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Thermo-electro-mechanical vibration analysis of size-dependent nanobeam resting on elastic medium under axial preload in presence of surface effect

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Abstract In the present manuscript, a nonclassical beam theory is developed to analyze free vibration of piezoelectric nanobeam by considering surface effects resting on Winkler–Pasternak elastic medium and thermal loading with axial preload. The nonclassical Eringen theory is utilized to incorporate the length-scale parameter to account for the small-scale effect, while the Gurtin–Murdoch model is employed to inject the surface effects including surface elasticity, surface stress and surface density. The governing equations are derived using Hamilton's principle in the framework of Euler–Bernoulli beam theory. The governing partial differential equations of motions of system are reduced to a set of algebraic equations with the help of differential transformation method as a semi-analytical–numerical. The mathematical derivations and numerical results are presented in detail for various boundary conditions. Some numerical examples are illustrated in order to investigate the effect of several parameters such as the nonlocal parameter, piezoelectric voltage, surface effects, temperature change, axial preload and elastic medium parameters. Moreover, it is also indicated that the numerical results have good agreement with previous studies.

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1 Introduction

With the advance of energy harvesting from ambient energy sources to generate other forms of energy such as electricity, MEM and NEM sensors are employed as energy harvesters in different fields. As an example, in tire pressure monitoring systems (TPMS) which directly affect vehicle's handling [1], one approach for harvesting is to provide sustainable power for wireless sensors, while it will be possible by piezoelectric materials. The piezoelectric materials when subjected in electrical loads will produce mechanical deformations for their intrinsic electro-mechanical coupling effect [2]. Many studies have been done around the macroscopic piezoelectric materials such as works done in Refs. [3–5].

Because of high sensitivity of MEMs/NEMs, investigating their mechanical properties and behavior is crucial to design and manufacture of these structures. It is observed that the piezoelectric nanostructures have different mechanical, electrical and also physical and chemical properties than their bulk. Among all nanostructures, investigating piezoelectric nanobeams' vibrational behavior is important according to their wide range of application such as nanosensors, actuators, generators, transistors and diodes [6]. Pan et al. [7] reported the ZnO piezoelectric nanostructures among other piezoelectric nanomaterials such as ZnS, PZT, GaN and $BaTiO_3$, and studying nanostructures has received attention from researchers such as those in [8, 9].

To date, studying buckling and vibration characteristics of nanobeams has been theoretically investigated in the content of size-dependent beam analysis by several researchers. However, it is known that the classical continuum mechanics cannot predict and explain the size-dependent behavior of nanostructures. Therefore, in order to incorporate the size effects in continuum mechanics, there are several higher-order continuum theories, such as the couple stress theory [34], strain gradient theory [10], nonlocal elasticity theory [11], micropolar theory [12] and surface elasticity [13]. But, among all of these theories, the nonlocal elasticity of Eringen [11] is proved to be capable for different analyses on nanostructures. The simplicity in the application of nonlocal elasticity of Eringen resulted in rapid extension of this theory in different static and dynamic analyses for various nanostructures such as those in [14, 15].

As the experimental and atomistic simulations show, the ratio of surface to volume has undeniable role in nanoscale problems [16]. Gurtin and Murdoch [17] represented the surface elasticity theory and also its applications in nanostructures which have a good agreement with experimental measurements [18]. Recently, surface effects on static and dynamic analyses of elastic materials have been extensively studied by researchers, such as works done in [19–22]. Further, the nonlocal and surface effects are two inevitable fields which studied simultaneously. For instance, Hosseini-Hashemi et al. [6] studied the static and dynamic analyses with considering both surface and nonlocal effects on piezoelectric functionally graded nanobeams. In a similar work, they also studied the nonlinear free vibration of piezoelectric functionally graded nanobeams in the framework of the Euler–Bernoulli beam theory [23].

Also, the study of surface and nonlocal effects on buckling and vibrational characteristics of Al and Si nanotubes is done by Ebrahimi et al. [24]. They also used DTM for the first time to investigate the nondimensional natural frequency and buckling loads. In a similar work [25], the influence of various surface effects on fundamental natural frequencies was mentioned.

In addition, investigating the thermal effect in high-temperature conditions has considerable effect on dynamic behavior of nanobeams. Afterward, the effect of temperature changing is studied in different researches, such as Ebrahimi and Salari [26], whom studied thermo-electrical buckling characteristics of functionally graded piezoelectric (FGP) nanobeams based on Timoshenko theory subjected to in-plane thermal loads and applied electrical voltage, while the motion equations have been solved by Navier-type solution. Furthermore, Ansari et al. [27] studied thermo-electro-mechanical vibration of postbuckled piezoelectric nanobeams based on the nonlocal elasticity theory in the framework of Timoshenko beam theory. In a similar work, thermo-electric-mechanical vibration of the piezoelectric nanobeams is done by Ke and Wang [2] based on the nonlocal theory and Timoshenko beam theory. Moreover, Ebrahimi and Barati [28] studied the dynamic modeling of a magneto-thermo-piezoelectrically actuated nanobeam based on the higher-order shear deformation beam theory. And also, study of the electro-mechanical coupling behavior of piezoelectric nanowires in the presence of both surface and small-scale effects based on Euler–Bernoulli beam theory is analyzed by Wang and Wang [29]. So, it can be concluded that it is necessity to consider the thermal effects in vibration analysis of nanostructures and should not be ignored in dynamic analysis.

It is known that studying the mechanical behavior of nanostructures rested in elastic medium is also achieved attention from researchers and is considered in analysis. It should be noted that the Winkler elastic foundation model consists of closely spaced elastic springs which are integrated to the bottom of beam surface. But this model cannot incorporate the continuousness of the elastic environment. Besides, Pasternak model consists of two parameters elastic foundation which is known as Winkler–Pasternak model. This model comprises a Winkler-type elastic spring besides the transverse shear stress as a result of the shear deformation in the medium, while the Winkler model incorporates the normal pressure from surrounding medium [30, 31]. A few investigations have been carried out in the open literature dealing with the elastic medium in dynamic analysis of nanostructures such as [15, 18, 32–34].

Lots of literature works presented the thermo-electro-mechanical vibration of size-dependent piezoelectric nanobeams, whereas investigating both surface and nonlocal effects in Winkler–Pasternak medium with preload is rather limited. This motivated the current work which is concerned with theoretical modeling on thermo-electro-mechanical analysis of piezoelectric nanobeams which is complemented with size-dependent model based on the extended theory of piezoelectricity and Euler–Bernoulli beam theory. The Gurtin–Murdoch model is used to incorporate the surface effects including surface elasticity, surface stress and surface density. The governing motion equations are obtained by Hamilton's principle using the nonlocal elasticity of Eringen within the framework of Euler–Bernoulli beam theory. The differential transformation method (DTM) is used as a semi-analytical–numerical technique which is simpler with high precision in comparison with other methods. However, implementing the DTM to solve similar works is also rather limited, which is used to solve the current motion equations for various boundary conditions for the first time. Next, the resultant natural frequencies are validated for different boundary conditions as it is shown that they are in good agreement with well-known literature. Finally, through some numerical examples, the influence of various parameters such as nonlocality, surface effects, temperature change, preload and elastic medium is investigated.

2 Governing equations

2.1 Nonlocal effects

The nonlocal elasticity theory introduces information about the forces between atoms in the bulk of material and the internal length scale, the constitutive equations will be obtained which will can used as the material parameters. This theory expresses that the stress at a reference point is a function of the stain at all points in the adjacent region. This assumption can explain some experimental observations of atomic and molecular scales, for example high-frequency vibration and wave dispersion [15]. For a homogeneous and nonlocal piezoelectric solid, the basic equations for stress tensor and electrical displacement at any point x in the bulk of material by neglecting the body forces can be expressed as [29]:

$$\sigma_{ij} - \mu^2\nabla^2\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \lambda_{ij}\Delta T \tag{1}$$

$$D_i - \mu^2\nabla^2 D_i = e_{ikl}\varepsilon_{kl} + \varepsilon_{ik}E_k + p_i\Delta T \tag{2}$$

where σ_{ij}, D_i are the component of the stress and electrical field, while the ε_{kl}, C_{ijkl}, e_{ikl}, λ_{ij} are the strain, elastic constant, piezoelectric constants and thermal moduli. ΔT and p_i are the temperature change and piezoelectric constants; $\mu = (e_0 a)^2$ is the nonlocal constant; moreover, $e_0 a$ is the scale length which takes the size effect into account on the response of nanostructures.

2.2 Surface effects

The energy which is associated by atoms in the surface layers affects the mechanical properties of nanostructures which have been extensively studied by researchers. So far, Gurtin and Murdoch [17] introduced the continuum framework which represented the influence of the surface elastic stress and strain field. Their model reflects the surface elastic, by assuming the surface layers as a two-dimensional film with zero thickness which is attached to the material body. Assuming that these surface layers are only because of modeling purpose and these layers do not actually exist, so the type of surface is not defined. For the surface of the piezoelectric materials, the local stresses and electrical displacement can be expressed as [29]:

$$\tau^{sl}_{\alpha\beta} = \tau^0_{\alpha\beta} + C^s_{\alpha\beta\gamma\delta}\varepsilon_{\gamma\delta} - e^s_{\alpha\beta k}E_k \tag{3}$$

$$D^{sl}_i = D^0_i + e^s_{\alpha\beta i}\varepsilon_{\alpha\beta} + k^s_{ij}E_j \tag{4}$$

where $C^s_{\alpha\beta\gamma\delta}$, $\varepsilon_{\gamma\delta}$, $e^s_{\alpha\beta k}$ and k^s_{ij} are surface elastic constant, surface strains, surface piezoelectric constants and surface dielectric constants, respectively. $\tau^{sl}_{\alpha\beta}$ and $\tau^0_{\alpha\beta}$ are the nonlocal stress tensor and residual surface stress tensor, respectively.

Assuming the same material properties in top and bottom layers, the relative stress–strain relations for surface layers will be expressed as:

$$\tau_{xx} = \tau_0 + E^s u_{x,x}, \quad E^s = 2\mu_0 + \lambda_0, \quad \tau_{nx} = \tau_0 u_{n,x} \tag{5}$$

The σ_{zz} which is often neglected in classical beam theories will be considered to satisfy the equilibrium equations and has linear relation with the beam thickness which can be obtained as:

$$\sigma_{zz} = \frac{2zv}{h}\left(\tau_o\frac{\partial^2 w}{\partial x^2} - \rho_o\frac{\partial^2 w}{\partial t^2}\right) \tag{6}$$

2.3 Problem formulation

Based on the nonlocal theory and surface effects of piezoelectric material proposed in previous section, the vibration of size-dependent piezoelectric nanobeam under thermo-electro-material in elastic foundation is analyzed. A piezoelectric nanobeam with length L $(0 \le x \le L)$, thickness h $(-h/2 \le z \le h/2)$ and width b $(-b/2 \le y \le b/2)$ is subjected to an applied voltage $\phi(x,z)$ and uniform temperature change ΔT. The Euler–Bernoulli beam theory is utilized to develop the piezoelectric nanobeam. Based on the Euler–Bernoulli beam model, the cross section of nanobeam is assumed to remain plane and normal to the deformed beam axis, and also in this study, the cross section of nanobeam is assumed to be constant along the length of the beam. The coordinate system is often supposed: The x-axis is taken along the length of the beam, the y-axis is taken along the width, while the z-axis is taken along the thickness of beam as shown in Fig. 1.

Following the Euler–Bernoulli beam model, the displacement can be expressed as follows:

$$u_1 = u - z\frac{\partial w}{\partial x}, \quad u_2 = 0, \quad u_3 = w(x,t) \tag{7}$$

where $u(x,t)$ and $w(x,t)$ are axial and lateral displacement components in mid-plane, and t is the time.

The strain relation according to Euler–Bernoulli beam theory is obtained as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial^2 x} \tag{8}$$

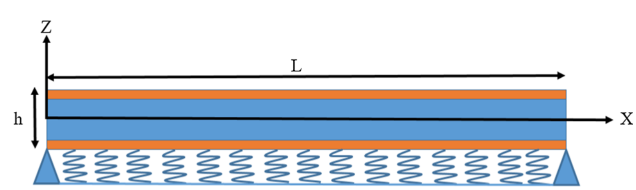

Fig. 1 Geometry of a nanobeam with length L and thickness h in elastic medium

Accordingly, the normal stress (σ_{xx}) and the surface stress (σ_s) are functions of surface strain ε_{xx} which is obtained as follows:

$$\begin{aligned} \sigma_{xx} &= E\varepsilon_{xz} + \upsilon\sigma_{zz} - e_{31}E_z \\ \sigma_s &= \tau_0 + E_s\varepsilon_{xx} \end{aligned} \tag{9}$$

where e_{31} and E_z are the piezoelectric coefficient and z-component of the electrical field, respectively. Also, τ_0 and E_s are the surface tension and surface Young's modulus.

The distribution field of the electrical potential should be assumed for the presented model. The electrical displacement can be defined as [35]:

$$\begin{aligned} &E_x = -\frac{\partial\phi}{\partial x}; \quad E_z = -\frac{\partial\phi}{\partial z}, \\ &D_x = \lambda_{11}E_x; \quad D_z = e_{31}\varepsilon_x + \lambda_{33}E_z, \\ &\frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} = 0 \end{aligned} \tag{10}$$

where λ_{11} and λ_{33} denote the dielectric constants; D_x and D_z are electrical displacements. Because λ_{11} and λ_{33} are in the same order and assuming $E_x \ll E_z$, so D_x can be ignored in comparison with D_z. Substituting Eq. (8) into Eq. (10), while the electrical boundary conditions are assumed $\phi(x,-h) = 0$, $\phi(x,h) = 2V$ the electrical potential is obtained as:

$$\phi(x,z) = -\frac{e_{31}}{\lambda_{33}}\left(\frac{z^2-h^2}{2}\right)\frac{\partial^2 w}{\partial x^2} + \left(1+\frac{z}{h}\right)V \tag{11}$$

So, in this case, the equivalent load to the piezoelectric layers will be obtained as:

$$P_{\text{electric}}(x,t) = b\int_{-h}^{h}\sigma_x^* \mathrm{d}z = 2V\mathrm{be}_{31} \tag{12}$$

with σ_x^* indicating the normal stress due to the piezoelectric property.

The governing motion equations and boundary conditions can be obtained by Hamilton's principle:

$$\int_0^t \delta(U - T + W_{\text{ext}})\mathrm{d}t = 0 \tag{13}$$

in which, U, T and W_{ext} indicate the strain energy, kinetic energy and work done by external forces.

The first variation of strain energy is obtained as:

$$U = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz} - D_x\delta E_x - D_z\delta E_z)\mathrm{d}z\,\mathrm{d}x \tag{14}$$

Combining Eq. (8) and Eq. (14) leads to:

$$\delta U = \int_0^l \int_{-h/2}^{h/2}\left[N\delta u - M\delta\left(\frac{\partial^2 w}{\partial x^2}\right) + D_x\delta\left(\frac{\partial\phi}{\partial x}\right) + D_z\delta\left(\frac{\partial\phi}{\partial z}\right)\right]\mathrm{d}x \tag{15}$$

where N the axial force and M the bending moment are determined through the following equations:

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_{xx}\mathrm{d}z, \quad M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_{xx}z\mathrm{d}z \tag{16}$$

The kinetic energy for piezoelectric nanobeam can be calculated as:

$$\begin{aligned} T &= \frac{1}{2}\rho\iint\left(\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2\right)\mathrm{d}A\cdot\mathrm{d}x \\ &= \frac{1}{2}\rho\int\left(I_1\left(\frac{\partial u}{\partial t}\right)^2 + I_2\left(\frac{\partial^2 w}{\partial x\partial t}\right)^2 + I_1\left(\frac{\partial w}{\partial t}\right)^2\right)\mathrm{d}x \end{aligned} \tag{17}$$

where I_1 and I_2 are defined:

$$I_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}}\rho\mathrm{d}z, \quad I_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}}\rho z^2\mathrm{d}z \tag{18}$$

Accordingly, the first variation of Eq. (17) can be obtained as:

$$\delta T = -\int_0^l\left(I_1\left(\frac{\partial^2 w}{\partial t^2}\right)\delta(w) - I_2\left(\frac{\partial^4 w}{\partial x^2\partial t^2}\right)\delta(w) + I_1\left(\frac{\partial^2 u}{\partial t^2}\right)\delta(u)\right)\mathrm{d}x \tag{19}$$

In this study, the axial load which is caused by the elastic medium based on the Winkler–Pasternak foundation is assumed in the following form:

$$f = -k_w w + k_p\frac{\partial^2 w}{\partial x^2} \tag{20}$$

And the work done by external forces is given by:

$$\delta W_{\text{ext}} = \int_0^t (q_w\delta(w) + q_u\delta(u)) \tag{21}$$

where q_w is calculated as:

$$\begin{aligned} q_w = &\left(H + N_p + N_T + P_{\text{electric}}\right)\left(\frac{\partial^2 W}{\partial x^2}\right) \\ &- K_w W + K_P\left(\frac{\partial^2 W}{\partial x^2}\right) \end{aligned} \tag{22}$$

In this equation, N_p and N_T are the normal forces which are induced by the biaxial force P_0 and temperature rise, and H is a constant which is obtained by residual surface stress and the shape of cross section.

$$\begin{aligned} N_P &= P_0 \\ N_T &= -\lambda_1 A\Delta T \\ H &= 2b\tau_0 \end{aligned} \tag{23}$$

Substituting Eqs. (15), (19) and (21) into Eq. (13) with setting the coefficient of δu and δw equal to zero, the following equations will be obtained:

$$\frac{\partial N}{\partial x} + q_{\mathrm{u}} - I_1\left(\frac{\partial^2 u}{\partial t^2}\right) = 0 \tag{24}$$

$$\frac{\partial^2 M}{\partial x^2} + q_{\mathrm{w}} + I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2}\right) - I_1\left(\frac{\partial^2 w}{\partial t^2}\right) = 0 \tag{25}$$

The bending moment for piezoelectric nanobeam with considering surface effects will be calculated by:

$$M = \int \sigma_{xx} z \mathrm{d}A + \int \tau_{xx} z \mathrm{d}A - \int e_{31}\phi_z z \mathrm{d}A \tag{26}$$

$$M = -(EI)^* \frac{\partial^2 w}{\partial x^2} + \frac{2Iv}{h}\left(\tau_0 \frac{\partial^2 w}{\partial x^2} - \rho_0 \frac{\partial^2 w}{\partial t^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^2 w}{\partial x^2} \tag{27}$$

where $(\mathrm{EI})^* = E(bh^3/12) + E_{\mathrm{s}}(h^3/6 + bh^2/2)$ is the effective bending stiffness. Using the nonlocal elasticity theory for bending moment:

$$M - \mu \frac{\partial^2 M}{\partial x^2} = -(\mathrm{EI})^* \frac{\partial^2 w}{\partial x^2} + \frac{2Iv}{h}\left(\tau_0 \frac{\partial^2 w}{\partial x^2} - \rho_0 \frac{\partial^2 w}{\partial t^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^2 w}{\partial x^2} \tag{28}$$

$$M = \mu\left(-I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2}\right) + I_1\left(\frac{\partial^2 w}{\partial t^2}\right) - q_w\right) - (\mathrm{EI})^* \frac{\partial^2 w}{\partial x^2} + \frac{2Iv}{h}\left(\tau_0 \frac{\partial^2 w}{\partial x^2} - \rho_0 \frac{\partial^2 w}{\partial t^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^2 w}{\partial x^2} \tag{29}$$

Substituting Eq. (29) into Eq. (25) the constitutive motion equation will be obtained:

$$\left(1 - \mu \frac{\partial^2}{\partial x^2}\right)\left(-I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2}\right) + I_1\left(\frac{\partial^2 w}{\partial t^2}\right) - q_w\right) - (EI)^* \frac{\partial^4 w}{\partial x^4} + \frac{2Iv}{h}\left(\tau_0 \frac{\partial^4 w}{\partial x^4} - \rho_0 \frac{\partial^4 w}{\partial x^2 \partial t^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^4 w}{\partial x^4} = 0 \tag{30}$$

Assuming the harmonic motion for the free vibration of nanobeam with natural frequency of ω, namely:

$$w(x,t) = W(x)\mathrm{e}^{i\omega t} \tag{31}$$

Substituting Eq. (31) into Eq. (30) leads to:

$$\left(1 - \mu \frac{\partial^2}{\partial x^2}\right)\left(\omega^2 I_2\left(\frac{\partial^2 W}{\partial x^2}\right) - \omega^2 I_1 W(x) - q_w\right) - (\mathrm{EI})^* \frac{\partial^4 W}{\partial x^4} + \frac{2Iv}{h}\left(\tau_0 \frac{\partial^4 W}{\partial x^4} + \rho_0 \omega^2 \frac{\partial^4 W}{\partial x^2}\right) + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^4 W}{\partial x^4} = 0 \tag{32}$$

2.4 Solution procedure

In addition, the governing motion equation for piezoelectric nanobeam and associated boundary conditions is discretized using DTM. This method is one of the most useful techniques for solving the differential equations which comes from Taylor's series expansion. Implementing DTM reduces the governing equations for various boundary conditions to algebraic equations, and finally, all the calculations turn into simple iterative process [36]. The basic definitions are defined as:

$$Y[k] = \frac{1}{k!}\left(\frac{\mathrm{d}^k y(x)}{\mathrm{d}x^k}\right)_{x=x_0} \tag{33}$$

$$y(x) = \sum_{k=0}^{\infty} (x - x_0)^k Y[k] \tag{34}$$

Some of the transformation functions in order to transform the constitutive equations and boundary conditions into algebraic equations are listed in Table 1.

Apply the mentioned equations in Table 1. Using the DTM to Eq. (32), the resultant equations are obtained as:

$$\left((\mathrm{EI})^* + \frac{2I_3 v}{h}\tau_0 - \frac{e_{31}^2}{\lambda_{33}} - \mu^2\left(N_{\mathrm{p}} + N_{\mathrm{T}} + P_{\mathrm{electric}} + H\right) - \mu^2 K_{\mathrm{P}}\right) \frac{(k+4)!}{k!} W[k+4] + \left(\frac{2I_3 v \rho_0 \omega^2}{h} + \left(N_{\mathrm{p}} + N_{\mathrm{T}} + P_{\mathrm{electric}} + H\right) + K_{\mathrm{P}} + \mu^2 K_{\mathrm{w}} - I_1 \mu^2 \omega^2\right) \frac{(k+2)!}{k!} W[k+2] - \left(K_w + I_1\omega^2\right) W[k] = 0 \tag{35}$$

Simplifying Eq. (35), the following relation can be obtained:

Also, the resultant equations for boundary conditions by using from Table 2 are given as:

$$W[k+4] = \frac{-\left(\frac{2I_3 v \rho_0 \omega^2}{h} + \left(N_{\mathrm{p}} + N_{\mathrm{T}} + P_{\mathrm{electric}} + H\right) + K_{\mathrm{P}} + \mu^2 K_{\mathrm{w}} - I_1 \mu^2 \omega^2\right) \frac{(k+2)!}{k!} W[k+2] + (K_{\mathrm{w}} + I_1\omega^2) W[k]}{\left((\mathrm{EI})^* + \frac{2I_3 v}{h}\tau_0 - \frac{e_{31}^2}{\lambda_{33}} - \mu^2\left(N_{\mathrm{p}} + N_{\mathrm{T}} + P_{\mathrm{electric}} + H\right) - \mu^2 K_{\mathrm{P}}\right) \frac{(k+4)!}{k!}} \tag{36}$$

Table 1 Some basic theorems of DTM for equations of motion

Original function	Transformed function
$f(x) = g(x) \pm h(x)$	$F(K) = G(K) \pm H(K)$
$f(x) = \lambda g(x)$	$F(K) = \lambda G(K)$
$f(x) = g(x)h(x)$	$F(K) = \sum_{l=0}^{K} G(K-l)H(l)$
$f(x) = \frac{d^n g(x)}{dx^n}$	$F(K) = \frac{(k+n)!}{k!} G(K+n)$
$f(x) = x^n$	$F(K) = \delta(K-n) = \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases}$

- Simply–simply supported:

$$W[0] = 0, \quad W[2] = 0$$

$$\sum_{k=0}^{\infty} W[k] = 0, \quad \sum_{k=0}^{\infty} k(k-1)W[k] = 0 \tag{37a}$$

- Clamped–clamped:

$$W[0] = 0, \quad W[1] = 0$$

$$\sum_{k=0}^{\infty} W[k] = 0, \quad \sum_{k=0}^{\infty} kW[k] = 0 \tag{37b}$$

- Clamped–simply:

$$W[0] = 0, \quad W[1] = 0$$

$$\sum_{k=0}^{\infty} W[k] = 0, \quad \sum_{k=0}^{\infty} k(k-1)W[k] = 0 \tag{37c}$$

The initial boundary conditions are calculated from the first four $W[k]$s, and also the higher quantities will be obtained by substituting these equations into recurrence Eq. (36). From Eq. (36) and transformed boundary conditions, the following expression will be obtained:

$$\begin{bmatrix} A_{11}(\omega) & A_{12}(\omega) \\ A_{21}(\omega) & A_{22}(\omega) \end{bmatrix} [C] = 0 \tag{38}$$

The eigenvalue equation is calculated from Eq. (38) as follows:

$$\begin{vmatrix} A_{11}(\omega) & A_{12}(\omega) \\ A_{21}(\omega) & A_{22}(\omega) \end{vmatrix} = 0 \tag{39}$$

Solving Eq. (39), we get $\omega = \omega_j^{(n)}$ which $\omega_j^{(n)}$ shows the jth natural frequency and for calculating n, the following equation is used:

$$\left| \omega_j^{(n)} - \omega_j^{(n-1)} \right| \leq \varepsilon \tag{40}$$

in which ε is the tolerance parameter which is assumed as 0.0001 which means the value of frequencies had four-digit precision. The constitutive equations are solved, and fundamental natural frequencies are obtained.

3 Numerical results and discussion

In this section, the natural frequencies of piezoelectric nanobeam based on the nonlocal Euler–Bernoulli beam theory under thermo-electro-mechanical resting on elastic medium with various boundary conditions are predicted. The Al material properties assumed are listed in Table 3 [28, 37].

The selected numerical results are investigated to present the influence of nonlocal parameter (μ), temperature change (ΔT), external electrical voltage (V_0) and preload (N_p) with various boundary condition including clamped–clamped (C–C), clamped–simply (C–S) and simply–simply supported (S–S).

Table 3 Al and Si material properties

Properties	Al [28, 37]
Young's modulus (E)	70 GPa
Poisson's ratio (v)	0.3
Mass density (ρ)	2700 kg m^{-3}
Residual surface tensions (τ^0)	0.9108 N m
Elasticity surface modules (E^s)	5.1882 N m
Density of surface layer (ρ^s)	5.46×10^{-7} kg m^2
Thermal coefficient (λ_1)	2.56×10^{-6} 1/k
Piezoelectric coefficient (e_{31})	-10 C m^2
Dielectric constants (λ_{33})	1.0275×10^{-8}

Table 2 Transformed boundary conditions (BC) based on DTM

$X = 0$		$X = L$	
Original BC	Transformed BC	Original BC	Transformed BC
$f(0) = 0$	$F[0] = 0$	$f(L) = 0$	$\sum_{k=0}^{\infty} F[k] = 0$
$\frac{df(0)}{dx} = 0$	$F[1] = 0$	$\frac{df(L)}{dx} = 0$	$\sum_{k=0}^{\infty} kF[k] = 0$
$\frac{d^2f(0)}{dx^2} = 0$	$F[2] = 0$	$\frac{d^2f(L)}{dx^2} = 0$	$\sum_{k=0}^{\infty} k(k-1)F[k] = 0$
$\frac{d^3f(0)}{dx^3} = 0$	$F[3] = 0$	$\frac{d^3f(L)}{dx^3} = 0$	$\sum_{k=0}^{\infty} k(k-1)(k-2)F[k] = 0$

Table 4 Comparison of the nondimensional fundamental frequency for a nanobeam with various nonlocal parameters with S–S boundary conditions

μ	S–S boundary condition		
	Present paper	Reddy [39]	Eltaher et al. [38]
0	9.8696	9.8696	9.8700
1	9.4158	9.4159	9.4162
2	9.0194	9.0195	9.0197
3	8.6691	8.6693	8.6695
4	8.3569	8.3569	8.3571
5	8.0760	8.0761	8.0762

In order to validate the numerical results and procedure, the comparative studies are presented in this section. The first case study is related to the dimensionless natural frequency; for S–S boundary condition, it is compared with ones presented in Refs. [38, 39] which are tabulated in Table 4. As it is indicated from Table 3, the results are in better agreement with Reddy [39] than Eltaher et al. [38], and this happens because the numerical results in [39] are obtained by analytical method which are more reliable in comparison with those presented in [38] which are obtained by finite element method (FEM). Table 5 also includes nondimensional natural frequencies for C–S and C–C boundary condition presented in Ref. [38], and in the present study, it can be concluded that results are in good agreement with those in [38] with a very slight difference due to solution method.

In the calculation of the natural frequencies by solving the governing motion equations with DTM, the value of natural frequencies after some iteration converges to the constant value. The convergences of first three natural frequencies are investigated in Table 6. As it can be observed from Table 6, the first natural frequency in S–S boundary condition converged after 15 iterations with four-digit precision, while the second natural frequency converged after 25 iterations.

Table 5 Comparison of the nondimensional fundamental frequency for a nanobeam with various nonlocal parameters with C–S boundary conditions

μ	C–S		C–C	
	Present paper	Eltaher et al. [38]	Present paper	Eltaher et al. [38]
0	15.4182	15.4189	22.3733	22.3744
1	14.9923	14.9929	21.1086	21.1096
2	14.5991	14.5997	20.0321	20.0330
3	14.2347	14.2353	19.1020	19.1028
4	13.8959	13.8965	18.2884	18.2890
5	13.5798	13.5803	17.5691	17.5696

Table 6 Convergence study of nanobeam for the first three natural frequencies ($L/h = 100$, $\mu = 2\ \text{nm}^2$)

k	C–C			C–S			S–S		
	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$
11	18.9537			14.4286			9.0384		
13	20.4543			14.6787			9.0181		
15	19.9907			14.5958			9.0194	27.9180	
17	20.0367			14.5995			9.0194	29.9999	
19	20.0318			14.5991	43.1861		9.0194	29.4611	
21	20.0322	43.6093		14.5991	41.6494		9.0194	29.5137	
23	20.0321	44.5424		14.5991	41.8090		9.0194	29.5086	
25	20.0321	44.3698		14.5991	41.7900		9.0194	29.5090	52.7212
27	20.0321	44.3893		14.5991	41.7918		9.0194	29.5090	53.3891
29	20.0321	44.3873		14.5991	41.7917	75.2351	9.0194	29.5090	53.2852
31	20.0321	44.3875	69.6921	14.5991	41.7917	74.7880	9.0194	29.5090	53.2962
33	20.0321	44.3875	70.1564	14.5991	41.7917	74.8396	9.0194	29.5090	53.2952
35	20.0321	44.3875	70.0864	14.5991	41.7917	74.8338	9.0194	29.5090	53.2952
37	20.0321	44.3875	70.0943	14.5991	41.7917	74.8344	9.0194	29.5090	53.2952
39	20.0321	44.3875	70.0935	14.5991	41.7917	74.8343	9.0194	29.5090	53.2952
41	20.0321	44.3875	70.0936	14.5991	41.7917	74.8343	9.0194	29.5090	53.2952
43	20.0321	44.3875	70.0936	14.5991	41.7917	74.8343	9.0194	29.5090	53.2952

Table 7 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects for S–S ($L = 20\,\mathrm{nm}$, $h = 0.1L$, $b = 0.5h$)

T (°C)	$\mu = 0$ nm^2			$\mu = 2$ nm^2			$\mu = 4$ nm^2		
	$V = -0.5$	$V = 0$	$V = 0.5$	$V = -0.5$	$V = 0$	$V = 0.5$	$V = -0.5$	$V = 0$	$V = 0.5$
0	20.0202	14.9643	6.8594	19.9887	14.7568	5.998	19.9584	14.5548	5.0347
15	20.0261	14.9722	6.8767	19.9948	14.7650	6.0184	19.9647	14.5634	5.0595
30	20.0321	14.9802	6.8940	20.0009	14.7733	6.0386	19.9709	14.5720	5.0842
45	20.0380	14.9881	6.9112	20.0070	14.7816	6.0588	19.9772	14.5806	5.1088
60	20.0439	14.9960	6.9284	20.0131	14.7898	6.0789	19.9835	14.5892	5.1333
75	20.0499	15.004	6.9455	20.0192	14.7981	6.9900	19.9897	14.5978	5.1577
90	20.0558	14.0119	6.9626	20.0253	14.8063	6.1190	19.996	14.6064	5.1819
105	20.0617	15.0198	6.9796	20.0314	14.8146	6.1389	20.0023	14.6150	5.2060
120	20.0677	15.0277	6.9966	20.0375	14.8228	6.1588	20.0850	14.6235	5.1100
135	20.0736	15.0356	7.0136	20.0436	14.8311	6.1786	20.0148	14.6321	5.2540
150	20.0795	15.0435	7.0305	20.0497	14.8393	6.1983	20.0211	14.6407	5.2778

Table 8 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects for C–S ($L = 20\,\mathrm{nm}$, $h = 0.1L$, $b = 0.5h$)

T (°C)	$\mu = 0$ nm^2			$\mu = 2$ nm^2			$\mu = 4$ nm^2		
	$V = -0.5$	$V = 0$	$V = 0.5$	$V = -0.5$	$V = 0$	$V = 0.5$	$V = -0.5$	$V = 0$	$V = 0.5$
0	26.8645	23.0550	18.4634	26.8552	22.7235	17.6356	26.8463	22.4011	16.8054
15	26.8692	23.0611	18.4704	26.8603	22.7295	17.6434	26.8517	22.4077	16.8142
30	26.8740	23.0666	18.4773	26.8654	22.7356	17.6512	26.8572	22.4143	16.8230
45	26.8787	23.0722	18.4843	26.8706	22.7416	17.6591	26.8627	22.4208	16.8317
60	26.8834	23.0777	18.4912	26.8757	22.7477	17.6669	26.8681	22.4274	16.8405
75	26.8882	23.0832	18.4982	26.8808	22.7538	17.6747	26.8736	22.4340	16.8493
90	26.8929	23.0888	18.5051	26.8859	22.7598	17.6825	26.8791	22.4405	16.8580
105	26.8977	23.0943	18.5120	26.8910	22.7659	17.6903	26.8845	22.4471	16.8668
120	26.9024	23.0999	18.5190	26.8961	22.7719	17.6981	26.8900	22.4537	16.8755
135	26.9072	23.1054	18.5259	26.9012	22.7780	17.7059	26.8955	22.4602	16.8843
150	26.9119	23.1109	18.5328	26.9064	22.7840	17.7137	26.9009	22.4668	16.8930

After studying convergence and validation, the influence of temperature and nonlocal parameters with changing voltage for piezoelectric nanobeam is represented in Tables 7, 8 and 9 for S–S, C–S and C–C boundary conditions in the presence of surface effects, respectively. And also according to the previous discussion, the mechanical properties are considered to be constant and all the results are calculated for Al in this section. As indicated in Tables 7, 8 and 9, increasing temperature causes an increase in natural frequency; the reason is that increase in temperature change brings in more increase in the nanobeam stiffness and hence leads to rise in natural frequencies, while increasing the nonlocal parameter and voltage from negative to positive amount decreases the value of natural frequencies. Cause of reducing frequency with growing nonlocal parameter is that the presence of the nonlocal effect tends to decrease the stiffness of nanostructures and decreases natural frequency. Also, the positive voltage tends to decrease natural frequency, while the negative voltage causes an increase in natural frequency. It happens for the fact that the axial compressive and tensile forces are generated in nanobeams by applied positive and negative voltage, respectively. It should be noted that when the nonlocal parameter assumed to be zero, the resultant natural frequencies correspond to classical beam theory.

Tables 10, 11 and 12 list the effect of voltage and temperature on elastics medium. Increasing both linear and shear stiffness constants (k_w, k_p) tends to decrease natural frequency. This happens because the foundation elastic affects the stiffness of nanobeams and increasing the value

Table 9 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects for C–C ($L = 20\,\mathrm{nm}$, $h = 0.1L$, $b = 0.5h$)

T (°C)	$\mu = 0$ nm^2			$\mu = 2$ nm^2			$\mu = 4$ nm^2		
	$V = -0.5$	$V = 0$	$V = 0.5$	$V = -0.5$	$V = 0$	$V = 0.5$	$V = -0.5$	$V = 0$	$V = 0.5$
0	36.0621	33.2374	30.1437	36.0831	32.7515	29.0349	36.1029	32.2804	27.9343
15	36.0658	33.2413	30.148	36.0873	32.7562	29.0402	36.1078	32.2858	27.9406
30	36.0694	33.2453	30.1524	36.0916	32.7609	29.0455	36.1127	32.2913	27.9469
45	36.0731	33.2492	30.1568	36.0959	32.7656	29.0508	36.1175	32.2967	27.9532
60	36.0767	33.2532	30.1612	36.1001	32.7703	29.0561	36.1224	32.3021	27.9595
75	36.0804	33.2572	30.1655	36.1440	32.7750	29.0614	36.1272	32.3076	27.9658
90	36.0840	33.2611	30.1699	36.1087	32.7797	29.0667	36.1321	32.3130	27.9720
105	36.0876	33.2651	30.1743	36.1129	32.7844	29.0721	36.1370	32.3185	27.9783
120	36.0913	33.2690	30.1786	36.1172	32.7892	29.0774	36.1418	32.3239	27.9846
135	36.0949	33.2730	30.1830	36.1215	32.7939	29.0827	36.1467	32.3293	27.9909
150	36.0986	33.2769	30.1874	36.1257	32.7986	29.0880	36.1515	32.3348	27.9972

Table 10 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects for S–S ($L = 20\,\mathrm{nm}$, $h = 0.1L$, $b = 0.5h$, $\mu = 2\,\mathrm{nm}^2$)

k_w	k_p	$\Delta T = 0$			$\Delta T = 50$			$\Delta T = 100$		
		$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$
0	0	17.0434	14.7568	12.0436	17.0672	14.7843	12.0773	17.0911	14.8118	12.1110
	4	16.5285	14.1590	11.3032	16.5531	14.1877	11.3392	16.5777	14.2164	11.375
	8	15.9970	13.5348	10.5108	16.0224	13.5648	10.5494	16.0478	13.5948	10.5880
4	0	16.9919	14.6973	11.9707	17.0158	14.6973	11.9707	17.0398	14.7526	12.0385
	4	16.4754	14.0970	11.2254	16.5001	14.1258	11.2617	16.5248	14.1546	11.2978
	8	15.9422	13.4699	10.4271	15.6770	13.5001	10.4661	15.9932	13.5302	10.5094
8	0	16.9403	14.6376	11.8973	16.9643	14.6654	11.9314	16.9883	14.6931	11.9655
	4	16.4222	14.0347	11.1472	16.4469	14.0637	11.1836	16.4717	14.0926	11.2200
	8	15.8871	13.4047	10.3428	15.9128	13.4351	10.4212	15.9383	13.4654	10.42212

Table 11 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects for C–S ($L = 20\,\mathrm{nm}$, $h = 0.1L$, $b = 0.5h$, $\mu = 2\,\mathrm{nm}^2$)

k_w	k_p	$\Delta T = 0$			$\Delta T = 50$			$\Delta T = 100$		
		$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$
0	0	24.4612	22.7235	20.8395	24.4799	22.7437	20.8615	24.4987	22.7639	20.8836
	4	24.0596	22.2902	20.3657	24.0787	22.3108	20.3883	24.0977	22.3314	20.4108
	8	23.6511	21.8482	19.8805	23.6705	21.8692	19.9036	23.6899	21.8903	19.9267
4	0	24.4287	22.6886	20.8015	24.4475	22.7088	20.8236	24.4663	22.7291	20.8457
	4	24.0266	22.2547	20.3269	24.0457	22.2753	20.3495	24.0648	22.2959	20.3721
	8	23.6176	21.8120	19.8407	23.6370	21.8331	19.8639	23.6564	21.8541	19.8870
8	0	24.3963	22.6533	20.7635	24.4151	22.6739	20.7856	24.4338	22.6942	20.8077
	4	23.9936	22.2191	20.2880	24.0127	22.2397	20.3106	24.0318	22.2604	20.3332
	8	23.5840	21.7757	19.8009	23.6035	21.7968	19.8241	23.6229	21.8179	19.8473

Table 12 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects for C–C ($L = 20\,\text{nm}$, $h = 0.1L$, $b = 0.5h$, $\mu = 2\,\text{nm}^2$)

k_w	k_p	$\Delta T = 0$			$\Delta T = 50$			$\Delta T = 100$		
		$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$
0	0	34.1237	32.7515	31.3185	34.1388	32.7672	31.3349	34.1538	32.7829	31.3513
	4	33.8026	32.4166	30.9679	33.8178	32.4325	30.9845	33.8330	32.4484	31.0011
	8	33.4784	32.0782	30.6133	33.4937	32.0943	30.6301	33.5091	32.1103	30.6469
4	0	34.1015	32.7284	31.2943	34.1165	32.7441	31.3107	34.1316	32.7598	31.3271
	4	33.7802	32.3933	30.9435	33.7954	32.4092	30.9601	33.8106	32.4250	30.9767
	8	33.4558	32.0546	30.5886	33.4711	32.0707	30.6054	33.4865	32.0867	30.6222
8	0	34.0792	32.7052	31.2701	34.0943	32.7209	31.2866	34.1094	32.7367	31.3030
	4	33.7577	32.3699	30.9190	33.7730	32.3858	30.9357	33.7882	32.4017	30.9523
	8	33.4331	32.0310	30.5639	33.4485	32.0471	30.5807	33.4638	32.0631	30.5975

Table 13 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects ($L = 20\,\text{nm}$, $h = 0.1L$, $b = 0.5h$)

N_p	$\mu = 0\,\text{nm}^2$			$\mu = 2\,\text{nm}^2$			$\mu = 4\,\text{nm}^2$		
	$V = -0.5$	$V = 0$	$V = 0.5$	$V = -0.5$	$V = 0$	$V = 0.5$	$V = -0.5$	$V = 0$	$V = 0.5$
S–S									
−10	18.941	13.4864	2.2368	18.8768	13.2116	–	18.815	12.9425	–
−5	19.4881	14.2445	5.1017	19.4407	14.0055	3.7906	19.3951	13.7723	1.7836
0	20.0202	14.9643	6.8594	19.9887	14.7568	5.998	19.9584	14.5548	5.0347
5	20.5386	15.651	8.2508	20.5221	15.4716	7.5884	20.5062	15.2974	6.8931
10	21.0442	16.3089	9.4392	21.0419	16.1548	8.8989	21.0397	16.0055	8.3476
C–S									
−10	24.0104	22.0518	17.1899	25.9335	21.6238	16.1899	25.8596	21.2063	15.1716
−5	24.4409	22.5593	17.8382	26.3984	22.1805	16.9283	26.3576	21.812	16.0095
0	26.8645	23.0555	18.4634	26.8552	22.7235	17.6356	26.8463	22.4011	16.8054
5	27.2813	23.5411	19.068	27.3042	23.2536	18.3153	27.3261	22.9751	17.565
10	27.6917	24.0167	19.6538	27.7459	23.7717	18.9705	27.7976	23.5349	18.2928
C–C									
−10	35.4115	32.529	29.3595	35.3199	31.9077	28.0781	35.2321	31.3024	26.7966
−5	35.7383	32.8851	29.7542	35.7036	32.3324	28.5605	35.6702	31.7952	27.3714
0	36.0621	33.2374	30.1437	36.0831	32.7515	29.0349	36.1029	32.2804	27.9343
5	36.383	33.5858	30.5281	36.4586	33.1653	29.5015	36.5305	32.7583	28.4859
10	36.7011	33.9306	30.9077	36.8302	33.5739	29.9607	36.953	33.2293	29.027

of medium parameters reduces natural frequency. Also, it is clear that the Winkler foundation has less effective than the Pasternak foundation on natural frequency.

Effects of voltage and nonlocal parameter via axial preload are shown in Table 13. It can be seen that when the axial comprehensive force is applied, the lower the natural frequency becomes, and for axial tensile force, natural frequency increases. Its reason is because the axial tensile force strengthens the nanobeam stiffness and comprehensive force weakens its stiffness.

Tables 14, 15 and 16 show nonlocal parameter and voltage effects via length of nanobeam. As the length of nanobeam increases, natural frequency decreases. The influence of surface effects on natural frequencies diminishes from nanoscale to macro-scale.

Moreover, Fig. 2 presents the variation of first natural frequency for piezoelectric nanobeam as a function of voltage for different nonlocal parameters in the presence of surface effects by ignoring the Winkler–Pasternak elastic medium and thermal effects. For all values of nonlocal parameter, a descending trend can be observed, while by increasing the voltage from negative to positive amount the difference between curves increases. The piezoelectric field shows increasing effect for negative voltage and decreasing

Table 14 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects for S–S ($L = 20\,\mathrm{nm}$, $h = 0.1L$, $b = 0.5h$, $\Delta T = 50\,°\mathrm{C}$)

L	$\mu = 0\,\mathrm{nm}^2$			$\mu = 2\,\mathrm{nm}^2$			$\mu = 4\,\mathrm{nm}^2$		
	$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$
10×10^{-9}	15.4764	13.415	10.9724	15.2794	12.8744	9.9016	15.0988	12.3657	8.8231
50×10^{-9}	16.9035	14.9523	12.7048	16.8682	14.9066	12.6443	16.8331	14.8612	12.5841
100×10^{-9}	15.3207	13.8049	12.1006	15.3104	13.7927	12.0859	15.3	13.7806	12.0713
1×10^{-6}	10.8608	10.5594	10.2491	10.8607	10.5593	10.249	10.8606	10.5592	10.2489
10×10^{-6}	9.9778	9.9440	9.9101	9.9778	9.9440	9.9101	9.9778	9.9440	9.9101
100×10^{-6}	9.8805	9.8771	9.8736	9.8805	9.8771	9.8736	9.8805	9.8771	9.8736
1×10^{-3}	9.8707	9.8703	9.8700	9.8707	9.8703	9.8700	9.8707	9.8703	9.8700
1×10^{-2}	9.8697	9.8696	9.8696	9.8697	9.8696	9.8696	9.8697	9.8696	9.8696

Table 15 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects for C–S ($L = 20\,\mathrm{nm}$, $h = 0.1L$, $b = 0.5h$, $\Delta T = 50\,°\mathrm{C}$)

L	$\mu = 0\,\mathrm{nm}^2$			$\mu = 2\,\mathrm{nm}^2$			$\mu = 4\,\mathrm{nm}^2$		
	$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$
10×10^{-9}	21.8909	20.4139	18.8206	21.606	19.5884	17.3367	21.3464	18.8151	15.8846
50×10^{-9}	24.8353	23.4014	21.8703	24.7771	23.3238	21.7704	24.7194	23.2469	21.6713
100×10^{-9}	22.8764	21.7541	20.5677	22.8588	21.7328	20.5422	22.8411	21.7115	20.5168
1×10^{-6}	16.7974	16.5725	16.3442	16.7972	16.5723	16.344	16.797	16.5721	16.3438
10×10^{-6}	15.5688	15.5436	15.5183	15.5688	15.5436	15.5183	15.5688	15.5436	15.5183
100×10^{-6}	15.4334	15.4309	15.4283	15.4334	15.4309	15.4283	15.4334	15.4309	15.4283
1×10^{-3}	15.4197	15.4195	15.4192	15.4197	15.4195	15.4192	15.4197	15.4195	15.4192
1×10^{-2}	15.4184	15.4183	15.4183	15.4184	15.4183	15.4183	15.4184	15.4183	15.4183

Table 16 First natural frequency for piezoelectric nanobeam by considering surface and nonlocal effects for C–C ($L = 20\,\mathrm{nm}$, $h = 0.1L$, $b = 0.5h$, $\Delta T = 50\,°\mathrm{C}$)

L	$\mu = 0\,\mathrm{nm}^2$			$\mu = 2\,\mathrm{nm}^2$			$\mu = 4\,\mathrm{nm}^2$		
	$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$	$V = -0.2$	$V = 0$	$V = 0.2$
10×10^{-9}	30.352	29.2851	28.1776	29.9574	28.1002	26.1109	29.5985	26.9922	24.1054
50×10^{-9}	35.0092	33.9611	32.8782	34.9232	33.8447	32.7293	34.838	33.7293	32.5814
100×10^{-9}	32.494	31.6707	30.8241	32.4674	31.6382	30.7853	32.4408	31.6058	30.7467
1×10^{-6}	24.2621	24.0965	23.9297	24.2618	24.0962	23.9294	24.2616	24.0959	23.9291
10×10^{-6}	22.5796	22.561	22.5424	22.5796	22.561	22.5424	22.5796	22.561	22.5424
100×10^{-6}	22.3941	22.3922	22.3903	22.3941	22.3922	22.3903	22.3941	22.3922	22.3903
1×10^{-3}	22.3754	22.3752	33.375	22.3754	22.3752	33.375	22.3754	22.3752	33.375
1×10^{-2}	22.3735	22.3735	22.3735	22.3735	22.3735	22.3735	22.3735	22.3735	22.3735

influence for positive voltage amount on fundamental natural frequencies of nanobeam.

Next, the variation of natural frequency versus the voltage for various piezoelectric nanobeam lengths is shown in Fig. 3 by considering surface and nonlocal effects in the absence of Winkler–Pasternak elastic medium effect, while the temperature change is considered to be 50 °C. The numerical results imply that the influence of surface and nonlocal effects decreases by increasing voltage in negative region, while they have different manner for positive voltage. And also, it can be discovered that by increasing the beam size the nonlocal and surface with piezoelectric effects have less influence on the natural frequencies.

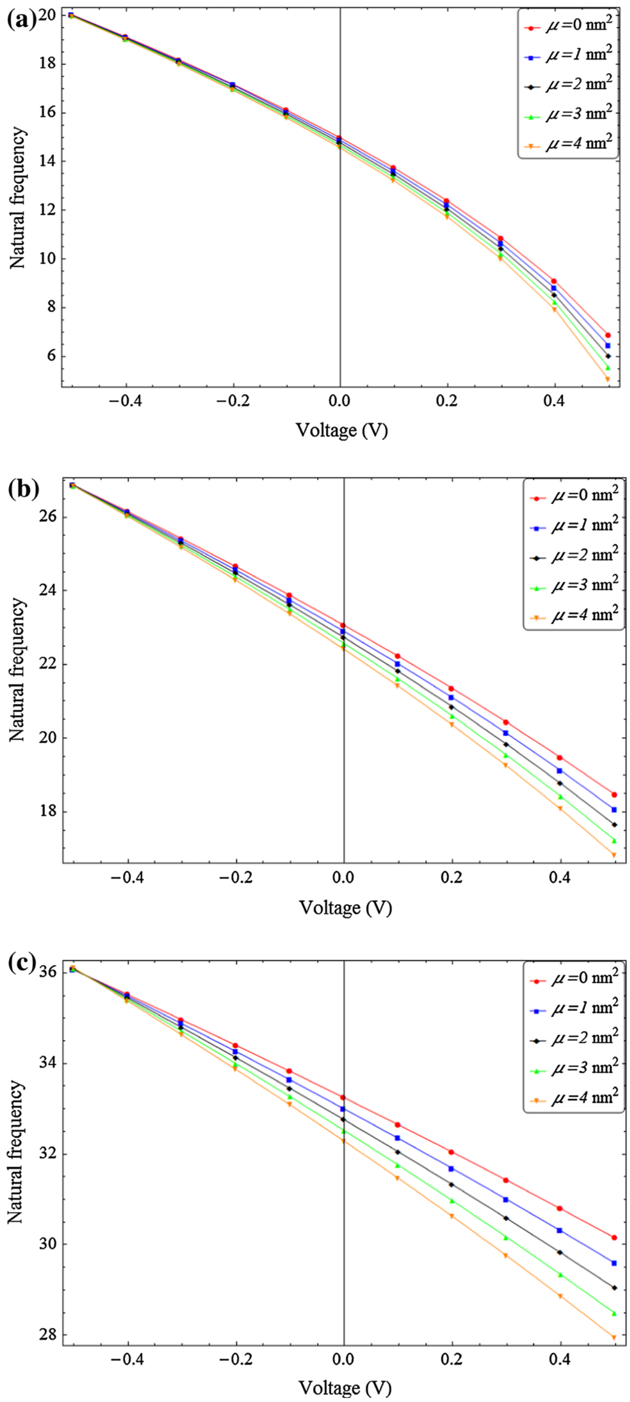

Fig. 2 Variation of first natural frequency of S–S for **a** S–S, **b** C–S and **c** C–C piezoelectric nanobeam for different nonlocal parameters in the presence of surface effects for different nonlocal parameters

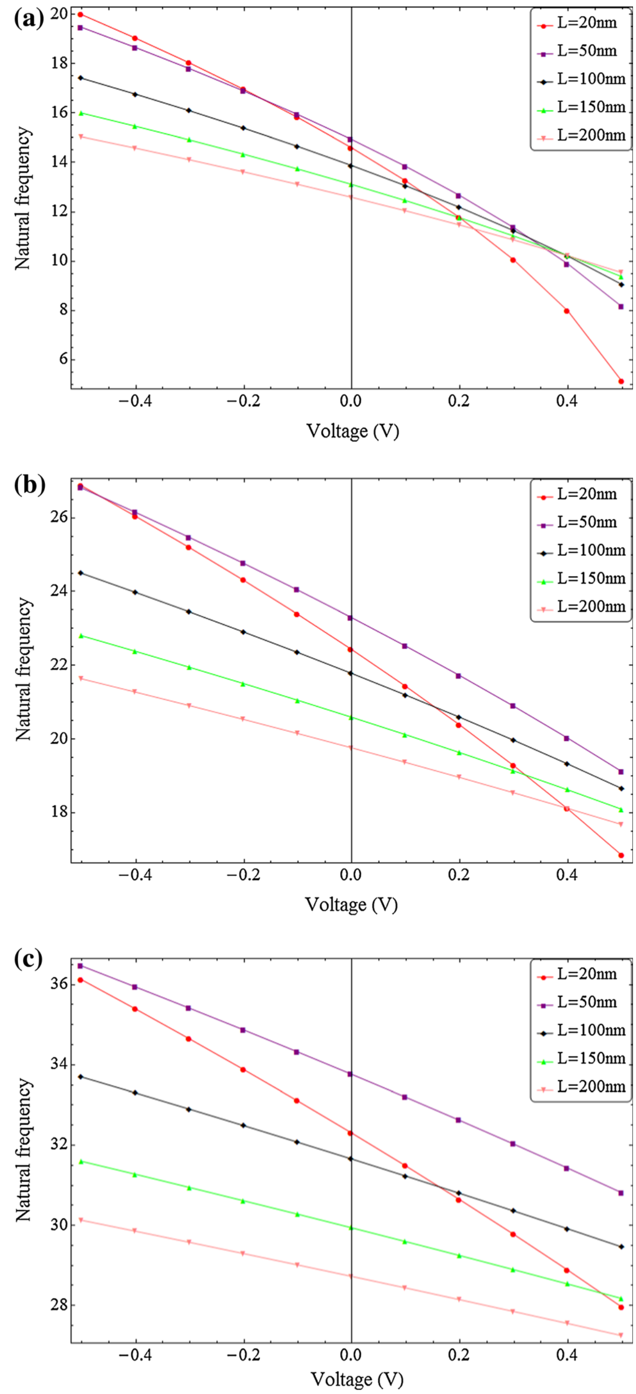

Fig. 3 Variation of first natural frequency of S–S for **a** S–S, **b** C–S and **c** C–C piezoelectric nanobeam for different nanobeam lengths in the presence of surface effects

4 Conclusion

In the current study, a semi-analytical method of nonlocal piezoelectric surface effect on thermal free vibration of nanobeam lying on Winkler–Pasternak for various boundary conditions with axial preload is presented. The main motivations of this paper are investigating influences of thermal, preload and elastic medium in the presence of electric potential field. As it is presented, governing equation is derived with using Hamilton's principle for Euler–Bernoulli beam; then, DTM as an efficient and accurate numerical tool was applied to solve vibration of nanobeam subjected to different boundary condition. After presenting the rate of convergence for first three natural frequencies and its validation, some were clarified the influences of piezoelectric, nonlocal and surface effects in combination with other parameters on the vibration of nanobeam. Based on the presented numerical results:

1. For all values of nonlocal parameter, voltage and different boundary condition, the influence of surface effects on natural frequencies decreases significantly as the nanobeam length increases.
2. Natural frequency increases with increasing thermal effect.
3. Increasing voltage parameter tends to decrease fundamental frequency.
4. Increasing tension load causes the natural frequency decrease, and increasing comprehension load tends to rise natural frequency .
5. Rising elastic medium parameters including Winkler and Pasternak foundations significantly reduces frequency.

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