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Surface effects on vibration analysis of elastically restrained piezoelectric nanobeams subjected to magneto-thermo-electrical field embedded in elastic medium

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Abstract In the present study, a generalized nonlocal beam theory is utilized to study the magneto–thermo–mechanical vibration characteristic of piezoelectric nanobeam by considering surface effects rested in elastic medium for various elastic boundary conditions. The nonlocal elasticity of Eringen as well as surface effects, including surface elasticity, surface stress and surface density are implemented to inject size-dependent effects into equations. Using the Hamilton's principle and Euler–Bernoulli beam theory, the governing differential equations and associated boundary conditions will be obtained. The differential transformation method (DTM) is used to discretize resultant motion equations and related boundary conditions accordingly. The natural frequencies are obtained for the various elastic boundary conditions in detail to show the significance of nonlocal parameter, external voltage, temperature change, surface effects, elastic medium, magnetic field and length of nanobeam. Moreover, it should be noted that by changing the spring stiffness at each end, the conventional boundary conditions will be obtained which are validated by well-known literature.

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1 Introduction

In recent years, the tendency in studying the mechanical behavior of nanostructures has been grown. Among all nanostructures, the piezoelectric, for their unique mechanical properties, proved to be capable of designing the nano-electro-mechanical systems (NEMSs). It is apparent that these nanomaterials can be applicable for building blocks for nanodevices integrating mechanical and electrical functionality at nanoscale size [1]. So, according to their wide range of application such as nano-sensors, actuators, generators, transistors, and diodes, investigating the vibrational behavior of piezoelectric nanomaterials is significant [2].

It is known that the classical continuum mechanics cannot predict the size effects. Therefore, to inject the size-dependent response of nanostructures, several non-classical higher-order continuum theories have been employed, such as nonlocal elasticity theory [3], stress theory [4], strain gradient theory [5], surface elasticity [6], and micropolar theory [7]. Among all these theories, the nonlocal elasticity theory of Eringen [3] can be employed in a wide range of applications in the analysis of nanostructures. Due to simplicity and high computational efficiency of nonlocal elasticity theory, rapid extensions of this theory in various mechanical analysis for different nanostructures can be observed as [8–10].

One of the main size-dependent factors of nanostructures is surface effects, which are happening for increasing the surface-to-volume ratio in nanoscale. Due to high surface-to-volume ratio, the surface effects as well as the small scale effect become substantial, which leads to exceptional mechanical characteristics at the nanoscale. Therefore, as the surface layers energy is negligible compared with the bulk energy of material at the macroscale, the classical continuum mechanics is not applicable to predict the surface energy effect. Gurtin and Murdoch [11, 12], based on the continuum mechanics, developed a theoretical framework to take the surface energy effects into account. According to this theory, the surface layer is considered as an elastic two-dimensional membrane with zero thickness adhered to the underlying bulk material without slipping. It is also considered that the surface layers have distinct properties than the bulk of material which is characterized using the Lame constants of surface followed by surface residual stress. Many efforts have been done to investigate the surface effects [13–18] and nonlocal effects [19–27] on the mechanical properties of nanobeam separately, and simultaneously [2, 28–36].

Also, remarkable attention has been paid to investigate the mechanical characteristic of piezoelectric nanostructures in high-temperature conditions. So, the influence of temperature changing in the mechanical analysis is studied extensively. As for instance, Ebrahimi et al. [37] investigated the vibration characteristic of smart piezoelectrically actuated nanobeams in the magneto-electrical field and thermal environment. Further, Ke et al. [38] implemented the nonlocal elasticity theory into the thermo-electric-mechanical vibration analysis of piezoelectric nanobeam. Furthermore, Mohammadimehr et al. [39] studied the vibration and buckling analysis of triple-walled ZnO piezoelectric nanobeam based on the Timoshenko beam theory resting on Pasternak foundation under magneto-electro-thermo-mechanical loadings. In the work done by Marzbanrad et al. [40], the vibration behavior of size-dependent piezoelectric nanobeam resting on elastic under axial preload was studied by considering surface and thermal effects. Moreover, Ansari et al. [41] implemented the analytical solution to predict the postbuckling characteristics of FGM nanobeams subjected to thermal environment and surface stress effect.

It should be noted that all the mentioned works presented the numerical results for various mechanical properties of nanostructures for conventional boundary conditions (BCs) including Simply–Simply (S–S), Clamped–Clamped (C–C), Clamped–Simply (C–S) and Clamped–Free (C–F). In continuous systems, the type of BCs from their direct effect on vibration response of structures is so important. Mostly, in real systems, one of the mentioned BCs which has the nearest manner are chosen and assumed to satisfy the conditions exactly [42, 43]. Moreover, the rotational and transitional springs will substitute at the ends to introduce small deflections and moments. Further, Wattanasakulpong et al. [44] studied the linear and nonlinear vibration behavior of nanobeams which are elastically end-restrained. The numerical results are presented for Elastic–Elastic (E–E) and Simply–Elastic (S–E) boundary conditions. Besides, Zarepour et al. [45] investigated the electro-thermo-mechanical nonlinear characteristic of nanobeams resting on Winkler–Pasternak elastic medium for E–E and S–S BCs.

The present paper makes the first attempt to investigate the magneto-thermo-electric-mechanical vibration of piezoelectric nanobeam with elastic boundary condition by considering surface and nonlocal elasticity effects. Based on the Eringen's nonlocal constitutive relations and using Gurtin–Murdoch theory to incorporate the surface effects, equilibrium equations of piezoelectric nanobeam subjected in magnet and thermal field is achieved. The differential transformation method (DTM) with an iterative algorithm on the basis of Taylor series expansion is utilized to solve resultant motion equations for various BCs. The natural frequencies for various elastic boundary conditions are obtained, while, by choosing right values for spring stiffness at each end of nanobeam, the corresponding natural frequencies for classical BCs will be achieved. To validate the accuracy of motion equations and numerical results, the resultant natural frequencies are compared with well-known literature which are in excellent agreement. The results are obtained for various spring stiffness constants, voltage values of piezoelectric field, temperature changing, magnetic field effect, nonlocal parameter, elastic foundation including Pasternak and Winkler foundations and nanobeam length for various elastic boundary conditions. It is shown that making changes to spring stiffness value and surface effect of piezoelectric nanobeam are two main approaches to achieve desired natural frequencies.

2 Formulation and theories

2.1 Eringen's nonlocal elasticity theory

Among various types of nonlocal elasticity theory, Eringen's theory proved to be capable and easy to use. The essence of nonlocal elasticity is that the stress field at a reference point x in an elastic medium does not only depend on the strain at that point, but also on the strains at all other points in the bulk of material [3].

This theory is based on the atomic theory of lattice dynamics and also the experimental observations of atomic and molecular scales which come from the observations on phonon dispersion. In this theory, the internal size as a material parameter is used to incorporate the scale effects into the equations [46]. The most general form of nonlocal elasticity relations will be indicated as an integral over the whole body of material, but by neglecting the body forces, the basic equations for stress tensor and electric displacement will be obtained as [47]:

$$\sigma_{ij} - \mu^2\nabla^2\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \lambda_{ij}\Delta T , \qquad (1)$$

$$D_i - \mu^2 \nabla^2 D_i = e_{ikl}\varepsilon_{kl} + \varepsilon_{ik}E_k + p_i \Delta T \,, \tag{2}$$

where ∇^2 is the Laplace operator, σ_{ij}, D_i denote the component of the stress, electric field; ε_{kl}, C_{ijkl}, e_{ikl}, λ_{ij} are the strain, elastic constant, piezoelectric constants and thermal module. ΔT and p_i are the temperature changes and piezoelectric constants; and also, $\mu = (e_0 a)^2$ represent the nonlocal parameter; moreover, $e_0 a$ denotes the scale length coefficient revealing the size effect in obtaining the response of nanostructures.

2.2 Surface effects

For the fact that the inter-atomic distance plays an important role in elastic constant of crystals, the bound contraction in the surface layers has a dominant influence than that in the bulk. To increase the surface-to-bulk ratio in nanoscale, the surface effects will become more significant and cannot be neglected. As this ratio increases, the surface effects role in response of nanobeam become more dominant. Accordingly, the energy which is produced by the atoms located in surface layers affects the mechanical properties of nanostructures which have been studied extensively by researchers. Gurtin et al. [11] proposed a continuum model which considered the surface layers as a zero-thickness film which subjected on the material body. In other words, the body and surface layers in the nanobeam are assumed such as a composite beam which is composed of a solid core with the bulk modulus and the surface shell with surface modulus as depicted in Fig. 1. The surface layer is assumed to be a two-dimensional thin film which attached perfectly to the bulk. Besides, the surface layers and the bulk material considered to be bonded, accordingly, the displacement field is continuous for both parts across the interface. It should be noted that this assumption is only for the modeling purpose which means that these layers do not actually exist; for this reason, the type of surface is not defined. For isotropic surfaces, the local stresses and electric displacement will be defined for piezoelectric nanobeam based on Gurtin model as [29]:

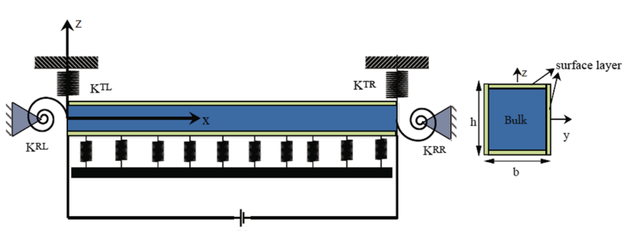


Fig. 1 Schematic of the nanobeam with elastic boundary conditions with length *L*, width *b* and height *h*

$$\tau^{sl}_{\alpha\beta} = \tau^0_{\alpha\beta} + C^s_{\alpha\beta\gamma\delta}\varepsilon_{\gamma\delta} - e^s_{\alpha\beta k}E_k \,, \tag{3}$$

$$D^{\mathrm{sl}}_i = D^0_i + e^s_{\alpha\beta i}\varepsilon_{\alpha\beta} + k^s_{ij}E_j \,, \tag{4}$$

where $C^s_{\alpha\beta\gamma\delta}$, $\varepsilon_{\gamma\delta}$,$e^s_{\alpha\beta k}$ and k^s_{ij} express the surface elastic, surface strains, surface piezoelectric and surface dielectric constants, respectively. $\tau^{sl}_{\alpha\beta}$, $\tau^0_{\alpha\beta}$ denote the nonlocal stress tensor and residual surface stress tensor, respectively.

By considering the same material properties for both top and bottom layers, the constitutive stress–strain relations for surface layers will be obtained as:

$$\tau_{xx} = \tau_0 + E^s u_{x,x} \,, \quad E^s = 2\mu_0 + \lambda_0, \;\; \tau_{nx} = \tau_0 u_{n,x} \,, \tag{5}$$

while σ_{zz} is often neglected in classical beam theories, which is assumed to satisfy the equilibrium equations by its linear relation with the beam thickness:

$$\sigma_{\mathbf{zz}} = \frac{2zv}{h}\left(\tau_o \frac{\partial^2 w}{\partial x^2} - \rho_o \frac{\partial^2 w}{\partial t^2}\right) \tag{6}$$

2.3 Problem formulation

The vibration analysis of piezoelectric nanobeam embedded in elastic medium using the nonlocal and surface effects under the magnet and thermal environment with various elastic boundary conditions will be presented. The piezoelectric nanobeam with length L $(0 \le x \le L)$, thickness h $(-h/2 \le z \le h/2)$, and width b $(-{}^{b}\!/_{2} \le y \le {}^{b}\!/_{2})$ subjected to an applied voltage $\phi(x, z)$ and uniform temperature change ΔT is depicted in Fig. 1. The Euler–Bernoulli beam theory is utilized to obtain the motion equations. Following the Euler–Bernoulli beam theory (EBT), the displacement field for nanobeam at any arbitrary point is [48]:

$$u_1 = u(x,\,t) - z\frac{\partial w(x,t)}{\partial x}, \;\; u_2 = 0, \;\; u_3 = w(x,\,t)\,, \tag{7}$$

where $u(x,\ t)$ and $w(x,\ t)$ denote the axial and transverse components of displacement, respectively.

The only nonzero strain component which can be defined based on EBT is:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial^2 x} \tag{8}$$

And also, the electric displacement field for piezoelectric nanobeam can be expressed as [14]:

$$\begin{gathered} E_x = -\tfrac{\partial \phi}{\partial x};\, E_z = -\tfrac{\partial \phi}{\partial z}, \\ D_x = \lambda_{11}E_x;\, D_z = e_{31}\varepsilon_x + \lambda_{33}E_z, \\ \tfrac{\partial D_x}{\partial x} + \tfrac{\partial D_z}{\partial z} = 0 \end{gathered} \tag{9}$$

where λ_{11} and λ_{33} are the dielectric constants, while $D_x$and $D_z$express the electric displacements.

For the fact that $\lambda_{11}$and λ_{33} are in the same order, $E_x << E_z$ should be considered, D_x in compare with D_z should be ignored. The electrical BCs will be assumed as $\phi(x, -h/2) = 0$, $\phi(x, h/2) = 2V$combining with Eqs. (8) and (9), the electrical potential can be obtained as:

$$\phi(x, z) = -\frac{e_{31}}{\lambda_{33}}\left(\frac{z^2 - h^2}{2}\right)\frac{\partial^2 w}{\partial x^2} + \left(1 + \frac{z}{h}\right)V \tag{10}$$

Furthermore, the equivalent piezoelectric load can be expressed as follows:

$$P_{\text{electric}}(x, t) = b\int_{-h}^{h} \sigma_x^* \, dz = 2\,V\,b\,e_{31} \tag{11}$$

Accordingly, using Hamilton's principle, the governing motion equations and relative BCs will be obtained as [48]:

$$\int_0^t \delta(U - T + W_{\text{ext}})\,dt = 0\,, \tag{12}$$

where U, T and W_{ext} are the strain energy, kinetic energy and the work done by external forces, respectively. The first variation of strain energy for piezoelectric nanobeams is:

$$\delta U = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\begin{array}{l} \sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz} - D_x\delta E_x - D_z\delta E_z + K_{\text{TL}}\,w(0,t)\delta w(0,t) \\ +K_{\text{RL}}\frac{\partial w(0,t)}{\partial x}\delta(\frac{\partial w(0,t)}{\partial x}) + K_{\text{TR}}\,w(L,t)\delta w(L,t) \\ +K_{\text{RR}}\frac{\partial w(L,t)}{\partial x}\delta(\frac{\partial w(L,t)}{\partial x}) \end{array}\right) dzdx\,, \tag{13}$$

where K_{RL}, K_{TL},$K_{\text{RR}}$and K_{TR} express the corresponding rotational and translational spring constants at the left and right ends, respectively. Using Eq. (8) and Eq. (13) gives:

$$\delta U = \int_0^l \int_{-h/2}^{h/2} \left[\begin{array}{l} N\delta u - M\delta\left(\frac{\partial^2 w}{\partial x^2}\right) + D_x\delta\left(\frac{\partial \phi}{\partial x}\right) + D_z\delta\left(\frac{\partial \phi}{\partial z}\right) \\ +K_{\text{TL}}\,w(0,t)\delta w(0,t) + K_{\text{RL}}\frac{\partial w(0,t)}{\partial x}\delta(\frac{\partial w(0,t)}{\partial x}) \\ +K_{\text{TR}}\,w(L,t)\delta w(L,t) + K_{\text{RR}}\frac{\partial w(L,t)}{\partial x}\delta(\frac{\partial w(L,t)}{\partial x}) \end{array}\right] dx \tag{14}$$

Accordingly, the axial force N and bending moment force M are defined as:

$$N = \int \sigma_{xx}dz\,,\; M = \int \sigma_{xx}zdz \tag{15}$$

After that, the kinetic energy will be determined as:

$$\begin{aligned} T &= \frac{1}{2}\rho \iint (\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2)\,dA.dx \\ &= \frac{1}{2}\rho \int \left(I_1\left(\frac{\partial u}{\partial t}\right)^2 + I_2\left(\frac{\partial^2 w}{\partial x \partial t}\right)^2 + I_1\left(\frac{\partial w}{\partial t}\right)^2\right) dx \end{aligned} \tag{16}$$

Thus, the first variation of kinetic energy from Eq. (16) can be written as:

$$\delta T = -\int_0^l \left(I_1\left(\frac{\partial^2 w}{\partial t^2}\right)\delta(w) - I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2}\right)\delta(w) + I_1\left(\frac{\partial^2 u}{\partial t^2}\right)\delta(u)\right) dx \tag{17}$$

where:

$$I_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho\,dz, \quad I_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho\,z^2\,dz \tag{18}$$

The axial load which is obtained by the elastic medium based on the Winkler–Pasternak foundation is considered to be as:

$$f = -k_{\text{w}}w + k_{\text{p}}\frac{\partial^2 w}{\partial x^2} \tag{19}$$

where k_{w} and k_{p} are the Winkler and Pasternak elastic medium constants.

The first variation of the work done by external forces will be determined by:

$$\delta W_{\text{ext}} = \int_0^t (f\delta(u) + q\delta(w))dt\,, \tag{20a}$$

where q is defined as:

$$q = \left(H + N_T + P_{\text{electric}} + K_P + q_z\right)\left(\frac{\partial^2 W}{\partial x^2}\right) - K_w\,W\,, \tag{20b}$$

$$N_T = -E\lambda_1\,A\,\Delta T \tag{21a}$$

$$f_z = \eta\,H_x^2\left(\frac{\partial^2 w}{\partial x^2}\right) \tag{21b}$$

$$q_z = \int_A f_z\,dz = \eta\,A\,H_x^2\left(\frac{\partial^2 w}{\partial x^2}\right) \tag{21c}$$

$$H = 2b\tau_0 \tag{21d}$$

Substituting Eqs. (14), (17) and (19) into Hamilton's principle [Eq. (12)], the motion equations are obtained as:

$$\frac{\partial N}{\partial x} + f - I_1\left(\frac{\partial^2 u}{\partial t^2}\right) = 0 \tag{22a}$$

$$\frac{\partial^2 M}{\partial x^2} + q + I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2}\right) - I_1\left(\frac{\partial^2 w}{\partial t^2}\right) = 0 \tag{22b}$$

The bending moment and spring constants based on the obtained BCs from Hamilton's principle will be described as:

$$-M - K_{\mathrm{RL}} \frac{\partial w}{\partial x} = 0, \quad \frac{\partial M}{\partial x} - K_{\mathrm{TL}}\, w = 0 \text{ at } x = 0 \tag{23a}$$

$$M - K_{\mathrm{RR}} \frac{\partial w}{\partial x} = 0, \quad -\frac{\partial M}{\partial x} - K_{\mathrm{TR}}\, w = 0 \text{ at } x = L \tag{23b}$$

The bending moment with considering surface and nonlocal effects for piezoelectric nanobeam will be obtained as:

$$M = \int \sigma_{xx} z dA + \int \tau_{xx} z dA - \int e_{31} \phi_z z dA \tag{24}$$

$$M = -(EI)^* \frac{\partial^2 w}{\partial x^2} + \frac{2I\nu}{h}\left(\tau_0 \frac{\partial^2 w}{\partial x^2} - \rho_0 \frac{\partial^2 w}{\partial t^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^2 w}{\partial x^2}, \tag{25}$$

where the effective bending stiffness of nanobeam will be determined as $(EI)^* = E\left({}^{bh^3}\!/_{12}\right) + E_s\left({}^{h^3}\!/_{6} + {}^{bh^2}\!/_{2}\right)$. The bending moment using the nonlocal elasticity will be obtained as:

$$M - \mu \frac{\partial^2 M}{\partial x^2} = -(EI)^* \frac{\partial^2 w}{\partial x^2} + \frac{2I\nu}{h}\left(\tau_0 \frac{\partial^2 w}{\partial x^2} - \rho_0 \frac{\partial^2 w}{\partial t^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^2 w}{\partial x^2} \tag{26}$$

$$\begin{aligned} M =& \mu\left(-I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2}\right) + I_1\left(\frac{\partial^2 w}{\partial t^2}\right) - q\right) - (EI)^* \frac{\partial^2 w}{\partial x^2} \\ &+ \frac{2I\nu}{h}\left(\tau_0 \frac{\partial^2 w}{\partial x^2} - \rho_0 \frac{\partial^2 w}{\partial t^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^2 w}{\partial x^2} \end{aligned} \tag{27}$$

Substituting Eqs. (27) into (22), the constitutive motion equation is as:

$$\begin{aligned} &\left(1 - \mu \frac{\partial^2}{\partial x^2}\right)\left(-I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2}\right) + I_1\left(\frac{\partial^2 w}{\partial t^2}\right) - q_w\right) - (EI)^* \frac{\partial^4 w}{\partial x^4} \\ &+ \frac{2I\nu}{h}\left(\tau_0 \frac{\partial^4 w}{\partial x^4} - \rho_0 \frac{\partial^4 w}{\partial x^2 \partial t^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^4 w}{\partial x^4} = 0 \end{aligned} \tag{28}$$

For the free vibration response of piezoelectric nanobeam, a harmonic motion is assumed with the natural frequency of ω as:

$$w(x,\ t) = W(x) e^{i\omega t} \tag{29}$$

Substituting Eqs. (29) into (28) resulted to:

$$\begin{aligned} &\left(1 - \mu \frac{\partial^2}{\partial x^2}\right)\left(\omega^2 I_2\left(\frac{\partial^2 W}{\partial x^2}\right) - \omega^2 I_1 W(x) - q_w\right) - (EI)^* \frac{\partial^4 W}{\partial x^4} \\ &+ \frac{2I\nu}{h}\left(\tau_0 \frac{\partial^4 W}{\partial x^4} + \rho_0 \omega^2 \frac{\partial^4 W}{\partial x^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^4 W}{\partial x^4} = 0 \end{aligned} \tag{30}$$

Table 1 Some basic theorems of DTM for equations of motion

Original function	Transformed function
$f(x) = g(x) \pm h(x)$	$F(K) = G(K) \pm H(K)$
$f(x) = \lambda g(x)$	$F(K) = \lambda G(K)$
$f(x) = g(x)\, h(x)$	$F(K) = \sum_{l=0}^{K} G(K-l) H(l)$
$f(x) = \frac{d^n g(x)}{dx^n}$	$F(K) = \frac{(k+n)!}{k!} G(K+n)$
$f(x) = x^n$	$F(K) = \delta(K-n) = \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases}$

2.4 Solution procedure

To derive an analytical solution for Eq. (30) due to the nature homogeneity is relatively difficult. In this condition, the DTM is utilized to translate the governing equations into ordinary equation. The manner of differential transform method is explained briefly in the following. In this method, differential transformation of k^{th} derivative function $y(x)$ and inverse of differential transformation of $Y(k)$ are explained as [49]:

$$Y(k) = \frac{1}{k!}\left[\frac{\mathrm{d}^k}{\mathrm{d}x^k} y(x)\right]_{x=0}, \tag{31a}$$

$$y(x) = \sum_{k=0}^{\infty} x^k Y(k), \tag{31b}$$

where $y(x)$ is the original function and $Y(k)$ is the transformed function. Equations (31) can be explored as:

$$y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}\left[\frac{\mathrm{d}^k}{\mathrm{d}x^k} y(x)\right]_{x=0} \tag{32}$$

The theory of the differential transformation is derived from Taylor's series expansion that can be deduced from Eq. (31a). The function $y(x)$ in Eq. (31b) can be written in a finite form as:

$$y(x) = \sum_{k=0}^{N} x^k Y(k) \tag{33}$$

From the definitions of DTM in Equations (31), fundamental theorems of differential transforms method can be utilized that are listed in Table 1 and in Table 2 tabulated the differential transformation of boundary conditions. Applying the DTM into the equation of motion resulted as:

Table 2 Transformed boundary conditions (BC) based on DTM

$X=0$		$X=L$	
Original BC	Transformed BC	Original BC	Transformed BC
$f(0)=0$	$\mathrm{F}[0]=0$	$f(L)=0$	$\sum_{k=0}^{\infty} F[k]=0$
$\frac{\mathrm{d}f(0)}{\mathrm{d}x}=0$	$F[1]=0$	$\frac{\mathrm{d}f(L)}{\mathrm{d}x}=0$	$\sum_{k=0}^{\infty} k\,F[k]=0$
$\frac{\mathrm{d}^2f(0)}{\mathrm{d}x^2}=0$	$F[2]=0$	$\frac{\mathrm{d}^2f(L)}{\mathrm{d}x^2}=0$	$\sum_{k=0}^{\infty} k(k-1)\,F[k]=0$
$\frac{\mathrm{d}^3f(0)}{\mathrm{d}x^3}=0$	$F[3]=0$	$\frac{\mathrm{d}^3f(L)}{\mathrm{d}x^3}=0$	$\sum_{k=0}^{\infty} k(k-1)(k-2)F[k]=0$

$$\begin{aligned}&\left((EI)^* + \frac{2I_3\nu}{h}\tau_0 - \frac{e_{31}^2}{\lambda_{33}} - \mu^2\left(N_T + P_{\text{electric}} + H + K_P + q_z\right)\right)\frac{(k+4)!}{k!}W[k+4]\\&+\left(\frac{2I_3\nu\rho_0\omega^2}{h} + \left(N_T + P_{\text{electric}} + H + K_P + q_z\right) + \mu^2K_w - I_1\mu^2\omega^2\right)\frac{(k+2)!}{k!}W[k+2]\\&-\left(K_w + I_1\omega^2\right)W[k]=0\end{aligned} \tag{34}$$

By simplifying Eq. (34), the following relation will be obtained:

$$W[k+4] = \frac{-\left(\frac{2I_3\nu\rho_0\omega^2}{h} + \left(N_T + P_{\text{electric}} + H + K_P + q_z\right) + \mu^2K_w - I_1\mu^2\omega^2\right)\frac{(k+2)!}{k!}W[k+2] + \left(K_w + I_1\omega^2\right)W[k]}{\left((EI)^* + \frac{2I_3\nu}{h}\tau_0 - \frac{e_{31}^2}{\lambda_{33}} - \mu^2\left(N_T + P_{\text{electric}} + H + K_P + q_z\right)\right)\frac{(k+4)!}{k!}} \tag{35}$$

Besides, applying the Table 2 relations to boundary conditions results:

Clamped–Elastic supported (C–E):

$$\begin{aligned}&W[0]=0,\quad W[1]=0,\quad W[2]=C_1,\quad W[3]=C_2\\&\sum_{k=0}^{\infty} EI_s k(k-1)\,W[k] - \sum_{k=0}^{\infty} K_{\mathrm{RR}}k\,W[k]=0\\&\sum_{k=0}^{\infty} EI_s k(k-1)(k-2)\,W[k] + \sum_{k=0}^{\infty} K_{\mathrm{TR}}\,W[k]=0\end{aligned} \tag{36a}$$

Simply–Elastic supported (S–E):

$$\begin{aligned}&W[0]=0,\quad W[2]=C_1,\quad W[1]=0,\quad W[3]=C_2\\&\sum_{k=0}^{\infty} EI_s k(k-1)\,W[k] - \sum_{k=0}^{\infty} K_{\mathrm{RR}}k\,W[k]=0\\&\sum_{k=0}^{\infty} EI_s k(k-1)(k-2)\,W[k] + \sum_{k=0}^{\infty} K_{\mathrm{TR}}\,W[k]=0\end{aligned} \tag{36b}$$

Elastic–Elastic supported (E–E):

$$\begin{aligned}&W[0]=C_1,\quad W[1]=C_2,\quad W[2]=-\frac{K_{\mathrm{RL}}C_2}{2EI_s},\quad W[3]=\frac{K_{\mathrm{TL}}C_1}{6EI_s}\\&\sum_{k=0}^{\infty} EI_s k(k-1)\,W[k] - \sum_{k=0}^{\infty} K_{\mathrm{RR}}k\,W[k]=0\\&\sum_{k=0}^{\infty} EI_s k(k-1)(k-2)\,W[k] + \sum_{k=0}^{\infty} K_{\mathrm{TR}}\,W[k]=0\end{aligned} \tag{36c}$$

It should be noted that the transitional and rotational spring constants at each end of nanobeam will be expressed in the terms of moment of inertia and the Young's modulus as:

$$K_{\mathrm{TL}} = \frac{\beta_{\mathrm{TL}}E_cI}{L^3},\quad K_{\mathrm{RL}} = \frac{\beta_{\mathrm{RL}}E_cI}{L},\quad K_{\mathrm{TR}} = \frac{\beta_{\mathrm{TR}}E_cI}{L^3},\quad K_{\mathrm{RR}} = \frac{\beta_{\mathrm{RR}}E_cI}{L}, \tag{37}$$

where β denotes the spring constant factor.

3 Numerical results and discussion

This section is dedicated to results obtained for analysis of megneto-thermo-mechanical vibration behavior of piezoelectric nanobeam incorporating nonlocal parameter, surface effect, elastic foundation for various elastic boundary conditions based on the Euler–Bernoulli beam theory. The material properties of nanobeam made of AL are given in Table 3.

It should be noted that in the case in which the spring constants at each end in Elastic–Elastic boundary condition set to be a high value as 10^6, the resultant natural frequencies correspond to the clamped boundary condition. Moreover, other conventional boundary conditions will be obtained for various values of spring constants,

Table 3 Al material properties

Properties		Al [52, 53]
Young's modules	E	70 GPa
Poisson's ratio	ν	0.3
Mass density	ρ	2700 $\mathrm{Kg/m^3}$
Residual surface tensions	τ^0	0.9108 N m
Elasticity surface modules	E^s	5.1882 N m
Density of surface layer	ρ^s	5.46×10^{-7} $\mathrm{Kg\ m^2}$
Thermal coefficient	λ_1	2.56×10^{-6} 1/k
Piezoelectric coefficient	e_{31}	-10 $\mathrm{C\ m^2}$
Dielectric constants	λ_{33}	1.0275×10^{-8}

Table 4 Comparison of the non-dimensional fundamental frequency for a nanobeam with various nonlocal parameters for simply-supported boundary condition ($L=10$ nm, $h/b=2$)

L/h	μ	Reddy [48]	Present
10	0	9.8696	9.8695
	1	9.4159	9.4158
	2	9.0195	9.0194
	3	8.6693	8.6692
	4	8.3569	8.3568
	5	8.0761	8.0760
20	0	9.8696	9.8695
	1	9.4159	9.4158
	2	9.0195	9.0194
	3	8.6693	8.6692
	4	8.3569	8.3568
	5	8.0761	8.0760
100	0	9.8696	9.8695
	1	9.4159	9.4158
	2	9.0195	9.0194
	3	8.6693	8.6692
	4	8.3569	8.3568
	5	8.0761	8.0760

for instance, in Simply–Elastic (S–E) boundary condition by substituting $\beta_{\mathrm{RR}} = 10^{-6}$, $\beta_{\mathrm{TR}} = 10^{6}$ is corresponding to S–S case and $\beta_{\mathrm{RR}} = \beta_{\mathrm{TR}} = 10^{6}$ related to S-C case. And also for clamped–elastic (C–E) boundary condition, by substituting $\beta_{\mathrm{RR}} = \beta_{\mathrm{TR}} = 10^{-6}$ is equivalent to C-F boundary condition and if by assuming $\beta_{\mathrm{RR}} = \beta_{\mathrm{TR}} = 10^{6}$, the obtained boundary condition is C–C. To validate the accuracy of the numerical results, comparison between the present results and available results obtained by Reddy [48] for simply-supported boundary condition and Eltaher [50] for clamped–clamped, simply–clamped and clamped–free boundary condition are tabulated in Tables 4 and 5, respectively. As it is indicated from Tables 4 and 5, an excellent agreement is obtained for all classical boundary conditions. And also, for the fact that the experimental tests do not exist in detail for various conditions, in this study, the first dimensional frequency versus aspect ratio is also compared with the results which are obtained by molecular dynamic simulation respresented in Ansari and Sahmani [51] and showed to be in acceptable agreement which is depicted in Fig. 2.

After that, the convergence study is performed to determine the minimum number of iterations required to obtain stable and accurate results for classical boundary condition as it is mentioned above in Table 6. As it can be observed for C–F boundary condition, the first three natural frequencies converge after 17th, 23rd and 35th iterations with four digit precisions, respectively. And also for S–S case, these natural frequencies converge after 17th, 27th, and 35th iterations, while for C-S, they converge after 19th, 29th, and 37th iterations, and at last, for C–C boundary condition, the resultant natural frequencies converge after 23th, 33th, and 41st iterations. Therefore, the number of iterations is selected as $k=25$ for the results reported here for first natural frequencies.

First, natural frequencies of nanobeam are presented in Table 7 for various elastic boundary condition with different nonlocal parameter and spring constant factors. It can be found from the results that by incorporating the surface effects, the natural frequencies corresponding to all values of nonlocal parameters increase which indicates the fact that by considering the surface effects, the stiffness of nanobeam will be increased. Also, it is observed from Table 7 that for spring constant factor between 10^{-6} and 1, increasing nonlocal parameter causes an increase in natural frequencies for C–E and E–E boundary conditions. Also,

Table 5 Comparison of the non-dimensional fundamental frequency for a nanobeam with various nonlocal parameters for Clamped–Simply, Clamped–Clamped, Clamped–Free boundary condition ($L=10$ nm, $h/b=2$)

μ	C–S		C–C		C–F	
	Present paper	Eltaher [50]	Present paper	Eltaher [50]	Present paper	Eltaher [50]
0	15.4177	15.4189	22.3724	22.3744	3.5160	3.5161
1	14.5988	14.9929	21.1083	21.1096	3.5312	3.5314
2	13.8959	14.5997	20.0323	20.0330	3.5469	3.5470
3	13.2841	14.2353	19.1025	19.1028	3.5629	3.5630
4	12.7456	13.8965	18.2891	18.2890	3.5794	3.5795
5	12.2669	13.5803	17.5699	17.5696	3.5962	3.5963

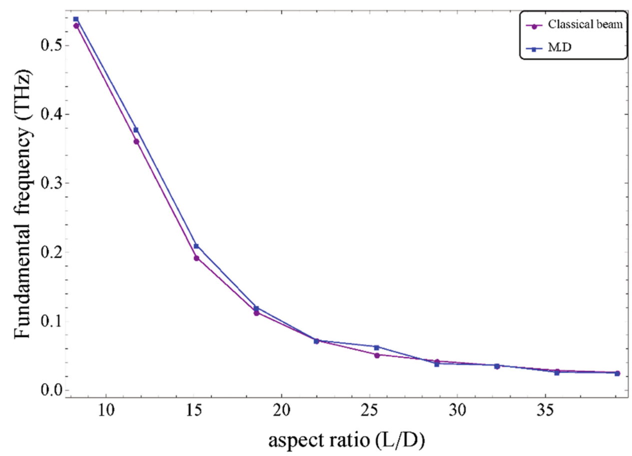


Fig. 2 The variation of the first frequency of classical beam versus aspect ratio in compare with the molecular dynamic results represented in Ref [51]

for spring constant factor between 1 and 10^6, the increase in nonlocal parameter tends to decrease the natural frequency in C–E and E–E boundary conditions and also for all values of spring stiffness in S-E boundary condition.

Table 8 illustrates the effect of the natural frequencies for various elastic boundary conditions with different length and spring constant factors. As shown in Table 8, the surface effect is very sensitive to beam length and thickness. At the nanoscale, the fundamental frequency of the nanobeam by considering surface effect is approximately 3–4 orders higher than that of a classical beam including nonlocal parameters. At the microscale, the surface effect decreases and it is approximately 1.5–2 times greater than the classical. At a macroscale, surface effects are completely ignored and natural frequency of beam is similar to the classical beam.

Table 9 implies the influences of elastic foundation including Winkler and Pasternak foundation on the natural frequencies. It can be readily observed that the value of the natural frequency has a direct relation with the stiffness of the elastic foundation. By increasing the value of Winkler–Pasternak foundation coefficients, the natural frequency increases which indicates the fact that by considering the elastic foundation, the stiffness of nanobeam will be increased, and hence, natural frequency will be grown.

In Table 10, effects of temperature change on the natural frequency of nanobeam are tabulated. With the increase of temperature change, generally, the natural frequencies

Table 6 Convergence study of nanobeam for the first three natural frequencies by considering surface effects ($L/h = 10,\ L = 20$ nm, $\mu = 2$ nm^2, $h/b = 2$)

k	C–F			S–S			C–S			C–C		
	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$
11	9.8174	–	–	14.7937	–	–	23.1375	–	–	–	–	–
13	9.8738	27.2969	–	14.7544	–	–	22.6873	–	–	–	–	–
15	9.8705	28.8027	–	14.7568	35.5376	–	22.7261	–	–	32.2324	63.7672	–
17	9.8707	28.4702	–	14.7567	38.2431	–	22.7231	–	–	32.8312	45.1801	–
19	9.8707	28.5145	–	14.7567	37.4262	–	22.7233	45.6402	–	32.7439	75.0133	–
21	9.8707	28.5099	–	14.7567	37.4976	53.8799	22.7233	46.3445	–	32.7519	52.4444	–
23	9.8707	28.5102	–	14.7567	37.4906	3755.63	22.7233	46.2342	–	32.7513	56.3325	–
25	9.8707	28.5102	–	14.7567	37.4912	58.6594	22.7233	46.2452	–	32.7513	54.7216	–
27	9.8707	28.5102	58.6377	14.7567	37.4911	59.4117	22.7233	46.2442	–	32.7513	54.8436	–
29	9.8707	28.5102	59.9023	14.7567	37.4911	59.2893	22.7233	46.2443	68.2207	32.7513	54.8293	–
31	9.8707	28.5102	59.6625	14.7567	37.4911	59.3021	22.7233	46.2443	68.4173	32.7513	54.8307	76.5658
33	9.8707	28.5102	59.6865	14.7567	37.4911	59.3021	22.7233	46.2443	68.3926	32.7513	54.8306	78.7267
35	9.8707	28.5102	59.6842	14.7567	37.4911	59.3010	22.7233	46.2443	68.3951	32.7513	54.8306	78.1387
37	9.8707	28.5102	59.6844	14.7567	37.4911	59.3010	22.7233	46.2443	68.3949	32.7513	54.8306	78.1989
39	9.8707	28.5102	59.6844	14.7567	37.4911	59.3010	22.7233	46.2443	68.3949	32.7513	54.8306	78.1919
41	9.8707	28.5102	59.6844	14.7567	37.4911	59.3010	22.7233	46.2443	68.3949	32.7513	54.8306	78.1926
43	9.8707	28.5102	59.6844	14.7567	37.4911	59.3010	22.7233	46.2443	68.3949	32.7513	54.8306	78.1926

Table 7 Non-dimensional fundamental frequency corresponding to first mode obtained with various elastic boundary condition ($L = 20$ nm, $L/h = 10$, $T = 20$, $V = 0.02$)

β	μ=0		μ=1		μ=2		μ=3		μ=4		μ=5	
	NSE*	NE*	NSE*	NE*	NSE*	NE*	NSE*	NE*	NSE*	NE*	NSE*	NE*
C–E												
10^{-6}	9.9778	3.9528	9.9796	3.9463	9.9815	3.9397	9.9835	3.9332	9.9857	3.9267	9.9881	3.9202
10^{-5}	9.9778	3.9529	9.9796	3.9463	9.9815	3.9397	9.9835	3.9332	9.9858	3.9267	9.9881	3.9202
10^{-4}	9.9780	3.9529	9.9798	3.9464	9.9817	3.9398	9.9837	3.9333	9.9859	3.9267	9.9883	3.9202
10^{-3}	9.9798	3.9536	9.9815	3.9471	9.9834	3.9405	9.9854	3.9339	9.9876	3.9274	9.9899	3.9208
10^{-2}	9.9977	3.9608	9.9991	3.9540	10.0006	3.9473	10.0022	3.9405	10.0040	3.9338	10.0059	3.9271
10^{-1}	10.1661	4.0300	10.1639	4.0215	10.1618	4.0129	10.1599	4.0044	10.1581	3.9959	10.1563	3.9874
1	11.2675	4.5629	11.2432	4.5418	11.2189	4.5207	11.1947	4.4996	11.1705	4.4784	11.1463	4.4572
10^{1}	15.6067	6.97205	15.5388	6.9166	15.4713	6.86153	15.4041	6.8066	15.3373	6.7520	15.2708	6.6976
10^{2}	29.4752	14.7494	29.3060	14.6149	29.1381	14.4814	28.9714	14.3487	28.8059	14.2169	28.6416	14.0860
10^{3}	32.9045	20.6253	32.6549	20.2668	32.4089	19.9189	32.1665	19.5812	31.9275	19.2531	31.6918	18.9344
10^{4}	33.1037	21.3705	32.8487	20.9584	32.5976	20.5618	32.3502	20.1798	32.1065	19.8113	31.8662	19.4557
10^{5}	33.1225	21.4439	32.8670	21.0262	32.6154	20.6246	32.3676	20.2381	32.1233	19.8655	31.8827	19.5061
10^{6}	33.1243	21.4512	32.8688	21.0330	32.6172	20.6309	32.3693	20.2439	32.1250	19.8709	31.8843	19.5111
S–E												
10^{-6}	0.0051	0.0012	0.0051	0.0016	0.0051	0.0011	0.0051	0.0011	0.0051	0.0011	0.0050	0.0011
10^{-5}	0.0162	0.0037	0.0162	0.0037	0.0161	0.0036	0.0160	0.0035	0.0160	0.0034	0.0160	0.0033
10^{-4}	0.0512	0.0118	0.0511	0.0116	0.0509	0.0113	0.0508	0.0111	0.0506	0.0108	0.0505	0.0106
10^{-3}	0.1619	0.0374	0.1615	0.0366	0.1610	0.0358	0.1605	0.0351	0.1600	0.0343	0.1596	0.0335
10^{-2}	0.5110	0.1184	0.5095	0.1160	0.5080	0.1136	0.5065	0.1111	0.5050	0.1086	0.5035	0.1060
10^{-1}	1.5821	0.3787	1.5773	0.3715	1.5724	0.3642	1.5674	0.3567	1.5625	0.34906	1.5576	0.3412
1	4.3380	1.2819	4.3213	1.26392	4.3047	1.24581	4.28815	1.22752	4.2715	1.20905	4.2549	1.19038
10^{1}	10.3349	4.3342	10.2882	4.29522	10.2417	4.25632	10.1955	4.21753	10.1494	4.17885	10.1036	4.14027
10^{2}	20.6655	10.7741	20.5323	10.6509	20.4004	10.5289	20.2698	10.4082	20.1405	10.2886	20.0125	10.1702
10^{3}	22.7096	13.7547	22.5386	13.5090	22.3701	13.2710	22.2038	13.0404	22.0399	12.8166	21.8783	12.5994
10^{4}	22.8789	14.1004	22.7045	13.8342	22.5327	13.5775	22.3634	13.3297	22.1965	13.0901	22.0319	12.8583
10^{5}	22.8954	14.1350	22.7207	13.8667	22.5486	13.6081	22.3790	13.3584	22.2118	13.1172	22.0469	12.8839
10^{6}	22.8970	14.1384	22.7223	13.8700	22.5502	13.6111	22.3805	13.3613	22.2133	13.1199	22.0484	12.8865
E–E												
10^{-6}	10.3705	4.0853	10.3731	4.0785	10.3759	4.0716	10.3789	4.0649	10.3820	4.0581	10.3854	4.0513
10^{-5}	10.0155	3.9656	10.0173	3.9590	10.0193	3.9524	10.0215	3.9458	10.0238	3.9393	10.0262	3.9328
10^{-4}	9.9817	3.9542	9.9835	3.9476	9.9854	3.9411	9.9875	3.9345	9.9897	3.9280	9.9921	3.9215
10^{-3}	9.9802	3.9538	9.9819	3.9472	9.9838	3.9406	9.9858	3.9340	9.9879	3.9275	9.9903	3.9210
10^{-2}	9.9977	3.9608	9.9991	3.9540	10.0006	3.9473	10.0023	3.9405	10.0040	3.9338	10.0060	3.9271
10^{-1}	10.1661	4.0300	10.1639	4.0215	10.1619	4.0129	10.1599	4.0044	10.1581	3.9959	10.1563	3.9874
1	11.2675	4.5629	11.2432	4.5418	11.2189	4.5207	11.1947	4.4996	11.1705	4.4784	11.1463	4.4572
10^{1}	15.6067	6.9720	15.5388	6.9166	15.4713	6.8615	15.4041	6.8066	15.3373	6.7520	15.2708	6.6976
10^{2}	29.4752	14.7494	29.3060	14.6149	29.1381	14.4814	28.9714	14.3487	28.8059	14.2169	28.6416	14.0860
10^{3}	32.9045	20.6253	32.6549	20.2668	32.4089	19.9189	32.1665	19.5812	31.9275	19.2531	31.6918	18.9344
10^{4}	33.1037	21.3705	32.8487	20.9584	32.5976	20.5618	32.3502	20.1798	32.1065	19.8113	31.8662	19.4557
10^{5}	33.1225	21.4439	32.8670	21.0262	32.6154	20.6246	32.3676	20.2381	32.1233	19.8655	31.8827	19.5061
10^{6}	33.1243	21.4512	32.8688	21.0330	32.6172	20.6309	32.3693	20.2439	32.1250	19.8709	31.8843	19.5111

*NE nonlocal effect, NSE coupling nonlocal and surface effects

Table 8 The natural frequency of nanobeam with various length and spring constant factor by considering surface effect ($L/h = 10,\ b/h = 2,\ \Delta T = 20°C,\ \ V = 0.02,\ \ \mu = 2\ \mathrm{nm}^2$)

β	L								
	10×10^{-9}	20×10^{-9}	50×10^{-9}	100×10^{-9}	10^{-6}	10×10^{-6}	100×10^{-6}	10^{-3}	10^{-2}
C–E									
10^{-6}	13.4097	9.9815	7.5566	6.1365	3.8915	3.5393	3.5015	3.4977	3.4973
10^{-5}	13.4096	9.9815	7.5566	6.1365	3.8915	3.5393	3.5015	3.4977	3.4974
10^{-4}	13.4093	9.9817	7.5568	6.1367	3.8917	3.5395	3.5017	3.4979	3.4975
10^{-3}	13.4056	9.9834	7.5590	6.1388	3.8934	3.5411	3.5034	3.4996	3.4992
10^{-2}	13.3695	10.0006	7.5805	6.1591	3.9105	3.5578	3.5200	3.5162	3.5158
10^{-1}	13.0865	10.1618	7.7847	6.3522	4.0723	3.7152	3.6770	3.6732	3.6728
1	12.4306	11.2189	9.14832	7.6521	5.1416	4.7466	4.7045	4.7002	4.6998
10^{1}	15.1948	15.4713	13.6616	11.7843	8.2242	7.6398	7.5772	7.5709	7.5702
10^{2}	26.5341	29.1381	27.0504	23.6660	16.5468	15.3288	15.1975	15.1842	15.1829
10^{3}	27.8308	32.4089	33.2265	30.8225	23.1742	21.6626	21.4971	21.4804	21.4787
10^{4}	27.9042	32.5976	33.6978	31.5010	24.0240	22.5080	22.3415	22.3246	22.3230
10^{5}	27.9112	32.6154	33.7423	31.5657	24.1076	22.5915	22.4249	22.4081	22.4064
10^{6}	27.9119	32.6172	33.7467	31.5722	24.1159	22.5999	22.4332	22.4164	22.4147
S–E									
10^{-6}	0.0055	0.0050	0.0042	0.0036	0.0027	0.0026	0.0026	0.0026	0.0026
10^{-5}	0.0175	0.0161	0.0134	0.0115	0.0087	0.0083	0.0082	0.0082	0.0082
10^{-4}	0.0555	0.0509	0.0424	0.0366	0.0275	0.0262	0.0261	0.0261	0.0261
10^{-3}	0.1757	0.1610	0.1341	0.1157	0.0872	0.0831	0.0826	0.0826	0.0826
10^{-2}	0.5539	0.5080	0.4234	0.3653	0.2753	0.2623	0.2609	0.2608	0.2608
10^{-1}	1.6985	1.5724	1.3171	1.1377	0.8570	0.8162	0.8119	0.8115	0.8114
1	4.4276	4.3047	3.7075	3.2195	2.4058	2.2837	2.2708	2.2695	2.2694
10^{1}	10.0144	10.2417	9.0876	7.8838	5.6281	5.2638	5.2249	5.2210	5.2206
10^{2}	18.2118	20.4004	19.6030	17.4727	12.5413	11.6730	11.5792	11.5697	11.5688
10^{3}	19.2834	22.3701	22.8599	21.2239	16.1294	15.1323	15.0233	15.0123	15.0112
10^{4}	19.3704	22.5327	23.1578	21.6015	16.5533	15.5510	15.4412	15.4302	15.4290
10^{5}	19.3789	22.5486	23.1870	21.6387	16.5957	15.5929	15.4831	15.4720	15.4709
10^{6}	19.3797	22.5502	23.1899	21.6424	16.5999	15.5971	15.4873	15.4762	15.4751
E–E									
10^{-6}	13.7989	10.3759	8.3227	7.5799	–	–	–	–	–
10^{-5}	13.4472	10.0193	7.6248	6.2480	4.8471	–	–	–	–
10^{-4}	13.4130	9.9855	7.5636	6.1477	3.9641	–	–	–	–
10^{-3}	13.4059	9.9838	7.5597	6.1399	3.9005	3.6076	–	–	–
10^{-2}	13.3695	10.0006	7.5807	6.1593	3.9113	3.5644	3.5859	–	–
10^{-1}	13.0865	10.1619	7.7848	6.3523	4.0724	3.7159	3.6835	3.7395	–
1	12.4306	11.2189	9.1483	7.6521	5.1416	4.7468	4.7052	4.7073	4.7727
10^{1}	15.1948	15.4713	13.6616	11.7843	8.2243	7.6399	7.5773	7.5718	7.5790
10^{2}	26.5341	29.1381	27.0504	23.6660	16.5468	15.3288	15.1975	15.1844	15.1845
10^{3}	27.8308	32.4089	33.2265	30.8225	23.1742	21.6626	21.4971	21.4804	21.4791
10^{4}	27.9042	32.5976	33.6978	31.5010	24.0240	22.5080	22.3415	22.3246	22.3230
10^{5}	27.9112	32.6154	33.7423	31.5657	24.1076	22.5915	22.4249	22.4081	22.4064
10^{6}	27.9119	32.6172	33.7467	31.5722	24.1159	22.5999	22.4332	22.4164	22.4147

Table 9 The natural frequency of nanobeam with various spring constant factor by considering surface effect and nonlocal effect for differenr values of elastic foundation ($L = 20$ nm, $L/h = 10$, $b/h = 2$, $\Delta T = 20\,°C$, $V = 0.02$, $\mu = 2$ nm^2)

β	Kp = 0			Kp = 10			Kp = 100		
	Kw = 0	Kw = 10	kw = 100	Kw = 0	Kw = 10	kw = 100	Kw = 0	Kw = 10	kw = 100
C–E									
10^{-6}	9.9815	10.5671	14.9464	9.3370	9.9407	14.3494	6.0093	6.8243	11.8765
10^{-5}	9.9815	10.5671	14.9464	9.3370	9.9407	14.3494	6.0095	6.8245	11.8766
10^{-4}	9.9817	10.5672	14.9464	9.3373	9.9410	14.3494	6.0112	6.8260	11.8774
10^{-3}	9.9834	10.5687	14.9461	9.3405	9.9438	14.3503	6.0289	6.8415	11.8856
10^{-2}	10.0006	10.5832	14.9436	9.3719	9.9718	14.3590	6.2021	6.9931	11.9663
10^{-1}	10.1618	10.7200	14.9237	9.6631	10.2319	14.4416	7.6455	8.2893	12.7029
1	11.2189	11.6451	14.9766	11.3734	11.7884	15.0420	13.5899	13.9213	16.6140
10^{1}	15.4713	15.7106	17.7202	16.1267	16.3563	18.2940	21.1391	21.3147	22.8342
10^{2}	29.1381	29.2385	30.1202	29.7701	29.8694	30.7425	34.7129	34.8040	35.6097
10^{3}	32.4089	32.4693	33.0072	33.2230	33.2820	33.8080	39.7753	39.8255	40.2739
10^{4}	32.5976	32.6558	33.1748	33.4222	33.4790	33.9858	40.0767	40.1244	40.5508
10^{5}	32.6154	32.6734	33.1906	33.4410	33.4976	34.0026	40.1048	40.1523	40.5767
10^{6}	32.6172	32.6752	33.1922	33.4429	33.4995	34.0043	40.1076	40.1550	40.5793
S–E									
10^{-6}	0.0050	3.1623	10.0000	0.0065	3.1623	10.0000	0.0132	3.1623	10.0000
10^{-5}	0.0161	3.1623	10.0000	0.0205	3.1623	10.0000	0.0416	3.1626	10.0001
10^{-4}	0.0509	3.1627	10.0000	0.0647	3.1629	10.0001	0.1317	3.1650	10.0008
10^{-3}	0.1610	3.1660	10.0002	0.2045	3.1686	10.0010	0.4163	3.1893	10.0077
10^{-2}	0.5080	3.1995	10.0019	0.6450	3.2242	10.0102	1.3127	3.4212	10.0766
10^{-1}	1.5724	3.5031	10.0211	1.9883	3.7091	10.0978	4.0364	5.1105	10.7022
1	4.3048	5.2255	10.3029	5.2583	6.0368	10.7456	10.2991	10.7212	13.9560
10^{1}	10.2417	10.5997	13.3917	11.1536	11.4841	14.1100	17.1797	17.3993	19.2617
10^{2}	20.4004	20.5274	21.6265	21.3239	21.4471	22.5162	27.9311	28.0352	28.9500
10^{3}	22.3701	22.4606	23.2587	23.4159	23.5027	24.2683	31.2475	31.3138	31.9034
10^{4}	22.5327	22.6206	23.3959	23.5878	23.6718	24.4145	31.5131	31.5765	32.1417
10^{5}	22.5486	22.6362	23.4093	23.6045	23.6883	24.4288	31.5387	31.6019	32.1648
10^{6}	22.5502	22.6378	23.4106	23.6062	23.6900	24.4302	31.5413	31.6044	32.1671
E–E									
10^{-6}	10.3759	10.9448	15.2464	9.7270	10.3119	14.6374	6.3514	7.1301	12.0698
10^{-5}	10.0193	10.6032	14.9752	9.3742	9.9761	14.3768	6.0413	6.8528	11.8943
10^{-4}	9.9855	10.5708	14.9493	9.3410	9.9445	14.3522	6.0144	6.8288	11.8792
10^{-3}	9.9838	10.5690	14.9464	9.3409	9.9441	14.3506	6.0292	6.8417	11.8858
10^{-2}	10.0006	10.5832	14.9437	9.3720	9.9718	14.3590	6.2021	6.9932	11.9663
10^{-1}	10.1619	10.7200	14.9237	9.6631	10.2319	14.4416	7.6455	8.2893	12.7029
1	11.2189	11.6451	14.9766	11.3734	11.7884	15.0420	13.5899	13.9213	16.6140
10^{1}	15.4713	15.7106	17.7202	16.1267	16.3563	18.2940	21.1391	21.3147	22.8342
10^{2}	29.1381	29.2385	30.1202	29.7701	29.8694	30.7425	34.7129	34.8040	35.6097
10^{3}	32.4089	32.4693	33.0072	33.2230	33.2820	33.8080	39.7753	39.8255	40.2739
10^{4}	32.5976	32.6558	33.1748	33.4222	33.4790	33.9858	40.0767	40.1244	40.5508
10^{5}	32.6154	32.6734	33.1906	33.4410	33.4976	34.0026	40.1048	40.1523	40.5767
10^{6}	32.6172	32.6752	33.1922	33.4429	33.4995	34.0043	40.1076	40.1550	40.5793

Table 10 The natural frequency of nanobeam by considering surface and nonlocal effects for various spring constant factor and temperature change ($L = 20$ nm, $L/h = 10$, $h/b = 0.2$, $V = 0.2$, $\mu = 2\,\text{nm}^2$)

β	ΔT						
	0	10	20	50	100	150	200
C–E							
10^{-6}	11.2168	11.2132	11.2096	11.1989	11.1811	11.1634	11.1458
10^{-5}	11.2168	11.2132	11.2096	11.1989	11.1811	11.1634	11.1458
10^{-4}	11.2167	11.2132	11.2096	11.1989	11.1811	11.1633	11.1457
10^{-3}	11.2161	11.2125	11.2089	11.1982	11.1805	11.1628	11.1452
10^{-2}	11.2097	11.2062	11.2027	11.1922	11.1747	11.1574	11.1401
10^{-1}	11.1550	11.1521	11.1491	11.1404	11.1259	11.1114	11.0970
1	11.0787	11.0789	11.0790	11.0795	11.0804	11.0813	11.0822
10^{1}	14.4191	14.4218	14.4245	14.4326	14.4461	14.4595	14.4730
10^{2}	28.1328	28.1354	28.1379	28.1456	28.1584	28.1712	28.1839
10^{3}	31.1273	31.1305	31.1338	31.1435	31.1597	31.1759	31.1921
10^{4}	31.3003	31.3036	31.3069	31.3167	31.3331	31.3495	31.3658
10^{5}	31.3167	31.3199	31.3232	31.3331	31.3495	31.3659	31.3823
10^{6}	31.3183	31.3216	31.3248	31.3347	31.3511	31.3675	31.3839
S–E							
10^{-6}	0.0008	0.0009	0.0009	0.0010	0.0012	0.0013	0.0014
10^{-5}	0.0026	0.0027	0.0029	0.0032	0.0037	0.0041	0.0044
10^{-4}	0.0083	0.0087	0.0091	0.0101	0.0116	0.0129	0.0141
10^{-3}	0.0264	0.0276	0.0287	0.0319	0.0366	0.0408	0.0445
10^{-2}	0.0870	0.0906	0.0941	0.1038	0.1182	0.1310	0.1426
10^{-1}	0.3623	0.3704	0.3784	0.4013	0.4368	0.4696	0.5003
1	2.0900	2.0984	2.1069	2.1321	2.1734	2.2139	2.2537
10^{1}	8.6518	8.6561	8.6604	8.6734	8.6950	8.7165	8.7380
10^{2}	18.8807	18.8846	18.8886	18.9004	18.9202	18.9399	18.9596
10^{3}	20.6738	20.6782	20.6826	20.6957	20.7176	20.7394	20.7612
10^{4}	20.8232	20.8277	20.8321	20.8453	20.8673	20.8893	20.9113
10^{5}	20.8379	20.8423	20.8467	20.8599	20.8819	20.9039	20.9259
10^{6}	20.8393	20.8437	20.8481	20.8614	20.8834	20.9054	20.9274
E–E							
10^{-6}	11.6153	11.6117	11.6081	11.5974	11.5796	11.5618	11.5442
10^{-5}	11.2554	11.2518	11.2482	11.2375	11.2197	11.2020	11.1843
10^{-4}	11.2206	11.2170	11.2134	11.2027	11.1849	11.1672	11.1495
10^{-3}	11.2165	11.2129	11.2093	11.1986	11.1809	11.1632	11.1456
10^{-2}	11.2098	11.2062	11.2027	11.1922	11.1748	11.1574	11.1401
10^{-1}	11.1550	11.1521	11.1491	11.1404	11.1259	11.1114	11.0970
1	11.0787	11.0789	11.0790	11.0795	11.0804	11.0813	11.0822
10^{1}	14.4191	14.4218	14.4245	14.4326	14.4461	14.4595	14.4730
10^{2}	28.1328	28.1354	28.1379	28.1456	28.1584	28.1712	28.1839
10^{3}	31.1273	31.1305	31.1338	31.1435	31.1597	31.1759	31.1921
10^{4}	31.3003	31.3036	31.3069	31.3167	31.3331	31.3495	31.3658
10^{5}	31.3166	31.3199	31.3232	31.3331	31.3495	31.3659	31.3823
10^{6}	31.3183	31.3216	31.3248	31.3347	31.3511	31.3675	31.3839

Table 11 The natural frequency of nanobeam by considering surface and nonlocal effects for various spring constant factor and voltage external ($L = 20$ nm, $L/h = 10$, $h/b = 2$, $\mu = 2\text{nm}^2$, $\Delta T = 20^0$C)

β	V								
	−0.2	−0.15	−0.1	−0.05	0	0.05	0.1	0.15	0.2
C–E									
10^{-6}	8.8600	9.0887	9.3313	9.5896	9.8656	10.1619	10.4817	10.8291	11.2096
10^{-5}	8.8601	9.0887	9.3313	9.5896	9.8656	10.1619	10.4817	10.8291	11.2096
10^{-4}	8.8605	9.0891	9.3316	9.5899	9.8658	10.1621	10.4818	10.8291	11.2096
10^{-3}	8.8649	9.0929	9.3348	9.5924	9.8678	10.1634	10.4825	10.8292	11.2089
10^{-2}	8.9084	9.1304	9.3664	9.6180	9.8874	10.1769	10.4897	10.8299	11.2027
10^{-1}	9.3075	9.4763	9.6588	9.8564	10.0709	10.3044	10.5596	10.8397	11.1491
1	11.5322	11.4510	11.3750	11.3051	11.2420	11.1868	11.1403	11.1039	11.0790
10^{1}	16.6647	16.4009	16.1329	15.8605	15.5835	15.3016	15.0147	14.7224	14.4245
10^{2}	30.2918	30.0357	29.7761	29.5129	29.2460	28.9752	28.7003	28.4213	28.1379
10^{3}	33.8995	33.5668	33.2307	32.8909	32.5474	32.2000	31.8488	31.4934	31.1338
10^{4}	34.1080	33.7707	33.4300	33.0858	32.7378	32.3861	32.0305	31.6708	31.3069
10^{5}	34.1276	33.7899	33.4488	33.1041	32.7558	32.4037	32.0476	31.6875	31.3232
10^{6}	34.1295	33.7918	33.4507	33.1060	32.7576	32.4054	32.0493	31.6892	31.3248
S–E									
10^{-6}	0.0074	0.0070	0.0065	0.0059	0.0054	0.0047	0.0039	0.0028	0.0009
10^{-5}	0.0235	0.0220	0.0205	0.0188	0.0169	0.0148	0.0122	0.0089	0.0029
10^{-4}	0.0742	0.0697	0.0648	0.0595	0.0535	0.0467	0.0387	0.0282	0.0091
10^{-3}	0.2346	0.2203	0.2049	0.1880	0.1692	0.1478	0.1223	0.0892	0.0287
10^{-2}	0.7397	0.6947	0.6461	0.5930	0.5338	0.4664	0.3861	0.2821	0.0941
10^{-1}	2.2774	2.1400	1.9918	1.8301	1.6506	1.4465	1.2050	0.8959	0.3784
1	5.9427	5.6157	5.2666	4.8905	4.4805	4.0259	3.5093	2.8970	2.1069
10^{1}	11.8694	11.5217	11.1619	10.7890	10.4013	9.9971	9.5742	9.1298	8.6604
10^{2}	22.0699	21.7054	21.3325	20.9507	20.5596	20.1585	19.7468	19.3237	18.8886
10^{3}	24.2692	23.8513	23.4257	22.9919	22.5495	22.0980	21.6368	21.1652	20.6826
10^{4}	24.4492	24.0273	23.5977	23.1600	22.7137	22.2584	21.7935	21.3183	20.8321
10^{5}	24.4667	24.0444	23.6144	23.1764	22.7298	22.2741	21.8088	21.3332	20.8467
10^{6}	24.4685	24.0461	23.6161	23.1780	22.7314	22.2757	21.8103	21.3347	20.8481
E–E									
10^{-6}	9.2458	9.4766	9.7212	9.9815	10.2593	10.5573	10.8785	11.2270	11.6081
10^{-5}	8.8967	9.1256	9.3685	9.6270	9.9033	10.1999	10.5199	10.8675	11.2482
10^{-4}	8.8642	9.0928	9.3353	9.5936	9.8696	10.1658	10.4856	10.8330	11.2134
10^{-3}	8.8653	9.0932	9.3352	9.5928	9.8682	10.1638	10.4829	10.8296	11.2093
10^{-2}	8.9084	9.1304	9.3664	9.6181	9.8874	10.1769	10.4897	10.8299	11.2027
10^{-1}	9.3075	9.4763	9.6588	9.8564	10.0709	10.3044	10.5596	10.8397	11.1491
1	11.5322	11.4510	11.3750	11.3051	11.2420	11.1868	11.1403	11.1039	11.0790
10^{1}	16.6647	16.4009	16.1329	15.8605	15.5835	15.3016	15.0147	14.7224	14.4245
10^{2}	30.2918	30.0357	29.7761	29.5129	29.2460	28.9752	28.7003	28.4213	28.1379
10^{3}	33.8995	33.5668	33.2307	32.8909	32.5474	32.2001	31.8488	31.4934	31.1338
10^{4}	34.1080	33.7707	33.4300	33.0858	32.7378	32.3861	32.0305	31.6708	31.3069
10^{5}	34.1276	33.7899	33.4488	33.1041	32.7558	32.4037	32.0476	31.6875	31.3232
10^{6}	34.1295	33.7918	33.4507	33.1060	32.7576	32.4054	32.0493	31.6892	31.3248

Table 12 The natural frequency of nanobeam by considering surface and nonlocal effects for various spring constant factor and magnetic field ($L = 20\,\text{nm}$, L/h = 10, b = 0.5 h, $\Delta T = 0°\text{C}$, $\mu = 2\,\text{nm}^2$)

β	H									
	1	2	3	4	5	6	7	8	9	10
C–E										
10^{-6}	9.8692	9.8647	9.8571	9.8466	9.8331	9.8167	9.7973	9.7751	9.7501	9.7223
10^{-5}	9.8692	9.8647	9.8572	9.8466	9.8331	9.8167	9.7974	9.7752	9.7501	9.7224
10^{-4}	9.8694	9.8649	9.8574	9.8468	9.8333	9.8169	9.7976	9.7754	9.7504	9.7226
10^{-3}	9.8714	9.8669	9.8593	9.8488	9.8354	9.8190	9.7997	9.7775	9.7526	9.7249
10^{-2}	9.8909	9.8865	9.8791	9.8688	9.8557	9.8396	9.8207	9.7991	9.7746	9.7475
10^{-1}	10.0737	10.0701	10.0642	10.0560	10.0455	10.0326	10.0175	10.0002	9.9807	9.9592
1	11.2413	11.2422	11.2438	11.2460	11.2489	11.2524	11.2566	11.2614	11.2670	11.2733
10^{1}	15.5799	15.5844	15.5917	15.6021	15.6153	15.6315	15.6507	15.6727	15.6976	15.7255
10^{2}	29.2426	29.2468	29.2539	29.2639	29.2766	29.2922	29.3106	29.3319	29.3559	29.3827
10^{3}	32.5430	32.5485	32.5576	32.5704	32.5868	32.6068	32.6305	32.6578	32.6887	32.7232
10^{4}	32.7334	32.7390	32.7482	32.7611	32.7778	32.7981	32.8220	32.8497	32.8810	32.9159
10^{5}	32.7514	32.7569	32.7662	32.7791	32.7958	32.8161	32.8401	32.8678	32.8991	32.9341
10^{6}	32.7532	32.7587	32.7680	32.7809	32.7975	32.8179	32.8419	32.8696	32.9009	32.9359
S–E										
10^{-6}	0.0054	0.0054	0.0054	0.0054	0.0054	0.0055	0.0055	0.0056	0.0056	0.0057
10^{-5}	0.0169	0.0170	0.0170	0.0171	0.0172	0.0173	0.0174	0.0176	0.0178	0.0179
10^{-4}	0.0536	0.0537	0.0538	0.0541	0.0543	0.0547	0.0551	0.0556	0.0562	0.0567
10^{-3}	0.1693	0.1696	0.1702	0.1709	0.1718	0.1730	0.1743	0.1758	0.1775	0.1794
10^{-2}	0.5342	0.5352	0.5368	0.5391	0.5421	0.5456	0.5498	0.5546	0.5600	0.5659
10^{-1}	1.6516	1.6546	1.6596	1.6666	1.6755	1.6863	1.6991	1.7136	1.7299	1.7480
1	4.4827	4.4896	4.5009	4.5167	4.5369	4.5615	4.5904	4.6235	4.6607	4.7019
10^{1}	10.4034	10.4097	10.4201	10.4347	10.4534	10.4763	10.5032	10.5342	10.5691	10.6081
10^{2}	20.5617	20.5680	20.5784	20.5930	20.6118	20.6347	20.6618	20.6929	20.7282	20.7675
10^{3}	22.5519	22.5590	22.5708	22.5873	22.6085	22.6344	22.6649	22.7001	22.7399	22.7844
10^{4}	22.7161	22.7233	22.7352	22.7518	22.7732	22.7993	22.8301	22.8656	22.9058	22.9506
10^{5}	22.7321	22.7393	22.7512	22.7679	22.7893	22.8154	22.8462	22.8818	22.9220	22.9668
10^{6}	22.7337	22.7409	22.7528	22.7695	22.7909	22.8170	22.8478	22.8834	22.9236	22.9684
E–E										
10^{-6}	10.2630	10.2584	10.2508	10.2402	10.2266	10.2101	10.1906	10.1683	10.1431	10.1151
10^{-5}	9.9069	9.9024	9.8948	9.8843	9.8708	9.8543	9.8350	9.8128	9.7877	9.7599
10^{-4}	9.8732	9.8686	9.8611	9.8506	9.8371	9.8206	9.8013	9.7791	9.7541	9.7263
10^{-3}	9.8718	9.8672	9.8597	9.8492	9.8357	9.8194	9.8001	9.7779	9.7530	9.7252
10^{-2}	9.8909	9.8865	9.8792	9.8689	9.8557	9.8396	9.8208	9.7991	9.7747	9.7476
10^{-1}	10.0737	10.0701	10.0642	10.0560	10.0455	10.0326	10.0175	10.0002	9.9808	9.9592

Table 12 (continued)

β	H									
	1	2	3	4	5	6	7	8	9	10
1	11.2413	11.2422	11.2438	11.2460	11.2489	11.2524	11.2566	11.2614	11.2670	11.2733
10^1	15.5799	15.5844	15.5917	15.6021	15.6153	15.6315	15.6507	15.6727	15.6976	15.7255
10^2	29.2426	29.2468	29.2539	29.2639	29.2766	29.2922	29.3106	29.3319	29.3559	29.3827
10^3	32.5430	32.5485	32.5576	32.5704	32.5868	32.6068	32.6305	32.6578	32.6887	32.7232
10^4	32.7334	32.7390	32.7482	32.7611	32.7778	32.7981	32.8220	32.8497	32.8810	32.9159
10^5	32.7514	32.7569	32.7662	32.7791	32.7958	32.8161	32.8401	32.8678	32.8991	32.9341
10^6	32.7532	32.7587	32.7680	32.7809	32.7975	32.8179	32.8419	32.8696	32.9009	32.9359

increase; the reason is that the increase in temperature change brings in more increase in the nanobeam stiffness and finally tends to grow in natural frequency. This behavior is right just for spring constant factor between 1 and 10^6 and for S–E completely. For C–E and E–E boundary condition, the natural frequency decreases in the range of 10^{-6}–1.

Table 11 presents the effect of the external voltage on the first natural frequency of nanobeam for various elastic boundary conditions. As it is observed, the positive voltage decreases the natural frequency and negative voltage increases the natural frequency generally. This is due to the fact that positive voltage weakens the nanobeam stiffness and negative voltage strengthens the nanobeam stiffness. But for C–E and E–E, this is opposite in the range of 10^{-6}–1 that natural frequency increases.

The effects of magnetic field on the natural frequency are examined in Table 12. It is found that natural frequency of nanobeam increases with increasing the value of magnetic potential. Obviously, the effect of external voltage is opposite to the magnetic potential. For this analysis, the behaviors of C–E and E–E boundary conditions are opposite to S–E within 10^{-6}–1 that the natural frequency decreases.

According to Tables 13, 14 and 15, the influences of each rotational and transitional spring constant factor change on the natural frequencies for C–E, S–E, and E–E boundary conditions are listed, respectively. As indicated from numerical results which are presented in detail, for a constant value of transitional spring, as the rotational spring increases, the natural frequencies increase. Moreover, for a constant value of transitional spring, with the increase in the value of rotational spring, the natural frequencies tend to increase.

The effects of external voltage and temperature change on the natural frequencies are investigated in Table 16. It can be seen that by increasing external voltage, for the spring stiffness within 10^{-6}–1 in C–E and E–E, the natural frequencies increase and for S–E, natural frequency declines. Also, in the range of 1–10^6, by growing external voltage, for all boundary condition, the natural frequency decreases. Also, by rising temperature change for C–E and E–E within 10^{-6}–1, the natural frequency decreases, whereas for S–E, natural frequency rises. Also, in the range of 1–10^6, growing temperature changes, natural frequency increases.

Depicted in Figs. 3, 4 and 5 are the variation of natural frequency for C–E, S–E, E–E boundary conditions by incorporating coupling of surface effect and nonlocal effect and just by considering nonlocal effect without surface effects, corresponding to various values of nonlocal parameters and spring constant factor. It can be observed that by considering the surface effect and by increasing the value of spring constant factor, natural frequency increases. It

Table 13 The natural frequency of nanobeam for C–E boundary condition ($L = 20$ nm, $L/h = 10$, $h/b = 2$, $\mu = 2\text{nm}^2$, $\Delta T = 20$)

β_{TR}	β_{RR}						
	1	10	10^2	10^3	10^4	10^5	10^6
1	10.6732	10.0973	9.9625	9.9472	9.9456	9.9455	9.9455
10	15.8327	14.6525	14.3156	14.2754	14.2713	14.2709	14.2709
10^2	22.2031	26.0375	28.4937	28.8950	28.9378	28.9421	28.9425
10^3	23.1836	28.4689	31.2748	31.6534	31.6924	31.6963	31.6967
10^4	23.2793	28.6625	31.4314	31.7980	31.8357	31.8395	31.8398
10^5	23.2888	28.6813	31.4462	31.8116	31.8491	31.8529	31.8533
10^6	23.2898	28.6831	31.4477	31.8130	31.8505	31.8542	31.8546

Table 14 First natural frequency of nanobeam for S-E boundary condition ($L = 20$ nm, $L/h = 10$, $h/b = 2$, $\mu = 2\,\text{nm}^2$, $\Delta T = 20^0$C)

β_{TR}	β_{RR}						
	1	10	10^2	10^3	10^4	10^5	10^6
1	4.4805	4.7923	4.8636	4.8716	4.8724	4.8725	4.8725
10	10.4749	10.4013	10.3787	10.3760	10.3757	10.3756	10.3756
10^2	15.4602	18.8116	20.5596	20.8124	20.8389	20.8415	20.8418
10^3	16.2193	20.3275	22.2874	22.5496	22.5766	22.5793	22.5796
10^4	16.2963	20.4674	22.4274	22.6870	22.7138	22.7164	22.7167
10^5	16.3040	20.4812	22.4411	22.7004	22.7271	22.7298	22.7300
10^6	16.3047	20.4826	22.4424	22.7017	22.7284	22.7311	22.7314

Table 15 First natural frequency of P-FG nanobeam for E-E boundary condition ($L = 20$ nm, $L/h = 10$, $h/b = 2$, $\mu = 2\,\text{nm}^2$, $\Delta T = 20$)

$\beta_{TL} - \beta_{TR}$	$\beta_{RR} - \beta_{RL}$						
	1	10	10^2	10^3	10^4	10^5	10^6
1	10.6732	10.0973	9.9625	9.9472	9.9456	9.9455	9.9455
10	15.8327	14.6525	14.3156	14.2754	14.2713	14.2709	14.2709
10^2	22.2031	26.0375	28.4937	28.8950	28.9378	28.9421	28.9425
10^3	23.1836	28.4689	31.2748	31.6534	31.6924	31.6963	31.6967
10^4	23.2793	28.6625	31.4314	31.7980	31.8357	31.8395	31.8398
10^5	23.2888	28.6813	31.4462	31.8116	31.8491	31.8529	31.8533
10^6	23.2898	28.6831	31.4477	31.8130	31.8505	31.8542	31.8546

means that surface energy effects and spring constant factor play more important role than the nonlocal parameter.

And also, the efficiency of piezoelectric material is described as follows:

$$\text{Efficiency of piezoelectric material} = \frac{\text{Natural frequency of piezoelectric nanobeam}}{\text{Natural frequency of nanobeam}}$$

The variation of efficiency of piezoelectric material versus voltage is depicted in Fig. 6 for various classical boundary conditions. As it can be observed, the negative voltage tends to increase the natural frequency of nanobeam, while the positive voltage decreases the value of natural frequencies. Also, for the C–F boundary condition, the voltage sign has inverse influence on the natural frequencies in comparison with other boundary conditions.

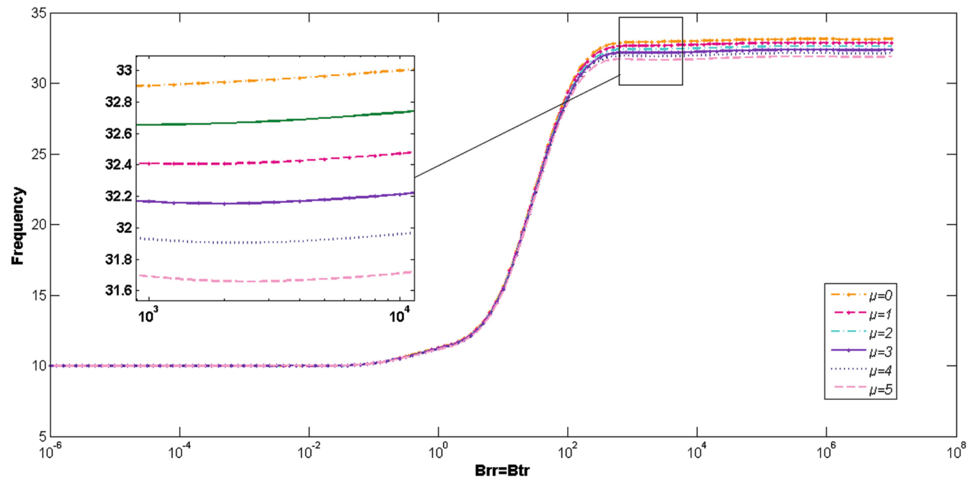
4 Conclusion

In the present study, the magneto-thermo-mechanical vibration analysis of piezoelectric nanobeam rested in elastic medium is studied by considering surface and nonlocal

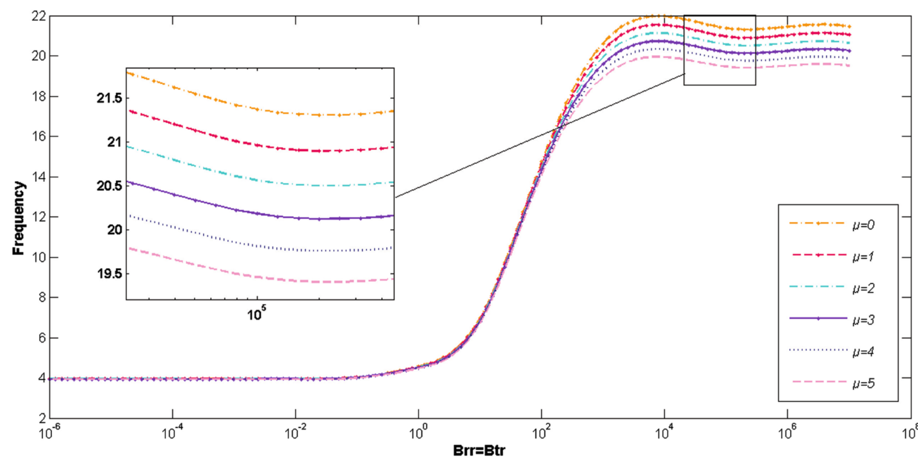
Table 16 The natural frequency of nanobeam with various spring constant factor by considering surface effect and nonlocal effect ($L = 20$ nm, $L/h = 10$, $h/b = 2$, $\mu = 2$ nm^2)

β	$V=-0.2$			$V=0$			$V=0.2$		
	$T=0$	$T=25$	$T=50$	$T=0$	$T=25$	$T=50$	$T=0$	$T=25$	$T=50$
C–E									
10^{-6}	8.8640	8.8590	8.8541	9.8707	9.8643	9.8579	11.2168	11.2079	11.1989
10^{-5}	8.8641	8.8591	8.8541	9.8707	9.8643	9.8579	11.2168	11.2079	11.1989
10^{-4}	8.8645	8.8595	8.8546	9.8709	9.8645	9.8581	11.2167	11.2078	11.1989
10^{-3}	8.8689	8.8639	8.8590	9.8729	9.8665	9.8601	11.2161	11.2072	11.1982
10^{-2}	8.9122	8.9074	8.9026	9.8924	9.8861	9.8799	11.2097	11.2009	11.1922
10^{-1}	9.3104	9.3068	9.3031	10.0749	10.0699	10.0649	11.1550	11.1477	11.1404
1	11.5307	11.5326	11.5345	11.2410	11.2423	11.2436	11.0787	11.0791	11.0795
10^{1}	16.6600	16.6659	16.6717	15.5785	15.5847	15.5910	14.4191	14.4259	14.4326
10^{2}	30.2873	30.2930	30.2987	29.2412	29.2472	29.2532	28.1328	28.1392	28.1456
10^{3}	33.8936	33.9010	33.9084	32.5412	32.5489	32.5567	31.1273	31.1354	31.1435
10^{4}	34.1019	34.1095	34.1170	32.7316	32.7394	32.7472	31.3003	31.3085	31.3167
10^{5}	34.1215	34.1291	34.1366	32.7495	32.7574	32.7652	31.3166	31.3249	31.3331
10^{6}	34.1235	34.1310	34.1385	32.7513	32.7592	32.7670	31.3183	31.3265	31.3347
S–E									
10^{-6}	0.0074	0.0074	0.0074	0.0053	0.0054	0.0054	0.0008	0.0009	0.0010
10^{-5}	0.0234	0.0235	0.0235	0.0169	0.0169	0.0170	0.0026	0.0029	0.0032
10^{-4}	0.0741	0.0742	0.0743	0.0534	0.0536	0.0537	0.0083	0.0092	0.0101
10^{-3}	0.2343	0.2346	0.2349	0.1689	0.1693	0.1698	0.0264	0.0293	0.0319
10^{-2}	0.7389	0.7399	0.7409	0.5327	0.5341	0.5355	0.0870	0.0958	0.1038
10^{-1}	2.2750	2.2779	2.2809	1.6472	1.6514	1.6557	0.3623	0.3823	0.4013
1	5.9370	5.9441	5.9512	4.4728	4.4824	4.4920	2.0900	2.1111	2.1321
10^{1}	11.8633	11.8710	11.8786	10.3942	10.4031	10.4120	8.6518	8.6626	8.6734
10^{2}	22.0634	22.0715	22.0796	20.5525	20.5614	20.5702	18.8807	18.8906	18.9004
10^{3}	24.2618	24.2711	24.2804	22.5415	22.5515	22.5615	20.6738	20.6848	20.6957
10^{4}	24.4417	24.4511	24.4604	22.7057	22.7158	22.7259	20.8232	20.8343	20.8453
10^{5}	24.4592	24.4686	24.4780	22.7217	22.7318	22.7419	20.8379	20.8489	20.8599
10^{6}	24.461	24.4703	24.4797	22.7233	22.7334	22.7435	20.8393	20.8503	20.8614
E–E									
10^{-6}	9.2498	9.2448	9.2398	10.2645	10.2580	10.2516	11.6153	11.6063	11.5974
10^{-5}	8.9007	8.8957	8.8908	9.9084	9.9020	9.8956	11.2554	11.2464	11.2375
10^{-4}	8.8682	8.8632	8.8582	9.8747	9.8683	9.8619	11.2206	11.2116	11.2027
10^{-3}	8.8692	8.8643	8.8593	9.8733	9.8669	9.8605	11.2165	11.2075	11.1986
10^{-2}	8.9123	8.9074	8.9026	9.8924	9.8862	9.8799	11.2098	11.2010	11.1922
10^{-1}	9.3104	9.3068	9.3032	10.0749	10.0699	10.0649	11.1550	11.1477	11.1404
1	11.5307	11.5326	11.5345	11.2410	11.2423	11.2436	11.0787	11.0791	11.0795
10^{1}	16.6600	16.6659	16.6717	15.5785	15.5847	15.5910	14.4191	14.4259	14.4326
10^{2}	30.2873	30.2930	30.2987	29.2412	29.2472	29.2532	28.1328	28.1392	28.1456
10^{3}	33.8936	33.9010	33.9084	32.5412	32.5489	32.5567	31.1273	31.1354	31.1435
10^{4}	34.1019	34.1095	34.1170	32.7316	32.7394	32.7472	31.3003	31.3085	31.3167
10^{5}	34.1215	34.1291	34.1366	32.7495	32.7574	32.7652	31.3166	31.3249	31.3331
10^{6}	34.1235	34.1310	34.1385	32.7513	32.7592	32.7670	31.3183	31.3265	31.3347

Fig. 3 The variation of the first dimensionless frequency of nanobeam with spring constant factor for different nonlocal parameters for C–E boundary condition ($L = 20$ nm, $L/h = 10$, $h/b = 2$)

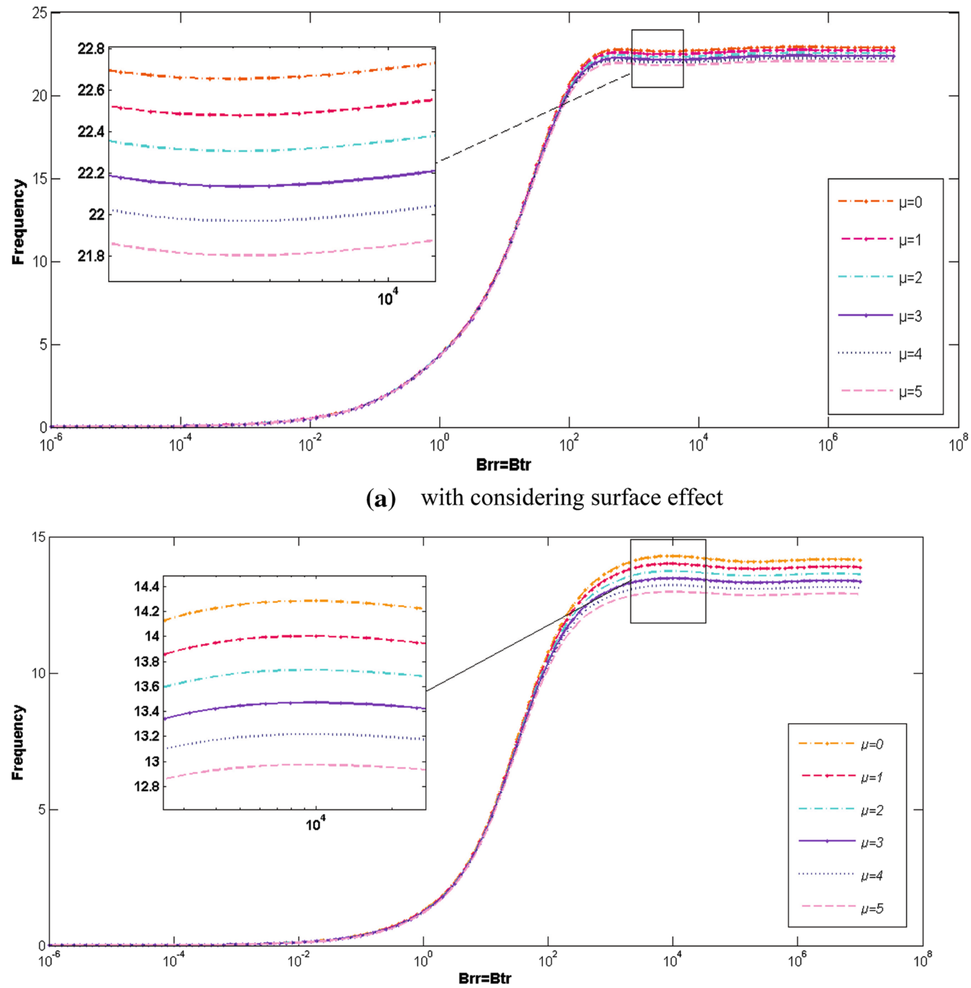


(a) with considering surface effect



(b) without considering surface effect

effects for various elastic boundary conditions. To assume the elastic boundary condition, the rotational and transitional springs at each end are located which are used to introduce small deflections and moments. The main goal of the presented study is to investigate the influence of nonlocal parameter, piezoelectric voltage, temperature change, surface effects, elastic medium, magnetic field and length of nanobeam natural frequencies for different values of spring constants in elastic supports. As it is shown, the Hamilton's principle is used to derive the motion equations based on the Euler–Bernoulli beam model. Then, DTM as an efficient and accurate numerical tool was implemented to solve vibration equations of piezoelectric nanobeam with elastic boundary condition. The convergence study and validation were also presented as well as the numerical results which clarified the influence of nonlocal parameter, piezoelectric voltage, temperature change, surface effects, elastic medium, magnetic field, length of nanobeam and spring constants on the first dimensionless natural frequency of piezoelectric nanobeam. Based on the presented numrecal results:



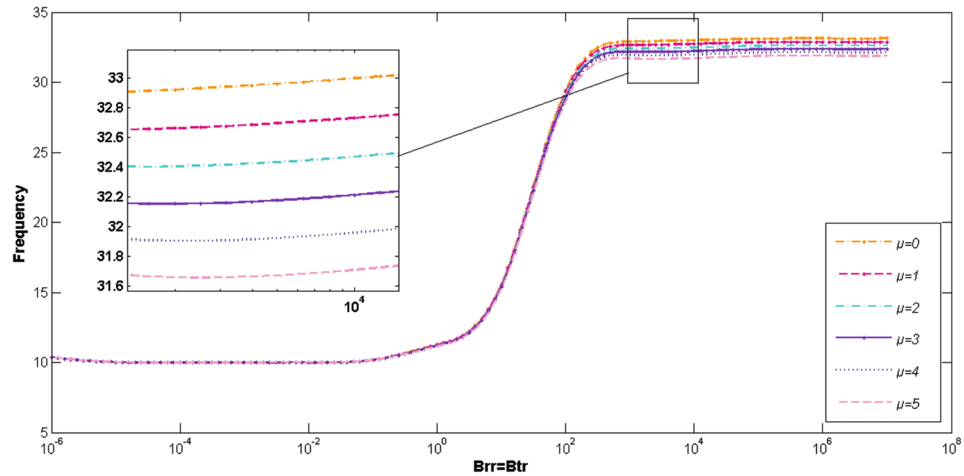
(a) with considering surface effect

(b) without considering surface effect

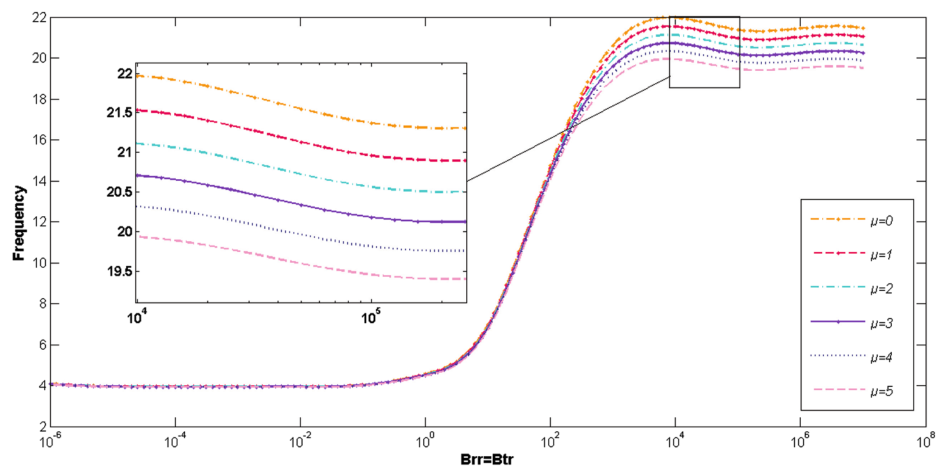
Fig. 4 The variation of the first dimensionless frequency of nanobeam with spring constant factor for different nonlocal parameters with considering surface effect for S–E boundary condition ($L = 20$ nm, $L/h = 10$, $h/b = 2$)

1. Increasing the spring constants leads to increase in the natural frequencies for all named boundary conditions.
2. For the variation of spring constant from 10^{-6} to 1 in C–E and E–E, the obtained result has the manner near the C–F boundary condition which shows different behavior in each case.
3. As the temperature change increases, the natural frequencies increase.
4. Increasing the voltage parameter from negative to positive amount tends to decrease the fundamental natural frequencies.
5. The natural frequencies decrease in the case that the nonlocal parameter increases.

Fig. 5 The variation of the first dimensionless frequency of nanobeam with spring constant factor for different nonlocal parameters with considering surface effect for E–E boundary condition ($L = 20$ nm, $L/h = 10$, $h/b = 2$)



(a) with considering surface effect



(b) without considering surface effect

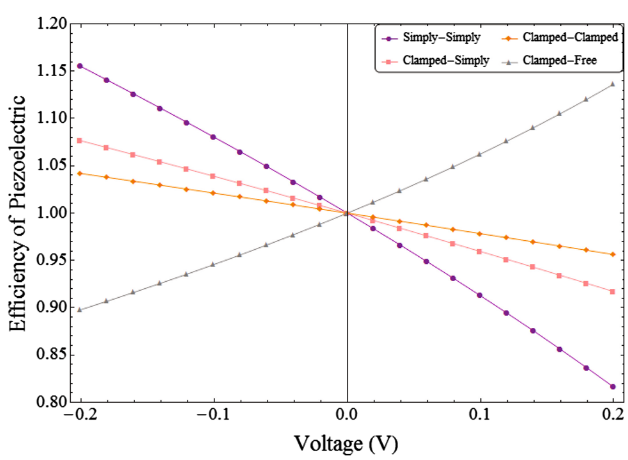


Fig. 6 Variation of efficiency of piezoelectric material versus voltage for various boundary conditions

6. As the length of nanobeam increases from the nanoscale to macroscale, the natural frequencies decrease and reach the classical value.
7. The surface effects tend to increase the natural frequencies which means the stiffness of nanobeam increases.

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