperature T and atomic mass M. Then the full width at half maximum is given by $2W_{\mathrm{D}}=2(\ln 2)^{1/2}ku$. However, in our analysis, for the sake of simplicity of the analytic expressions, we adopt a lorentzian distribution with FWHM of $2W_{\mathrm{D}}$, instead of a gaussian distribution, so that

$$f(kv)=\frac{W_{\mathrm{D}}/\pi}{W_{\mathrm{D}}^2+(kv)^2}. \tag{28}$$

The two distributions are shown in Fig. 2, and there we see that a gaussian distribution with the same width (FWHM $2W_{\mathrm{D}}$) has a maximum larger than that of a lorentzian distribution by a factor of $(\pi\ln 2)^{1/2}$. Hence, if we multiply the factor $(\pi\ln 2)^{1/2}$ in (28), the central distribution becomes very similar to that of gaussian as illustrated in Fig. 2c. In Fig. 3 the absorption profiles are described numerically by using the two different distributions. We note that the two distributions give an almost identical result when the factor $(\pi\ln 2)^{1/2}$ is taken into account; see Fig. 3c.

Now using the distribution of (28), (24) may be considered as a contour integration in the complex plane. We find three poles in the upper half plane, at

$$kv=\frac{\Delta(\Omega^2-\gamma_{bc}^2-\Delta^2)+\mathrm{i}(\Delta^2\gamma+\gamma_{bc}^2\gamma+\gamma_{bc}\Omega^2)}{\gamma_{bc}^2+\Delta^2},$$
$$\mathrm{i}W_{\mathrm{D}},\quad \mathrm{i}\sqrt{\frac{\Omega^2\gamma}{2\gamma_{bc}}}, \tag{29}$$

and two poles in the lower half plane, at

$$kv=-\mathrm{i}W_{\mathrm{D}},\quad -\mathrm{i}\sqrt{\frac{\Omega^2\gamma}{2\gamma_{bc}}}. \tag{30}$$

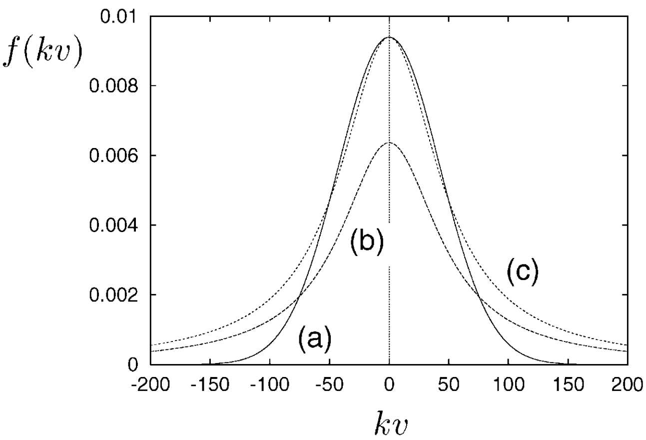

FIGURE 2 Velocity distribution of FWHM $2W_{\mathrm{D}}=100\gamma$ as a function a kv in units of γ, with (a) a gaussian profile of (27), (b) a lorentzian profile of (28), (c) the plot of (28) multiplied by a factor $(\pi\ln 2)^{1/2}$

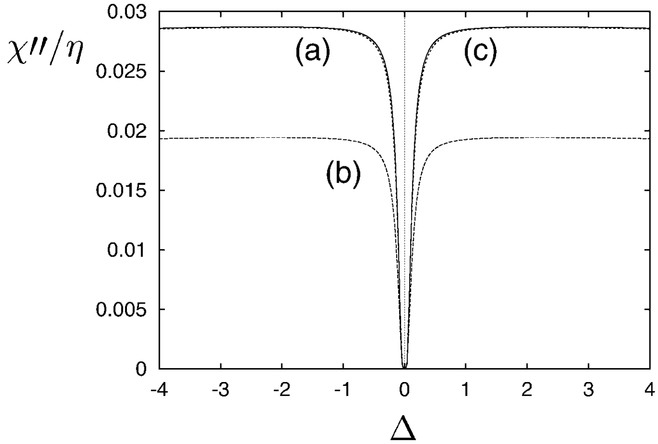

FIGURE 3 Absorption profiles (χ''/η) as a function of probe field detuning (Δ in units of γ) for $2W_{\mathrm{D}}=100\gamma$, $\gamma_{bc}=10^{-3}\gamma$, and $\Omega=2\gamma$ using (a), (b), (c) of Fig. 2, respectively

We can see that one pole is from the expression for Y, two poles ($\pm\mathrm{i}(\Omega^2\gamma/2\gamma_{bc})^{1/2}$) are from the expression for D in (26), and two poles ($\pm\mathrm{i}W_{\mathrm{D}}$) are from the velocity distribution function [1]. Let us take the contour in the lower half plane and denote

$$\chi=\chi_1+\chi_2, \tag{31}$$

where the χ_i are the contributions from the two poles at $-\mathrm{i}W_{\mathrm{D}}$ and $-\mathrm{i}(\Omega^2\gamma/2\gamma_{bc})^{1/2}$, respectively. For the pole at $kv=-\mathrm{i}W_{\mathrm{D}}$, we obtain

$$\chi_1=\frac{-\mathrm{i}\eta}{2Z_1A}\left[(B_1-\Delta^2)-\mathrm{i}\Delta W_{\mathrm{D}}\right]\left[C_1-\mathrm{i}\Delta D_1\right], \tag{32}$$

where A is given by

$$A=-2\gamma_{bc}W_{\mathrm{D}}^2+\Omega^2\gamma, \tag{33}$$

and

$$\begin{aligned}
Z_1&=(\gamma_{bc}W_{\mathrm{D}}+\Omega^2-\Delta^2)^2+\Delta^2W_{\mathrm{D}}^2,\\
B_1&=\gamma_{bc}W_{\mathrm{D}}+\Omega^2,\\
C_1&=2\gamma_{bc}W_{\mathrm{D}}(\gamma_{bc}W_{\mathrm{D}}+\Omega^2)-2\gamma_{bc}\Omega^2\gamma,\\
D_1&=-2\gamma_{bc}W_{\mathrm{D}}^2+2\Omega^2\gamma.
\end{aligned} \tag{34}$$

For the pole at $kv=-\mathrm{i}(\Omega^2\gamma/2\gamma_{bc})^{1/2}$, we have

$$\chi_2=\frac{\mathrm{i}\eta\Omega^2\gamma W_{\mathrm{D}}}{2Z_2Ay}\left[(B_2-\Delta^2)-\mathrm{i}\Delta y\right][C_2-\mathrm{i}\Delta], \tag{35}$$

where $y=(\Omega^2\gamma/2\gamma_{bc})^{1/2}$, and

$$\begin{aligned}
Z_2&=(\gamma_{bc}y+\Omega^2-\Delta^2)^2+\Delta^2y^2,\\
B_2&=\gamma_{bc}y+\Omega^2,\\
C_2&=-\gamma_{bc}+\Omega^2/y.
\end{aligned} \tag{36}$$

Note that we have assumed $\Omega^2\gg\gamma_{bc}\gamma$, $W_{\mathrm{D}}\gg\Omega$, $\gamma\gg\gamma_{bc}$.

[1] Note that the expression $-\Omega^2 4\gamma X\gamma_{bc}/(\gamma+\gamma_{bc}/2+\mathrm{i}kv)$ in (26) does not have a pole as we recall the original form of X in (9)

3.2 *Absorption and dispersion at EIT resonance*

The absorption profile is now obtained by the imaginary parts of (32,35):

$$\chi_1'' = \frac{-\eta}{2Z_1 A}\left[(B_1-\Delta^2)C_1 - \Delta^2 W_D D_1\right],$$
$$\chi_2'' = \frac{\eta\Omega^2\gamma W_D}{2Z_2 A y}\left[(B_2-\Delta^2)C_2 - \Delta^2 y\right]. \quad (37)$$

Taking $\Delta = 0$, we found

$$\chi_1''(\Delta=0) = \frac{-\eta}{A}\left[\gamma_{bc} W_D - \frac{\gamma_{bc}\Omega^2\gamma}{+\gamma_{bc}W_D + \Omega^2}\right],$$
$$\chi_2''(\Delta=0) = \frac{\eta\gamma_{bc}W_D}{A}\left[1 - \frac{2\gamma_{bc}y}{\gamma_{bc}y+\Omega^2}\right], \quad (38)$$

which gives the minimum value of absorption at the EIT line center:

$$\chi''(\Delta=0) = \frac{\eta\gamma_{bc}}{\gamma_{bc}W_D + \Omega^2}\left[\frac{\sqrt{x}}{1+\sqrt{x}}\right], \quad (39)$$

where

$$x = \frac{\Omega^2\gamma}{2\gamma_{bc}W_D^2}. \quad (40)$$

We note that when $x \ll 1$,

$$\chi''\big|_{\Delta=0} \Longrightarrow \quad \frac{\eta\gamma_{bc}}{\gamma_{bc}W_D+\Omega^2}\sqrt{x} < \frac{\eta\sqrt{x}}{W_D} \ll \frac{\eta}{W_D}, \quad (41)$$

and when $x \gg 1$,

$$\chi''\big|_{\Delta=0} \Longrightarrow \quad \frac{\eta\gamma_{bc}}{\gamma_{bc}W_D+\Omega^2} < \frac{\eta\gamma_{bc}}{\Omega^2} \ll \frac{\eta\gamma}{W_D^2}. \quad (42)$$

In both cases the EIT can be achieved, i.e.,

$$\chi''\big|_{\Delta=0} \ll \eta/W_D.$$

Therefore, the condition for EIT is still $\Omega^2 \gg \gamma\gamma_{bc}$, the same as in the homogeneously broadened system.

One interesting quantity here is the slope of the real part of the susceptibility, which is important in precision magnetometry, and also governs the group velocity of the probe light. From (32,35) the real part of the susceptibility is found to be

$$\chi_1' = \frac{-\eta\Delta}{2Z_1 A}\left[W_D C_1 + D_1(B_1-\Delta^2)\right],$$
$$\chi_2' = \frac{\eta\Omega^2\gamma W_D}{2Z_2 A y}\Delta\left[C_2 y + (B_2-\Delta^2)\right], \quad (43)$$

and its derivative at resonance is given by

$$\frac{\partial\chi_1'}{\partial\Delta}\bigg|_{\Delta=0} = -\frac{\eta\gamma}{A}, \quad \frac{\partial\chi_2'}{\partial\Delta}\bigg|_{\Delta=0} = \frac{\eta\sqrt{2\gamma_{bc}\gamma W_D^2/\Omega^2}}{A}. \quad (44)$$

Hence, we obtained the slope of χ' at $\Delta = 0$; it was

$$\frac{d\chi'}{d\Delta}\bigg|_{\Delta=0} = -\frac{\eta}{\Omega^2}\frac{\sqrt{x}}{1+\sqrt{x}}. \quad (45)$$

Therefore, when $x \gg 1$, it approaches η/Ω^2 and when $x \ll 1$, it goes as $(\eta/\Omega^2)(x^{1/2})$. We note that, under the EIT condition $\Omega^2 \gg \gamma_{bc}\gamma$, $(\eta/\Omega^2)(x^{1/2})$ is still much larger than $(\eta/\Omega^2)(\gamma/W_D)$. We have

$$\frac{d\chi'}{d\Delta}\bigg|_{\Delta=0} \Longrightarrow -\frac{\eta}{\Omega}\frac{1}{\sqrt{2\gamma_{bc}\gamma}}\left(\frac{\gamma}{W_D}\right). \quad (46)$$

3.3 *Transmission width of EIT resonance*

In order to estimate the linewidth of EIT we follow the same procedure as in Sect. 2. First, we find that the maximum of χ'' $\chi_{max} \approx \eta/W_D$ at $\Delta \approx \pm\Omega$. Then, we evaluate Δ which defines Γ_{EIT} as

$$\chi''(\Delta = \Gamma_{EIT}) = \eta/2W_D. \quad (47)$$

By (37) it readily gives the following equation:

$$\Delta^4 - \frac{2\gamma_{bc}\Omega^2}{\gamma}\frac{2\gamma_{bc}W_D^2+\Omega^2\gamma}{2\gamma_{bc}W_D^2}\Delta^2 - \frac{2\gamma_{bc}\Omega^2}{\gamma}\Omega^4 W_D^2 = 0, \quad (48)$$

which yields the half width of the EIT for the Doppler broadened system:

$$\Gamma_{EIT}^2 = \frac{\gamma_{bc}}{\gamma}\Omega^2(1+x)\left[1+\left\{1+\frac{4x}{(1+x)^2}\right\}^{1/2}\right],$$
$$\approx \frac{2\gamma_{bc}}{\gamma}\Omega^2(1+x), \quad (49)$$

where $x = \Omega^2\gamma/2\gamma_{bc}W_D^2$ is given by (40). Now if we define the saturation intensity by

$$\Omega_s^2 = \frac{2\gamma_{bc}W_D^2}{\gamma}, \quad (50)$$

the linewidth expression can be written

$$\Gamma_{EIT} \approx \left[\sqrt{\frac{2\gamma_{bc}}{\gamma}}\Omega\right]\sqrt{1+\frac{\Omega^2}{\Omega_s^2}}. \quad (51)$$

Here we can see that in the limit $\Omega \ll \Omega_s$ Γ_{EIT} is proportional to the Rabi frequency of the driving field. Such a linewidth was predicted by Feld and Javan in the study of laser-induced line narrowing [23]. On the other hand, in the limit $\Omega \gg \Omega_s$ Γ_{EIT} is proportional to the intensity of the driving field (Ω^2/W_D). This power broadening feature is shown in Fig. 4.

The expression of (51) shows a reminiscence of the power broadening factor in the description of hole burning [27]. In place of the homogeneous linewidth in the expression of hole burning, here we have an effective width which is determined by the spectral packet involved in population trapping [26].

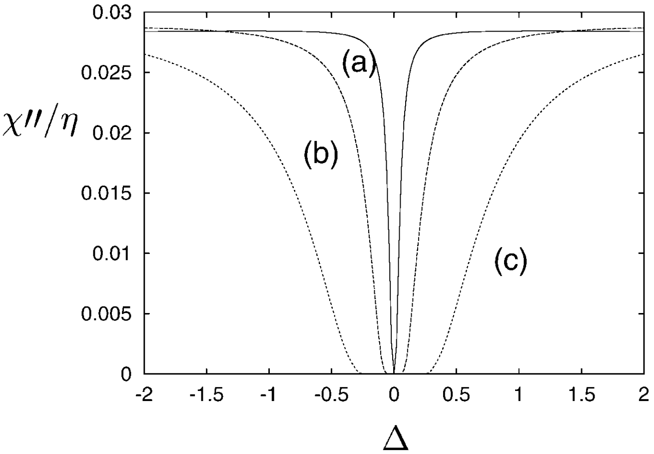

FIGURE 4 Absorption profile for $2W_D = 100\gamma$, $\gamma_{bc} = 10^{-3}\gamma$. (a) $\Omega = \gamma$, (b) $\Omega = 3\gamma$, (c) $\Omega = 6\gamma$. Note that $(\gamma_{bc}\gamma)^{1/2} \sim 0.03\gamma$ and $\Omega_s \sim 4.5\gamma$

3.4 *The role of optical pumping*

We have seen that the parameter $x = \Omega/\Omega_s$ plays an important role in the case of an inhomogeneously broadened medium. Let us here examine the physical meaning of this parameter.

Suppose the system is homogeneously broadened. When the driving is in resonance, the optical pumping rate from the level c is then of the order of Ω^2/γ, as given in (10). A complete optical pumping within the homogeneous linewidth is then possible if this rate is larger than the pumping from level b to c: $\Omega^2/\gamma \gg \gamma_{bc}$. This, in turn, gives the EIT condition. When we have the driving field detuned by Δ_0, the optical pumping rate decreases by a factor of $\gamma^2/(\gamma^2+\Delta_0^2)$. Again for a complete optical pumping we need $\Omega^2\gamma/[\gamma^2+\Delta_0^2] \gg \gamma_{bc}$.

If we now assume that we have the resonant driving field, then $\Delta_0 = 0$ again, and, instead, the atoms are moving. Then, for atoms with velocity v, the optical pumping rate becomes $\Omega^2\gamma/[\gamma^2+(kv)^2]$. Then, on the average, to have complete optical pumping in a Doppler broadened system we need to require $\Omega^2\gamma/(\gamma^2+W_D^2) \gg \gamma_{bc}$, which corresponds to $x \gg 1$ (assuming $W_D \gg \gamma$), i.e. $\Omega \gg \Omega_s \equiv 2\gamma_{bc}W_D^2/\gamma$. Hence, the parameter $x = \Omega^2/\Omega_s^2$ represents the degree of saturation in the $b \leftrightarrow c$ transition, or the degree of optical pumping from level c to b within the inhomogeneous linewidth.

4 Comparison with an open system description

In this section we examine the case of an open system and show that the result is essentially the same as our model of a closed system. The open system is modeled for the atoms that are coming in and out of the interaction (with the radiation fields) region. Although in such a case all the levels have the same decay rate (say, γ_{bc}), the upper level can decay much faster than the time of flight through the interaction region (for example, radiative decay or collisional decay). Hence we assume that the lower levels b and c decay with rate γ_{bc} and the upper level a decays with rate γ_a which is much larger than γ_{bc} (see Fig. 5).

Furthermore, for simplicity, we assume that the atoms are coming into the interaction region with a same rate for the lower levels. Under these assumption, the equation of motion

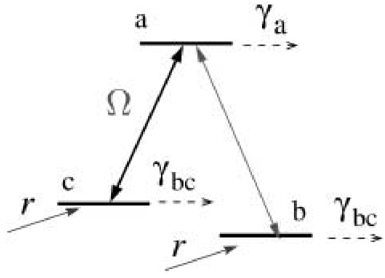

FIGURE 5 Model scheme of the open system. The upper level a decays with rate γ_a, the lower levels b and c decay with the same rate γ_{bc}. Atoms are pumped at a rate r equally to the lower levels

for the density matrix elements can be written

$$\dot{\varrho}_{ab} = -\Gamma_{ab}\varrho_{ab} - \mathrm{i}\alpha(\varrho_{aa}-\varrho_{bb}) + \mathrm{i}\Omega\varrho_{cb}, \tag{52a}$$
$$\dot{\varrho}_{cb} = -\Gamma_{cb}\varrho_{cb} - \mathrm{i}\alpha\varrho_{ca} + \mathrm{i}\Omega\varrho_{ab}, \tag{52b}$$
$$\dot{\varrho}_{ac} = -\Gamma_{ac}\varrho_{ac} - \mathrm{i}\alpha\varrho_{bc} - \mathrm{i}\Omega(\varrho_{aa}-\varrho_{cc}), \tag{52c}$$
$$\dot{\varrho}_{aa} = -\gamma_a\varrho_{aa} - \mathrm{i}\alpha(\varrho_{ab}-\varrho_{ba} - \mathrm{i}\Omega(\varrho_{ac}-\varrho_{ca}), \tag{52d}$$
$$\dot{\varrho}_{bb} = r - \gamma_{bc}\varrho_{bb} - \mathrm{i}\alpha(\varrho_{ab}-\varrho_{ba}) - \mathrm{i}\Omega(\varrho_{ac}-\varrho_{ca}), \tag{52e}$$
$$\dot{\varrho}_{cc} = r - \gamma_{bc}\varrho_{cc} - \mathrm{i}\Omega(\varrho_{ca}-\varrho_{ac}). \tag{52f}$$

Here the notations are the same as in (2). Note that now we have $\Gamma_{ac} = (\gamma_a+\gamma_{bc})/2 + \mathrm{i}\Delta_0$, which gives

$$X' \equiv \frac{\left[(\gamma_a/2+\gamma_{bc}/2)^2+\Delta_0^2\right]}{2(\gamma_a/2+\gamma_{bc}/2)}. \tag{53}$$

Again if we assume that $\gamma_a \gg \gamma_{bc}$, the populations are found to be

$$\varrho_{aa}^{(0)} - \varrho_{cc}^{(0)} \approx -\frac{\gamma_{bc}\gamma_a X'}{2D'}, \quad \varrho_{bb}^{(0)} \approx \frac{\gamma_{bc}\gamma_a X' + \Omega^2\gamma_a}{2D'}, \tag{54}$$

where

$$X' \approx \frac{(\gamma_a/2)^2+\Delta_0^2}{\gamma_a}, \quad D' \approx \gamma_{bc}\gamma_a X' + \Omega^2\gamma_a. \tag{55}$$

Comparing (54) with (12), we can see that the population distribution is almost identical to the one for the model of the closed system.

Furthermore, the expression for $\varrho_{ab}^{(1)}$ is identical to the one for the closed system given in (4). Let us then recall (33) saying that $A = -2\gamma_{bc}W_D^2 + \Omega^2\gamma$, which is obtained by putting $-\mathrm{i}W_D$ to Δ_0 in the expression of D in (11). The sign of A determines whether the crucial parameter x is > 1 or < 1. Similarly, here for the open system, when we put $\Delta_0 = -\mathrm{i}W_D$, we can define A' as $A' \approx -\gamma_{bc}W_D^2 + \Omega^2\gamma_a$ such that we have $x' = \Omega\gamma_a/\gamma_{bc}W_D^2$ as the parameter which plays the same role as x in (13). Hence, by replacing $\gamma_a \Rightarrow 2\gamma$, we have an open system description almost identical to the description for our model scheme of the closed system. A detailed analysis of the open system will be presented elsewhere [28].

5 Summary

In this paper, we have studied the transmission width of EIT in a three-level Λ system. The Doppler averaged susceptibility is found by using a lorentzian velocity distribution rather than the gaussian distribution. Then we have shown the requirement for achieving EIT, and the analytic expression of the EIT linewidth. The saturation intensity Ω_s^2 defines the degree of optical pumping by Ω^2/Ω_s^2, and represents the *condition* under which the broadening is either linear or quadratic in the Rabi frequency of the driving field.

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