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Nonlinear and nonlocal effects on dispersion properties of coupled surface plasmon polaritons in linear/wire-medium/nonlinear dielectric structures

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Abstract We analytically investigate the nonlinear and nonlocal effects on dispersion properties of coupled surface plasmon polariton (SPP) modes at the interfaces of the linear/wire-medium/nonlinear dielectric with a Kerr nonlinearity. By employing a "first integral" method, we obtain the dispersion relation of coupled nonlinear SPP modes in the nonlinear waveguide system. Numerical results show that there exist two branches of SPP modes in the asymmetric multilayer structure, and both the nonlinearity and the nonlocality have a great impact on dispersion properties. We demonstrate that the focusing and defocusing nonlinearity can lead the SPP frequency to shift downward and upward, respectively, and there is no cutoff frequency for the nonlinear SPP mode when spatial nonlocality is taken into account, evidently different from those in the absence of spatial nonlocality. In addition, the nonlocality of optical response can also induce the SPP frequency to have a blueshift, but the nonlocal effect would be weakened with the nonlinearity enhanced. These interesting nonlinear SPP properties in metamaterial waveguides have potential applications in optoelectronic devices.

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1 Introduction

Surface plasmon polaritons (SPPs) are electromagnetic exciting modes propagating or located at the interface between any two materials where the real part of the dielectric function changes sign across the interface such as at the metal-dielectric interface, whose electromagnetic fields decay exponentially with distance from the surface in its perpendicular direction. In 1941, U. Fano firstly explained the implicit properties of surface waves in Ref. [1], which emphasized the existence of polarized quasi-stationary waves, which represent an energy current rolling along the surface of a metal [1]. Because SPPs can concentrate the electromagnetic energy into subnanometer volumes giving rise to a strong local energy density, it is possible that excitation of SPPs can overcome the diffraction limit and offer a promising approach to control and manipulate light propagation and dispersion properties at subnanometer scales.

In the early years, a significant amount of experimental and theoretical studies have concerned SPPs at metallic surfaces, but few people pay attention to wire medium at optical frequency because nanostructures are difficult to be realized. With the development of nanotechnology, the size of materials can become smaller and smaller, which makes the production of metallic nanostructures such as wire medium become possible. The wire medium is a kind of artificial metamaterials composed of periodic arrays of thin metallic nanorods embedded into a dielectric matrix [2–4]. In the last few years, SPP modes in metallic nanostructures with novel geometries (such as rods, tapered tips, and shells) have been attracting extensive attention due to their novel optical properties [5–10].

Recently, ample attention has been devoted to deriving dispersion equations of SPP modes in left-handed electromagnetic media [11–13], nonlinear dielectric/metal interface [14, 15], and linear-metal-nonlinear dielectric waveguide [16]. However, the properties of SPPs at the nonlinear dielectric/wire-medium interface have not been investigated so far. Here, we have a great interest in nonlinear SPP modes at the interfaces of the multilayer structure constructed by the linear/wire-medium/nonlinear dielectric with a Kerr nonlinearity. In this paper, our motivation is to clarify the dispersion properties of nonlinear SPP modes existing at the interfaces of the waveguide system and demonstrate the influence of both the Kerr nonlinearity and the nonlocality on the dispersion properties of nonlinear SPP modes. It is expected that the effects of the nonlinearity and nonlocality on nonlinear SPP modes would interact with each other in the metamaterial waveguide system [17–19].

This paper is structured as follows. In Sect. 2, we derive the dispersion relation for nonlinear SPP modes in the asymmetric multilayer structure constructed by the linear/wire-medium/nonlinear dielectric with a Kerr nonlinearity in the case of considering the nonlocality of optical response in wire-medium metamaterials. In Sect. 3, we discuss mainly the dispersion curves, the characteristics of cutoff frequency, and the shift of the SPP frequency in both local and nonlocal cases. In addition, we demonstrate further the interaction between the nonlinearity and nonlocality and their influence on dispersion properties of SPP modes in the metamaterial waveguide system in detail. And a brief summary is given in Sect. 4.

2 Dispersion properties of nonlinear SPP modes

We consider a plasmonic waveguide with the integral geometry as shown in Fig. 1. In this waveguide structure, a nonlinear dielectric layer and a linear dielectric layer occupy the space of $x > 0$ (region 1) and $x < -d$ (region 3),

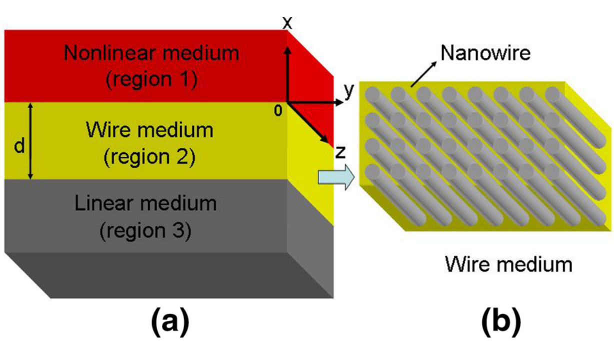


Fig. 1 **a** The system consists of wire medium separating two semi-infinite dielectric regions, one of which is linear and the other of Kerr nonlinearity. The coordinate origin of cartesian coordinate system *xyz* is placed at the nonlinear dielectric/wire-medium interface, and the TM-polarized modes propagating along the *z*-direction. **b** The geometry of wire medium: a lattice of parallel ideally conducting nanorods directed along the *z*-direction

respectively. Here, we consider the nonlinear dielectric of a kerr nonlinearity, and its **D**–**E** relation is of the form $\mathbf{D} = \varepsilon\mathbf{E} + \alpha|\mathbf{E}|^2\mathbf{E} = \varepsilon_{nl}(\mathbf{E})\mathbf{E}$, where ε is the linear part of the dielectric function $\varepsilon_{nl}(\mathbf{E})$, α is a nonlinear coefficient of the kerr material, characterizing nonlinear type and size. When $\alpha > 0$, the kerr material is of focusing nonlinearity, while it is of defocusing nonlinearity when $\alpha < 0$. If $\alpha = 0$, the material becomes linear. In the middle of this structure is the wire-medium metamaterial, which is formed by a regular lattice of ideally conducting nanorods embedded into a host medium. Compared with the wavelength of incident light, the size of lattice is so small that the wire medium can be approximately seen as a homogeneous medium. It is well known that the wire medium could be modeled as a uniaxial dielectric with the following permittivity dyadic [20, 21]:

$$\bar{\bar{\varepsilon}} = \varepsilon_{zz}\hat{\mathbf{e}}_z\hat{\mathbf{e}}_z + \varepsilon_h(\hat{\mathbf{e}}_x\hat{\mathbf{e}}_x + \hat{\mathbf{e}}_y\hat{\mathbf{e}}_y), \tag{1}$$

with

$$\varepsilon_{zz}(\omega, k_z) = \varepsilon_h\left(1 - \frac{k_p^2}{\varepsilon_h k_0^2 - \beta k_z^2}\right), \tag{2}$$

where $k_0 = \omega/c, k_p = \omega_p/c, \varepsilon_h$ is the relative permittivity of the host medium, ω_p is an equivalent plasma frequency, and c is the speed of light in vacuum. Here k_z is the z-component of the wave vector **k** in the wire medium, and β is a spatial dispersion parameter, $\beta = 1$ or 0, corresponding to consider the nonlocality ($\beta = 1$, nonlocal case) [20, 21] or omit the nonlocality ($\beta = 0$, local case) [22] of optical response in the wire-medium metamaterial, respectively.

In order to investigate nonlinear SPP modes, we consider the wave fields have a simple form as

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2}[E_x(x)\mathbf{e}_x + iE_z(x)\mathbf{e}_z]e^{i(k_z z - \omega t)} + c.c., \tag{3}$$

$$\mathbf{H}(\mathbf{r}, t) = \frac{1}{2}H_y(x)\mathbf{e}_y e^{i(k_z z - \omega t)} + c.c., \tag{4}$$

where k_z is the z-component of the wave vector, which is also called propagation constant in the waveguide system. Here only the transverse magnetic (TM) mode is considered, whose magnetic field is polarized along y-direction and propagates along z-direction, as shown in Fig. 1. According to the Maxwell's equations $\nabla \times \mathbf{H} = \partial\mathbf{D}/\partial\mathbf{t}$ and $\nabla \times \mathbf{E} = -\mu_0\partial\mathbf{H}/\partial\mathbf{t}$, we can obtain the field equations in the following form:

$$\frac{\partial}{\partial x}E_z = k_z E_x - \mu_0\omega H_y, \tag{5}$$

$$H_y = \frac{\omega\varepsilon_{xx}\varepsilon_0}{k_z}E_x, \tag{6}$$

$$E_z = \frac{1}{k_z\varepsilon_{zz}}\frac{\partial}{\partial x}(\varepsilon_{xx}E_x). \tag{7}$$

Here, ε_{xx} and ε_{zz} are the components of tensor dielectric constant. Particularly, we have the relation $\varepsilon_{xx} = \varepsilon_{zz} = \varepsilon_{nl}$, and ε_l in the nonlinear and linear dielectrics, respectively.

In wire-medium region (region 2), the general form of the **E**-field can be assumed as

$$E_{x2}(x) = E_{x2}^{-}(0)e^{q_m x} + E_{x2}^{+}(0)e^{-q_m x}, \tag{8}$$

where the amplitudes are those at the value of x specified inside the parentheses. Substituting Eq. (8) into Eqs. (6) and (7), we can obtain the expression of $H_{y2}(x)$ and $E_{z2}(x)$ in the wire medium. Here $q_m = \sqrt{k_z^2 \varepsilon_{zz}/\varepsilon_h - \varepsilon_{zz}\omega^2/c^2}$, which follows from Eqs. (5). According to the character of evanescent waves, we set $E_{x3} = E_{x3}(-d)e^{q_l(x+d)}$ in linear region (region 3). Similarly, we have $E_{z3}(x) = (q_l/k_z)E_{x3}(-d)e^{q_l(x+d)}$ and $H_{y3}(x) = (\omega\varepsilon_l\varepsilon_0/k_z)E_{x3}(-d)$ $e^{q_l(x+d)}$, where $q_l = \sqrt{k_z^2 - \varepsilon_l\omega^2/c^2}$, and ε_l is the relative permittivity of the linear dielectric. Due to the anisotropy of the wire medium, we can notice that there is a different expression for q_m, compared with q_l.

In the nonlinear region (region 1), we treat the fields by employing the method of "first integral," which was first shown by Mihalache et al. [23]. Eqs. (5) and (7) have a first integral that can be written as

$$\left(\frac{\mathrm{d}E_{z1}(x)}{\mathrm{d}x}\right)^2 = \left(k_z^2 - \frac{\varepsilon\omega^2}{c^2}\right)E_{x1}^2(x) - \frac{\varepsilon\omega^2}{c^2}E_{z1}^2(x) - \frac{1}{2}\alpha\frac{\omega^2}{c^2} \times \left[E_{x1}^2(x) + E_{z1}^2(x)\right]^2, \tag{9}$$

where $E_{x1}(x)$ and $E_{z1}(x)$ are the components of electric field in nonlinear region (region 1). Applying $\frac{\mathrm{d}E_{z1}(x)}{\mathrm{d}x}$ to $x = 0^+$, we obtain

$$\left(\frac{\mathrm{d}E_{z1}(x)}{\mathrm{d}x}\right)^2_{z=0^+} = \left(k_z - \frac{\varepsilon_{nl}\omega^2}{k_z c^2}\right)^2 E_{x1}^2(x). \tag{10}$$

Substituting Eq. (10) into Eq. (9), we can get the relation between $E_{x1}(0)$ and $E_{z1}(0)$ as

$$2\left(\frac{\varepsilon_{nl}^2\omega^2}{k_z^2c^2} - 2\varepsilon_{nl}\right)E_{x1}^2(0) + (\varepsilon_{nl} + \varepsilon)E_0^2 = 0, \tag{11}$$

here, we have set $E_0^2 = E_{x1}^2(0) + E_{z1}^2(0)$. With the continuity of E_z and H_y at the interfaces of $x = 0$ and $x = -d$, we can obtain

$$E_{x1}(0) = \frac{1}{2q_m\varepsilon_{nl}}\left[(q_l\varepsilon_{zz} + q_m\varepsilon_l)e^{q_m d} - (q_l\varepsilon_{zz} - q_m\varepsilon_l)e^{-q_m d}\right] \times E_{x3}(-d), \tag{12}$$

$$E_{z1}(0) = \frac{1}{2k_z\varepsilon_{zz}}\left[(q_l\varepsilon_{zz} + q_m\varepsilon_l)e^{q_m d} + (q_l\varepsilon_{zz} - q_m\varepsilon_l)e^{-q_m d}\right] \times E_{x3}(-d). \tag{13}$$

Finally, substituting Eqs. (12) and (13) in Eq. (11), the dispersion relation for the nonlinear SPP modes can be derived as follows

$$2\left(\frac{\varepsilon_{nl}^2\omega^2}{k_z^2c^2} - 2\varepsilon_{nl}\right) + (\varepsilon_{nl} + \varepsilon)\left(1 + \frac{P^2q_m^2\varepsilon_{nl}^2}{Q^2k_z^2\varepsilon_{zz}^2}\right) = 0, \tag{14}$$

where $P = q_l\varepsilon_{zz} + q_m\varepsilon_l \tanh(q_m d)$, and $Q = q_m\varepsilon_l + q_l\varepsilon_{zz}\tanh(q_m d)$. It is worth noting that by taking $P = Q(q_m d \to \infty)$, the coupled nonlinear SPP modes at the interfaces of the linear/wire-medium/nonlinear dielectric structure will degenerate into two independent surface waves at the wire-medium/linear dielectric and wire-medium/nonlinear dielectric interface, respectively.

3 Results and discussions

In this paper, the relative dielectric constant of the host matrix for the wire medium is assumed as $\varepsilon_h = 4.4$, the linear dielectric is assumed as a vacuum without loss of generality, and we fix the linear part of the dielectric constant of the Kerr medium at $\varepsilon = 2.4$. According to the dispersion relation Eq. (14), we have plotted the dispersion curves of the waveguide system in the case of not considering the nonlinearity for two different cases of gap width d with the nonlocal effect ($\beta = 1$) in Fig. 2. There are two branches for a given gap width d; the upper branch and lower branch denote the characters of the SPP mode at the linear dielectric/wire-medium interface and the nonlinear dielectric/wire-medium interface in the waveguide system, respectively. It must be noted that both q_l and q_m

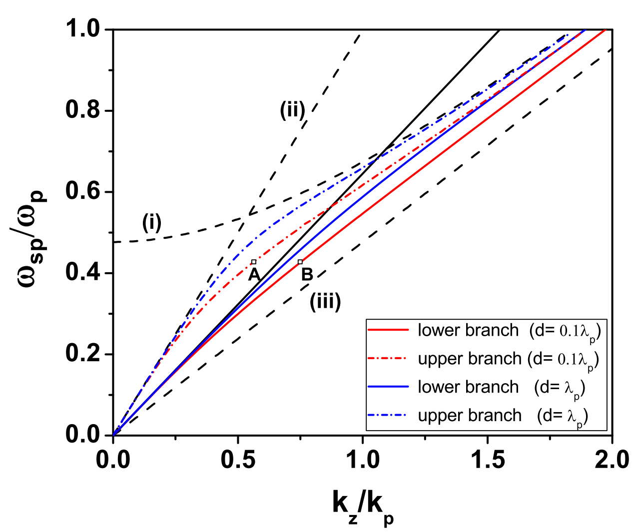


Fig. 2 Dispersion curves of SPP modes in the case of not considering the nonlinearity for two different gap widths $d = 0.1\,\lambda_p$ and $d = \lambda_p$, with the nonlocal effect ($\beta = 1$)

should be real in order to guarantee the existence of SPP modes, in other words, two inequalities should be fulfilled simultaneously: $k_z^2 - \varepsilon_l(\omega^2/c^2) > 0$ and $(\varepsilon_{zz}/\varepsilon_h)k_z^2 - \varepsilon_{zz}(\omega^2/c^2) > 0$. Solving the two inequalities, we can find that the region, in which SPP modes can exist, is surrounded by the three dashed lines, denoted by (i), (ii), and (iii). Dashed line (i) denotes $\omega/\omega_p = \sqrt{[1+\beta(k_z/k_p)^2]/\varepsilon_h}$, where the spatial dispersion parameter $\beta = 0$ and $\beta = 1$ correspond to the optical effects of locality and nonlocality, respectively. Dashed line (ii) is $\omega/\omega_p = (k_z/k_p)/\sqrt{\varepsilon_l}$, and dashed line (iii) represents $\omega/\omega_p = (k_z/k_p)/\sqrt{\varepsilon_h}$. In addition, we have plotted the light line ($\omega/\omega_p = (k_z/k_p)/\sqrt{\varepsilon}$, black solid line) in region 1 in Fig. 2. Seen from Fig. 2, the mode, whose dispersion curve lies to the left of the light line, is radiative (leaky). However, the mode, whose dispersion curve lies to the right of the light line, is nonradiative. For example, the point A (left of the light line, radiative mode) cannot guarantee the inequality $k_z^2 - \varepsilon(\omega^2/c^2) > 0$, which leads to the case that the electromagnetic field in region 1 corresponds to a plane wave radiating away from the wire-medium boundary. Due to very less dependence of the upper branch of dispersion curves on the nonlinear parameter αE_0^2, we only discuss the influence of the coupled effect of the Kerr nonlinearity and nonlocality on lower branch in the following analysis.

In order to clarify the character of waveguide system, we have numerically plotted the dispersion curves of SPP modes for two different widths of wire medium in mid-layer: $d = 0.1\,\lambda_p$ (in Fig. 3a, c) and $d = \lambda_p$ (in Fig. 3b, d) in both local ($\beta = 0$) and nonlocal ($\beta = 1$) cases, where $\lambda_p = 2\pi c/\omega_p$. In Fig. 3a–d, we have shown three lines: (1) red line ($\alpha|E_0|^2 = 0$), the dielectric in region 1 is linear; (2) green line ($\alpha|E_0|^2 = 1$), the dielectric in region 1 is of focusing nonlinearity; (3) blue line ($\alpha|E_0|^2 = -1$), the dielectric in region 1 is of defocusing nonlinearity. It must be emphasized that both nonlinearity and nonlocality of optical response have an evident influence on the dispersion curves of the nonlinear SPP modes (lower branch) in the metamaterial waveguide system. In the case of local response ($\beta = 0$), we can find that there exists a cutoff frequency ω_c for each dispersion curve of SPP modes in three cases of different nonlinearity. This indicates that the SPP modes cannot exist and propagate at the interface of the wire-medium waveguide when the SPP frequency $\omega_{sp} > \omega_c$ in the local case. In Fig. 3a, we can see that the cutoff frequency decreases obviously with the nonlinear parameter $\alpha|E_0|^2$ increasing. Different from Fig. 3a, the dispersion curves in Fig. 3b approach to the same cutoff frequency ($\omega_c = 0.465\,\omega_p$) for different values of nonlinear parameter when $d = \lambda_p$. In order to illustrate clearly the point, we have plotted the dependence of the cutoff frequency ω_p on the mid-layer width d for three different values of nonlinear parameter αE_0^2 in Fig. 4. Seen from Fig. 4, we can find the cutoff frequency ω_p increases monotonically with the change of the width d. In small width d region ($0.1\,\lambda_p < d < 1.5\,\lambda_p$), the effect of Kerr nonlinearity has an large impact on the cutoff frequency ω_p. In large width d region ($d > 1.5\,\lambda_p$), the curves for different values of nonlinear parameter αE_0^2 almost overlap together, meaning that the effect of Kerr nonlinearity on SPP modes become weaker and weaker with the width d increasing.

Compared with the dispersion curves of linear-metal-nonlinear dielectric structure [15], we easily find that there exist significant difference in present dispersion curves. The reason is that when SPP frequency $\omega < \omega_p$, $q_m = \sqrt{k_z^2 - \varepsilon_{zz}(\omega^2/c^2)}$ is a real number without any additional conditions in metal, which induces a consequence that there is no cutoff frequency ω_c in the linear-metal-nonlinear dielectric structure for the case of local response. For more accurate treatment, the nonlocality of optical response for TM modes in the wire medium should be taken into account. Comparison with the local case ($\beta = 0$), the dispersion curves in the nonlocal case ($\beta = 1$) have evident different properties: (1) the dispersion curves tends asymptotically to the dashed line (i) with the wave vector k_z increasing in Fig. 3c, d, and the larger the width d is, the more evident the asymptotic behavior will be. (2) there exist no any cutoff frequencies for nonlocal SPP modes.

From Fig. 3, we can also see that the SPP frequency in the presence of a kerr nonlinearity deviates from that in the absence of nonlinearity. To better clarify these properties, we illustrate the deviations

$$\Delta_\pm \equiv \omega_{sp}\,|_{\alpha E^2=\pm 1} - \omega_{sp}\,|_{\alpha E^2=0} \tag{15}$$

as a function of normalized wave vector k_z/k_p for three different widths d in Fig. 5: $d = 0.1\,\lambda_p$ [(a) and (b)]; $d = \lambda_p$ [(c) and (d)]; $d = 3.0\,\lambda_p$ [(e) and (f)] in the two cases of local ($\beta = 0$) and nonlocal ($\beta = 1$) responses, where the red and black lines correspond to Δ_+ and Δ_-, respectively. Slight deviations in the dispersion relation from the linear case ($\alpha = 0$) are exhibited, the positive (negative) Kerr coefficient results in SPP frequency ω_{sp} slightly smaller (larger) than the linear case in the present choice of values for αE_0^2. Compared with the case of local response ($\beta = 0$, see Fig. 5a, c, e), the range of k_z, in which the deviations $\Delta_\pm$ are evident, suggesting the nonlinearity has a great influence, is broaden considerably to $2.0\,k_p$ in the nonlocal case ($\beta = 1$) as shown in Fig. 5b, d, f. It must be mentioned that we only show the relative narrow range of k_z/k_p due to the existence of cutoff frequency for SPP modes in the case

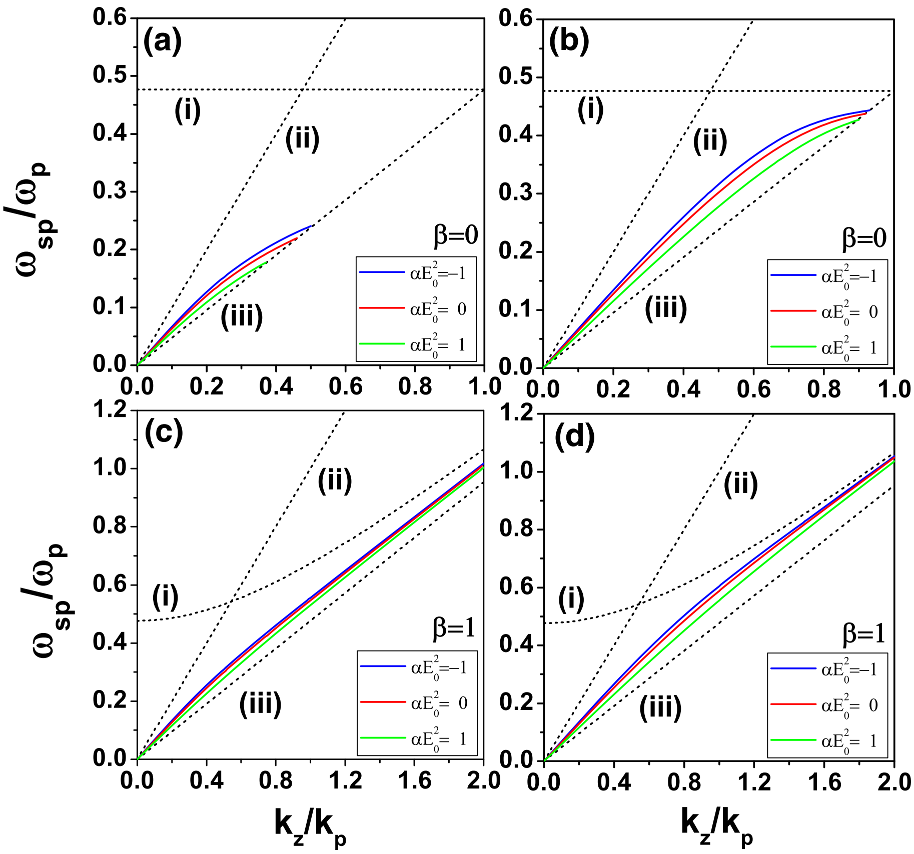


Fig. 3 Dispersion curves of SPP modes for the two values of wire-medium width d in three different cases of defocusing nonlinearity $\alpha|E|^2 = -1$ (*blue line*), linearity $\alpha|E|^2 = 0$ (*red line*), and focusing nonlinearity $\alpha|E|^2 = 1$ (*green line*). **a** the width d $= 0.1\,\lambda_p, \beta = 0$ (local case), **b** the width $d = \lambda_p, \beta = 0$ (local case), **c** the width $d = 0.1\,\lambda_p, \beta = 1$ (nonlocal case), **d** the width $d = \lambda_p, \beta = 1$ (nonlocal case)

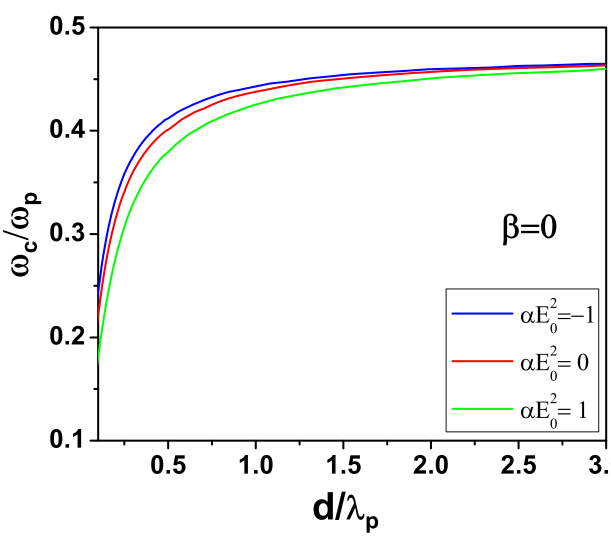


Fig. 4 The dependence of the normalized cutoff frequency ω_c/ω_p on the mid-layer width d for three different values of nonlinear parameter αE_0^2: defocusing nonlinearity $\alpha E_0^2 = -1$ (*blue line*), linearity $\alpha E_0^2 = 0$ (*red line*), and focusing nonlinearity $\alpha E_0^2 = 1$ (*green line*)

of local response, as shown in Fig. 5a, c, e. In addition, comparison with a thin width d (see Fig. 5b, $d = 0.1\,\lambda_p$), the absolute values of deviations $\Delta_\pm$ for thick wire medium are larger (see Fig. 5d, $d = \lambda_p$). However, the tendency of curves is almost same with the width d increasing further (see Fig. 5d, f). When the width $d = 3\,\lambda_p$ in Fig. 5f, there is almost no significant difference for the deviations $\Delta_\pm$ compared with Fig. 5d. These observed tendency indicates the nonlinearity and nonlocality interact with each other, which is stronger and stronger with the mid-layer width d increasing, then trends toward stabilization when the width $d > \lambda_p$.

In Fig. 6a, b, we further illustrate the effects of Kerr nonlinearity and nonlocality on the SPP modes by plotting the nonlinear parameter $|\alpha|E_0^2$ dependence of normalized k_z/k_p for two different values of SPP frequency ω_{sp}: (a) $\omega_{sp} = 0.2\omega_p$; (b) $\omega_{sp} = 0.4\,\omega_p$ with the width d fixed at λ_p, respectively, where the red and blue lines correspond to the local ($\beta = 0$) and nonlocal ($\beta = 1$) cases. From Fig. 6a, b, we can see that the variation rules of curves for both local ($\beta = 0$) and nonlocal ($\beta = 1$) cases have a similar behavior. Concretely speaking, in the case of focusing Kerr coefficient $\alpha > 0$ (dashed line), k_z increases linearly with nonlinear parameter $|\alpha|E_0^2$ enhanced, and in the case of defocusing Kerr coefficient $\alpha < 0$ (solid line), the value of k_z first decreases and then increases with nonlinear parameter $|\alpha|E_0^2$ increasing. However, it is worthy noting that when SPP frequency $\omega_{sp} = 0.4\,\omega_p$ (Fig. 6b), k_z varies significantly when the nonlocality of optical response in the

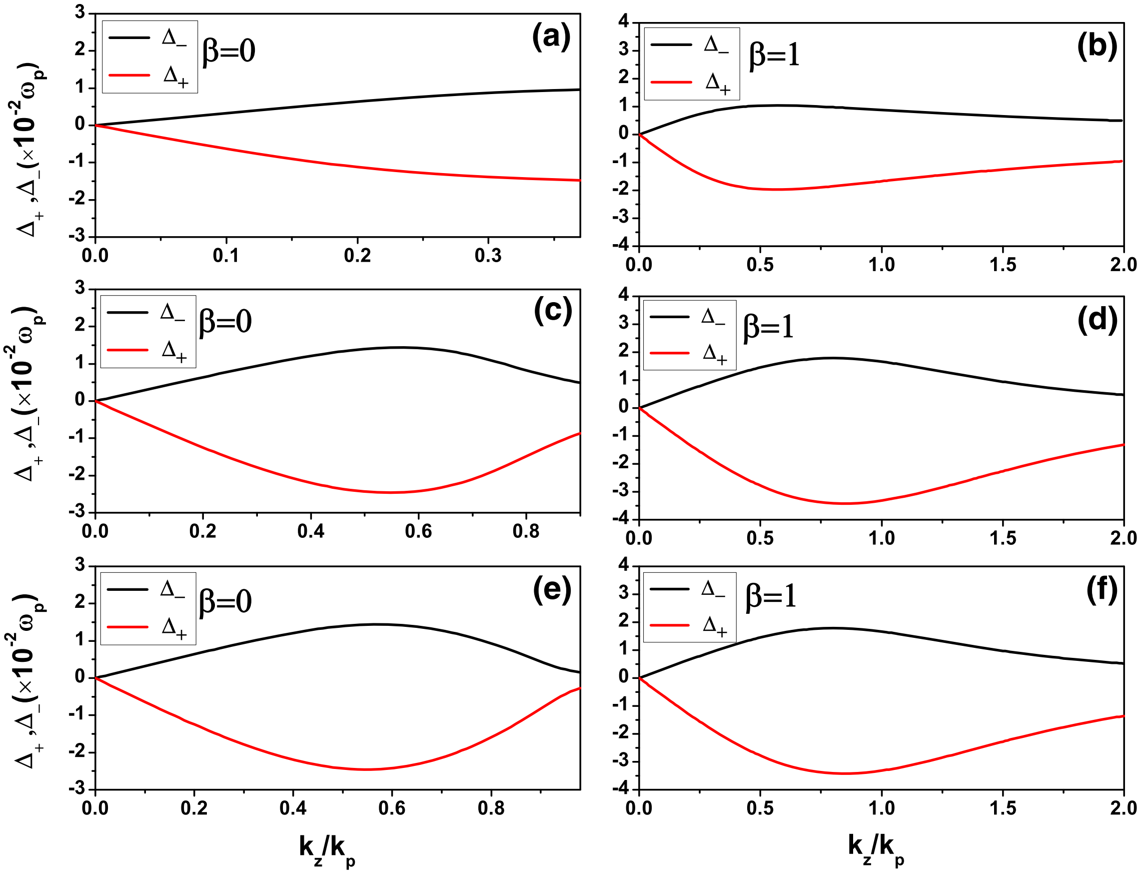
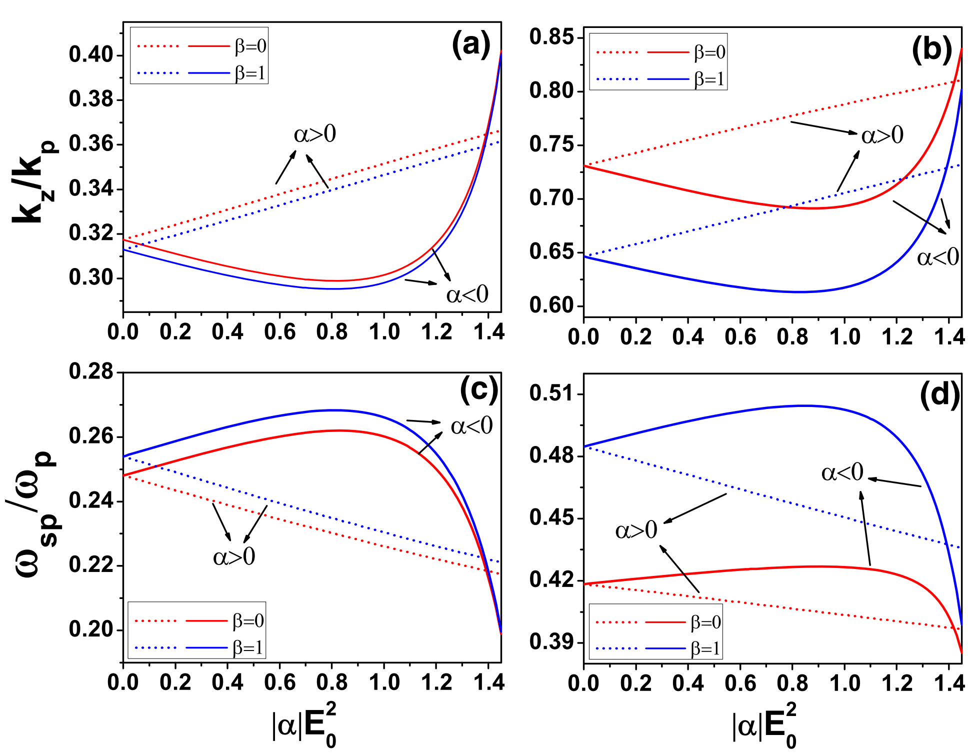


Fig. 5 Deviations of nonlinear SPP frequency ω_{sp} from the linear case, Δ_+ and Δ_- [defined by Eq. (15)], as a function of k_z/k_p for **a** width $d = 0.1\,\lambda_p, \beta = 0$; **b** width $d = 0.1\,\lambda_p$, $\beta = 1$; **c** width $d = \lambda_p, \beta = 0$; **d** width $d = \lambda_p, \beta = 1$; **e** width $d = 3\,\lambda_p, \beta = 0$; **f** width $d = 3\,\lambda_p, \beta = 1$

Fig. 6 The normalized k_z/k_p as a function of the nonlinear parameter $|\alpha|E_0^2$ for two different SPP frequencies: **a** $\omega_{sp} = 0.2\,\omega_p$; **b** $\omega_{sp} = 0.4\,\omega_p$. SPP frequency ω_{sp} versus the nonlinear parameter $|\alpha|E_0^2$ for two different values of k_z: **c** $k_z = 0.4\,k_p$; **d** $k_z = 0.8\,k_p$, with mid-layer width $d = \lambda_p$. *Red* and *blue lines* denote the local ($\beta = 0$) and nonlocal ($\beta = 1$) cases, respectively. *Dashed* and *solid lines* correspond to the focusing ($\alpha > 0$) and defocusing ($\alpha < 0$) nonlinearity, respectively



wire medium is taken into account, which indicates that the nonlocality has a great influence on the SPP modes in some SPP frequency range.

As a supplement, we also plot the nonlinear parameter $|\alpha|E_0^2$ dependence of SPP frequency ω_{sp} for two given values of k_z in Fig. 6c ($k_z = 0.4\,k_p$) and Fig. 6d ($k_z = 0.8\,k_p$). For $\alpha > 0$, the SPP frequency ω_{sp} decreases monotonically with respect to the change of $|\alpha|E_0^2$. For $\alpha < 0$, the SPP frequency ω_{sp} first increases to a certain maximum value and then deceases with $|\alpha|E_0^2$ increasing. Comparison with the case of local response ($\beta = 0$), the nonlocality of optical response in the wire-medium layer can lead the SPP frequency to shift upward (blue shift), which is very evident in certain ranges of k_z, such as $k_z = 0.8\,k_p$ (see Fig. 6d). In addition, the effects of the nonlocality on the SPP frequency will be weakened with nonlinear parameter $|\alpha|E_0^2$ enhanced.

At last, we compare these characters of the coupled waveguide system with the findings of Ref. [5]: (1) There exist two branches of SPP modes in the waveguide system constructed by the linear/wire-medium/nonlinear dielectric; however, there exists only one dispersion curve at the single metal–Kerr medium interface. (2) Although there exists some difference in the range of the deviations $\Delta_\pm$, the changing tendency of $\Delta_\pm$ with k_z is nearly similar to that of Ref. [5]. (3) With the nonlinear parameter $|\alpha|E_0^2$ increasing, the changes of the SPP frequency ω_{sp} (see Fig. 6c, d) of our waveguide system are more evident than that in single metal–Kerr medium interface mentioned in Ref. [5]. These interesting properties may be instructive for precise experiments and have potential applications in metamaterial waveguide devices.

4 Summary

In this paper, we investigate mainly the dispersion properties of coupled nonlinear SPP modes at the interfaces of the multilayer structure constructed by the linear/wire-medium/nonlinear dielectric with a Kerr nonlinearity. We derive the dispersion relations for nonlinear SPP modes by employing a first integral approach at the interface between a kerr nonlinear dielectric and a wire medium with strong nonlocality of optical response. We show that there exist two branches of SPP modes in the asymmetric multilayer structure, and the optical responses of the Kerr nonlinearity and nonlocality interact with each other in the metamaterial waveguide system, which have a significant influence on the dispersion properties and the cutoff frequency. The focusing nonlinearity can lead the SPP frequency to shift downward, whereas it is contrary in the case of defocusing nonlinearity. The nonlocality of optical response can lead the SPP frequency to have a blueshift. The nonlocal effect, however, would be weakened with the nonlinear effect increasing. In addition, the range of k_z, on which the nonlinearity has a major influence, can be broaden considerably when the nonlocality is taken into account. These properties may be instructive for precise experiments in future and practical application in artificial metamaterial optoelectronic devices at nanometer scales.

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