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Direct method for phase and amplitude determination of ultra-short light pulses

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ABSTRACT We present a new method for obtaining the phase of an ultra-short light pulse in the frequency domain. This technique is based on measuring the logarithmic derivative of the cross correlation of the pulse with a frequency-filtered version of it. It only requires sweeping the center frequency of the filter, while the time delay of the correlator is held fixed for measuring the group delay of the frequency components of the pulse. It is, to our knowledge, the first one-dimensional method to recover directly the phase information without resorting to a deconvolution or a procedure involving two Fourier transforms. Since it directly measures the delay operator we have called this technique direct group delay operator measurement. We present the theory, experimental results and a brief discussion of the best choice for the fixed setting of the time delay in order to improve the signal to noise ratio in this technique.

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1 Introduction

The state of the art of ultra-short-pulsed lasers is far beyond the time resolution of electronic detectors. As a diagnosis tool for improving design of laser cavities and pulse compressors, as well as the study of non-linear propagation, it is necessary to obtain accurate measurements of pulse shapes or their spectra, including amplitude and phase.

If the field spectral amplitude and phase are known, a simple Fourier transform can accurately retrieve the pulse shape. Taking the spectral amplitude is easy, but there is no simple and accurate technique for the direct measurement of the spectral phase.

The standard method employed to characterize these ultrashort pulses is intensity autocorrelation [1]. Pulse compression and dispersion-compensated laser cavities lead to the femtosecond range, but the pulses are often chirped, or present satellites or pedestals. These features are not revealed by the intensity autocorrelation, which only gives an approximated measurement of the pulse length. Interferometric autocorrelation [2] is affected by the chirp, but its measurement is not straightforward from this technique.

Alternative methods to retrieve the phase information from interferometric measurements have been reported [3, 4], but they require accurate and detailed measurement of the interference fringes and rely upon deconvolution procedures that are often numerically unstable and critically sensitive to measurement errors.

Other techniques developed during the last decade, such as frequency-resolved optical gating (FROG) [5–8] and spectral phase interferometry for direct electric-field reconstruction (SPIDER) [9, 10], have been successful in characterizing ultra-short laser pulses. The FROG technique gets a qualitative output with a simple and intuitive interpretation, but it resorts to deconvolution computations for the attainment of a quantitative answer to the problem. In addition, it is essentially a two-dimensional method that needs to acquire enough information for the simultaneous deconvolution of amplitude and phase in order to reconstruct the pulse shape. The SPIDER technique is a frequency-domain interferometric method that does not use an iterative algorithm as FROG does, but it still requires performing some mathematical manipulation (Fourier transforms) to get the phase of the pulse.

A few years ago, Chilla and Martínez developed a technique, called frequency domain phase measurement (FDPM) [11, 12], which is a method for direct measurement of phase and amplitude of an ultra-short pulse in the frequency domain. It is based on the measurement of the cross correlation of two pulses. The first is the pulse to be characterized and the second is a synthesized copy of the first pulse obtained by means of a Fourier-filtering scheme [13]. This filtering is performed in a zero-dispersion compressor using a mask consisting of a narrow slit placed near the back mirror of the compressor.

2 Basics of FDPM

The FDPM technique essentially consists in retrieving the phase of an ultra-short pulse by measuring the group delay of its different frequency components. This is done by sequentially performing cross correlations of a reference pulse with different frequency-filtered pulses, all obtained from the pulse to be characterized by means of a Fourier-filtering scheme. The output intensity of the filtered pulse can be expressed as [12]

$$I_{\text{filt}}(\omega_o, t) = \left| \int_{-\infty}^{\infty} |A_{\text{in}}(\omega)| e^{i\varphi(\omega)} M(\omega - \omega_0) e^{i\omega t} d\omega \right|^2 , \quad (1)$$

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where $M(\omega-\omega_o)$ is an effective filter centered at ω_o, constructed as a convolution of the transmittance mask that constitutes the filter with a Gaussian that accounts for the finite beam size, and $A_{in}(\omega)$ is the Fourier transform of the input pulse. If we assume $M(\omega-\omega_o)$ to be a narrow function of the phase of the incoming pulse around the filter central frequency ω_o, this equation may be rewritten as

$$\begin{aligned} I_{\text{filt}}(\omega_o, t) &= |A_{\text{in}}(\omega_o)|^2 |m\left(t+\varphi'_o\right)|^2 \\ &= |A_{\text{in}}(\omega_o)|^2 h\left(t+\varphi'_o\right), \end{aligned} \quad (2)$$

where $\varphi_o = \varphi(\omega_o)$ and $\varphi'_o = |\frac{d\varphi}{d\omega}|_{\omega_o}$ are the coefficients of the first-order expansion of the phase $\varphi(\omega)$ and $m(t+\varphi'_o) = \int_{-\infty}^{\infty} M(\omega-\omega_0)\, e^{i\varphi'_o(\omega-\omega_o)} e^{i(\omega-\omega_o)t} d\omega$ is the effect of the frequency filter in the time domain.

Equation 2 shows that the output of the filtering system is the square of the Fourier transform of the narrow function $m(t-\varphi'_o)$ shifted by a time equal to the derivative of the phase at the filter frequency ($\Delta t = -\varphi'(\omega_o)$), and with an amplitude that scales according to the spectral power corresponding to the filtered frequency ($|A_{\text{in}}(\omega_o)|^2$).

The cross-correlation signal of the filtered pulse, I_{filt}, with the reference pulse, I_{ref}, as a function of the relative timing, τ, is given by

$$S(\tau) = \int_{-\infty}^{\infty} I_{\text{ref}}(t) I_{\text{filt}}(t+\tau) dt. \quad (3)$$

Fig. 1a schematically represents the situation for five different frequencies. Substituting (2) into (3) gives

$$S(\tau) = |A_{\text{in}}(\omega_o)|^2 \int_{-\infty}^{\infty} I_{\text{ref}}(t) h\left(t+\varphi'_o+\tau\right) dt. \quad (4)$$

As demonstrated by this equation, and graphically shown in Fig. 1b, this cross-correlation function presents the same $-\varphi'(\omega_o)$ time shift and $|A_{\text{in}}(\omega_o)|^2$ amplitude dependence as the filtered pulse.

In the FDPM method the only information used to recover the phase of the pulse is the relative timing $\tau_{(M)}(\omega_o)$ that maximizes each cross correlation (Fig. 1b). It becomes clear that to complete measurement of all the correlation functions produces information in excess of the only purpose of the phase measurement, and the price to be paid for this redundancy is the time necessary to obtain the data.

It may be convenient, because of experimental reasons, to measure the derivative of the cross correlation instead of the cross correlation itself. This function is mathematically expressed as

$$S'(\tau) = |A_{\text{in}}(\omega_o)|^2 \int_{-\infty}^{\infty} I_{\text{ref}}(t) \frac{d}{d\tau} h\left(t+\varphi'_o+\tau\right) dt. \quad (5)$$

The time shift and amplitude behavior of $S'(\tau)$ are the same as (3) and (4). In this case $\tau_{(M)}(\omega_o)$(g)s the relative timing for which the derivative of the cross correlation vanishes (Fig. 1c).

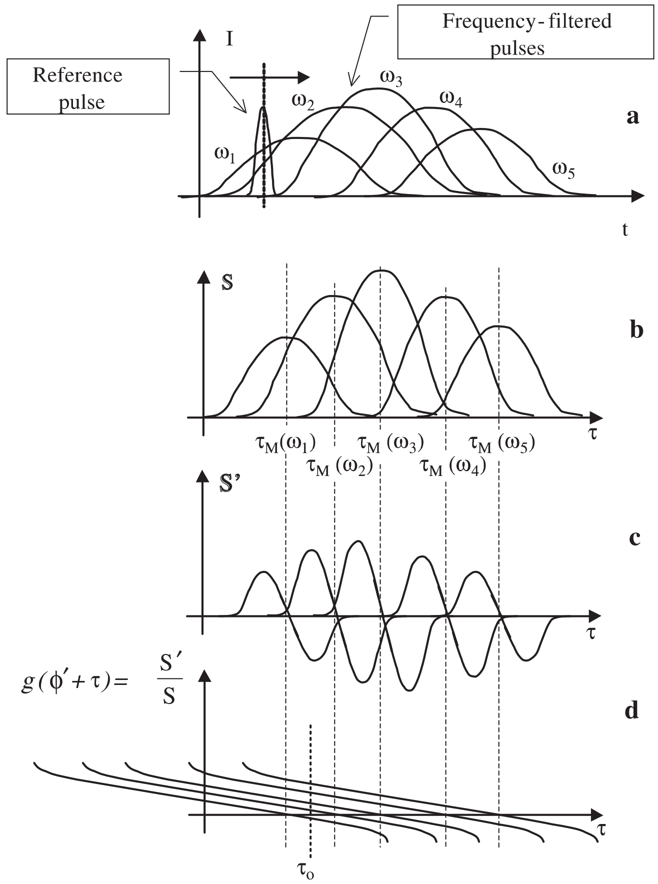

FIGURE 1 Representation of the cross correlations, the derivative and the logarithmic derivative as a function of the central frequency of the filter (ω_i). **a** Filtered pulses obtained from an ultra-short pulse by means of a Fourier-filtering scheme. All the pulses are shifted in time because of the group delay corresponding to each frequency. **b** Cross correlations ($S(\omega_o, g\tau)$) of the filtered pulses with the reference pulse shown in (**a**). $\tau_M(\omega_i)$ is the relative timing of the reference pulse for the maximum of the cross correlation. **c** Derivative of the cross correlations ($S'(\omega_o, \tau)$). **d** Logarithmic derivative of the cross correlations

3 Direct group delay operator measurement (DGDOM) technique

3.1 *Introduction to the technique*

As we mentioned before, a change in the central frequency of the filter does not modify the shapes of the cross correlation ($S(\tau)$) and its derivative ($S'(\tau)$) ; it only varies their amplitude and position. Since the variation for both functions is exactly the same (they are both shifted by $\varphi'(\omega_o)$ and scaled by $|A_{\text{in}}(\omega)|^2$), it is possible to get rid of the scaling factor while keeping the required information of the relative timing by simply dividing $S'(\tau)$ by $S(\tau)$. The resulting function is the logarithmic derivative of the cross correlation

$$\begin{aligned} \frac{d}{d\tau}\ln(S) = \frac{S'(\tau)}{S(\tau)} &= \frac{\int_{-\infty}^{\infty} I_{\text{ref}}(t) \frac{d}{d\tau} h\left(t+\varphi'_o+\tau\right) dt}{\int_{-\infty}^{\infty} I_{\text{ref}}(t) h\left(t+\varphi'_o+\tau\right) dt} \\ &= g\left(\varphi'_o+\tau\right). \end{aligned} \quad (6)$$

As the central frequency of the filter (ω_o) is varied, this function suffers a displacement along the timing axis according to the group delay $\varphi'(\omega_o)$, but no change in its amplitude is produced. In this way a set of identical curves is obtained with a displacement directly related to $\varphi'(\omega_o)$ (Fig. 1d). Another way to understand this is that for a fixed delay τ_o the variations of the value of the function g for each frequency ω_o are only related to the displacement produced because of the difference in the group delay $\varphi'(\omega_o)$. This is the key idea of the DGDOM technique, to retrieve the phase of an ultra-short pulse by calculating the group delay from the measurement of the value of the function g at some fixed τ_o.

The resolution of the system will depend on the width of the filtering slit. A narrower slit will improve the spectral resolution but it will decrease the signal level at the same time. Just as in a monochromator, a compromise between the desired resolution and the signal level that can be detected must be achieved.

3.2 *The shape of the logarithmic derivative*

In a Fourier-filtering scheme two limiting cases may arise: a beam spot narrower than the filtering slit and a slit narrower than the beam spot. For the first case a Gaussian profile of the effective filtering function may be assumed, so the logarithmic derivative will be a linear function of the timing. To obtain the phase derivative from the measurement is in this case very simple: it is only necessary to multiply the measured value by a constant. In this case, if $S(\omega_o, \tau) = A(\omega_o) \exp\left(\frac{\tau^2}{\tau_g^2}\right)$ is the cross correlation, its logarithmic derivative will be

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\ln(S) = -\frac{2\tau}{\tau_g}, \tag{7}$$

with τ_g defined as $\frac{1}{\tau_g} = \sigma\frac{\mathrm{d}\nu}{\mathrm{d}x}$, where σ is the width of the beam spot at the filter and $\frac{\mathrm{d}\nu}{\mathrm{d}x}$ is the dispersion of the grating. $1/\tau_g$ can be understood as the width of the focused beam spot measured in units of frequency.

In the limit of a beam spot much narrower than the slit width, the effective filtering function becomes a sinc function.

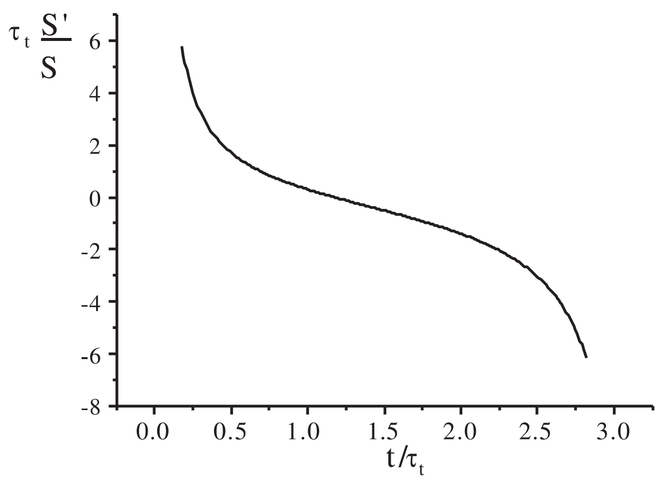

FIGURE 2 Logarithmic derivative of a sinc function

Its logarithmic derivative is

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left\{\ln\left[\mathrm{sinc}^2\left(\frac{\tau}{\tau_t}\right)\right]\right\} = \frac{2\frac{\tau}{\tau_t^2}\cos\left(\frac{\tau}{\tau_t}\right) - \frac{1}{\tau_t}\sin\left(\frac{\tau}{\tau_t}\right)}{\frac{\tau}{\tau_t}\sin\left(\frac{\tau}{\tau_t}\right)}, \tag{8}$$

$1/\tau_t$ being the slit width in units of frequency that comes, as in the other limiting case, from $\frac{1}{\tau_t} = d\frac{\mathrm{d}\nu}{\mathrm{d}x}$, where d is the slit width. In Fig. 2 a plot of this function is represented, showing that its most relevant portion can be very well approximated by a linear function in the region of interest.

Between both limits the filter is the convolution of the slit with a Gaussian function. The Fourier transform of the filter will then be given by $S(\tau) = s_1 s_2 = \mathrm{sinc}^2(\tau/\tau_t)\exp\left(\frac{\tau^2}{\tau_g^2}\right)$. It is easy to show that the first two terms of its logarithmic derivative are

$$-A\tau\left[1 + C\left(\frac{\tau}{\tau_t}\right)^2\right], \tag{9}$$

with

$$C = \left(\frac{\tau}{\tau_g}\right)^2\left[30 + 5\left(\frac{\tau}{\tau_g}\right)^2\right]^{-1} \leq 0.2\,.$$

As it can not be expected that the phase retrieval can be performed without appreciable distortion with a slit width comparable with the pulse spectral spread, it will be assumed that the time delays that are of interest for the measurement are less than $\tau/5$.

This yields $C\left(\frac{\tau}{\tau_t}\right)^2 < 8 \times 10^{-3}$. Hence it can be asserted within a very good approximation that the logarithmic derivative of the cross correlation is a linear function of the time delay. The fact that one does not actually measure the filtered function, but its correlation with a shorter pulse [12], not only does not change the linearity of the measurement, as long as the reference pulse is much shorter than the filtered pulse, but also makes the result completely independent of the reference pulse actual shape. The time average of the logarithmic derivative is proportional to the time average of the group delay because of the linear relation we have just demonstrated. The calibration that relates these two quantities can be measured by taking two values of the logarithmic derivative corresponding to two time delays of known difference. We have named this technique direct group delay operator measurement (DGDOM) because, as we have shown, a quantity directly proportional to the group delay is measured.

3.3 *Optimal arm length*

To accomplish the aim of obtaining an optimal signal to noise ratio, it is necessary to determine the correlator arm length (or the corresponding time delay) which yields the best results. A detailed theory of noise sources and their influence on the retrieved data is beyond the scope of this paper. If it is assumed that the dominant noise comes from the detector and its associated electronics, it can be asserted that noise is additive and there is no correlation between its DC and f_o spectral components or, in other words, errors in the signal and its derivative are not correlated. In this case, calling S the correlation signal and S' its time derivative, the error in the logarithmic derivative of the correlation will be

$$\left[\Delta\left(\frac{\mathrm{d}}{\mathrm{d}\tau}\ln(S)\right)\right]^2 = \left[\Delta\left(\frac{S'(\tau)}{S(\tau)}\right)\right]^2 = \frac{\left(\Delta S'\right)^2}{S^2} + \frac{S'^2\,(\Delta S)^2}{S^4}. \tag{10}$$

Hence the best place to set the mirror is where the correlation is maximized, i.e. the one which makes its derivative vanish. It can not be placed there a priori, because this position of the mirror depends on the filtered wavelength. The compromise is then to place the arm at null delay.

4 Experimental work

For the experimental work we have used a mode-locked Ti:sapphire laser as the source of ultra-short pulses. Our oscillator generates pulses at a repetition rate of 80 MHz, with about 500 mW of average power.

The experimental setup is shown in Fig. 3. The output of the laser enters a Michelson-like system, were a 1200 l/mm diffraction grating, positioned to have the first diffraction order exiting normal to it, acts as the beam splitter. The order one of diffraction goes through a 5 cm focal length lens, placed at a distance f from the grating, and is reflected back by a flat mirror placed at a distance f from the lens, with a 0.4-mm-wide movable slit positioned immediately before the mirror. This is the arm where the Fourier-filtering scheme is implemented. The zero order of the diffraction grating provides the reference pulse for the cross correlations. The light emerging from the grating is reflected back by a hollow retroreflector mounted on a piezoelectric crystal that dithers it so as to obtain the derivative of the cross correlation. This vibrating retroreflector was also mounted on a translation stage to scan the relative timing between pulses.

The beams returning from both arms of this arrangement are aligned to emerge from the grating parallel to each other and to the incoming beam. These outputs are redirected by mirrors 1 and 2 towards a lens that focuses them into a 0.1-mm-thick KDP crystal. The non-collinear second-harmonic beam generated by the crystal is recollimated by

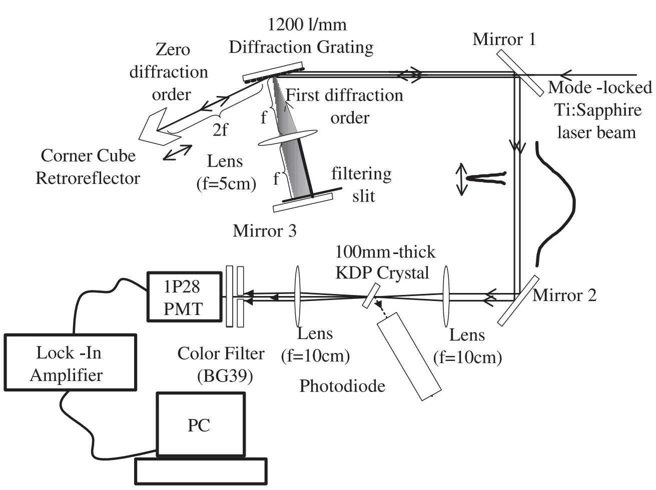

FIGURE 3 Schematics of the experimental setup

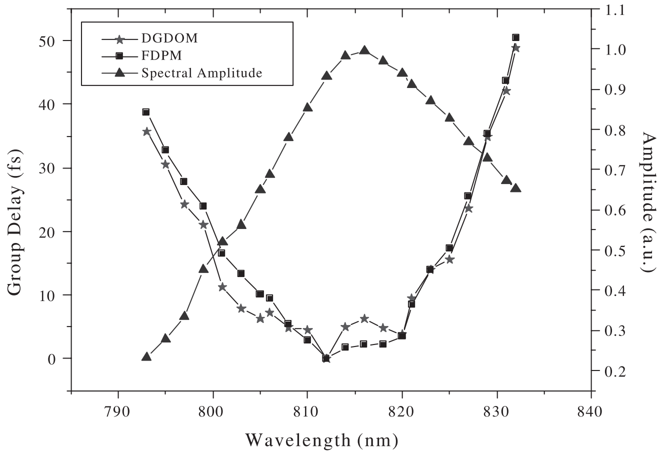

FIGURE 4 Measured spectral amplitude and group delays obtained by the FDPM (*squares*) and the DGDOM (*stars*) techniques

a second lens, filtered to eliminate the fundamental wavelength and detected with a photomultiplier tube (PMT). In order to control the non-linear crystal alignment, we registered the relative spectral intensity of the pulse, and measured simultaneously with a photodiode the intensity of the reflection of the filtered pulse at the KDP crystal.

To measure the group delay using the DGDOM technique we first fixed the position of the retroreflector and made it vibrate. Then, we positioned the filtering slit at one end of the spectrum and started to move it parallel to mirror 3 to scan the different frequencies. We measured the second-harmonic signal detected by the PMT with a lock-in amplifier linked to a computer, locked at the vibrating frequency of the piezoelectric crystal. With the lock-in amplifier we simultaneously registered the amplitude of the signal in AC (derivative) and DC (cross-correlation) modes, and the position of the filtering slit. The whole measurement took us just a few minutes and this time was limited by the speed of the scanning device used to move the slit.

To validate the DGDOM results we also implemented the well-established FDPM technique using the same setup in a different way. For the FDPM technique we registered the PMT signal with the lock-in amplifier in DC mode. To get the cross-correlation curve for each fixed position of the frequency-filtering slit we varied the delay of the testing pulse by moving the retroreflector on the translation stage. By varying the filtered frequency (i.e. moving the slit along the spectrum on mirror 3), we acquired all the data needed to retrieve the phase and frequency information of the pulse. In Fig. 4 we show the results obtained using both methods.

5 Conclusions

We have shown that the logarithmic derivative of the cross correlation of the pulse with a frequency-filtered version of it is proportional to the group delay.

We have theoretically demonstrated the DGDOM technique and presented its experimental validation. We have also discussed the best choice for the fixed setting of the time delay in order to improve the signal to noise ratio for implementing this technique.

DGDOM is, to our knowledge, the first method that directly measures the group delay of an ultra-short laser pulse.

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